# Theory Approaches to Heavy-Quark Exotic Hadrons: A Breezy Overview

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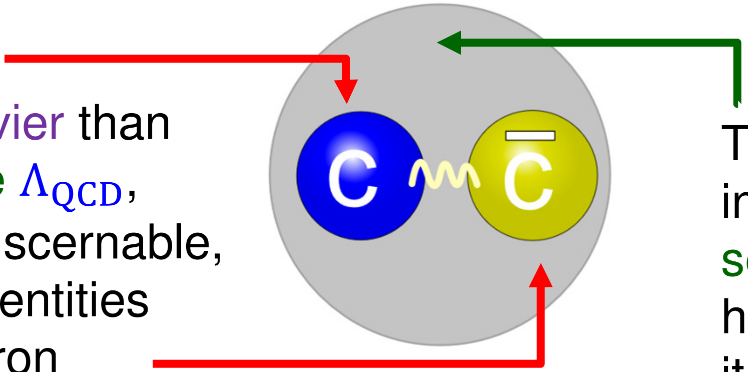
# How can it be so hard to figure out what the exotics are?

- When $J/\psi$ was discovered on November 11, 1974, it was immediately recognized to be this:

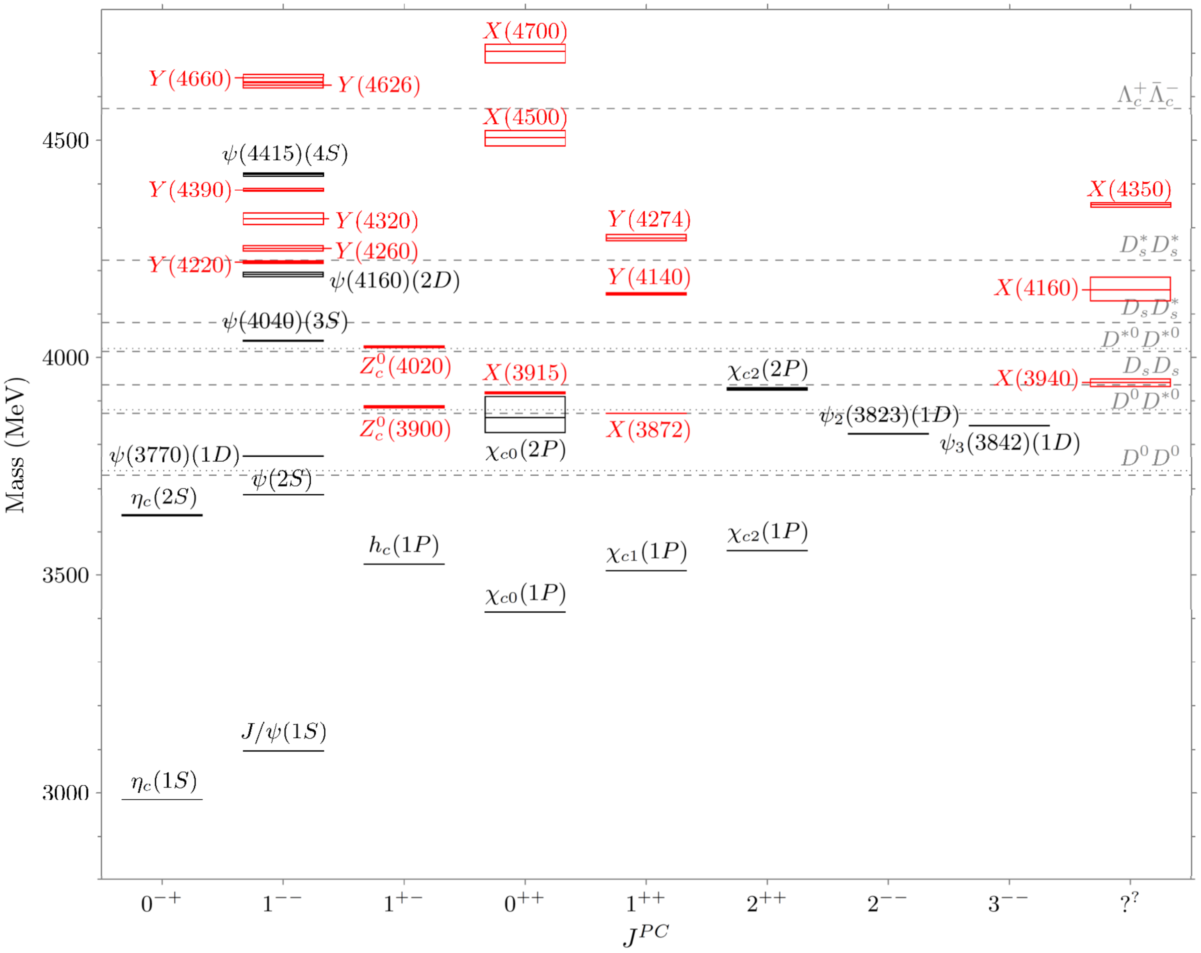


These quarks are much heavier than the QCD scale $\Lambda_{\mathrm{QCD}}$, & hence are discernable, nonrelativistic entities within the hadron

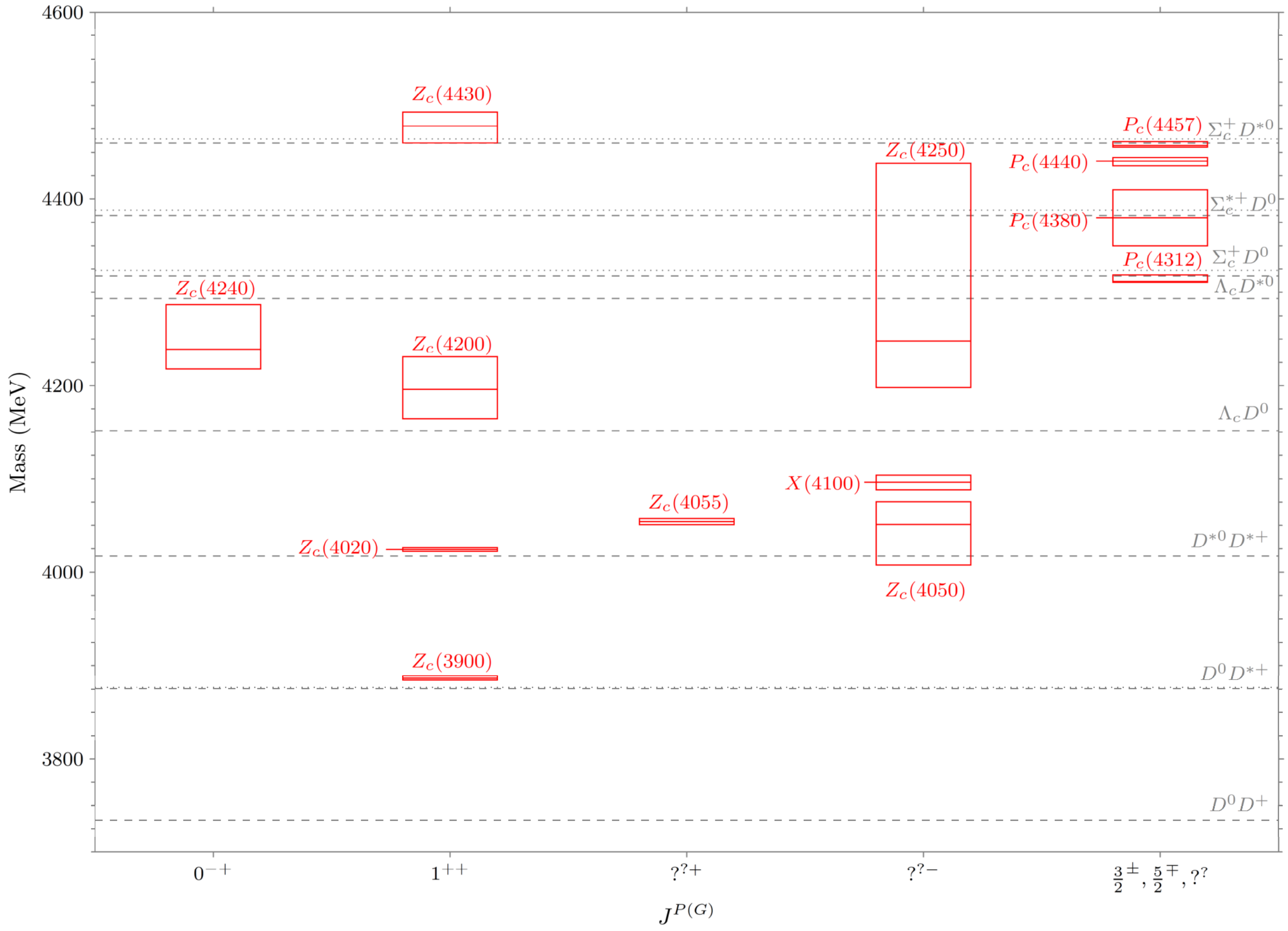
This part consists of innumerable gluons and sea $q\bar{q}$ pairs, but in the hadrons of lowest mass it just has $J^{PC} = 0^{++}$

