Small-*x* resummation effects in (NNLO) PDF fits

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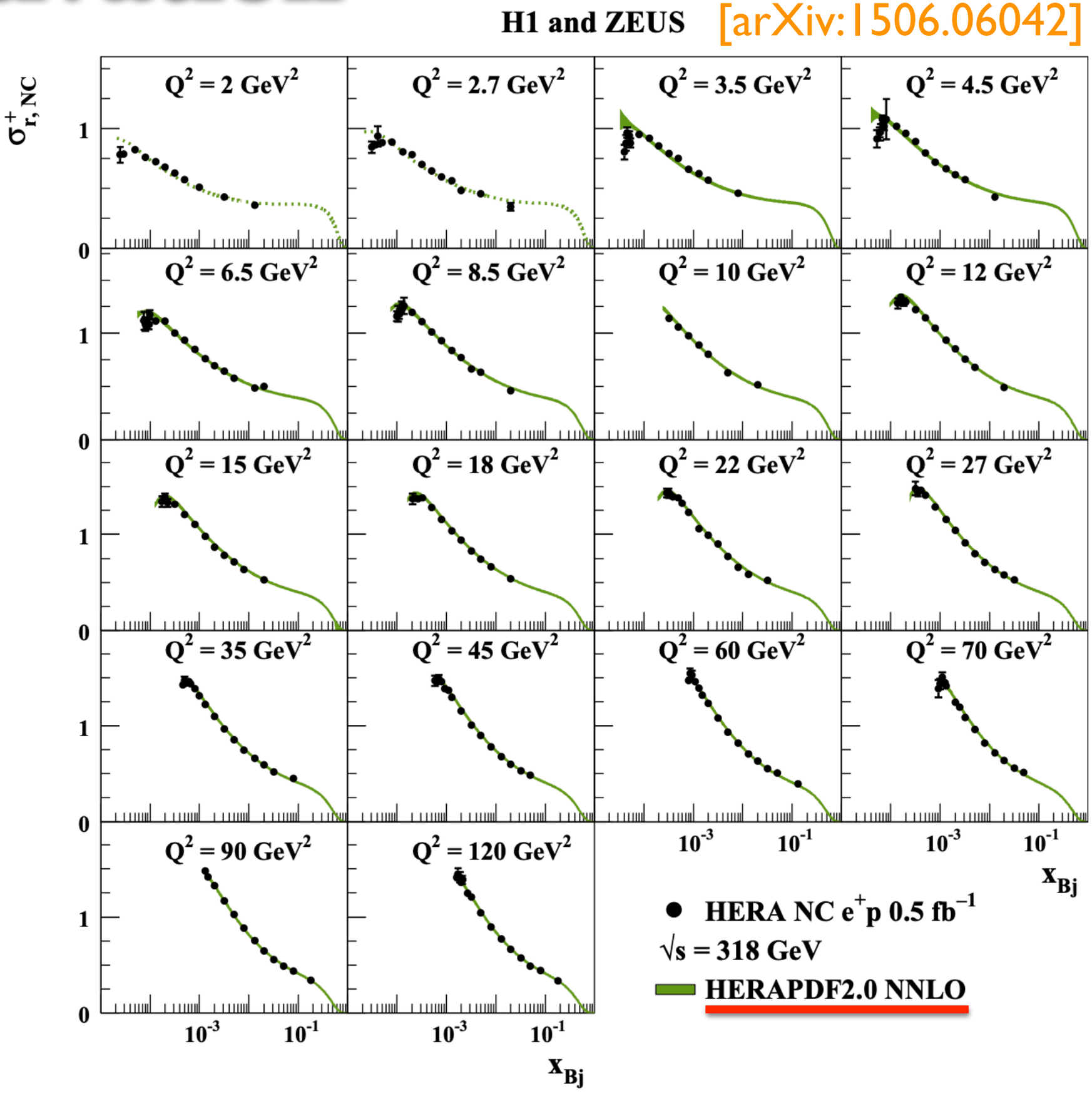
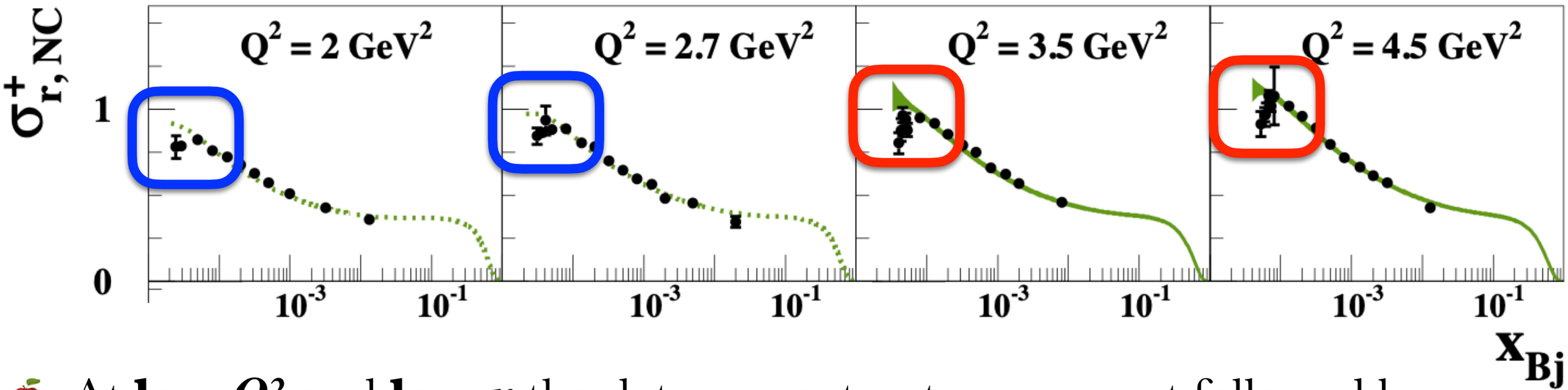
Motivation


H1 and ZEUS
[arXiv:1506.06042]
$\sigma^+_{r,NC}$
$Q^2 = 2$ GeV^2
$Q^2 = 2.7$ GeV^2
$Q^2 = 3.5$ GeV^2
$Q^2 = 4.5$ GeV^2
$Q^2 = 6.5$ GeV^2
$Q^2 = 8.5$ GeV^2
$Q^2 = 10$ GeV^2
$Q^2 = 12$ GeV^2
$Q^2 = 15$ GeV^2
$Q^2 = 18$ GeV^2
$Q^2 = 22$ GeV^2
$Q^2 = 27$ GeV^2
$Q^2 = 35$ GeV^2
$Q^2 = 45$ GeV^2
$Q^2 = 60$ GeV^2
$Q^2 = 70$ GeV^2
$Q^2 = 90$ GeV^2
$Q^2 = 120$ GeV^2
0
1
10^{-3}
10^{-1}
x_{Bj}
HERA NC e^+p 0.5 fb^{-1}
$\sqrt{s} = 318$ GeV
HERAPDF2.0 NNLO

Motivation



- At **low Q^2** and **low x** the data presents a turnover not followed by fixed-order (NNLO) predictions.

- **Enhancement of F_L** not predicted by fixed-order QCD:

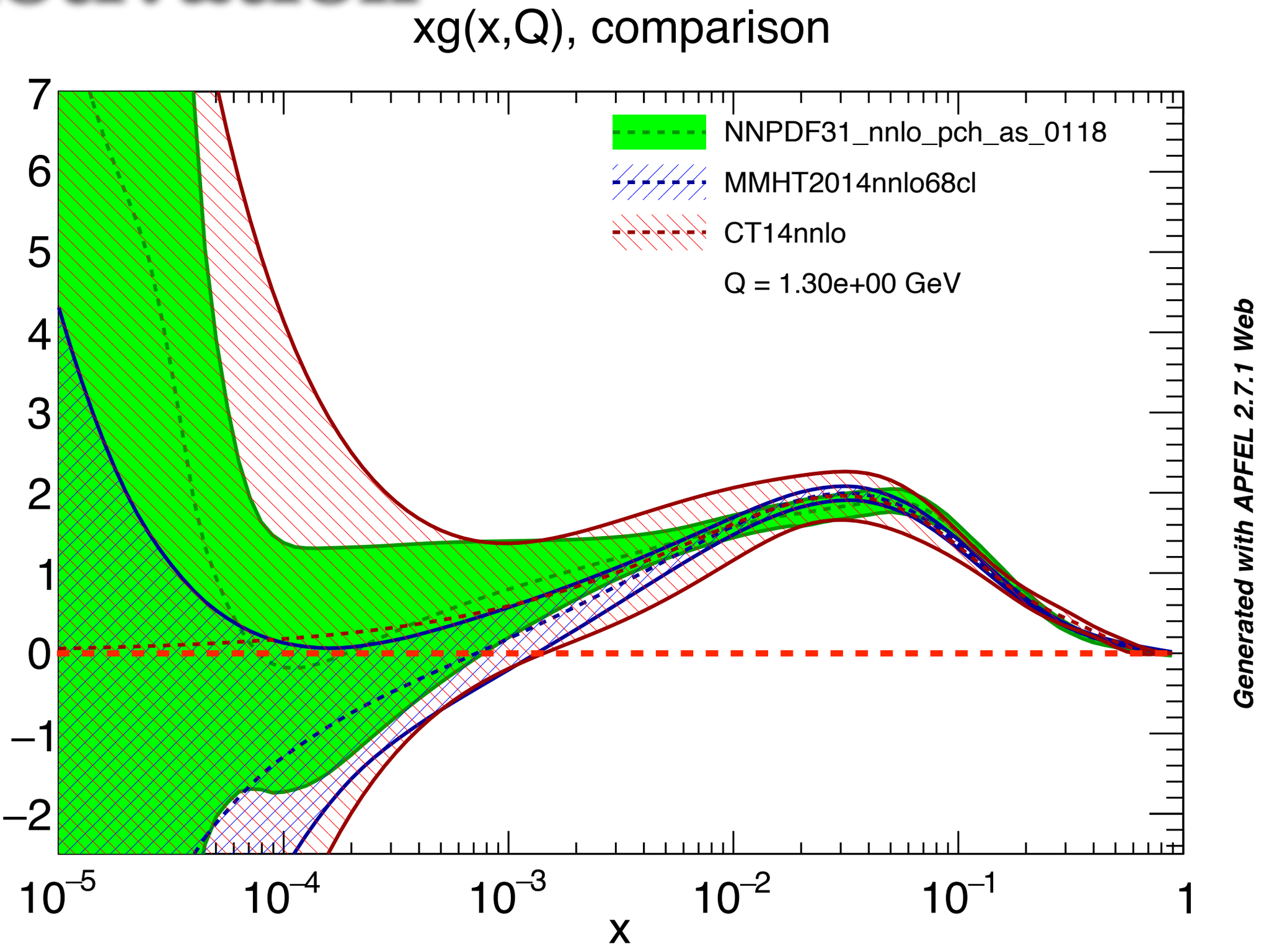
$$\sigma^{+}_{r,\,\mathrm{NC}} = F_2 - \frac{y^2}{1+(1-y)^2} F_L \qquad y = \frac{Q^2}{xs}$$

