$0\nu\beta\beta$ in effective field theory and lattice QCD

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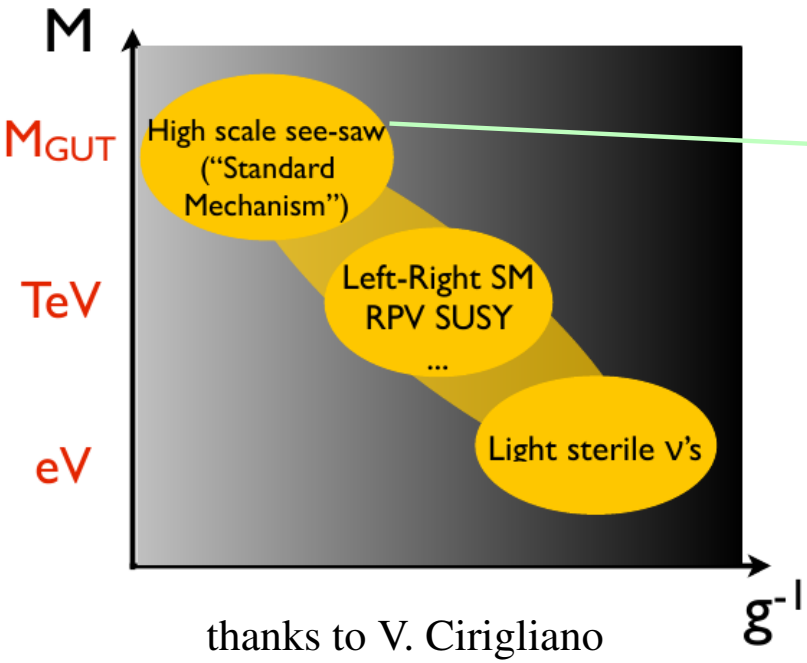
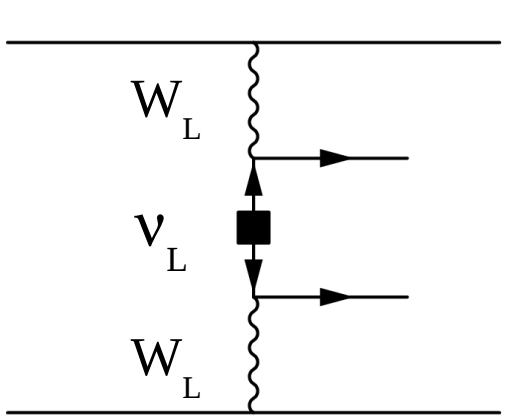
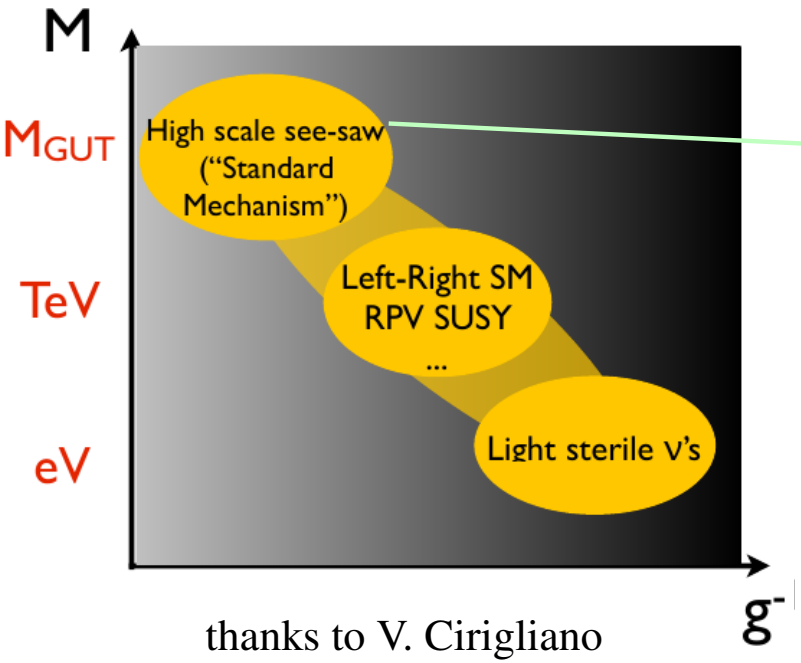


