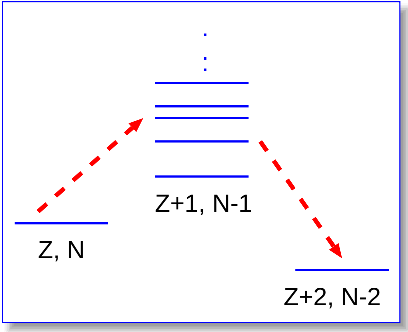
Double-Beta Decay and Nuclear Theory

J. Engel

July 22, 2020

Neutrinoless Double-Beta Decay

If energetics are right (ordinary beta decay forbidden)…



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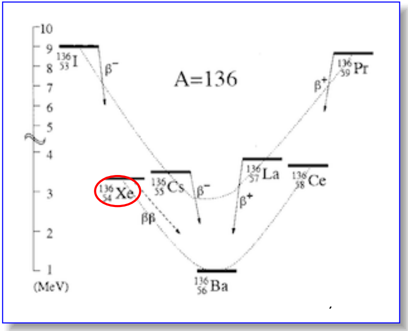

Example: ^{136}Xe
Others: ^{76}Ge, ^{130}Te, ^{150}Nd …

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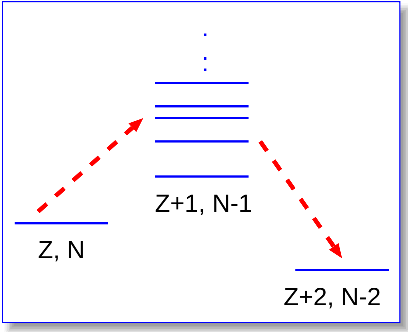
If energetics are right (ordinary beta decay forbidden)…

and neutrinos are their own antiparticles…



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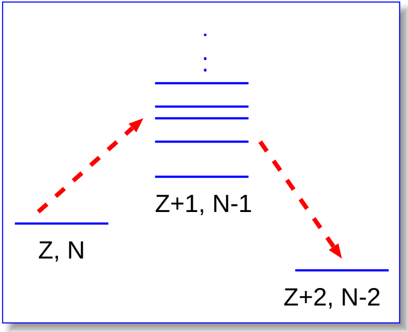
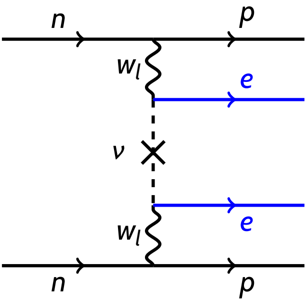


and neutrinos are their own antiparticles…

can observe two neutrons turning into protons, emitting two electrons and nothing else.



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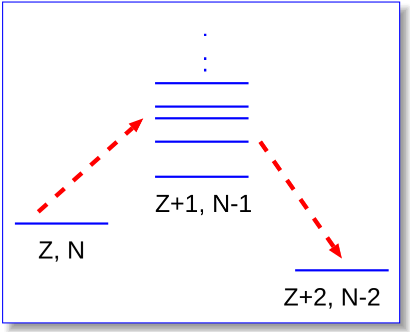

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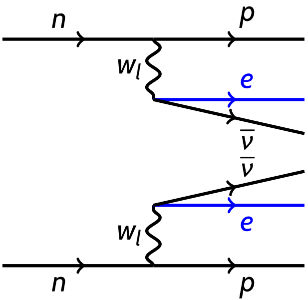

can observe two neutrons turning into protons, emitting two electrons and nothing else.

Different from already observed two-neutrino process.

Neutrino Physics in Neutrinoless Double-Beta Decay

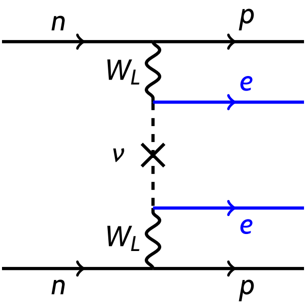

Diagram is proportional to effective “Majorana mass” of light neutrinos,

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \,,$$

so if $0\nu\beta\beta$ decay is seen neutrinos are their own antiparticles!

And if mass hierarchy is inverted, or maybe even if it’s normal, next generation of experiments should be able to see the decay.

