Theory Approaches to Heavy-Quark Exotic Hadrons: A Breezy Overview

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How can it be so hard to figure out what the exotics are?

• When $J/\psi$ was discovered on November 11, 1974, it was immediately recognized to be this:

These quarks are much heavier than the QCD scale $\Lambda_{\text{QCD}}$, & hence are discernable, nonrelativistic entities within the hadron.

This part consists of innumerable gluons and sea $q\bar{q}$ pairs, but in the hadrons of lowest mass it just has $J^{PC} = 0^{++}$.

• Thus quarkonium can be treated as a two-body problem in a simple scalar potential $V(r)$, and given also $m_c$, the Schrödinger equation predicts the entire spectrum [e.g., the Cornell potential, PRD 17 (1978) 3090, PRD 21 (1980) 203]
It works very well for $c\bar{c}$ states, but...
Charged charmoniumlike states don’t fit into this scheme either.
The problems with exotics modeling

• 4- and 5-quark states would be hard to model analytically even if all the quarks were heavy, and even if the QCD glue interactions were greatly simplified
  – Developing good methods to study multi-electron atoms took decades, and that’s just Coulomb interactions and quantum mechanics

• The most physically significant degrees of freedom in the exotics seem to vary from state to state
  – Stronger couplings to open-flavor hadrons or to quarkonium?

• Many have the same $J^{PC}$ as conventional hadrons & might mix

• The world data set is constantly improving
  – Models based upon hints from old data may not bear up under the scrutiny of superior data [e.g., Does the $Y(4260)$ actually exist?]
But having heavy quarks helps

- All of the common theoretical pictures for exotics rely in one way or another on $\Lambda_{QCD}/m_{c,b} \ll 1$:
  - $m_Q$ is nonrelativistic in the state (potential models, lattice simulations)
  - Its Compton wavelength $\hbar/m_Q c$ is smaller than the full hadronic size, making the heavy quarks “discernable” within the state (hybrids, molecular models, diquark models, hadroquarkonium)
  - The scale $m_Q$ is heavy enough to belong to the asymptotic freedom region of QCD, allowing for an operator expansion in powers of $1/m_Q$ (heavy-quark spin effective theory, QCD sum rules)
  - The two-hadron thresholds are spaced further apart than in the forest of overlapping states in the range 1-2 GeV (the reason that light-quark exotics are hard to identify)
The primary models for exotic mesons

Hybrid mesons

Any meson for which the quantum numbers of this part* are electrically neutral & isoscalar and not \( J^{PC} = 0^{++} \) (excited glue)

*assigning any orbital angular momentum to the \( c\bar{c} \) pair

- “Naïve” hybrid model: Light d.o.f. form a “constituent gluon”
- More sophisticated: Heavy, near-static \( c\bar{c} \) quarks connected by a color flux tube with nontrivial \( J^{PC} \)
- Light d.o.f. form Born-Oppenheimer approximation potentials with which the \( c\bar{c} \) pair interacts
- Simulated on the lattice multiple times over the decades [starting with Griffiths, Michael, Rakow, PLB 129 (1983) 351]
Hybrid mesons

- All simulations agree on ordering of multiplets in mass:
  First nontrivial multiplet consists of $J^{PC} = (0,1,2)^{-+}, 1^{--}$
- Note that $1^{-+}$ not allowed for quark-model mesons (“exotic”)
- Typical $1^{--}$ mass predictions sit right atop the $Y$ states
  [Hadron Spectrum Collaboration, JHEP 07 (2012) 126]:

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>Mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^{--}</td>
<td>4195(13)</td>
</tr>
<tr>
<td>1^{--}</td>
<td>4217(16)</td>
</tr>
<tr>
<td>1^{--}</td>
<td>4285(14)</td>
</tr>
<tr>
<td>2^{--}</td>
<td>4334(17)</td>
</tr>
</tbody>
</table>

- If $Y(4220)$ is a hybrid, why does it transition strongly to (non-hybrid) $Z_c(3900)$? [BESIII, 2004.13788]
- Hybrid decays should obey selection rules (e.g., no decays to two s-wave mesons [Page, PLB 402 (1997) 183]) that can help distinguish from other structures
Hadronic molecules

• Owing to the usual QCD rules of forming color singlets, multiquark states of any number of quarks > 1 might exist, and all can be expressed as combinations of mesons & baryons:
  • \((q\bar{q})(q\bar{q}), (q\bar{q})(q\bar{q})(q\bar{q}), (qqq)(qqq), \ldots\) (tetraquark, hexaquark, ...)
  • \((qqq)(q\bar{q}), (qqq)(q\bar{q})(q\bar{q}), \ldots\) (pentaquark, heptaquark, ...)