- but:

$$F_L(x, Q^2) \sim \alpha_s(Q) g(x, Q^2)$$

- Combined HERA I+II data is pointing to an **enhanced gluon PDF** at **low Q^2** and **low x**.

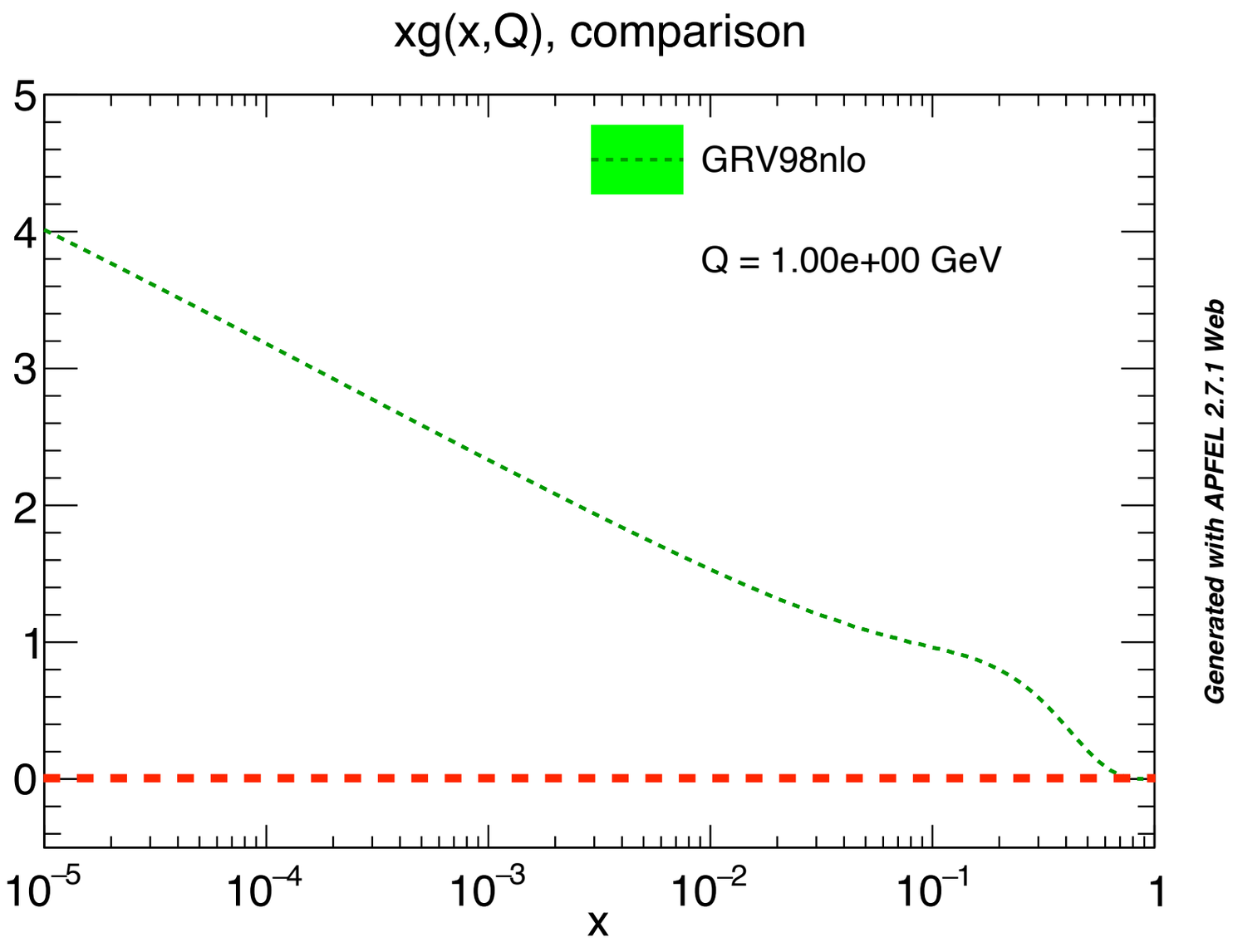
Motivation



- Actual and perhaps **unnatural** strong suppression of the NNLO gluon PDF at low Q^2 (= 1.3 GeV) and low x ($\lesssim 10^{-3}$).

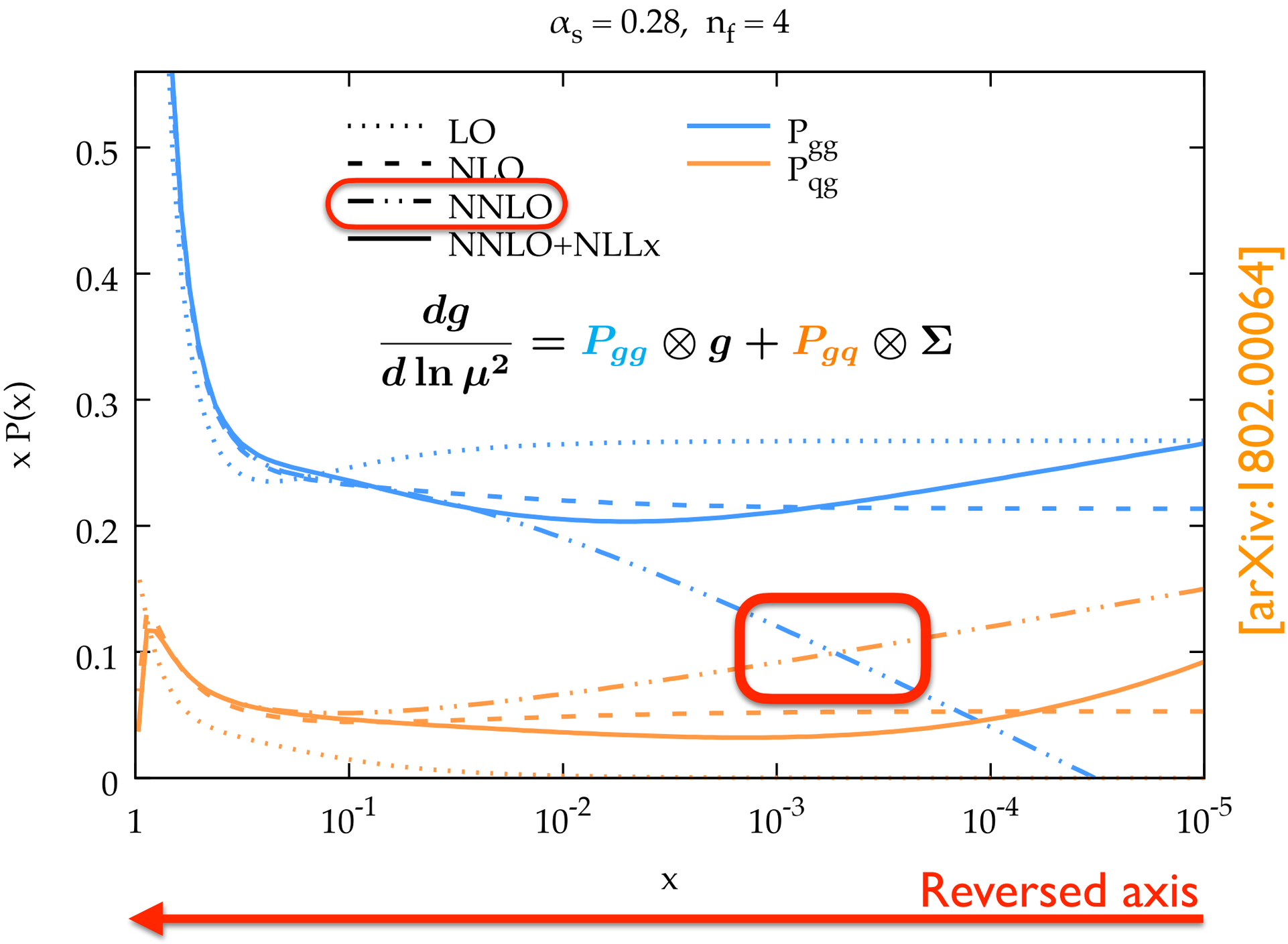
Digression

- Glück, Reya, and Vogt back in 1994 [*Z.Phys.C* 67 (1995) 433-448] assumed the gluon PDF to be mostly **dynamically** generated through DGLAP evolution from a low-scale valence-like configuration (~1600 citations).



