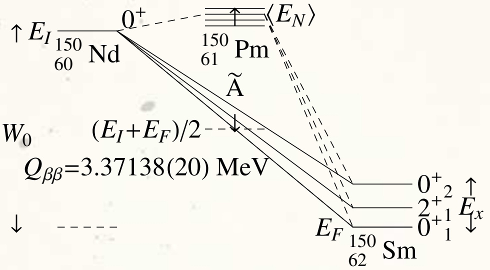
WHAT WE ARE DEALING WITH

- Example decay scheme: ^{150}Nd



- Reaction Q-value from experiments
- Wavefunctions for initial, intermediate and final states from theory
- Closure: $\langle \boldsymbol{E_N} \rangle$ is the average excitation energy of the intermediate nucleus
- Decays possible to ground state and excited states

WHAT WE ARE DEALING WITH

- Experiments measure half-life, so from theory we need G, effective g_A and M:
  - $2\nu\beta\beta$: $\left[\tau^{0\nu}_{1/2}\right]^{-1} = G_{2\nu} g_A^4 |M^{(2\nu)}|^2$
  - $0\nu\beta\beta$: $\left[\tau^{0\nu}_{1/2}\right]^{-1} = G_{0\nu} g_A^4 |M^{(0\nu)}|^2 |f(m_i, U_{ei})|^2$
  - $0\nu ECEC$: $\left[\tau^{0\nu}_{1/2}\right]^{-1} = G_{0\nu} g_A^4 \left|M^{0\nu}\right|^2 |f(m_i, U_{ei})|^2 \frac{(m_e c^2)\Gamma}{\Delta^2+\Gamma^2/4}$
- G: Phase space factor varies depending on the decaying nucleus, Q-value of the decay, and the scenario and mechanism of the decay
- M: Nuclear matrix element calculated using a chosen theoretical model. The model gives the wave functions of the initial and final states, and they are connected by proper transition operator, that varies depending on the scenario and mechanism of the decay
- g_A: Axial vector coupling constant, which effective value essentially model dependent
- $f(m_i, U_{ei})$ contains the physics beyond standard model and is different for different scenarios and mechanisms: exchange of light or heavy neutrino, emission of Majoron, exchange of sterile neutrino(s)...

NUCLEAR MODELS

NMEs are calculated in nuclear models, such as:

- The nuclear shell model (SM) is a well-established many-body method, used to describe the properties of medium mass and heavy nuclei.
  - Idea: nucleons near the Fermi level are the most important for low-energy nuclear properties, and all the correlations between these nucleons are relevant
  - uses a limited set of single-particle states, typically one harmonic-oscillator major shell or one nuclear major shell
  - in SM all possible many-nucleon configurations in a given single-particle space are formed, each configuration described by one Slater determinant, and the nuclear (residual) Hamiltonian is then diagonalized in the basis formed by these Slater determinants
  - many-body features are taken into account exactly but only in a restricted set of single-particle states, typically leaving out one or two spin-orbit-partner orbitals from the model space
  - A particular problem with the SM is to find a suitable (renormalized) nucleon-nucleon interaction to match the limited single-particle space
  - Since this space is small, the renormalization effects of the two-body interaction become substantial

NUCLEAR MODELS

NMEs are calculated in nuclear models, such as:

- The proton-neutron version of the QRPA (pnQRPA) uses two-quasiparticle excitations that are built from a proton and a neutron quasiparticle
  - constructs ground state correlations by iterating two-quasiparticle excitations on top of a BCS or HFB vacuum and quasiboson approximation is then imposed on the excitations
  - large valence space including several major shells => no problems associated with spin-orbit partner orbitals since they can easily be accommodated in the model space
  - restricted set of correlations
  - Hamiltonian is typically based on a realistic G matrix, but modified in the like-particle pairing and particle-hole channels to reproduce experimental pairing gaps and Gamow-Teller resonance energies
  - Results depend on fine-tuning of the interaction, especially near the spherical-deformed transition

NUCLEAR MODELS

NMEs are calculated in nuclear models, such as:

