

# Higgs and EW theory - lecture 3

Today: fermion masses + CKM matrix, and Higgs self-coupling

Need:  $\overline{\Psi} \Psi$  for Lorentz invariance  
 $\sim [m]^3$

$\Phi \sim \text{SU}(2)_L$  doublet  $\rightarrow$  couple to  $Q_L$  or  $L_L$   
 and a R. H. fermion

Electron:

$$\mathcal{L}_{\text{Yukawa}} \supset - \left[ y_e \bar{e}_R \overset{\dagger}{\Phi} L_L + y_e^* \bar{L}_L \overset{\dagger}{\Phi} e_R \right]$$

h.c. of 1st term

$$(\phi^{+*}, \phi^0*) \cdot \begin{pmatrix} v_L \\ e_L \end{pmatrix}$$

Absorb phase of  $y_e$  into  
 redefinition of  $e_R$  field

$$Y: \bar{e}_R \overset{\dagger}{\Phi} L_L + 1 - \frac{1}{2} - \frac{1}{2} = 0$$

In unitarity gauge:

$$\overset{\dagger}{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\overset{\dagger}{\Phi} L_L = \left( 0, \frac{v+h}{\sqrt{2}} \right) \begin{pmatrix} v_L \\ e_L \end{pmatrix} = \frac{v+h}{\sqrt{2}} e_L$$

$$\mathcal{L}_{\text{Yuk.}} \supset - y_e \frac{1}{\sqrt{2}} \left[ (v+h) \bar{e}_R e_L + (v+h) \bar{e}_L e_R \right]$$

$\uparrow p_L$        $\uparrow p_R$

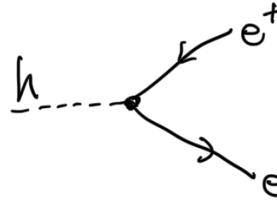
$$= -y_e \frac{1}{\sqrt{2}} (\sigma + h) \bar{e} e$$

$$= - \left( \frac{y_e v}{\sqrt{2}} \right) \bar{e} e - \frac{y_e}{\sqrt{2}} h \bar{e} e$$

$\underbrace{\hspace{10em}}$

$$-m_e \bar{e} e$$

$\underbrace{\hspace{10em}}$



$$-i \frac{y_e}{\sqrt{2}} = -i \frac{g}{\sqrt{2}}$$

$$y_e = \frac{\sqrt{2} m_e}{v}$$

$$h \rightarrow e^+ e^- \leftarrow 7 \times 10^{-3} \quad 2 \times 10^{-6}$$

$$h \rightarrow \mu^+ \mu^- \leftarrow 4 \times 10^{-4}$$

Quark masses + mixing

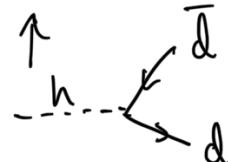
$$\mathcal{L}_{\text{Yuk.}} \supset - \left[ y_d \overline{d}_R \overset{+\frac{1}{3}}{\Phi} Q_L + y_d^* \overline{Q}_L \overset{-\frac{1}{3}}{\Phi} d_R \right]$$

h.c.

$$\overset{+\frac{1}{3}}{\Phi} Q_L = \begin{pmatrix} 0, & \frac{\sigma+h}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} d_L \\ \bar{d}_R \end{pmatrix} = \frac{\sigma+h}{\sqrt{2}} d_L$$

$$\mathcal{L}_{\text{Yuk.}} \supset - \left( \frac{y_d v}{\sqrt{2}} \right) \bar{d} d - \frac{y_d}{\sqrt{2}} h \bar{d} d$$

$\uparrow$   
 $m_d$



Nice feature of  $SU(2)$ :

"anti-doublet" transforms the same as doublet.

Conjugate doublet

$$\tilde{\Phi} \equiv i\tau^2 \bar{\Phi}^* = i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

↑  
Pauli matrix

unitarity gauge:

$$\tilde{\Phi} = \begin{pmatrix} (v+h)/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\tilde{\Phi}: v = -\frac{1}{2}$$

$$L_{Yuk.} \supset - \left[ y_u \bar{u}_R \tilde{\Phi} \overset{-2}{\cancel{\Phi}} \overset{+}{Q_L} + h.c. \right]$$

$$\tilde{\Phi}^+ Q_L = \left( \frac{v+h}{\sqrt{2}}, 0 \right) \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \frac{v+h}{\sqrt{2}} u_L$$

$$= - \left( \frac{y_u v}{\sqrt{2}} \right) \bar{u} u - \frac{y_u}{\sqrt{2}} h \bar{u} u$$

$$\begin{matrix} \uparrow \\ m_u \end{matrix}$$

3 generations of quarks:

$$Q_{Lj}, \bar{u}_{Rj}, \bar{d}_{Rj} \quad j=1,2,3 \text{ generation index}$$

$$L_{Yuk.}^q = - \sum_{i=1}^3 \sum_{j=1}^3 \left[ y_{ij}^u \bar{u}_{Ri} \tilde{\Phi}^+ Q_{Lj} + y_{ij}^d \bar{d}_{Ri} \tilde{\Phi}^+ Q_{Lj} + h.c. \right]$$

$$j\bar{j} \rightarrow \nu \bar{\nu}$$

Two  $3 \times 3$  complex matrices  $y_{ij}^u, y_{ij}^d$

Quark mass terms:  $\langle \bar{q} q \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\mathcal{L}_{\text{Yuk}} \supset - \left( \bar{u}_1, \bar{u}_2, \bar{u}_3 \right)_R \mathcal{M}^u \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L$$