- The gluon at low Q^2 and low x behaves much like a **sea** distribution.
- More **natural** but disfavoured within a (NNLO) fixed-order analysis.

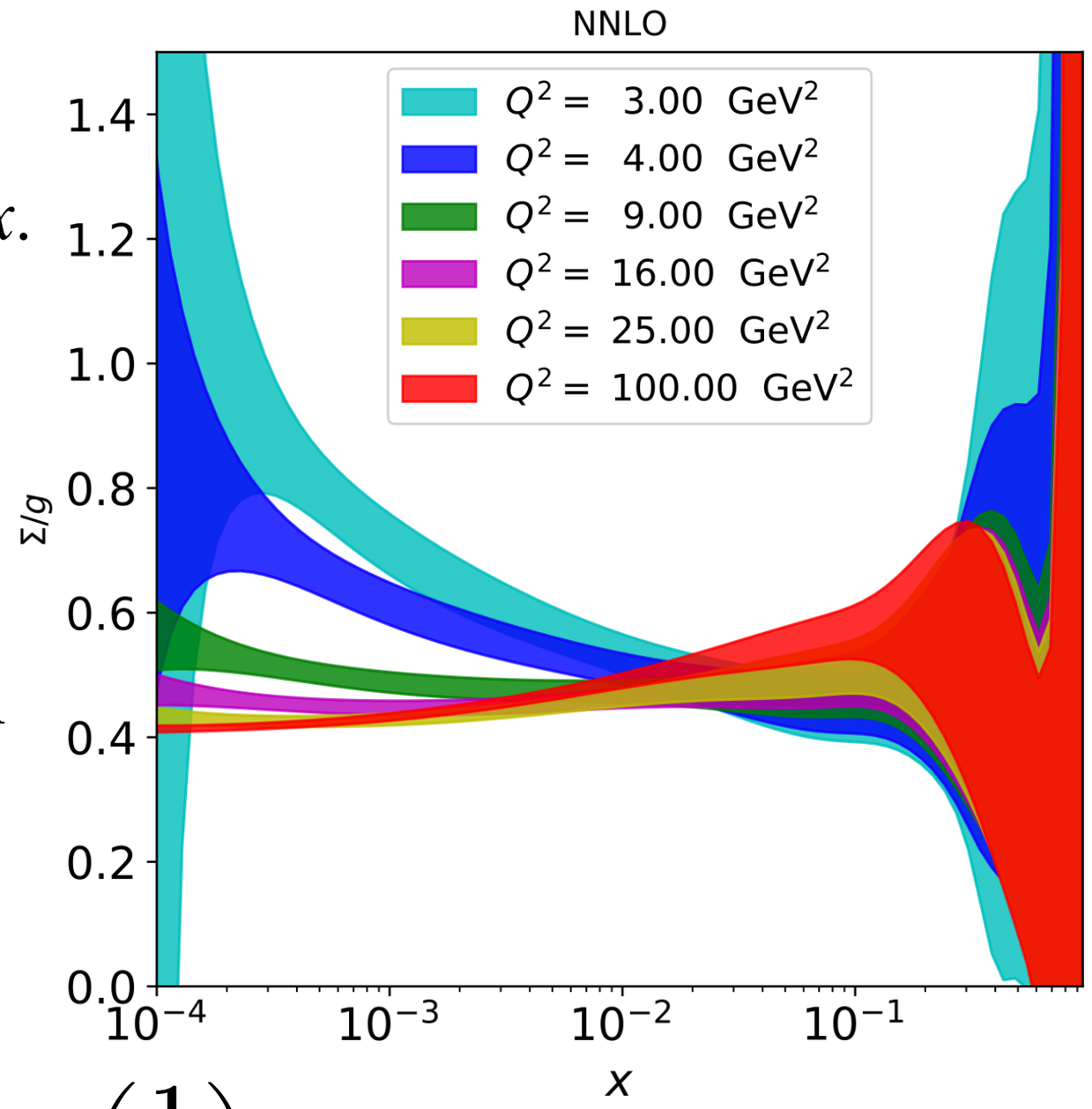
Fixed-order evolution



- At NNLO P_{gg} and P_{qg} have **opposite behaviour** at low x.
- Suppression of the low-x gluon in favour of the total singlet PDF.

Fixed-order evolution

- "Misbalance" of singlet (Σ) and gluon distributions at low Q^2 and x.
- Progressive **realignment** for larger values of Q^2 where most of the data is.
- Very **steep** evolution of the gluon PDF at low x (perhaps too steep).



- In the small-x regime:

$$P_{gi}^{(n)}(x) \underset{x\to 0}{\longrightarrow} \alpha_s^{n+1} \frac{1}{x} \sum_{k=0}^{n} \ln^k\left(\frac{1}{x}\right) P_{gi}^{(n,k)} \qquad i = g, q$$

- **Accidentally**:

$$P_{gi}^{(1,1)} = P_{gi}^{(2,2)} = 0$$

Small-x corrections to PDFs

- General idea: **match collinear** and **high-energy** factorisations:

$$\text{e.g. DIS:} \quad \sigma(x) = \begin{cases} C_i \otimes_x f_i & 0 \ll x \lesssim 1 \\ \mathcal{C}_g \otimes_{x,k_t} \mathcal{F}_g & x \ll 1 \end{cases}$$

- along with the "respective" **evolution equations**:

$$\begin{cases} \dfrac{df_i}{d\ln\mu^2} = P_{ij} \otimes_x f_i & \text{DGLAP} \\ \dfrac{d\mathcal{F}_g}{d\ln(1/x)} = \mathcal{K} \otimes_{k_t} \mathcal{F}_g & \text{BFKL} \end{cases}$$

- knowing the relation between integrated and unintegrated distributions:

$$\mathcal{F}_g = R \otimes_x \frac{df_i}{dk_t^2}$$

- and using the conformal *duality* of DGLAP and BFKL kernels.

See e.g. Bonvini, Marzani, Peraro [arXiv:1607.02153] and references therein.

Enabling PDF fits

- Matching of collinear and high-energy factorisations using the ABF approach [see *e.g.* arXiv:0802.0032] is currently implemented in the public code **HELL** (High-Energy Large Logarithm) developed by M. Bonvini and available at www.ge.infn.it/~bonvini/hell/.

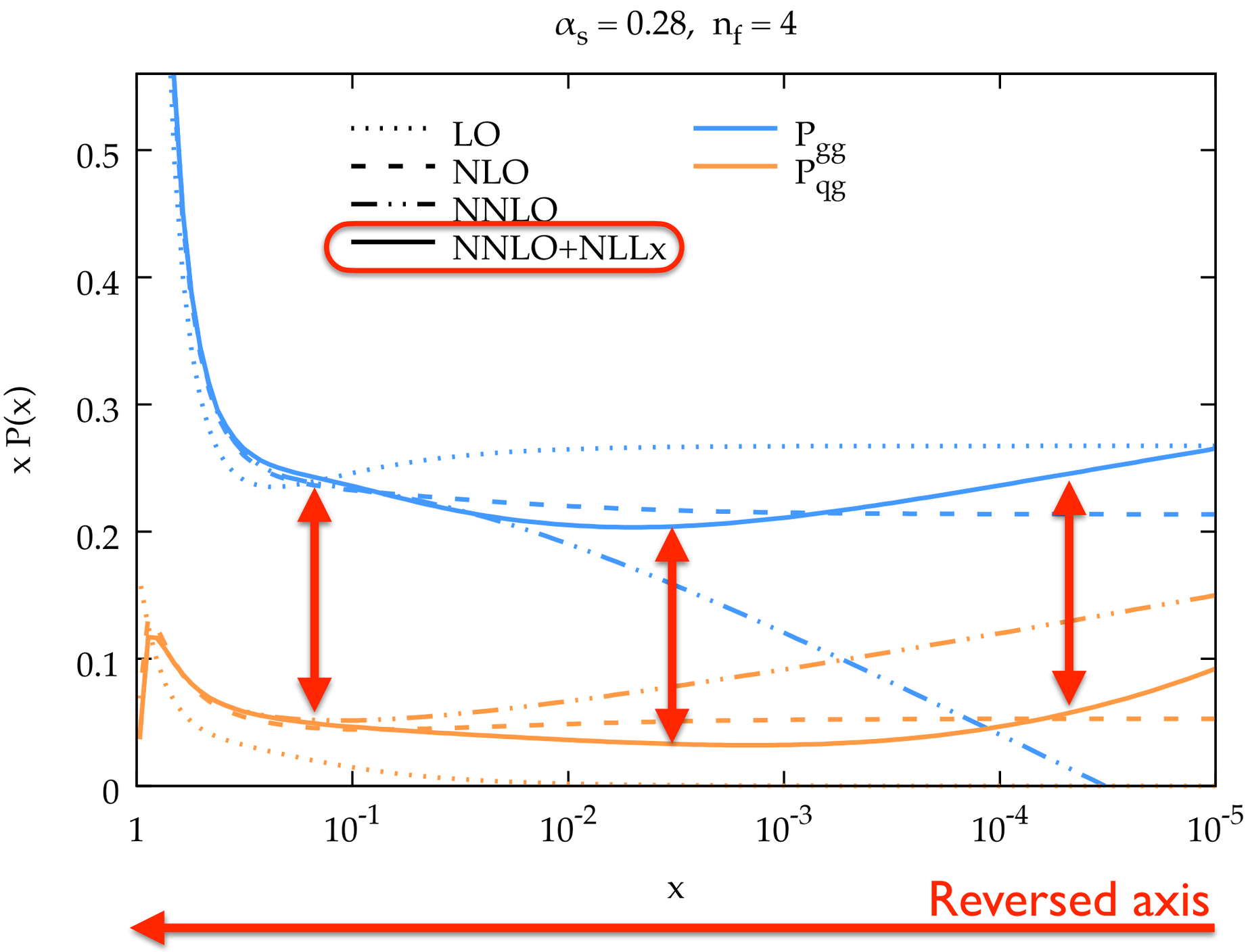
- HELLv3 provides small-x resummation corrections up to NLL accuracy (NLLx) to:
  - collinear **splitting functions** P_{ij},
  - massless and massive **DIS coefficient functions** C_{ij},
  - PDF **matching conditions** K_{ij}.