- The idea that inspires the microscopic interacting boson model, IBM-2, is a truncation of the very large shell model space to states built from pairs of nucleons with $J = 0$ and 2
  - These pairs are then assumed to be collective and are taken as bosons
  - IBM-2 has clear connections to both the shell model and the collective model of Bohr and Mottelson: On the one hand, the bosons represent nucleon pairs and on the other quadrupole phonons
  - The first correspondence is hard to make precise, Hamiltonians and effective operators are usually determined by fits to data
  - For $0\nu\beta\beta$ calculations it is necessary to relate the bosons to the underlying fermion model space through a mapping procedure
  - mapping is approximate: it involves only two- and four valence-nucleon states and schematic interaction
  - IBM-2 is known to be very successful in reproducing trends for spectra and E2 transitions involving collective states across isotopic and isotonic chains

NUCLEAR MODELS

NMEs are calculated in nuclear models, such as:

- Energy-density functional (EDF) theory refers to the process of minimizing an energy functional ϵ. Once the functional is obtained, minimizing it with respect to its arguments provides the exact ground state energy and densities
  - minimization can be formulated so that it looks like mean-field theory with one-body potentials and orbitals
  - The energy functional ϵ usually derived from the Hartree-Fock or HFB energy associated with Skyrme, Gogny or relativistic Walecka type, sometimes with additional modifications
  - parameters of the interaction or functional are then fit to ground state properties (masses, radii, etc) in a variety of nuclei and used without alteration all over the nuclear chart
  - method can be extended to EDF-based RPA or QRPA

TRANSITION OPERATOR (CLOSURE)

- Transition operator for $\beta\beta$-decay can be written in momentum space, including higher order corrections as

$$T(p) = H(p) f(m_i, U_{ei}),$$

  - (two-body) operator $H(p)$ can be written as

$$H(p) = \sum_{n,n'} \tau_n^\dagger \tau_{n'}^\dagger \left[-h^F(p) + h^{GT}(p) \vec{\sigma}_n \cdot \vec{\sigma}_{n'} + h^T(p) S^p_{nn'} \right]$$

  - tensor operator is defined as

$$S^p_{nn'} = 3 \left[(\vec{\sigma}_n \cdot \hat{p}) (\vec{\sigma}_{n'} \cdot \hat{p}) \right] - \vec{\sigma}_n \cdot \vec{\sigma}_{n'}$$

  - Fermi (F), Gamow-Teller (GT) and tensor (T) contributions are given by
    $h^F(p) = h^F_{VV}(p)$
    $h^{GT}(p) = h^{GT}_{AA}(p) + h^{GT}_{AP}(p) + h^{GT}_{PP}(p) + h^{GT}_{MM}(p)$
    $h^T(p) = h^T_{AP}(p) + h^T_{PP}(p) + h^T_{MM}(p)$
  - The terms AP-PP-MM are higher order corrections (HOC) arising from weak magnetism (M) and induced pseudoscalar terms (P) in the weak nucleon current

TRANSITION OPERATOR (CLOSURE)

- terms $h^{F,GT,T}(p)$ can be further factorized as

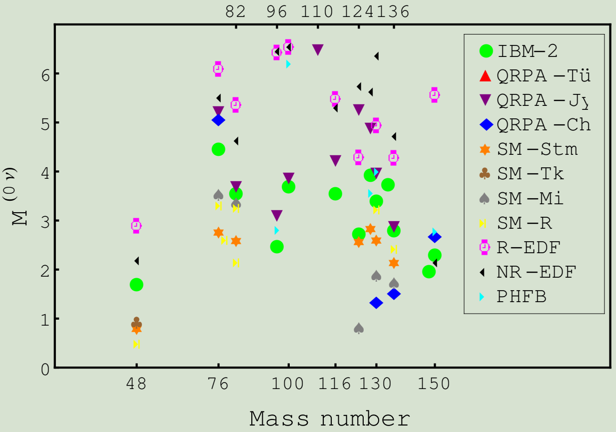
$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

where $v(p)$ is called the neutrino "potential" (m_ν : $v(p) = \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$) and $\tilde{h}^{F,GT,T}(p)$ the "form factors"