- Thus quarkonium can be treated as a two-body problem in a simple scalar potential $V(r)$, and given also $m_c$, the Schrödinger equation predicts the entire spectrum [*e.g.*, the Cornell potential, PRD **17** (1978) 3090, PRD **21** (1980) 203]

# It works very well for $c\bar{c}$ states, but…



# Charged charmoniumlike states don’t fit into this scheme either



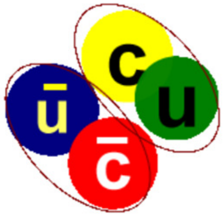
# The problems with exotics modeling

- $4$- and $5$-quark states would be hard to model analytically
  *even* if all the quarks were heavy, and
  *even* if the QCD glue interactions were greatly simplified
  - Developing good methods to study multi-electron atoms took decades, and that's just Coulomb interactions and quantum mechanics
- The most physically significant degrees of freedom in the exotics seem to vary from state to state
  - Stronger couplings to open-flavor hadrons or to quarkonium?
- Many have the same $J^{PC}$ as conventional hadrons & might mix
- The world data set is constantly improving
  - Models based upon hints from old data may not bear up under the scrutiny of superior data [*e.g.*, Does the $Y(4260)$ actually exist?]

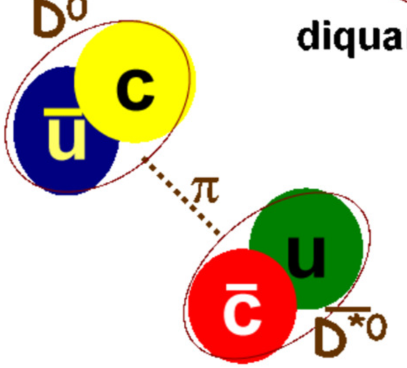
# But having heavy quarks helps

- All of the common theoretical pictures for exotics rely in one way or another on $\Lambda_{\text{QCD}}/m_{c,b} \ll 1$:
  - $m_Q$ is nonrelativistic in the state (potential models, lattice simulations)
  - Its Compton wavelength $\hbar/m_Q c$ is smaller than the full hadronic size, making the heavy quarks "discernable" within the state (hybrids, molecular models, diquark models, hadroquarkonium)
  - The scale $m_Q$ is heavy enough to belong to the asymptotic freedom region of QCD, allowing for an operator expansion in powers of $1/m_Q$ (heavy-quark spin effective theory, QCD sum rules)
  - The two-hadron thresholds are spaced further apart than in the forest of overlapping states in the range $1\text{-}2\ \text{GeV}$ (the reason that light-quark exotics are hard to identify)