- Corrections are delivered as *additive* factors to the fixed-order expressions:

$$\text{e.g.}\quad P_{ij}^{\mathrm{N^kLO+NLL}x} = P_{ij}^{\mathrm{N^kLO}} + \Delta_{\mathrm{k}} P_{ij}^{\mathrm{NLL}x}$$

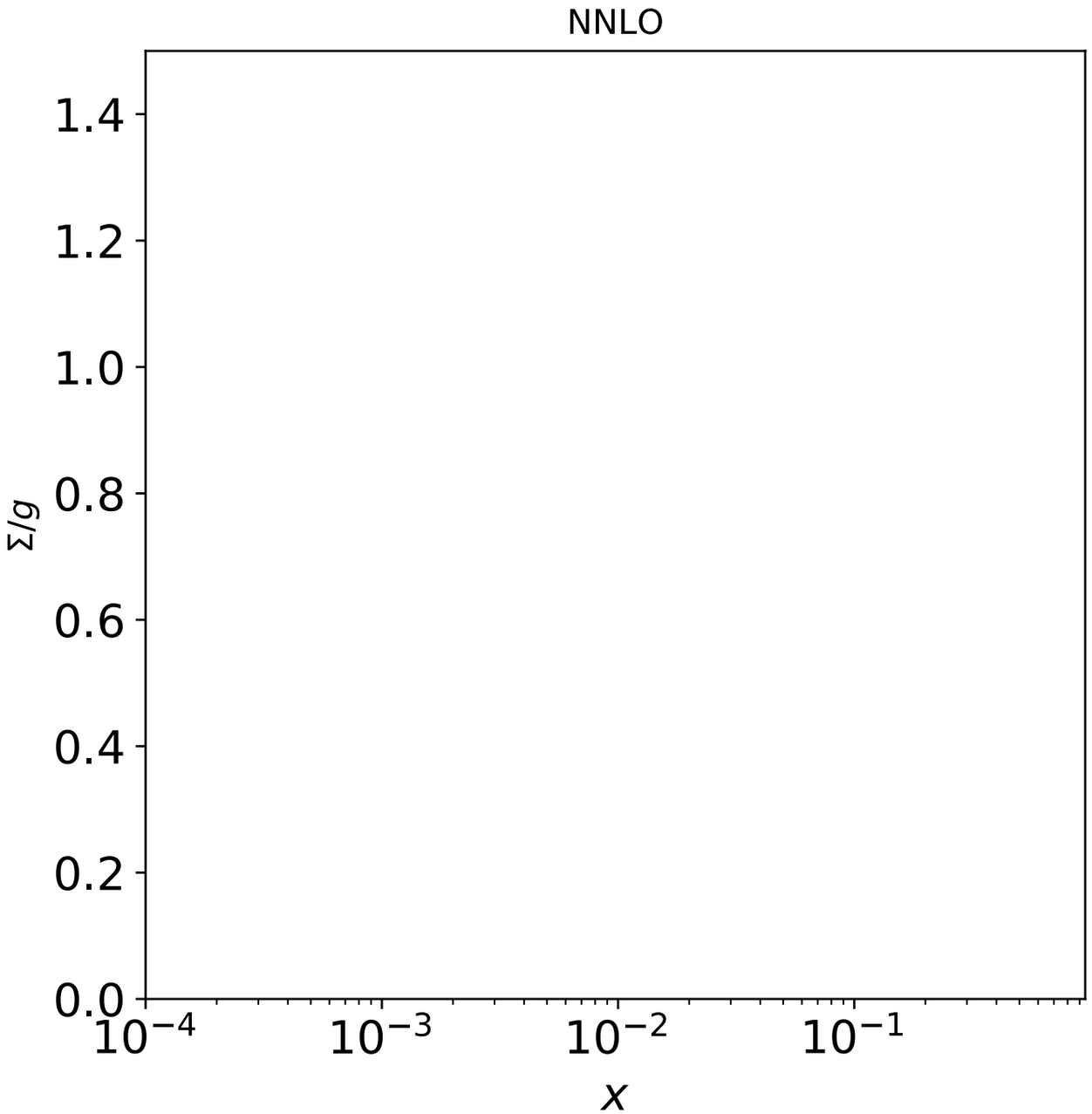
- HELL is interfaced to the public code **APFEL** (https://github.com/scarrazza/apfel):
  - **NNPDF3.1sx** analysis [arXiv:1710.05935],
  - **xFitter** analysis (https://www.xfitter.org/xFitter/) [arXiv:1802.00064].

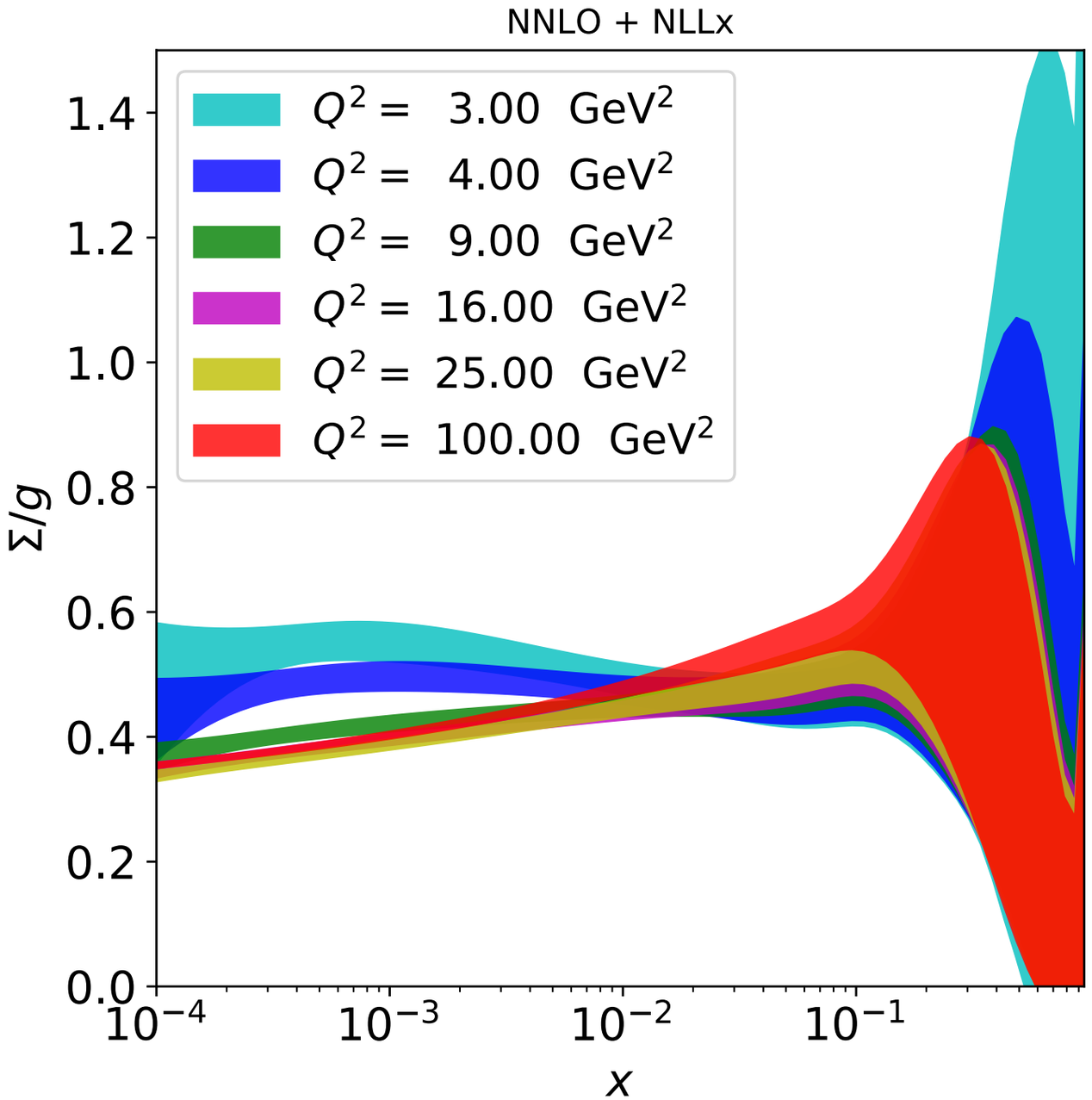
Effect of small-x corrections



- Restoration of the hierarchy of P_{gg} and P_{qg} at NNLO+NLLx.

Effect of small-x corrections



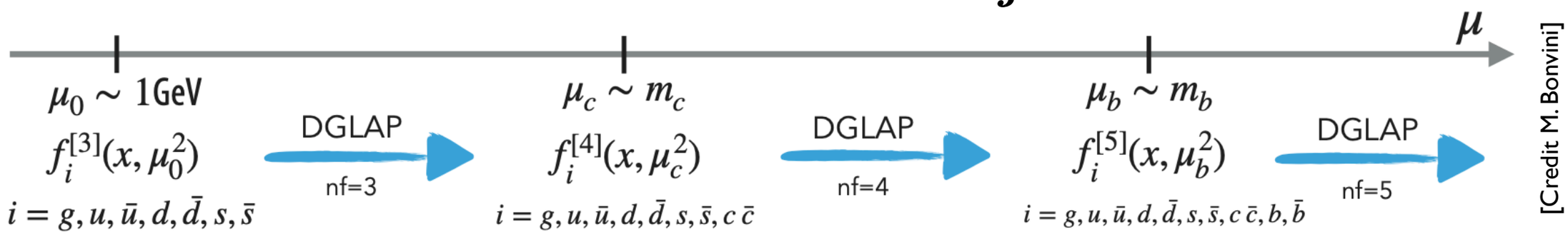


- Small-x evolution of gluon and singlet PDFs becomes very similar when including small-x resummation effects.