Term	$\tilde{h}(p)$
$\tilde{h}^{F}_{VV}$	$g_A^2\frac{(g_V^2/g_A^2)}{\left(1+p^2/M_V^2\right)^4}$
$\tilde{h}^{GT}_{AA}$	$\frac{g_A^2}{\left(1+p^2/M_A^2\right)^4}$
$\tilde{h}^{GT}_{AP}$	$g_A^2\left[-\frac{2}{3}\frac{1}{\left(1+p^2/M_A^2\right)^4}\frac{p^2}{p^2+m_\pi^2}(1-\frac{m_\pi^2}{M_A^2})\right]$
$\tilde{h}^{GT}_{PP}$	$g_A^2\left[\frac{1}{\sqrt{3}}\frac{1}{\left(1+p^2/M_A^2\right)^2}\frac{p^2}{p^2+m_\pi^2}\left(1-\frac{m_\pi^2}{M_A^2}\right)\right]^2$
$\tilde{h}^{GT}_{MM}$	$g_A^2\left[\frac{2}{3}\frac{g_V^2}{g_A^2}\frac{1}{\left(1+p^2/M_V^2\right)^4}\frac{\kappa_\beta^2 p^2}{4m_p^2}\right]$
$\tilde{h}^{T}_{AP}$	$-\tilde{h}^{GT}_{AP}$
$\tilde{h}^{T}_{PP}$	$-\tilde{h}^{GT}_{PP}$
$\tilde{h}^{T}_{MM}$	$\frac{1}{2}\tilde{h}^{GT}_{MM}$

NUCLEAR MATRIX ELEMENTS

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$



IBM-2: PRC91, 034304 (2015); QRPA-Tu: PRC87, 045501 (2013), PRC92, 044301 (2015); QRPA-Jy: PRC91, 024613 (2015); QRPA - Ch: PRC87, 064302 (2013);SM - StM: NPA 818 139 (2009), SM - Tk PRL116, 112502 (2016); SM - Mi RD93, 113014 (2016); SM-R: PRC 101, 044315 (2020); R - EDF: PRC91, 024316 (2015); NR - EDF: PRL111, 142501 (2013); PHFB: Front.Phys.7:64. (2019)

- Shell effects: The matrix elements are smaller at the closed shells than in the middle of the shell
- Deformation effects always decrease the matrix elements
- Isospin restoration reduces matrix elements

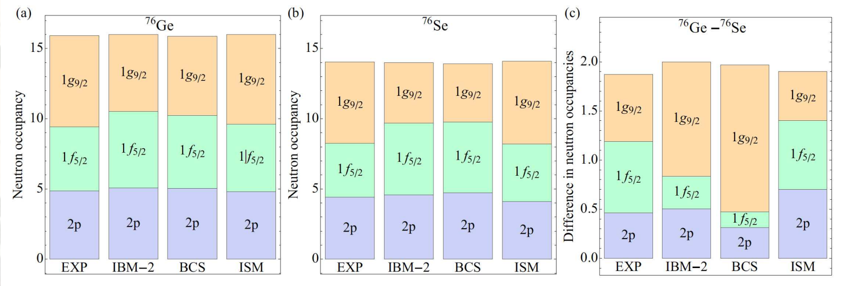
COMPARISONS AND TESTS

- The fact that $0\nu\beta\beta$-decay is a unique process, and there is no direct probe which connects the initial and final states other than the process itself makes the prediction challenging for theoretical models.
- The reliability of the used wave functions, and eventually $M^{(0\nu)}$, has to be then tested using other available relevant data.