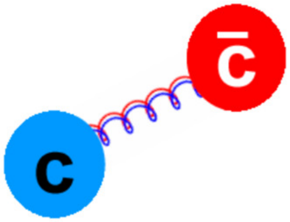
# The primary models for exotic mesons



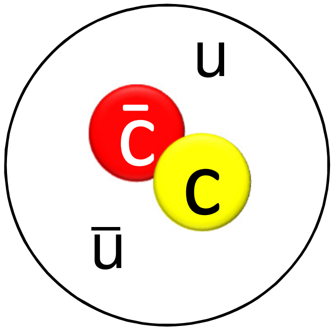
**diquark-diantiquark**



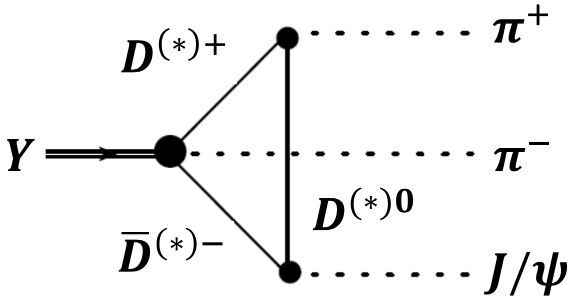
**$D^0 - \bar{D}^{*0}$ “molecule”**



**$q\bar{q}$-gluon“hybrid”**



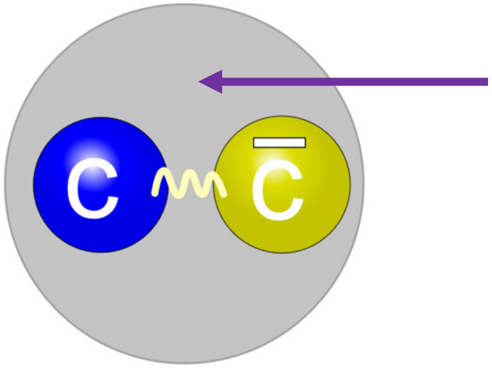
**hadrocharmonium**



**threshold/rescattering/cusp effect**

Image adapted from Godfrey & Olsen,
Ann. Rev. Nucl. Part. Sci. **58** (2008) 51

# Hybrid mesons



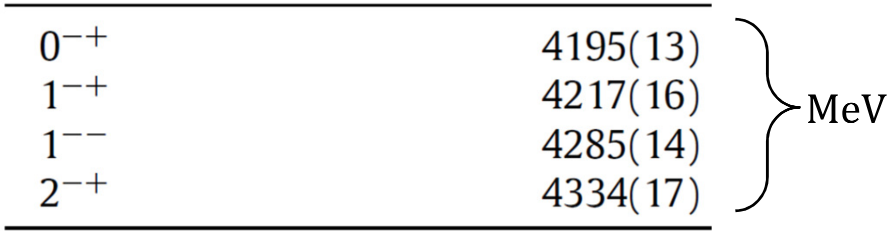
Any meson for which the quantum numbers of this part* are electrically neutral & isoscalar and **not** $J^{PC} = 0^{++}$ (excited glue)

*assigning any orbital angular momentum to the $c\bar{c}$ pair

- "Naïve" hybrid model: Light d.o.f. form a "constituent gluon"
- More sophisticated: Heavy, near-static $c\bar{c}$ quarks connected by a color flux tube with nontrivial $J^{PC}$
- Light d.o.f. form Born-Oppenheimer approximation potentials with which the $c\bar{c}$ pair interacts
- Simulated on the lattice multiple times over the decades [starting with Griffiths, Michael, Rakow, PLB **129** (1983) 351]

# Hybrid mesons

- All simulations agree on ordering of multiplets in mass: First nontrivial multiplet consists of $J^{PC} = (0,1,2)^{-+}, 1^{--}$
- Note that $1^{-+}$ not allowed for quark-model mesons ("exotic")
- Typical $1^{--}$ mass predictions sit right atop the $Y$ states [Hadron Spectrum Collaboration, JHEP **07** (2012) 126]:



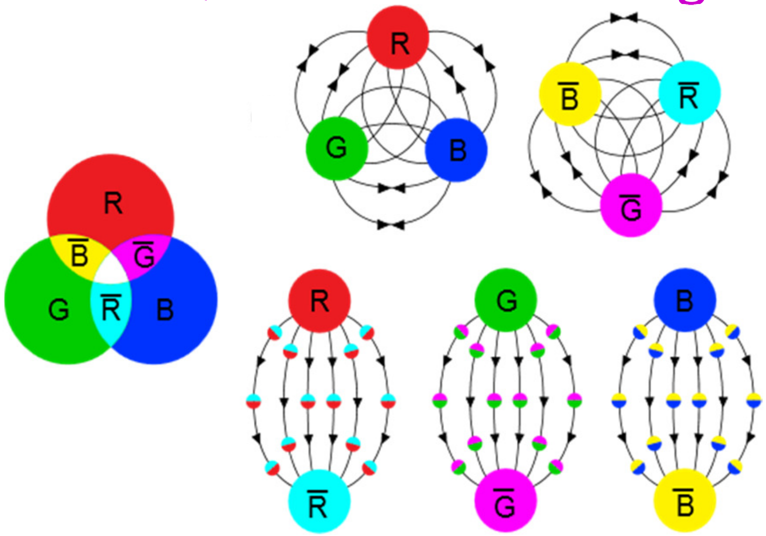
| | |
|---|---|
| $0^{-+}$ | 4195(13) |
| $1^{-+}$ | 4217(16) |
| $1^{--}$ | 4285(14) |
| $2^{-+}$ | 4334(17) |

MeV

- If $Y(4220)$ is a hybrid, why does it transition strongly to (non-hybrid) $Z_c(3900)$? [BESIII, 2004.13788]
- Hybrid decays should obey selection rules (*e.g.*, no decays to two *s*-wave mesons [Page, PLB **402** (1997) 183]) that can help distinguish from other structures

# Hadronic molecules

- Owing to the usual QCD rules of forming color singlets,



Gell-Mann & Zweig
knew about this,
even before they
knew about color!

multiquark states of *any number of quarks* $> 1$ might exist, and all can be expressed as combinations of mesons & baryons:

- $(q\bar{q})(q\bar{q})$, $(q\bar{q})(q\bar{q})(q\bar{q})$, $(qqq)(qqq)$, … (*tetraquark*, *hexaquark*, …)
- $(qqq)(q\bar{q})$, $(qqq)(q\bar{q})(q\bar{q})$, … (*pentaquark*, *heptaquark*, …)

# Hadronic molecules

- Most naïve idea, modeled on atomic nuclei:
  $2$ hadrons bound by meson (dominantly $\pi$) exchange
- Prototype: the $pn$ deuteron, binding energy $2.2\ \mathrm{MeV}$
- Charmed-meson molecules have been proposed since:
  Voloshin & Okun, JETP Lett. **23** (1976) 333
- With the 2003 Belle discovery of $X(3872)$ and

  $$m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.01 \pm 0.14\ \mathrm{MeV}$$ [LHCb, 2005.13419]

  how could $X(3872)$ *not* be a $\bar{D}^0 D^{*0}$ molecule?
- Several other exotics lie just below thresholds:
  $m_{X(3915)} - m_{D_s} - m_{D_s} \cong -18\ \mathrm{MeV}$, $m_{P_c(4312)} - m_{\Sigma_c^+} - m_{D^0} \cong -6\ \mathrm{MeV}$

  *but not all of them do*!