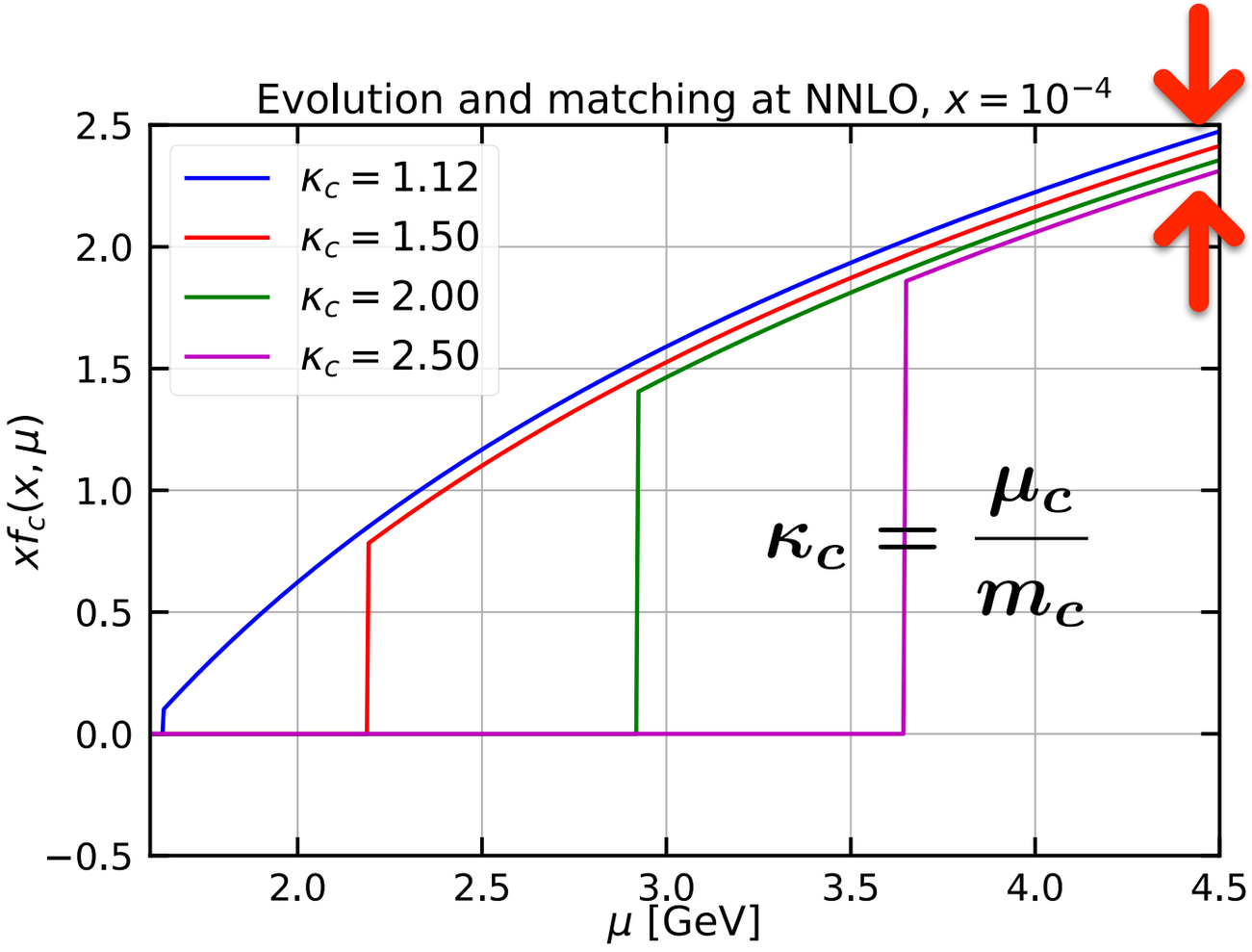
Effect of small-x corrections

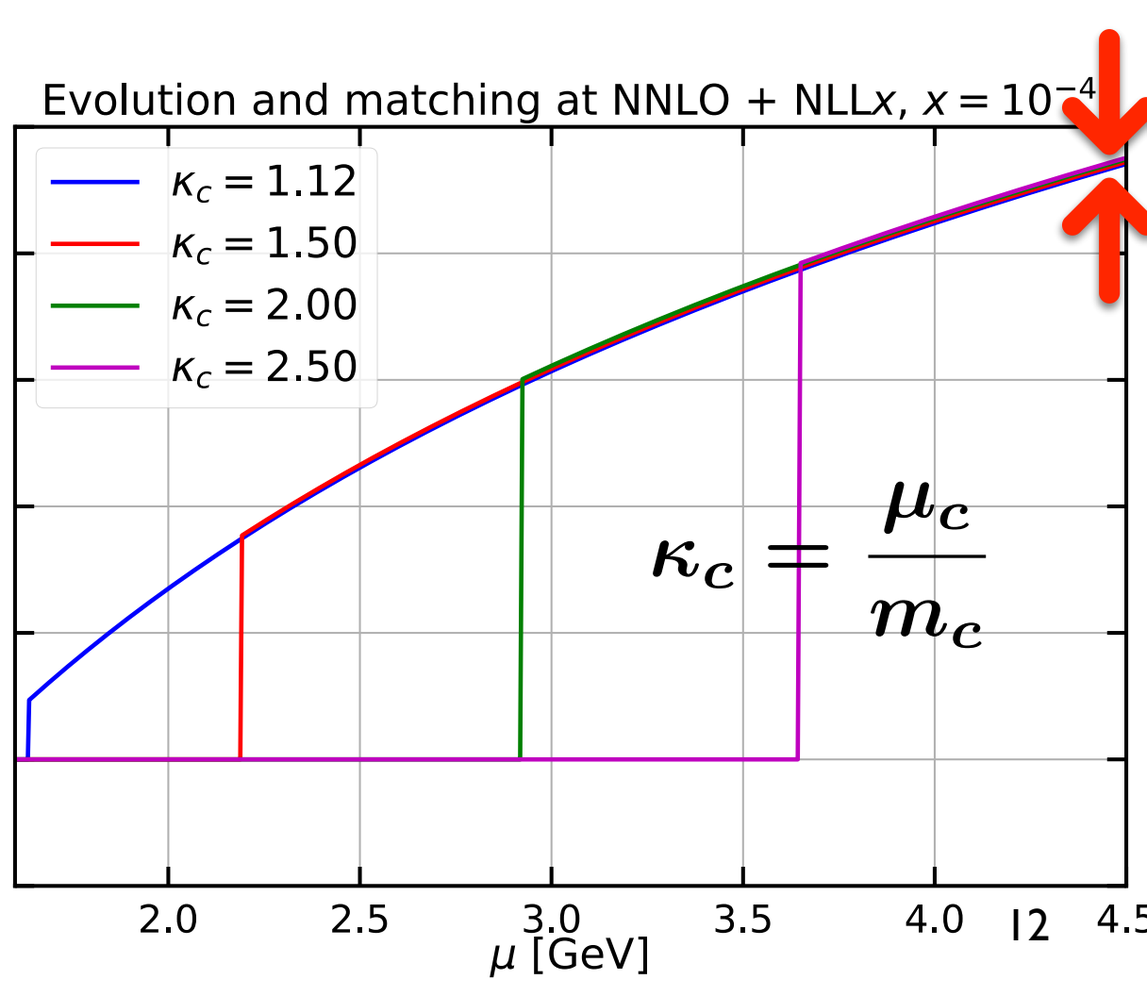
- In a variable-flavour-number scheme the number of *active* flavours n_f changes by one unit when crossing each heavy-quark threshold:

$$f_i^{[n_f+1]} = K_{ij} \otimes_x f_j^{[n_f]}$$

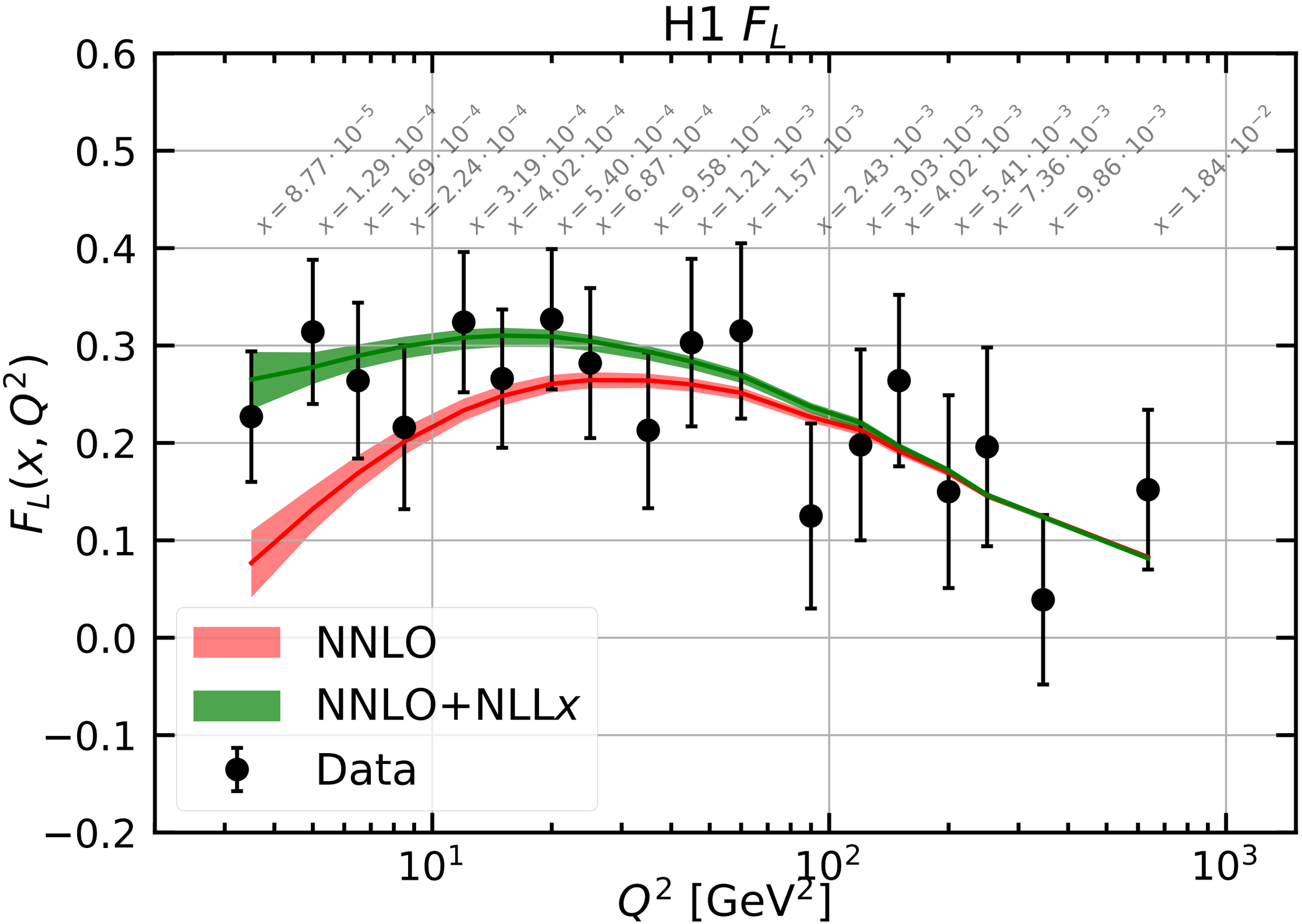


[Credit M. Bonvini]





Effect of small-x corrections



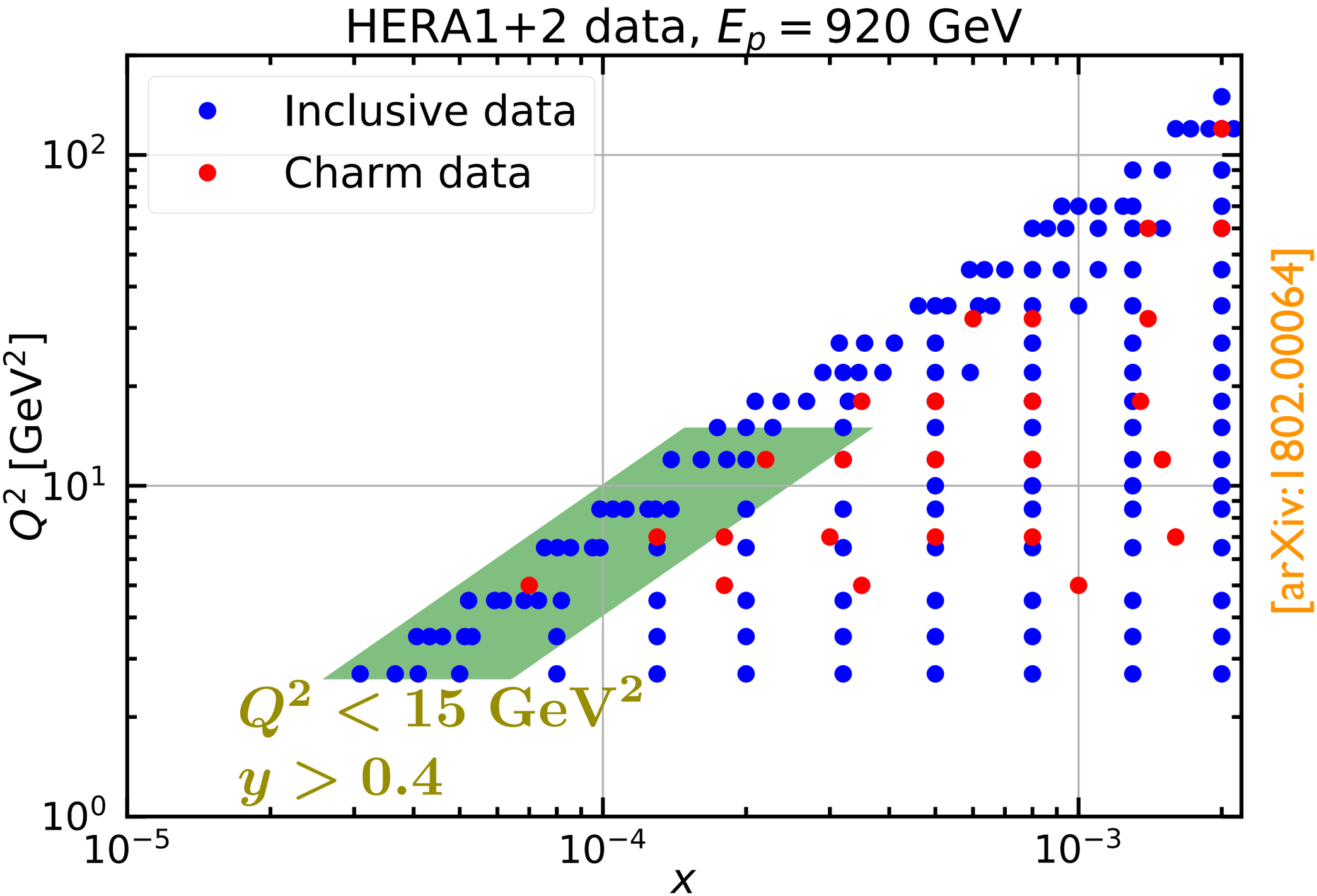
Small-x corrections in PDF fits

- As of today, there **three** PDF analyses including small-x corrections:
  - all based on **HELL + APFEL**,
  - different fitting **methodologies**,
  - different **datasets**,
  - small-x corrections to **evolution** and **DIS** cross sections only,
  - cuts on **hadronic data** (NNPDF) to exclude small-x sensitive data.

Analysis	**Param.**	**Dataset**	N_{pt}	χ^2 **NNLO**	χ^2 **NNLO+NLLx**	$\Delta\chi^2$
NNPDF3.1sx [arXiv:1710.05935]	Neural nets	DIS + (pN)	4052	4578	4457	**121**
xFitter (1) [arXiv:1802.00064]	Standard	HERA Inclusive+c+b	1221	1468	1394	**74**
xFitter (2) [arXiv:1902.11125]	Standard improved	HERA Inclusive	1145	1284	1312	**28**

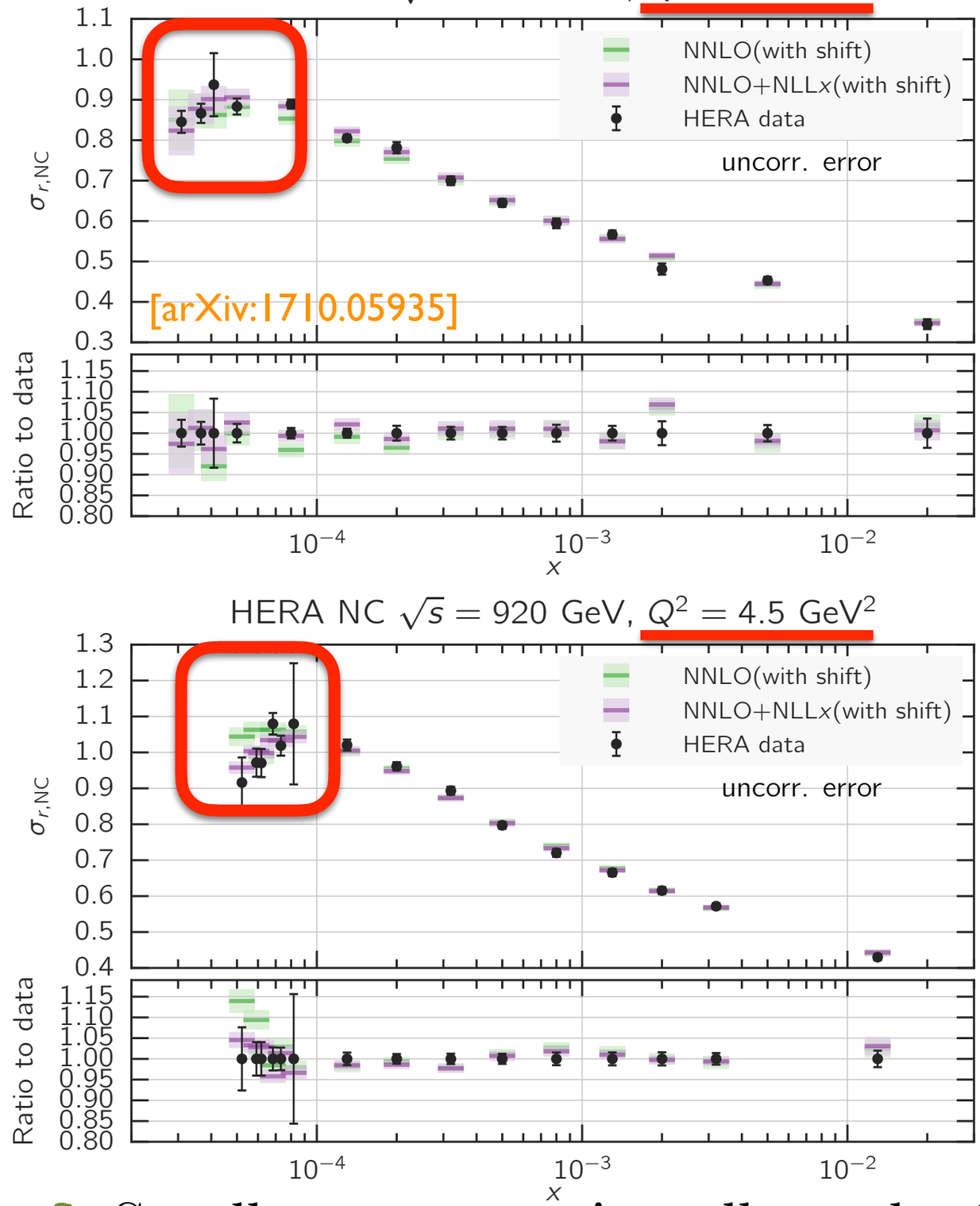
- Improvement driven by the low-Q^2 low-x **HERA data**.