COMPARISONS AND TESTS

Example of tests of wave functions:

Occupation probabilities: A=76 system, neutrons



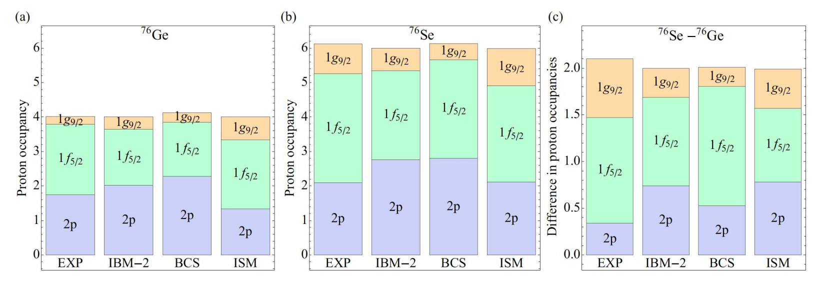
PRC 94, 034320 (2016)

- IBM-2: $\mathbf{2p, 1f_{5/2}}$ overfilled for both ^{76}Ge and ^{76}Se
- In IBM-2 change appears to be dominated by the $\mathbf{1g_{9/2}}$, experimentally $\mathbf{1g_{9/2}}$ and $\mathbf{1f_{5/2}}$
- ISM gives best correspondence

COMPARISONS and TESTS

Some examples of tests of wave functions

Occupation probabilities: A=76 system, protons



PRC 94, 034320 (2016)

- IBM-2: $\mathbf{1}f_{5/2}$ depletion of protons for both ^{76}Ge and ^{76}Se
- IBM gives slightly better correspondence than ISM

COMPARISONS and TESTS

Closure approximation:

TABLE I. NME for the $0\nu\beta\beta$ decay of ^{82}Se (light neutrino exchange) calculated within different approximations. All calculations were done with CD-Bonn SRC parametrization and for the average closure energy $\langle E \rangle = 10.08$ MeV. The difference between mixed and pure closure total NME is about 8%.

	Pure closure	Run. closure	Run. nonclosure	Mixed
$M^{0\nu}_{GT}$	2.750	2.664	2.898	2.983
$M^{0\nu}_{F}$	−0.607	−0.594	−0.620	−0.632
$M^{0\nu}_{T}$	−0.011	−0.008	−0.007	−0.011
$M^{0\nu}_{tot}$	3.127	3.035	3.285	3.377

R. A. Sen'kov et al. PRC89, 054304 (2014)

- CA avoids the explicit calculation of excited states of the intermediate odd-odd nucleus up to high energies
- Approaches that do allow non-closure calculation suggest that a sensible choice of $\langle E \rangle$ lead the closure approximation to reproduce the unapproximated $0\nu\beta\beta$ NME within 10%

COMPARISONS and TESTS

Short range correlations

Multipole decomposition and the total value of the matrix element $M_{GT}^{(0\nu)}$ for ^{48}Ca. The cases are: no short-range correlations included (bare), with Jastrow correlations and with UCOM correlations using Bonn-A and Argonne V18 parametrizations

J^π	Bare	Jastrow	UCOM	
			Bonn-A	AV18
1^+	−0.330	−0.305	−0.322	−0.319
2^+	−0.117	−0.092	−0.108	−0.104
3^+	−0.327	−0.246	−0.302	−0.293
4^+	−0.066	−0.035	−0.054	−0.051
5^+	−0.246	−0.121	−0.212	−0.199
6^+	−0.042	−0.008	−0.030	−0.027
7^+	−0.150	−0.029	−0.120	−0.107
Sum	−1.278	−0.835	−1.150	−1.101

Table 2
The same as Table 1 but for $M_F^{(0\nu)}$

J^π	Bare	Jastrow	UCOM	
			Bonn-A	AV18
1^+	0.000	0.000	0.000	0.000
2^+	0.185	0.145	0.174	0.169
3^+	0.000	0.000	−0.001	−0.001
4^+	0.116	0.061	0.102	0.096
5^+	0.000	0.000	−0.002	−0.002
6^+	0.061	0.012	0.050	0.045
7^+	0.000	0.000	−0.002	−0.002
Sum	0.367	0.221	0.324	0.308

M.
Kortelainen et al. PLB 647 (2007) 128

- Short-range correlations that are omitted by Hilbert-space truncation in most many-body calculations are usually taken into account by multiplying the operators inside the matrix elements a radial function $f(r_a b)$
- Several parameterizations of f have been proposed
- Even though the prescriptions differ from one another they seem have small effects on matrix elements