Gell-Mann & Zweig knew about this, even before they knew about color!
Hadronic molecules

- Most naïve idea, modeled on atomic nuclei:
  2 hadrons bound by meson (dominantly $\pi$) exchange
- Prototype: the $pn$ deuteron, binding energy 2.2 MeV
- Charmed-meson molecules have been proposed since:
  Voloshin & Okun, JETP Lett. 23 (1976) 333
- With the 2003 Belle discovery of $X(3872)$ and
  $$m_{X(3872)} - m_{D^*0} - m_{D^0} = -0.01 \pm 0.14 \text{ MeV}$$
  [LHCb, 2005.13419]
  how could $X(3872)$ not be a $\bar{D}^0 D^{*0}$ molecule?
- Several other exotics lie just below thresholds:
  $$m_{X(3915)} - m_{D_s} - m_{D_s} \approx -18 \text{ MeV},
  m_{P_c(4312)} - m_{\Sigma^+_c} - m_{D^0} \approx -6 \text{ MeV}$$
  but not all of them do!
Hadronic molecules

- Some are *just above* thresholds:
  \[
  m_{Z^+_c(3900)} - m_{D^{*+}} - m_{D^0} \cong +13 \text{ MeV},
  m_{Z^+_b(10610)} - m_{B^{*+}} - m_{B^0} \cong +3 \text{ MeV}
  \]

- Several are *nowhere near* obvious thresholds, and *not every threshold* exhibits a strong enhancement

- Naïve characteristic size of \(X(3872)\): \(\hbar / \sqrt{2\mu \Delta E} > 10 \text{ fm}\), yet lots of them produced promptly in collider experiments: Partly molecular, partly a strongly bound component?

- Weinberg criterion [PR 137 (1965) B672]: Line shape parameters (\(e.g.,\) scattering length) used to obtain parameter \(Z\) that measures compositeness (\(= 0 \ [1]\) for molecule [compact])

- Latest data [LHCb, 2005.13419]: \(Z > 15\%\)
Threshold/rescattering/cusp effects

- Presence of nearby thresholds has other important effects on how some exotic states are interpreted.
- Mainly arise due to complex analytic features of amplitudes like poles, branch points, Riemann sheets (1960s technology!)
- Simplest example: opening of on-shell channel creates branch point via imaginary part of amplitude $\Pi$, which by Cauchy’s theorem creates cusp in $\text{Re } \Pi$ resembling resonance

\[ \text{Im } \Pi(s) \quad \text{Re } \Pi(s) \]

If combined with a pole due to intrinsic state, observed pole is “dragged” toward threshold
Threshold/rescattering/cusp effects

• Important threshold effects also appear through exchanges of virtual particles: rescattering effects especially if some legs go on shell (e.g., triangle singularities), which create new analytic structures & change line shapes [first applied to exotics by Chen & Liu, PRD 84 (2011) 094003]

• Rescattering diagrams can create poles, either below or above threshold [like $Z_c(3900)$!]

• Depending upon where poles occur in Riemann sheets, called bound states, virtual states, or resonances [see Guo et al., RMP 90 (2018) 1, 015004]
Threshold/rescattering/cusp effects

- Important point: Weinberg’s $Z$ can also be applied to rescattering states
- A state containing a large component of hadrons $AB$ by this measure in modern terminology is still called an $AB$ molecule
- Thus, LHCb [2005.13419] showed that $X(3872)$ is mostly a $D^{*0}\bar{D}^0$ molecule, even though the traditional $\pi$-exchange picture might not be a useful depiction
- And $Z_c(3900)$ might simultaneously be both a virtual state resonance above the $D^{*}\bar{D}$ threshold and a molecule [JPAC Collaboration, PLB 772 (2017) 200]
Hadrocharmonium
[Dubyinskiy & Voloshin, PLB 666 (344) 2008]

• Originally developed to explain why some exotics, such as \( Y(4260), Y(4660), Z_c(4430) \), prefer to decay to specific charmonium states \( (J/\psi, \psi', \chi_c, h_c) \) rather than to open-charm \( D(\ast) \) meson pairs

• State has a quarkonium core in a specific spin state and radial excitation
  \( [\text{e.g., } s_{c\bar{c}} = 1 \text{ and } n = 2 \text{ in } Z_c(4430) \text{ leads to preferential decay into } \psi'] \)