# Hadronic molecules

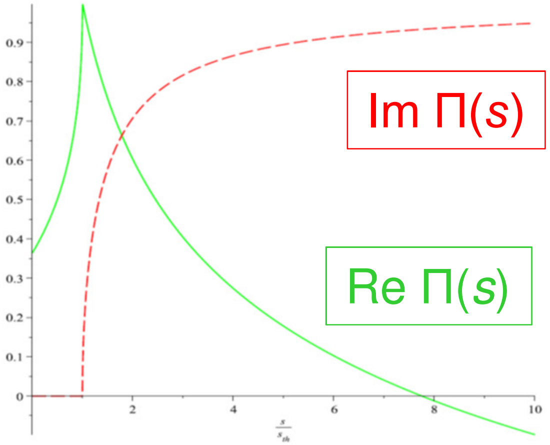
- Some are *just above* thresholds:

$$m_{Z_c^+(3900)} - m_{D^{*+}} - m_{D^0} \cong +13\ \mathrm{MeV},$$
$$m_{Z_b^+(10610)} - m_{B^{*+}} - m_{B^0} \cong +3\ \mathrm{MeV}$$

- Several are *nowhere near* obvious thresholds, and *not every threshold* exhibits a strong enhancement
- Naïve characteristic size of $X(3872)$: $\hbar/\sqrt{2\mu\,\Delta E} > 10\ \mathrm{fm}$, yet lots of them produced promptly in collider experiments: Partly molecular, partly a strongly bound component?
- Weinberg criterion [PR **137** (1965) B672]: Line shape parameters (*e.g.*, scattering length) used to obtain parameter $Z$ that measures compositeness ($= 0$ [$1$] for molecule [compact])
- Latest data [LHCb, 2005.13419]: $Z > 15\%$

# Threshold/rescattering/cusp effects

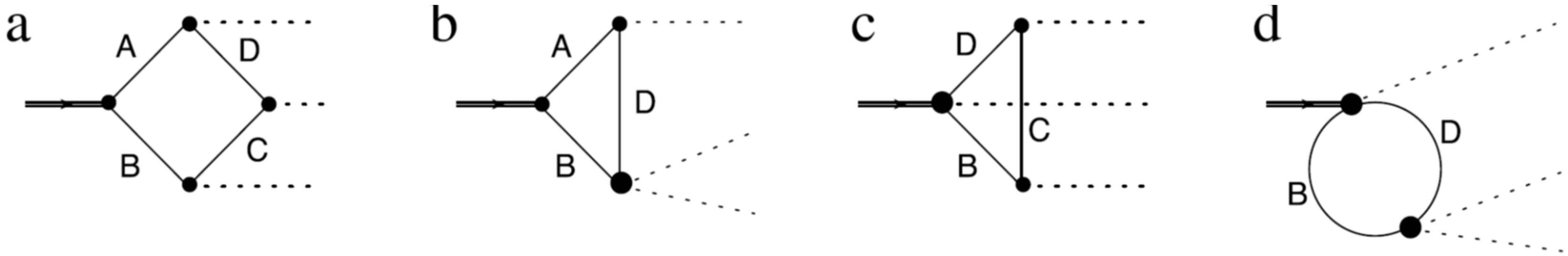
- Presence of nearby thresholds has other important effects on how some exotic states are interpreted
- Mainly arise due to complex analytic features of amplitudes like poles, branch points, Riemann sheets (1960s technology!)
- Simplest example: opening of on-shell channel creates branch point via imaginary part of amplitude $\Pi$, which by Cauchy's theorem creates cusp in $\mathrm{Re}\,\Pi$ resembling resonance



If combined with a pole due to intrinsic state, observed pole is "dragged" toward threshold
[Bugg, J. Phys. G **35** (2008) 075005]

# Threshold/rescattering/cusp effects

- Important threshold effects also appear through exchanges of virtual particles: rescattering effects



especially if some legs go on shell (*e.g.*, triangle singularities), which create new analytic structures & change line shapes
[first applied to exotics by Chen & Liu, PRD **84** (2011) 094003]

- Rescattering diagrams can create poles, either below or above threshold [like $Z_c(3900)$!]
- Depending upon where poles occur in Riemann sheets, called bound states, virtual states, or resonances
[see Guo *et al*., RMP **90** (2018) 1, 015004]