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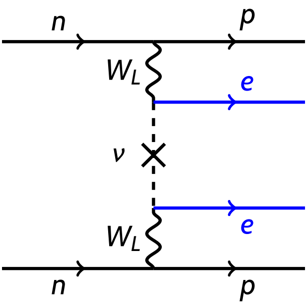

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And if mass hierarchy is inverted, or maybe even if it's normal, next generation of experiments should be able to see the decay.

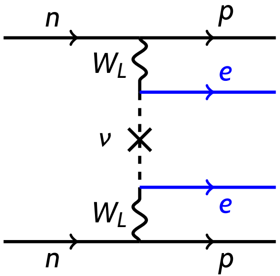
Whatever the hierarchy, the Majorana mass must come from somewhere, and the Standard Model by itself doesn't allow it.

Its presence implies new particles, which could make the low masses of neutrinos natural, and could also change $0\nu\beta\beta$ rate.

New Physics Can Contribute Directly to $\beta\beta$ Decay

If neutrinoless decay occurs then ν's are Majorana, no matter what,

but high-scale physics can contribute directly alongside light-neutrino exchange:



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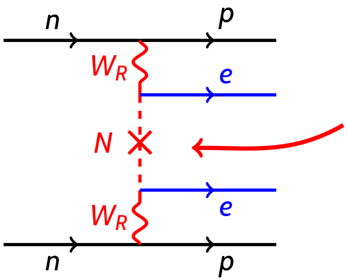

Exchange of heavy right-handed neutrino in left-right symmetric model.

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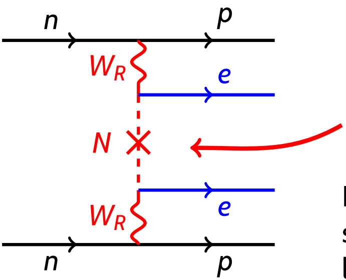
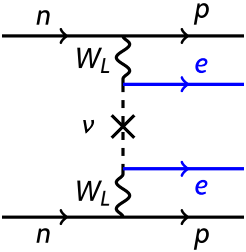


Exchange of heavy right-handed neutrino in left-right symmetric model.

Heavy-particle exchange can occur at the same rate as light-ν exchange (or even a larger rate) if $m_N \approx m_{W_R} \approx 1$ TeV.

New Physics at Hadronic Level

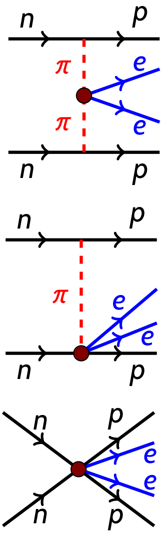
Light-ν exchange:



Effective field theory lists pion-nucleon-level operators and determines their importance.

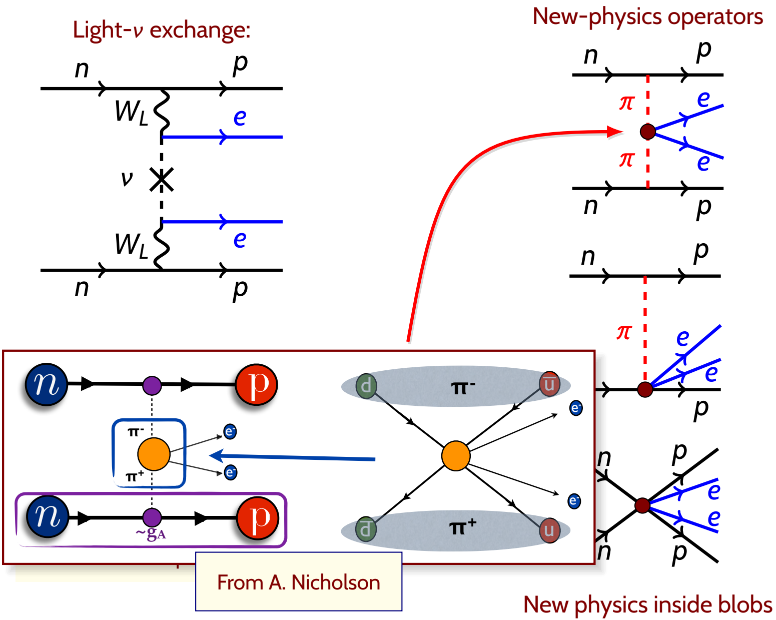
Lattice QCD can then compute dependence of blobs on new particle masses and couplings.