COMPARISONS and TESTS

ISOSPIN RESTORATION reduces matrix elements

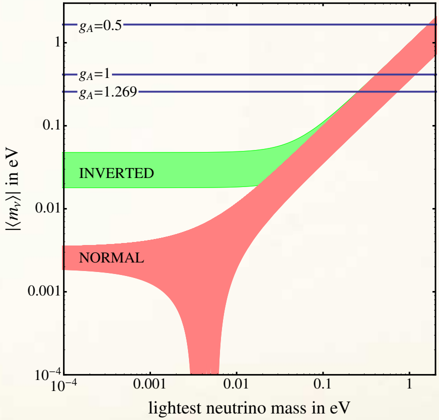
	$\chi_F = (g_V/g_A)^2 M_F^{(0\nu)} / M_{GT}^{(0\nu)}$		
Decay	IBM-2	QRPA	ISM
^{48}Ca	-0.10(-0.39)	-0.32(-0.93)	
^{76}Ge	-0.09(-0.37)	-0.21(-0.34)	-0.12
^{82}Se	-0.10(-0.40)	-0.23(-0.35)	-0.11
^{96}Zr	-0.08(-0.08)	-0.23(-0.38)	
^{100}Mo	-0.08(-0.08)	-0.30(-0.30)	
^{110}Pd	-0.07(-0.07)	-0.27(-0.33)	
^{116}Cd	-0.07(-0.07)	-0.30(-0.30)	
^{124}Sn	-0.12(-0.34)	-0.27(-0.40)	
^{128}Te	-0.12(-0.33)	-0.27(-0.38)	-0.15
^{130}Te	-0.12(-0.33)	-0.27(-0.39)	-0.15
^{136}Xe	-0.11(-0.32)	-0.25(-0.38)	-0.15
^{148}Nd	-0.12(-0.12)		
^{150}Nd	-0.10(-0.10)		
^{154}Sm	-0.09(-0.09)		
^{160}Gd	-0.07(-0.07)		
^{198}Pt	-0.10(-0.10)		
^{232}Th	-0.08(-0.08)		
^{238}U	-0.08(-0.08)		

χ_F (before restoration values in parentheses):

- Considerable reduction obtained!
- Isospin restored χ_F values very close to the ones obtained from ISM, where isospin is a good quantum number by construction
- Similar prescription has been used for QRPA (Simkovic *et al.*, PRC 87 045501 (2013) and Suhonen *et al.*, PRC 91 024613 (2015))

QUENCHING OF g_A

- The problematic question of effective value of g_A is still open. Three suggested scenarios are:
  - Free value: 1.269
  - Quark value: 1
  - Even stronger quenching: $g_{A,eff} < 1$



QUENCHING OF g_A

- It is well-known from single β decay/EC^* and $2\nu\beta\beta$ that g_A is renormalized in nuclei. Reasons:
  - Limited model space
  - Omission of non-nucleonic degrees of freedom($\Delta, N^*, \ldots$)
- The effective value of g_A in β decay/EC and $2\nu\beta\beta$ can be
  - defined as

$$M_{2\nu}^{eff} = \left(\frac{g_{A,eff}}{g_A}\right)^2 M_{2\nu}$$

$$M_{\beta/EC}^{eff} = \left(\frac{g_{A,eff}}{g_A}\right) M_{\beta/EC}$$

  - obtained by comparing the calculated and measured half-lives for β/EC and/or for $2\nu\beta\beta$

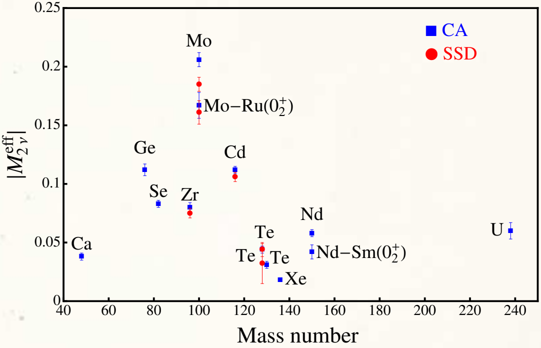
* J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965), D.H. Wilkinson. Nucl. Phys. A225, 365 (1974)