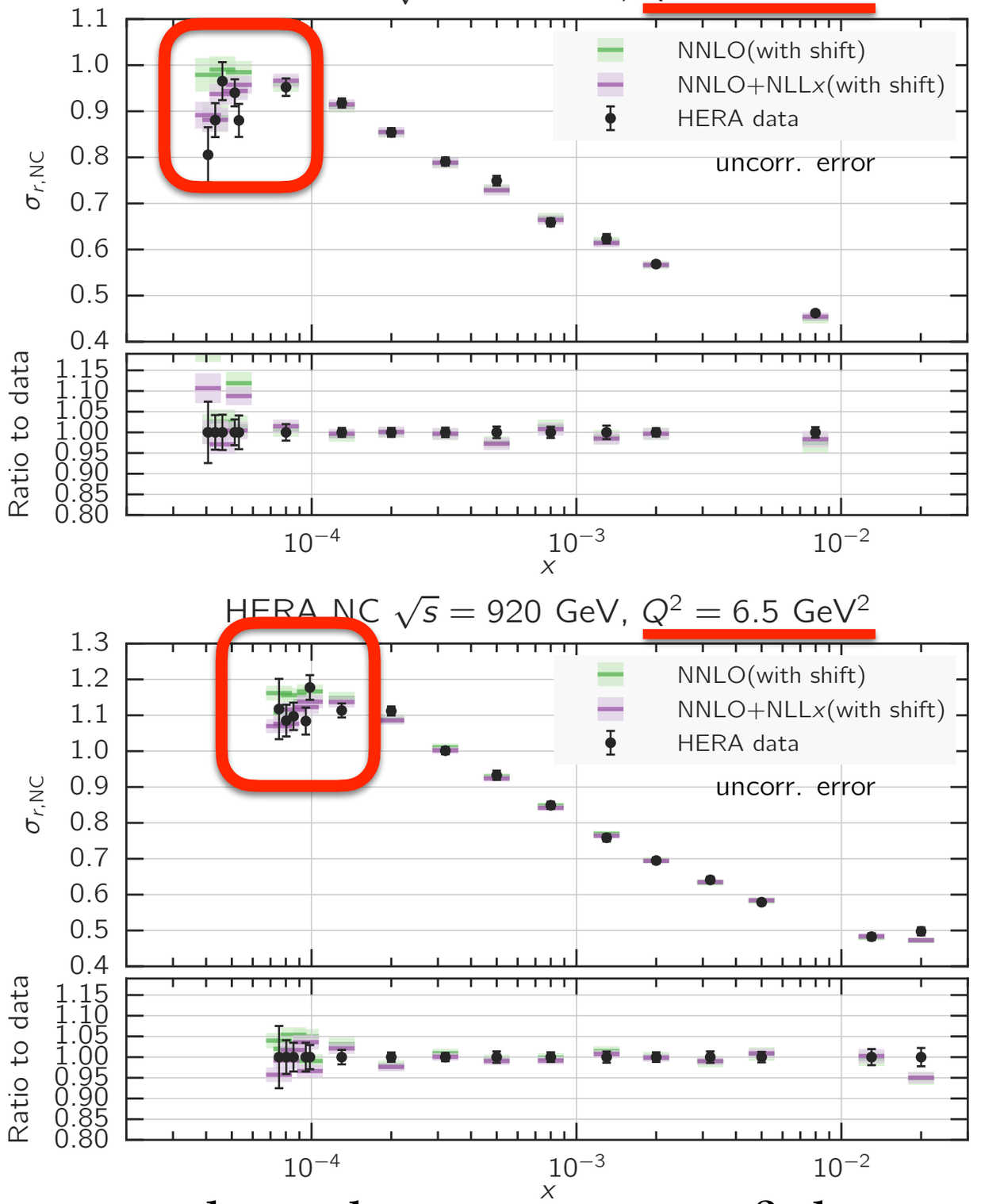
Small-x corrections in PDF fits



- Small-x resummation corrections are particularly relevant in the **green** region of the (x, Q^2) plane.

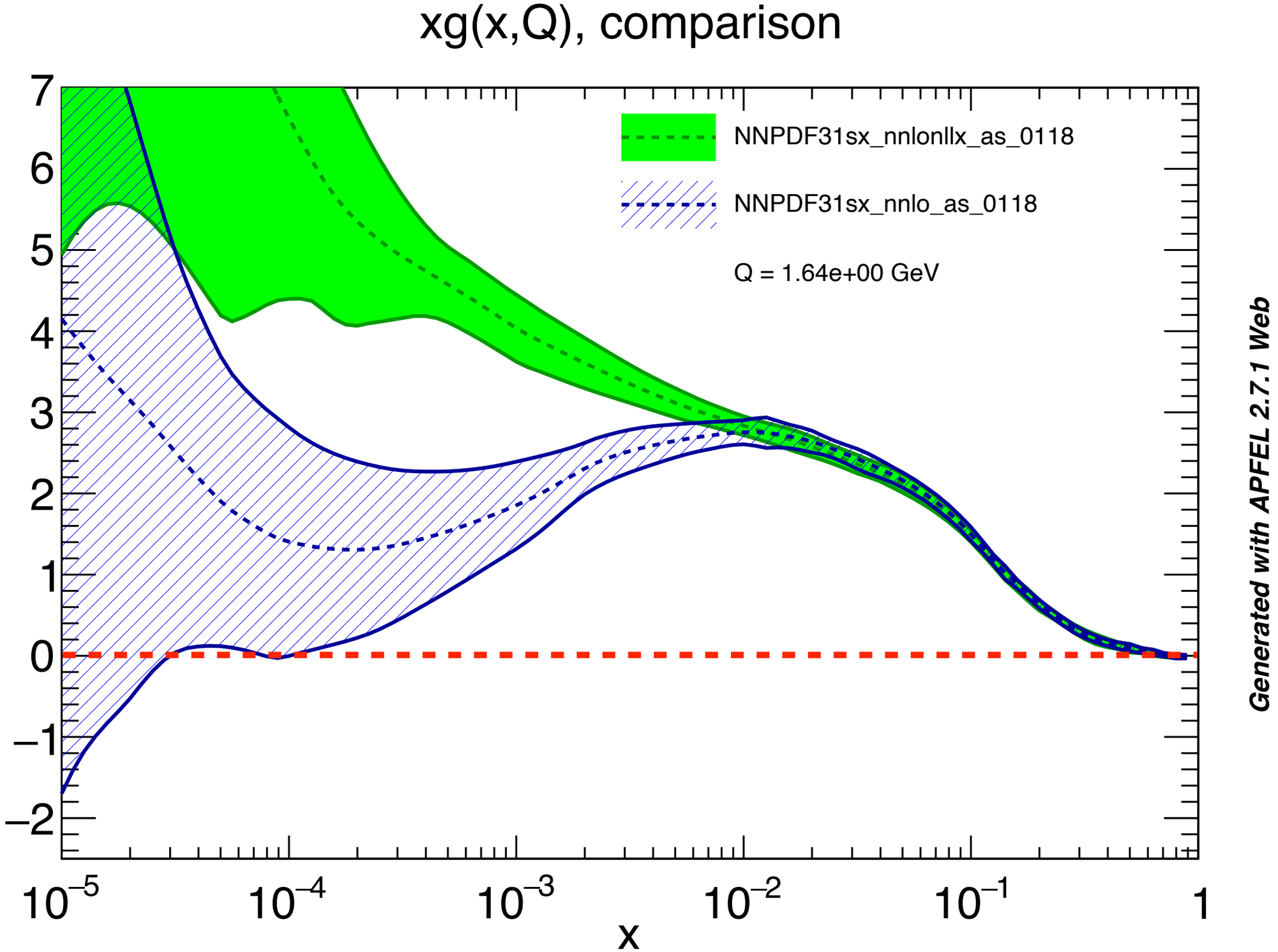
Small-x corrections in PDF fits





- Small-x resummation allows the fit to reproduce the **turnover** of the HERA data at low-Q^2 and low-x.