New-physics operators



New physics inside blobs

New Physics at Hadronic Level



Light-ν Exchange in a Nucleus

$$[T^{0\nu}_{1/2}]^{-1} = G(Z,N)\,|M_{0\nu}|^2\, m^2_{\beta\beta}$$

Phase-space factor → $G(Z,N)$

Nuclear matrix element → $M_{0\nu}$

"Traditional" part of matrix element:

$$M_{0\nu} = M^{GT}_{0\nu} - \frac{g_V^2}{g_A^2} M^F_{0\nu} + \ldots$$

with

$$M^{GT}_{0\nu} = \langle F| \sum_{i,j} H(r_{ij})\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\, \tau_i^+ \tau_j^+ \; |I\rangle + \ldots$$

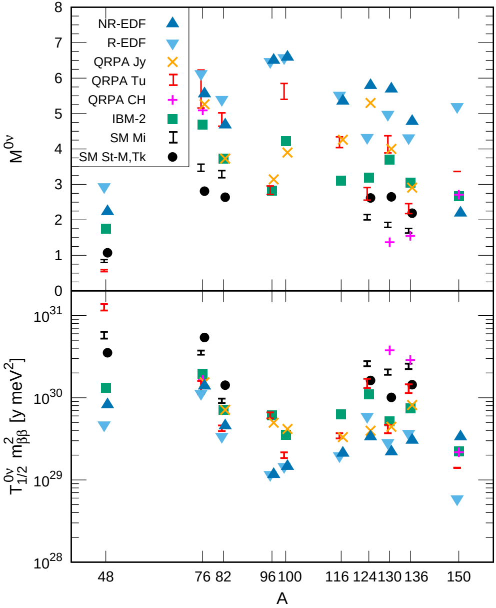
$$M^{F}_{0\nu} = \langle F| \sum_{i,j} H(r_{ij})\, \tau_i^+ \tau_j^+ \; |I\rangle + \ldots$$

$$H(r) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2} \quad \text{roughly} \propto 1/r$$

Corrections are from "forbidden" terms, weak nucleon form factors, many-body currents, other effects of high-energy physics that depend on framework.

Light-ν-Exchange Matrix Elements

Recent Values

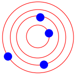


Significant spread. And all the models may miss important physics.

Uncertainty hard to quantify.

Nuclear Structure: Contrasting the Approaches

Starting point is always mean field(s)



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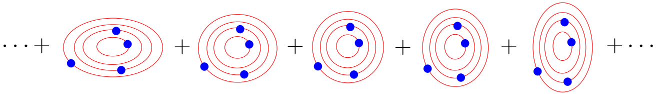

"Energy-Density Functional Theory" mixes many such states with different collective properties.

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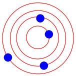

Other methods build on single independent-particle state.

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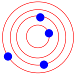

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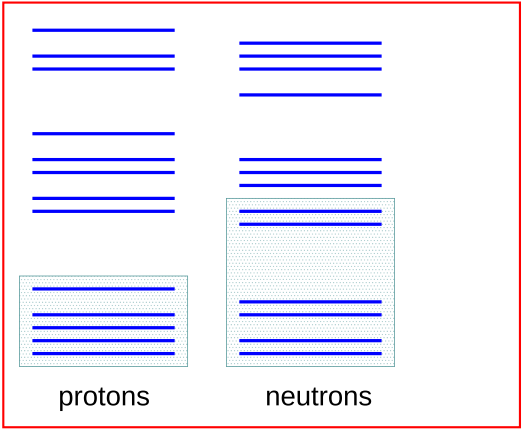

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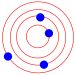

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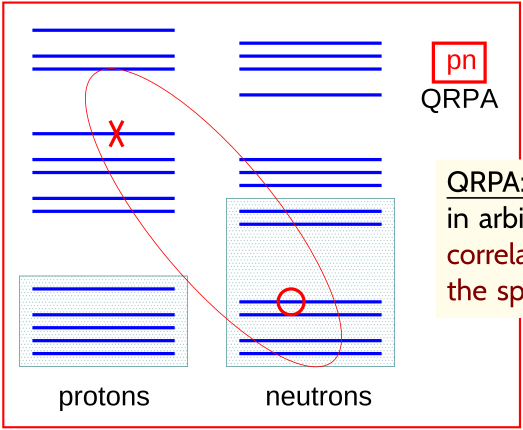

QRPA: Large single-particle spaces in arbitrary single mean field; simple correlations and excitations within the space.