QUENCHING OF g_A

Maximally quenched value from $2\nu\beta^-\beta^-$ experiments:

Nucleus	$\tau_{1/2}^{2\nu}(10^{18}$ yr) exp[&]
^{48}Ca	44^{+6}_{-5}
^{76}Ge	1650^{+140}_{-120}
^{82}Se	92 ± 7
^{96}Zr	23 ± 2
^{100}Mo	7.1 ± 0.4
^{100}Mo*	670^{+50}_{-40}
^{116}Cd	28.7 ± 1.3
^{128}Te	2000000 ± 300000
^{130}Te	690 ± 130
^{136}Xe	2110 ± 250
^{150}Nd	8.2 ± 0.9
^{150}Nd*	120^{+30}_{-20}
^{238}U	2000 ± 600

* transition to 0_2^+



$$|M_{2\nu}^{eff}|^2 = \left[\tau_{1/2}^{2\nu} \times G_{2\nu}\right]^{-1}$$

Smallest $M_{2\nu}^{eff}$ for ^{136}Xe, the newest one measured!

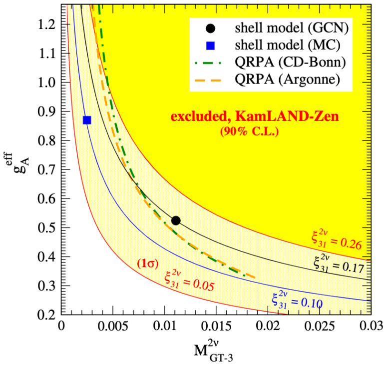
[&] A.S. Barabash, Nucl. Phys. A 935, 52 (2015).

QUENCHING OF g_A

Quenched value from $2\nu\beta\beta$ improved formalism (PRC 97 (2018) 034315):

$$(T^{2\nu}_{1/2})^{-1} \simeq (g_A^{\rm eff})^4 |(M^{2\nu}_{GT})^2 G^{2\nu}_0 + M^{2\nu}_{GT} M^{2\nu}_{GT-3} G^{2\nu}_2|$$
$$= (g_A^{\rm eff})^4 |M^{2\nu}_{GT-3}|^2 \frac{1}{|\xi^{2\nu}_{31}|^2} |G^{2\nu}_0 + \xi^{2\nu}_{31} G^{2\nu}_2|,$$

- $M^{2\nu}_{GT}$ is sensitive to contributions from high-lying states in the intermediate odd-odd nucleus
- for $M^{2\nu}_{GT-3}$ only the lowest-energy states are relevant due to rapid suppression in the energy denominator
- Thus $\xi^{2\nu}_{31}$ probes additional, complementary physics to the $2\nu\beta\beta$ half-life



PRL122,192501 (2019)

QUENCHING OF g_A

Effective value of g_A is a work in progress, since:

- Is the renormalization of g_A the same in $2\nu\beta\beta$ as in $0\nu\beta\beta$?
  - In $2\nu\beta\beta$ only the 1^+ (GT) multipole contributes. In $0\nu\beta\beta$ all multipoles 1^+, 2^-,...; 0^+, 1^-,... contribute. Some of which could be even unquenched.
  - This is a critical issue, since half-life predictions with maximally quenched g_A are > 6 times longer due to the fact that g_A enters the equations to the power of 4!
- Additional ways to study quenching of g_A:
  - Theoretical studies by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents)
  - Experimental and theoretical studies of single beta decay and single charge exchange reactions involving the intermediate odd-odd nuclei
  - Double charge exchange reactions

HEAVY NEUTRINO EXCHANGE $0\nu_h\beta\beta$

- Besides light neutrinos, $m_\nu < 1$eV there is the possibility of heavy neutrino double beta decay with $m_{\nu_h} \gg 1$GeV
- In heavy neutrino exchange scenario the transition operator has same form as for light neutrinos, but with

$$f \propto m_p \left\langle m_{\nu_h}^{-1} \right\rangle$$

$$\langle m_{\nu_h}^{-1} \rangle = \sum_{k=heavy} \left(U_{ek_h} \right)^2 \frac{1}{m_{k_h}}$$