thanks to V. Cirigliano



$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

1. LNV originates at very high scales
   direct connection between ν oscillations and $0\nu\beta\beta$



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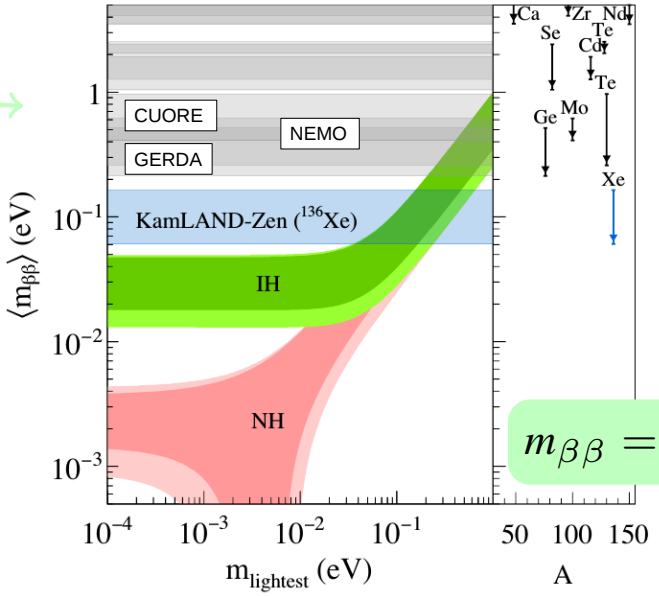
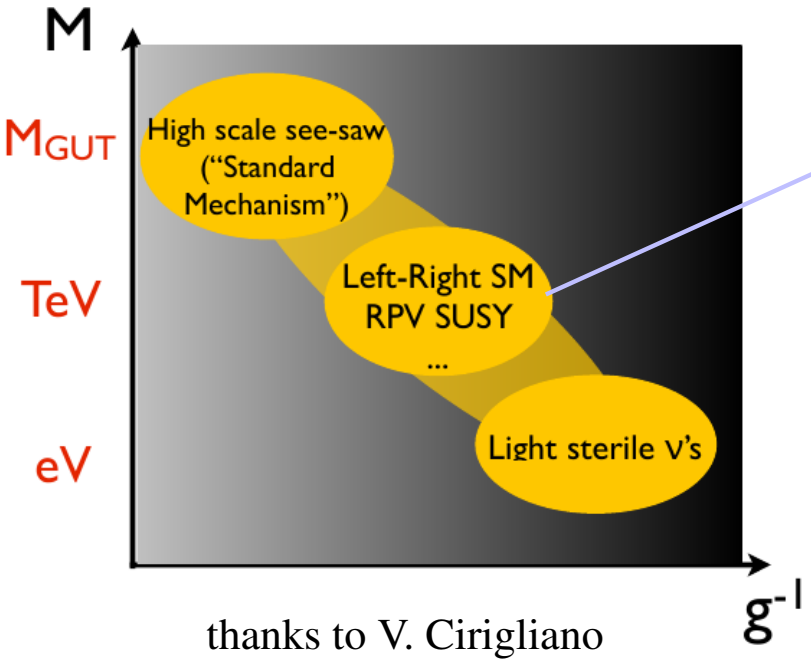


$$m_{\beta\beta} = U_{ei}^2 m_i$$

$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

1. LNV originates at very high scales
   direct connection between ν oscillations and $0\nu\beta\beta$
   clear interpretative framework and goals



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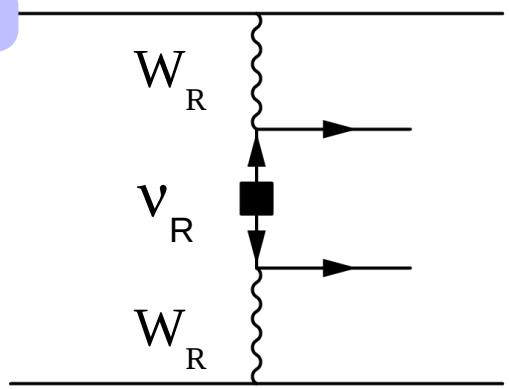
2.