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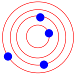

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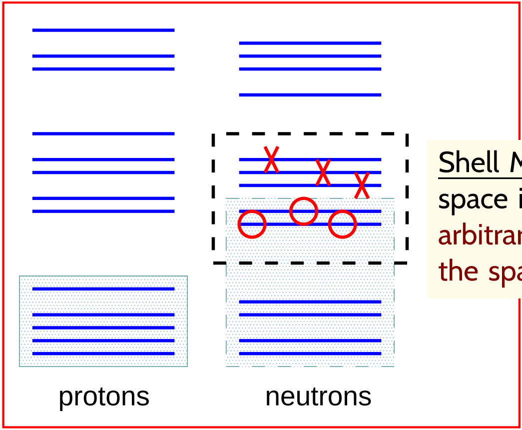

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

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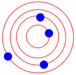

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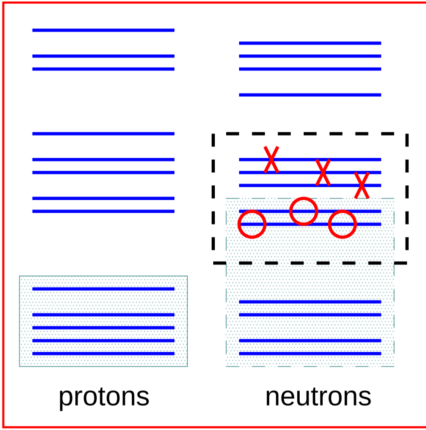

Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

Interacting boson model uses same space, with nucleon pairs as bosons.

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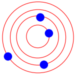

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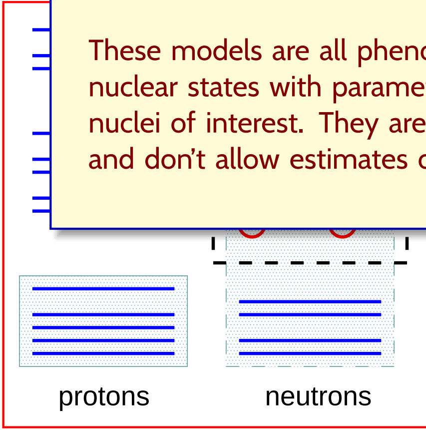
These models are all phenomenological representations of nuclear states with parameters that are fit to data in the nuclei of interest. They are not systematically improvable, and don't allow estimates of uncertainty.



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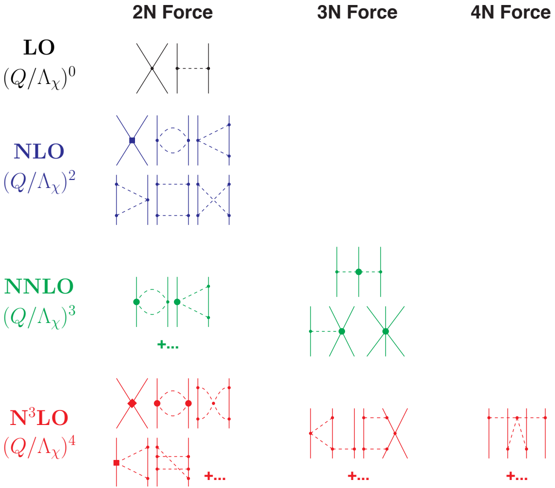
The Way Forward: Ab Initio Nuclear Theory

Starts with chiral effective field theory.

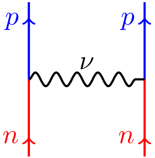
Nucleons, pions sufficient below chiral-symmetry breaking scale.