Effect of small-x corrections

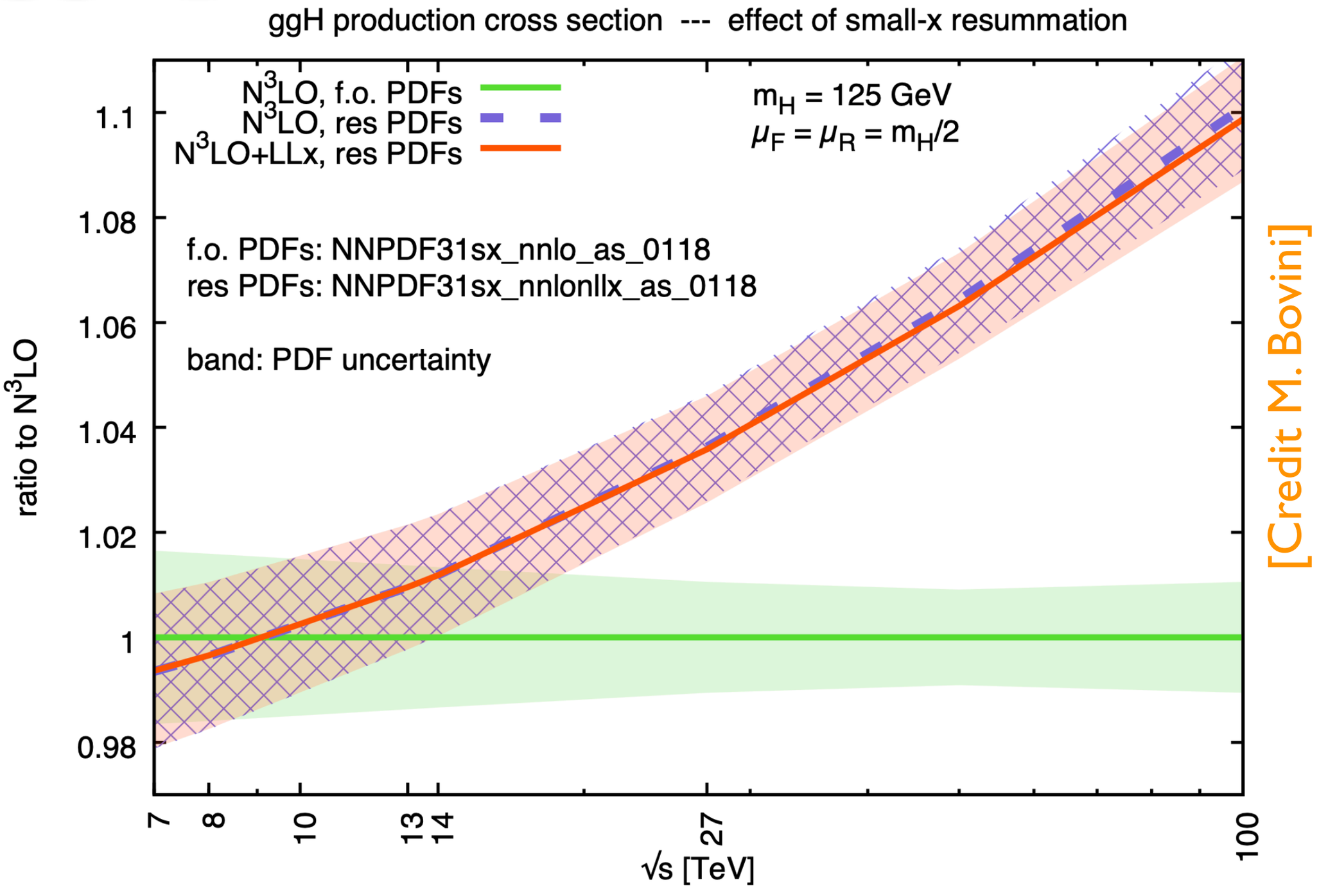


- Small-x resummation effects make the low-Q^2 and low-x gluon PDF look much like a *sea* distribution.

Hadron-hadron collisions

- Small-x corrections to **hadronic processes** typically included in PDF analyses are not (publicly) available as yet:
  - in the NNPDF3.1sx analysis hadronic-data predictions are computed at fixed-order but cuts are enforced to exclude kinematic regions sensitive to small-x resummation effects.
- Processes already available in HELL:
  - total cross section for $\boldsymbol{gg \to H}$ [Bovini, Marzani, 1802.07758] [Bonvini, 1805.08785].
- Processes being considered in HELL:
  - differential cross section for $\boldsymbol{gg \to H}$,
  - differential cross section for **heavy-quark-pair** production [Bonvini, Silvetti, in preparation],
  - differential cross section for **Drell-Yan** production [Bonvini, in preparation].
- Inclusion in PDF fits as soon as available through HELL + APFEL.

Higgs production



- As expected small-x resummation effects grow as the energy grows.
- Indication that PDF **evolution** is responsible for the bulk of the effect.

Conclusions

- Small-x resummation effects have very recently been included for the first time in PDF fits in a consistent way:
  - NLLx corrections included in splitting functions, PDF matching conditions, and DIS hard cross sections.
  - this was possible thanks to HELL [M. Bonvini] and APFEL [V. Bertone].
  - These corrections improve very significantly the fit quality allowing for a better description of the low-Q^2 and low-x HERA data.
  - Large impact on the resulting gluon PDF (resolving a longstanding *unnaturalness* at low x).
- Exploration of the hadronic processes has just started:
  - A number of relevant processes are already being considered.
  - Further impact of PDFs to be expected.

Backup

xFitter (2)

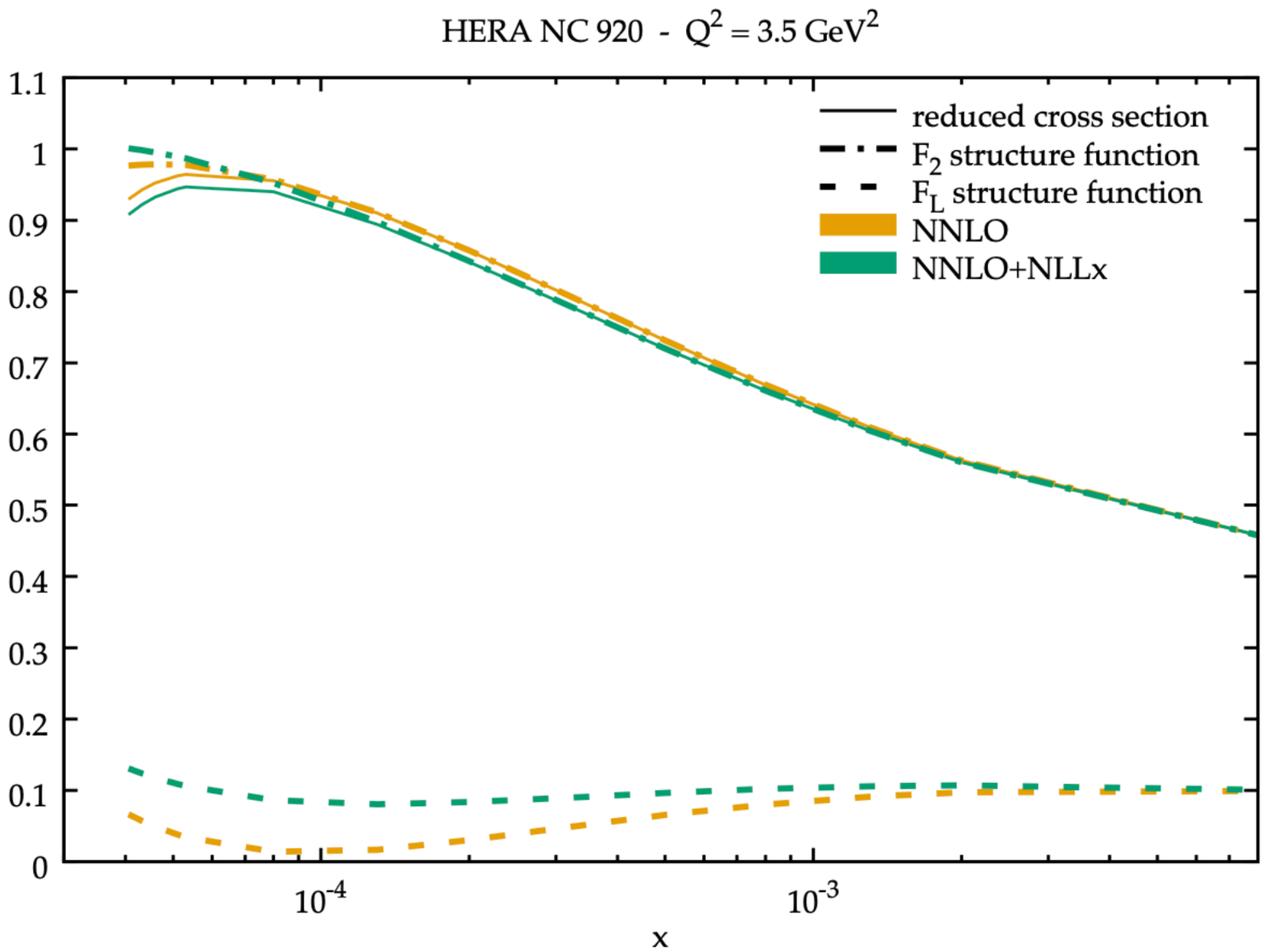
$$xg(x,\mu_0^2) = A_g\, x^{B_g}(1-x)^{C_g}\left[1 + F_g \log x + G_g \log^2 x\right]$$

$$xu_v(x,\mu_0^2) = A_{u_v}\, x^{B_{u_v}}(1-x)^{C_{u_v}}\left[1 + E_{u_v} x^2 + F_{u_v} \log x + G_{u_v} \log^2 x\right]$$

$$xd_v(x,\mu_0^2) = A_{d_v}\, x^{B_{d_v}}(1-x)^{C_{d_v}}$$

$$x\bar{u}(x,\mu_0^2) = A_{\bar{u}}\, x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}\left[1 + D_{\bar{u}} x + F_{\bar{u}} \log x\right]$$

$$x\bar{d}(x,\mu_0^2) = A_{\bar{d}}\, x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}}\left[1 + D_{\bar{d}} x + F_{\bar{d}} \log x\right],$$



[Bonvini, Giuli, arXiv:1902.11125]