- Also the neutrino "potential" is different:

$$v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}$$

- NMEs: Factor of $\sim$2 difference between IBM-2/ISM and QRPA-Jy
- The average inverse heavy neutrino mass is not constrained by experiments, and only model dependent limits can be set

MAJORON EMITTING $0\nu\beta\beta$

- Requires the emission of one or two additional bosons, Majorons, so it has similarities with $2\nu\beta\beta$
- There are many different models, where m, the number of emitted Majorons and n, the spectral index of the decay take different values: $\left[\tau_{1/2}^{0\nu}\right]^{-1} = g_A^4 G_{m\chi_0 n}^{(0)} \left|\left\langle g_{\chi_{ee}^M}\right\rangle\right|^{2m} \left|M_{0\nu M}^{(m,n)}\right|^2$
- Experimental limits on $\tau_{1/2,exp}^{0\nu M}$ give information about $\left\langle g_{ee}^M\right\rangle$, the majoron-neutrino coupling constant
- Ordinary Majoron decay $m = 1, n = 1$: If the Majoron couples only to light neutrino, the NME needed to calculate the half-life are the same as for light neutrino exchange
- There are cosmologic constraints on $\left\langle g_{ee}^M\right\rangle$, such as values $3 \times 10^{-7} \lesssim g_{ee}^M \lesssim 2 \times 10^{-5}$ or $g_{ee}^M \gtrsim 3 \times 10^{-4}$ are excluded by the observation of SN 1987A
  - The most stringent of the current limits are at these regions

STERILE NEUTRINOS

- Scenario, currently being extensively discussed, is the mixing of additional "sterile" neutrinos
- The NME for sterile neutrinos of arbitrary mass can be calculated using a transition operator as in ν_{light} and ν_{heavy} exchange but with

$$f = \frac{m_N}{m_e}, \qquad v(p) = \frac{2}{\pi}\frac{1}{\sqrt{p^2+m_N^2}\left(\sqrt{p^2+m_N^2}+\tilde{A}\right)},$$

where m_N is the mass of the sterile neutrino

- The product

$$fv(p) = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{\sqrt{p^2+m_N^2}\left(\sqrt{p^2+m_N^2}+\tilde{A}\right)}$$

has the limits: $m_N \to 0$ $\qquad fv(p) = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$

$m_N \to \infty$ $\qquad fv(p) = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{m_N^2} = \frac{2}{\pi}\frac{1}{m_e m_N}$

STERILE NEUTRINOS

- Several types of sterile neutrinos have been suggested both light and heavy.
- When the mass m_N is intermediate the factorization is not possible, and physics beyond the standard model is entangled with nuclear physics

$$[\tau^{0\nu}_{1/2}]^{-1} = G_{0\nu} \left| \sum_N (U_{eN})^2 M_{0\nu}(m_N) \frac{m_N}{m_e} \right|^2$$

- Approximated simple formula

$$[\tau^{0\nu}_{1/2}]^{-1} = G_{0\nu} g_A^4 \left| M^{(0\nu_h)} \right|^2 \left| m_p \sum_N (U_{eN})^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2 ,$$

with $\langle p^2 \rangle = \frac{M^{(0\nu_h)}}{M^{(0\nu)}} m_p m_e$, and $M^{(0\nu)}$ and $M^{(0\nu_h)}$ are calculated in the limits $m_N \to 0$ and $m_N \to \infty$

Conclusions

- We need matrix elements to obtain information about the absolute neutrino masses once a $0\nu\beta\beta$ decay lifetime is known.
- The reliability of nuclear matrix elements, as well as the quenching of g_A are becoming more and more important
- The uncertainty affects also the choice of material to be used in $0\nu\beta\beta$ decay searches, a choice that is a compromise between experimental advantages and the matrix element value
- We do not know what is the mechanisms of $0\nu\beta\beta$ -decay and several mechanisms may contribute with different relative phases

THANK YOU!

Image: The Royal Swedish Academy of Sciences