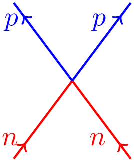
Decay operators in EFT

Recent realization (Cirigliano et al., PRL **120**, 202001 (2018)):

Usual light neutrino exchange,



must be supplemented, even at leading order in chiral EFT, by short-range operator (representing high-energy ν exchange):

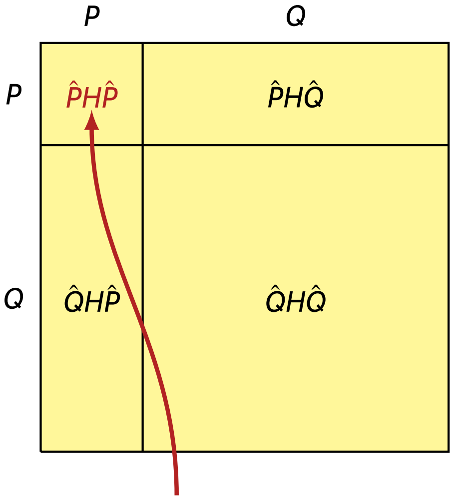


Coefficient of this leading-order term is also unknown. Results in uncertainty of order 100%

Higher-order corrections have also been worked out.

Ab Initio Many-Body Methods

Partition of Full Hilbert Space



Simpler calculation done here.

P = subspace you want
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing most important eigenvalues.

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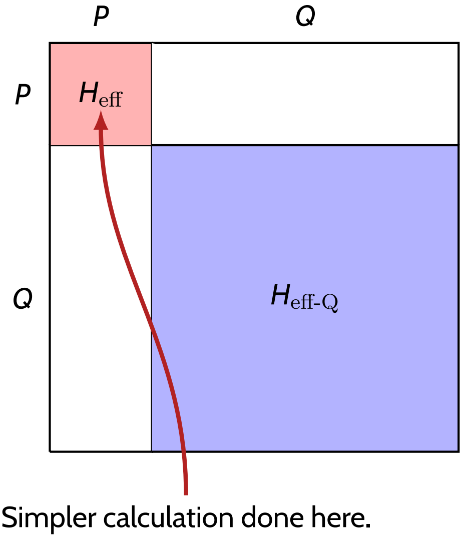

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Must must apply same unitary transformation to transition operator.

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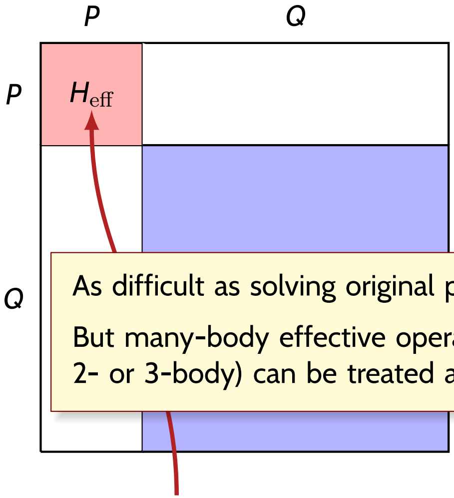

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As difficult as solving original problem.

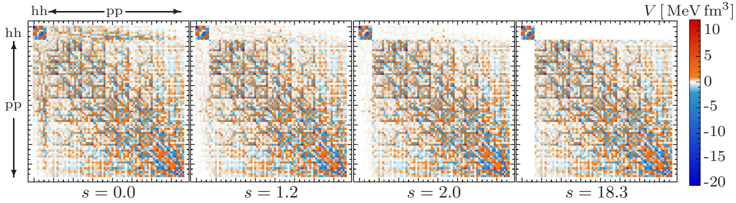
But many-body effective operators (beyond 2- or 3-body) can be treated approximately.

Simpler calculation done here.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples selected set of states.



from H. Hergert

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]\,, \qquad \eta(s) = [H_d(s), H_{od}(s)]\,, \qquad H(\infty) = H_{\text{eff}}$$

Trick is to keep all 1- and 2-body terms in H at each step *after normal ordering* (**IMSRG-2**, includes most important parts of 3, 4-body …terms).

If selected set is a single state, end up with g.s. energy. If it is a valence space, get effective shell-model interaction and operators.

Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T\,|\varphi_0\rangle \qquad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \ldots$$

Slater determinant

$m,n>F$ $\quad i,j<F$

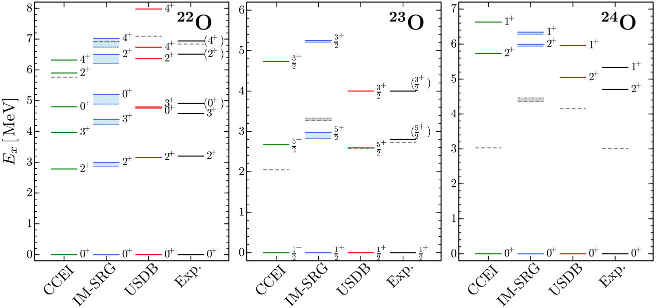
Here the Hamiltonian is transformed in a non-unitary way:

$$H \longrightarrow \tilde{H} \equiv e^{-T} H e^T$$

so that $|\varphi_0\rangle$ is its ground state. Must solve algebraic equations for the t's.