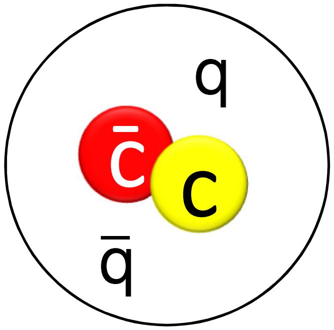
# Threshold/rescattering/cusp effects

- Important point: Weinberg's $Z$ can also be applied to rescattering states
- A state containing a large component of hadrons $AB$ by this measure in modern terminology is still called an $AB$ molecule
- Thus, LHCb [2005.13419] showed that $X(3872)$ is mostly a $D^{*0}\overline{D}^0$ molecule, even though the traditional $\pi$-exchange picture might not be a useful depiction
- And $Z_c(3900)$ might simultaneously be both a virtual state resonance above the $D^*\overline{D}$ threshold and a molecule [JPAC Collaboration, PLB **772** (2017) 200]

# Hadrocharmonium

[Dubyinskiy & Voloshin, PLB **666** (344) 2008]

- Originally developed to explain why some exotics, such as $Y(4260), Y(4660), Z_c(4430)$, prefer to decay to specific charmonium states ($J/\psi, \psi', \chi_c, h_c$) rather than to open-charm $D^{(*)}$ meson pairs
- State has a quarkonium core in a specific spin state and radial excitation [*e.g.*, $s_{c\bar{c}} = 1$ and $n = 2$ in $Z_c(4430)$ leads to preferential decay into $\psi'$]



- In the intervening years, some states like $Y(4220)$ have been seen to decay into $J/\psi$ ($s_{c\bar{c}} = 1$) and into $h_c$ ($s_{c\bar{c}} = 0$), and into open charm ($\pi^+ D^0 D^{*-}$) as well
- Using heavy-quark spin basis remains useful to understanding spin structure and decays in molecular and diquark states

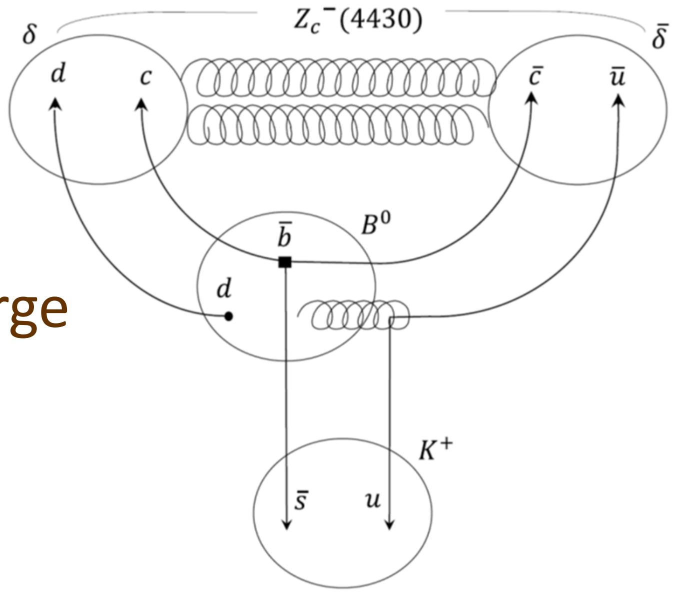
# Diquark-antidiquark hadrons

- $SU(3)_{\mathrm{color}}$ provides attraction between $qq$ in the channel $\mathbf{3} \otimes \mathbf{3} \rightarrow \overline{\mathbf{3}}$
- At short distances it is fully half as strong as the $q\bar{q}$ attraction $\mathbf{3} \otimes \overline{\mathbf{3}} \rightarrow \mathbf{1}$
- At long $(1/\Lambda_{\mathrm{QCD}})$ distances the $qq$ attractive channel needed for manifesting the baryon color structure $\mathbf{3} \otimes (\mathbf{3} \otimes \mathbf{3}) \rightarrow \mathbf{1}$
- Diquarks long considered as quasiparticles within baryons [Lichtenberg & Tassie, PR **155** (1967) 160]
- Peculiar features of scalar mesons like $a_0(980)$, $f_0(980)$ led to proposal of tetraquarks consisting of a diquark-antidiquark $(\delta\text{-}\bar{\delta})$ pair [Jaffe, PRD **15** (1977) 267]