↑ matrix

$$- \left( \bar{d}_1, \bar{d}_2, \bar{d}_3 \right)_R \mathcal{M}^d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + \text{h.c.}$$

$$\mathcal{M}_{ij}^u = \frac{v}{\sqrt{2}} y_{ij}^u \quad \mathcal{M}_{ij}^d = \frac{v}{\sqrt{2}} y_{ij}^d \quad \leftarrow$$

Diagonalize  $\mathcal{M}_{ij}^u$  and  $\mathcal{M}_{ij}^d$ :

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}$$

4 unitary transformations

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^{-1} M^u U_L = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

$$D_R^{-1} M^d D_L = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$h\bar{q}q : -i\frac{y_h}{\sqrt{2}} = -\frac{im_h}{v}$  + no off-diagonal  
(flavor-changing)  
Higgs couplings.

W couplings to quarks:

$$\begin{aligned}
 W_\mu^+ & \left[ \bar{u}_{L1} \gamma^\mu d_{L1} + \bar{u}_{L2} \gamma^\mu d_{L2} + \bar{u}_{L3} \gamma^\mu d_{L3} \right] \\
 \overbrace{J_L^{f\mu}} &= (\bar{u}_1 \bar{u}_2 \bar{u}_3) \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \\
 &= (\bar{u}, \bar{c}, \bar{t}) \bar{u}_L \gamma^\mu D_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} \\
 &= (\bar{u}, \bar{c}, \bar{t}) \gamma^\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}
 \end{aligned}$$

.. .. t

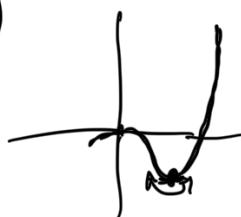
$$V_{CKM} = U_L V_L$$

$Z_\mu$  couplings

$$\text{coeff.} \left[ (\bar{u}_1 \bar{u}_2 \bar{u}_3)_{L,R} Y^\mu \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} \right] \\ = (\bar{u} \bar{c} \bar{t})_L U_L^+ Y^\mu U_L \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$


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$$\mathcal{L}_{V(\Phi)} = -V(\Phi) = \mu^2 \Phi^+ \Phi - \lambda (\Phi^+ \Phi)^2$$