$0\nu\beta\beta$ is the most sensitive probe of lepton number violation (LNV)

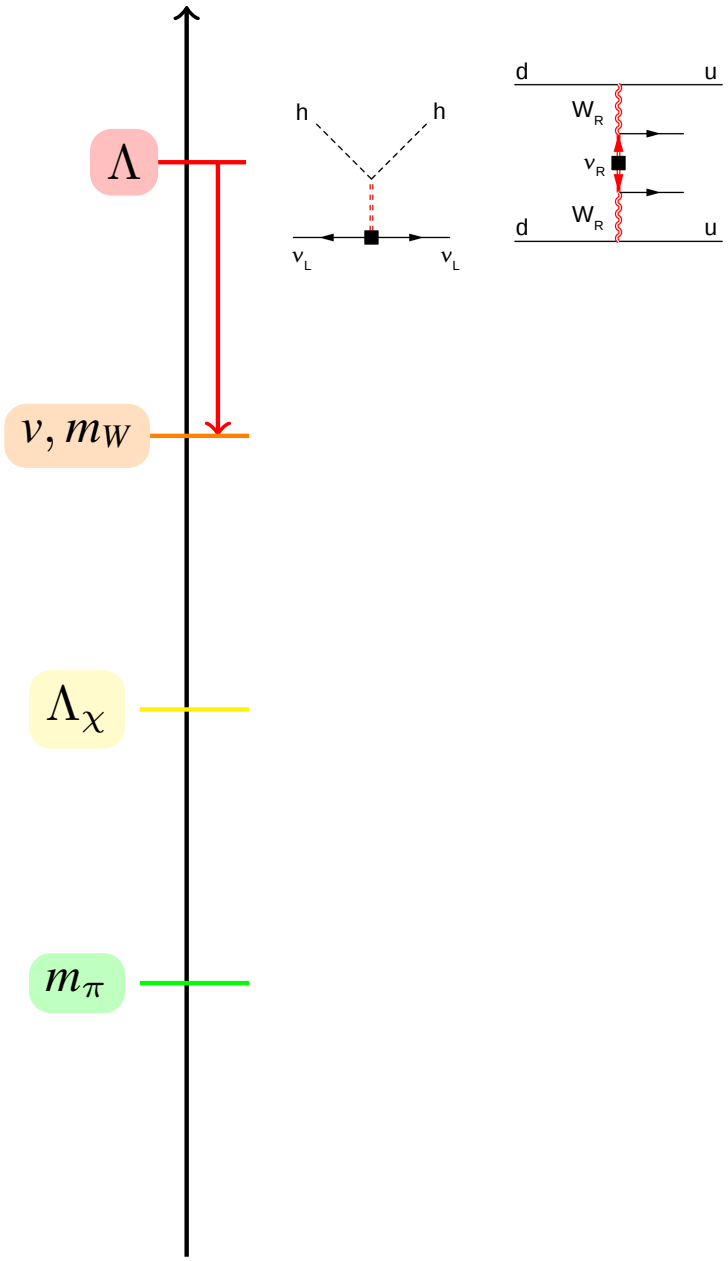
2. LNV at intermediate scales

   $0\nu\beta\beta$ is mediated by new particles, accessible at colliders?