• In the intervening years, some states like \( Y(4220) \) have been seen to decay into \( J/\psi (s_{c\bar{c}} = 1) \) and into \( h_c (s_{c\bar{c}} = 0) \), and into open charm \( (\pi^+ D^0 D^{*-}) \) as well

• Using heavy-quark spin basis remains useful to understanding spin structure and decays in molecular and diquark states
Diquark-antidiquark hadrons

- $SU(3)_{\text{color}}$ provides attraction between $qq$ in the channel $3 \otimes 3 \rightarrow \bar{3}$
- At short distances it is fully half as strong as the $q\bar{q}$ attraction $3 \otimes \bar{3} \rightarrow 1$
- At long $(1/\Lambda_{\text{QCD}})$ distances the $qq$ attractive channel needed for manifesting the baryon color structure $3 \otimes (3 \otimes 3) \rightarrow 1$
- Diquarks long considered as quasiparticles within baryons [Lichtenberg & Tassie, PR 155 (1967) 160]
- Peculiar features of scalar mesons like $a_0(980), f_0(980)$ led to proposal of tetraquarks consisting of a diquark-antidiquark ($\delta-\bar{\delta}$) pair [Jaffe, PRD 15 (1977) 267]
Diquark-antidiquark hadrons

• Diquark paradigm first applied to heavy-quark exotics in [Maiani et al., PRD 71 (2005) 014028]

• Issue: Different spin states like $X(3872)$ & $Z_c(3900)$ end up with wrong multiplicities & mass ordering

• Fix: Dominant spin couplings are those within each diquark [Maiani et al., PRD 89 (2014) 114010]

• Dynamical diquark picture [Brodsky et al., PRL 113 (2014) 112001]: In high-energy $B$-decay or collider processes, $\delta$ and $\bar{\delta}$ can form and separate before color recombination into mesons occurs, allowing identification of state as $\delta-\bar{\delta}$
Diquark-antidiquark hadrons

- Color flux tube forming between $\delta$-$\bar{\delta}$ pair brings them to rest (Born-Oppenheimer approximation) before decay to mesons
- Allows exotics to become spatially large but still strongly coupled
- Can be extended to pentaquarks as diquark-triquark $[\bar{c} \bar{\bar{d}} (ud)_{\bar{d}}]_3$ systems [RFL, PLB 749 (2015) 454]
- With introduction of isospin dependence between $\delta$-$\bar{\delta}$ pair, provides full spectrum of states (e.g., all 12 states in ground-state multiplet; do they all exist?) [Giron et al., JHEP 01 (2020) 124]
Lattice QCD

- Why not just stick the 4 (or 5) quarks on the lattice?
  1) Old-fashioned lattice simulations can’t handle unstable, above-threshold resonances
     But newer ones can (assuming only one dominant decay channel):
     Study scattering matrix in a finite box [Lüscher, NPB 364 (1991) 237]
  2) Calculations for exotics with $>1$ decay channels not yet performed
  3) Lattice calculations tricky for problems when there are several energy or length scales: large masses & small binding energies

- With these limitations in mind, what has been accomplished so far on the lattice?
Lattice QCD

- A state like $X(3872)$ appears, but requires both $c\bar{c}$ and $\bar{D}D^*$ (but not diquark) components, and has no charged partner
  [Padmanath et al., PRD 92 (2015) 034501]

- $Z_c(3900)$ evidence mixed; one group says it appears through coupling of $\bar{D}D^*$ & $J/\psi\pi$ channels
  [HAL QCD Collaboration, PRL 117 (2016) 242001]
  Another group, using Lüscher method, hasn’t found it yet

- $Z_b(10610)$ (correctly) found just above $\bar{B}B^*$ threshold
  [Prelovsek et al., PLB 805 (2020) 135467]

- No $P_c$ pentaquarks yet, but simulations so far only allow coupling to $J/\psi\ p$ channel
  [Skerbis & Prelovsek, PRD 99 (2019) 094505]
Closing thoughts

• It is becoming increasingly clear that no single theoretical paradigm explains all > 40 heavy-quark exotics

• Threshold effects seem to be essential in several cases, but even those seem to be insufficient if one allows for only a single component [like $\bar{D}D^*$ in $X(3872)$]

• Heavy-quark spin symmetry (most easily probed in decays of exotics to quarkonium) provide important clues on the underlying spin structure of the exotic

• Conventional quarkonium & diquark models provide specific spectra that can be verified or falsified for each observed state

• Transitions between exotic states [like $Y(4220) \rightarrow Z_c(3900)$] will be essential in determining which exotics have similar underlying structures