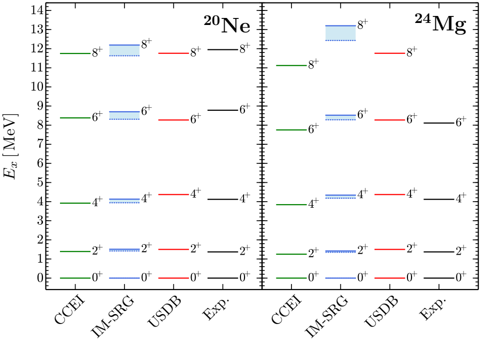
Excited states, states in closed-shell + a few nucleons, constructed from simple excitations of $|\varphi_0\rangle$.

Ab Initio Calculations of Spectra

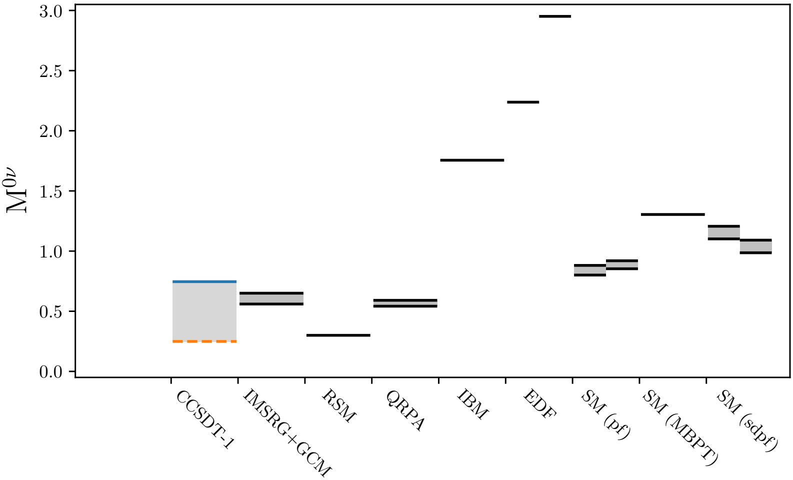


Neutron-rich oxygen isotopes

Deformed nuclei



^{48}Ca: Ab-Inito $0\nu\beta\beta$ Matrix Elements vs. Older Ones



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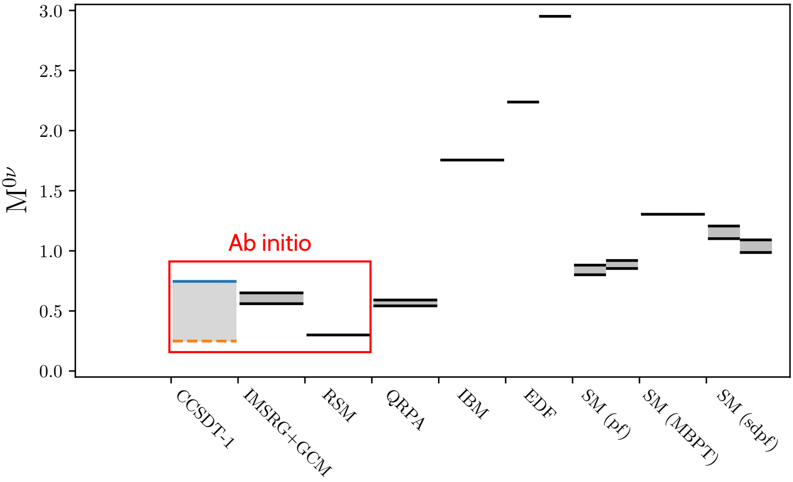

Parting Messages

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That's all; thanks. 

3. Chiral EFT implies short-range contribution to neutrino exchange at leading order with unknown coefficient. Need to to go beyond the naturalness estimate of the coefficient.