3. very light and weakly coupled new physics

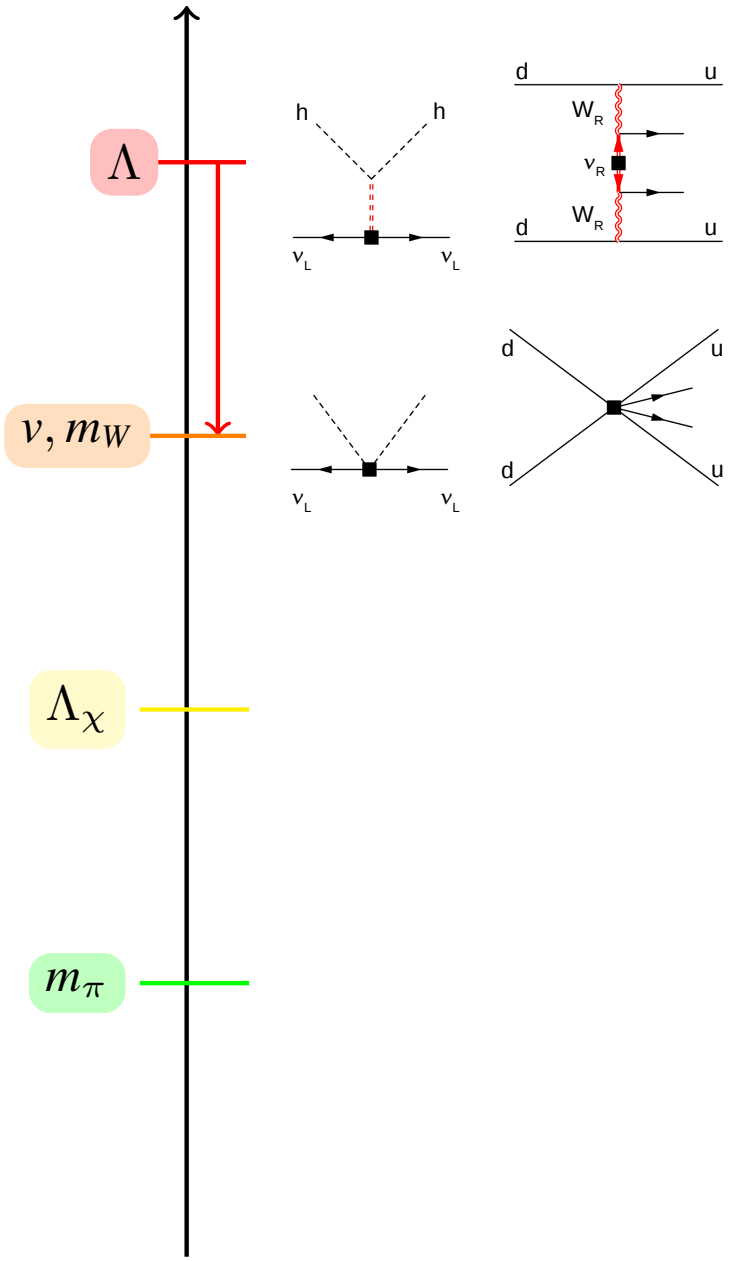
general framework to interpret $0\nu\beta\beta$ exp.?
with controlled uncertainties ?