$$\Phi^+ \Phi = \frac{1}{2} (v + h)^2 \quad \mu^2 = \lambda v^2$$

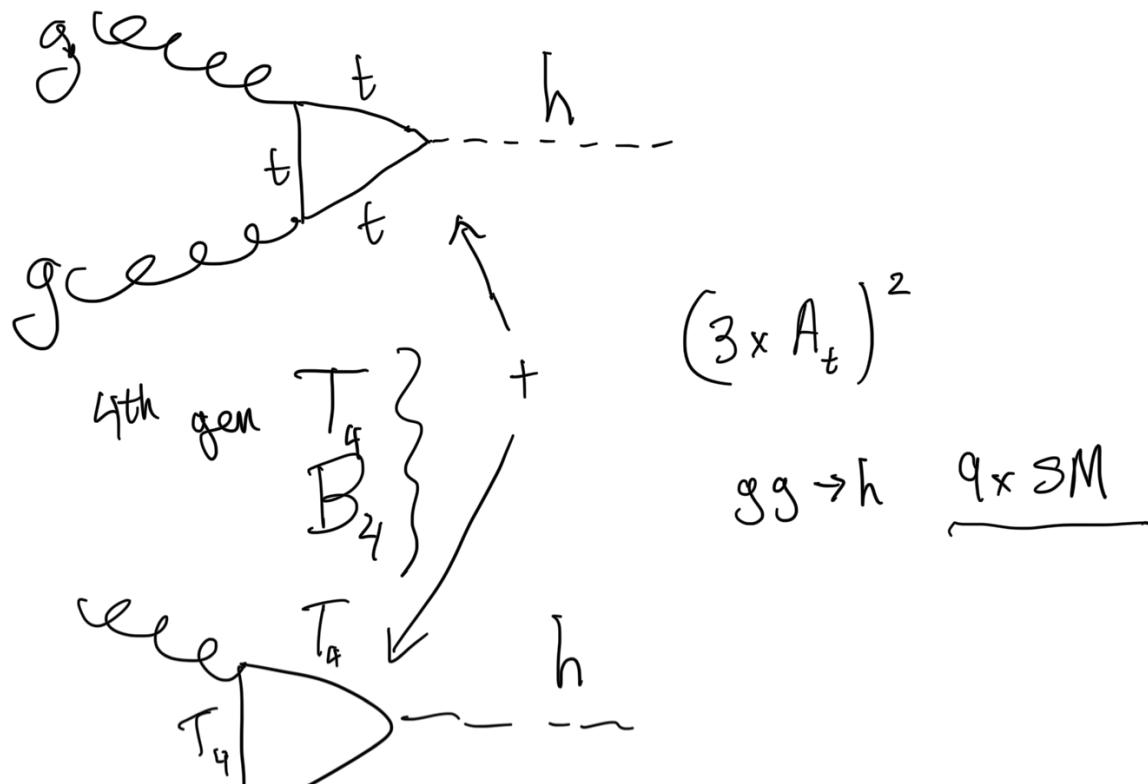
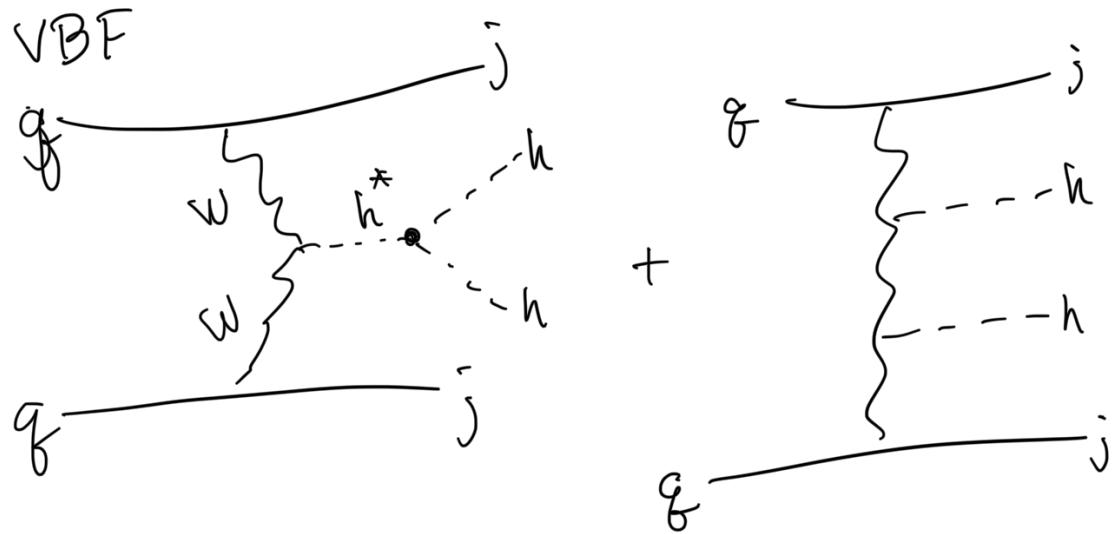
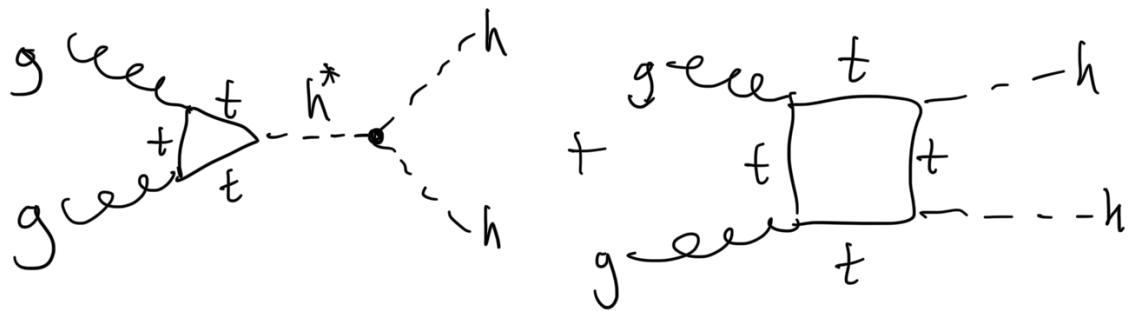
$$\mathcal{L}_v = \underbrace{-\lambda v^2 h^2}_{-\frac{1}{2} M_h^2} - \underbrace{\lambda v h^3}_{h \cdot h} - \underbrace{\frac{\lambda}{4} h^4}_{h \cdot h \cdot h \cdot h} + \text{const.}$$

$$\left. \begin{array}{l} v = 246 \text{ GeV} \\ M_h = 125 \text{ GeV} \end{array} \right\} \Rightarrow \begin{array}{l} \lambda = 0.129 \\ -\mu^2 = -(88.4 \text{ GeV})^2 \end{array}$$

$$Q = T^3 + Y$$

↑      ↑      ↑

LHC:



Elect T<sub>4</sub>