# Diquark-antidiquark hadrons

- Diquark paradigm first applied to heavy-quark exotics in [Maiani *et al.*, PRD **71** (2005) 014028]
- Issue: Different spin states like $X(3872)$ & $Z_c(3900)$ end up with wrong multiplicities & mass ordering
- Fix: Dominant spin couplings are those *within* each diquark [Maiani *et al.*, PRD **89** (2014) 114010]
  …which is expected if $\delta$ and $\bar{\delta}$ are somewhat separated within the hadron
- Dynamical diquark picture [Brodsky *et al.*, PRL **113** (2014) 112001]: In high-energy $B$-decay or collider processes, $\delta$ and $\bar{\delta}$ can form and separate before color recombination into mesons occurs, allowing identification of state as $\delta$-$\bar{\delta}$

# Diquark-antidiquark hadrons



- Color flux tube forming between $\delta$-$\bar{\delta}$ pair brings them to rest (Born-Oppenheimer approximation) before decay to mesons
- Allows exotics to become spatially large but still strongly coupled
- Can be extended to pentaquarks as diquark-*triquark* $[\bar{c}_{\bar{\mathbf{3}}}(ud)_{\bar{\mathbf{3}}}]_{\mathbf{3}}$ systems
  [RFL, PLB **749** (2015) 454]
- With introduction of isospin dependence between $\delta$-$\bar{\delta}$ pair, provides full spectrum of states (*e.g.*, all 12 states in ground-state multiplet; *do they all exist*?)
  [Giron *et al.*, JHEP **01** (2020) 124]

# Lattice QCD

- Why not just stick the $4$ (or $5$) quarks on the lattice?
  1) Old-fashioned lattice simulations can't handle unstable, above-threshold resonances
     But newer ones can (assuming only one dominant decay channel): Study scattering matrix in a finite box [Lüscher, NPB **364** (1991) 237]
  2) Calculations for exotics with $> 1$ decay channels not yet performed
  3) Lattice calculations tricky for problems when there are several energy or length scales: large masses & small binding energies [deuteron: NPLQCD Collaboration, PRD **85** (2012) 054511]
- With these limitations in mind,
  what has been accomplished so far on the lattice?

# Lattice QCD

- A state like $X(3872)$ appears, but requires both $c\bar{c}$ and $\bar{D}D^*$ (but not diquark) components, and has no charged partner
  [Padmanath *et al.*, PRD **92** (2015) 034501]
- $Z_c(3900)$ evidence mixed; one group says it appears through coupling of $\bar{D}D^*$ & $J/\psi\,\pi$ channels
  [HAL QCD Collaboration, PRL **117** (2016) 242001]
  Another group, using Lüscher method, hasn't found it yet
  [CLQCD Collaboration, Chin. Phys. C **43** (2019) 103103]
- $Z_b(10610)$ (correctly) found just above $\bar{B}B^*$ threshold
  [Prelovsek *et al*., PLB **805** (2020) 135467]
- No $P_c$ pentaquarks yet, but simulations so far only allow coupling to $J/\psi\ p$ channel
  [Skerbis & Prelovsek, PRD **99** (2019) 094505]

# Closing thoughts

- It is becoming increasingly clear that no single theoretical paradigm explains all $> 40$ heavy-quark exotics
- Threshold effects seem to be essential in several cases, but even those seem to be insufficient if one allows for only a single component [like $\bar{D}D^*$ in $X(3872)$]
- Heavy-quark spin symmetry (most easily probed in decays of exotics to quarkonium) provide important clues on the underlying spin structure of the exotic
- Conventional quarkonium & diquark models provide specific spectra that can be verified or falsified for each observed state
- Transitions between exotic states [like $Y(4220) \to Z_c(3900)$] will be essential in determining which exotics have similar underlying structures