Effective Field Theories approach to LNV



new physics $\Lambda \gg v$

Effective Field Theories approach to LNV

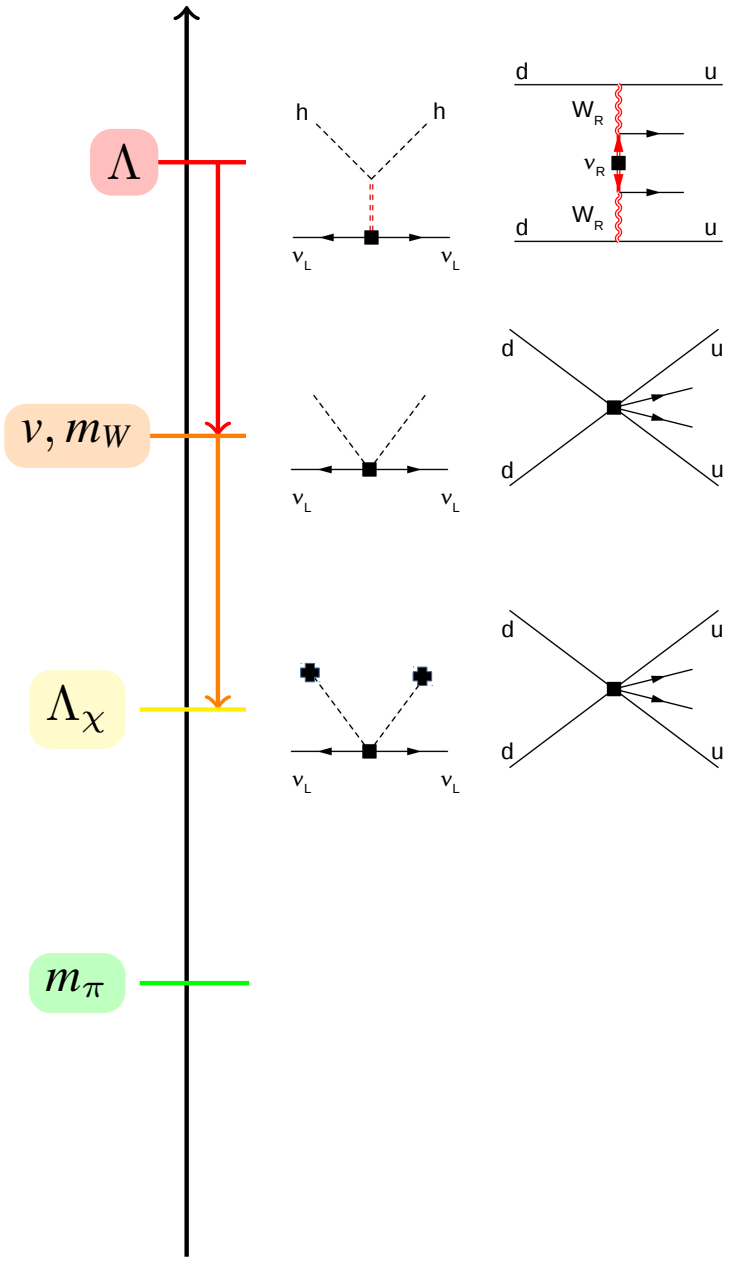


new physics $\Lambda \gg v$

SM-EFT operators

dim. 5, 7, 9 ...
can be extended with light ν_R

Effective Field Theories approach to LNV



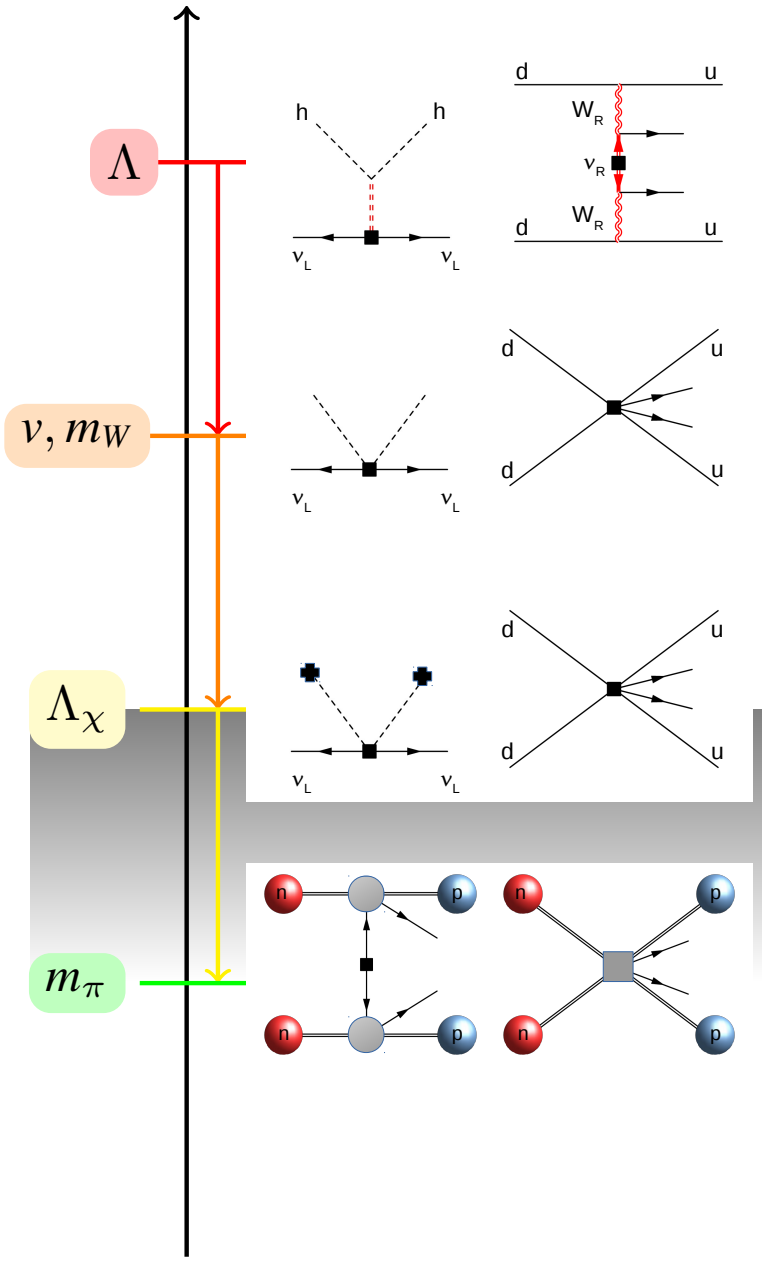
new physics $\Lambda \gg v$

SM-EFT operators

$SU(3)_c \times U(1)_{\text{em}}$ operators

perturbative matching
integrate out heavy SM d.o.f.

Effective Field Theories approach to LNV



new physics $\Lambda \gg v$

SM-EFT operators

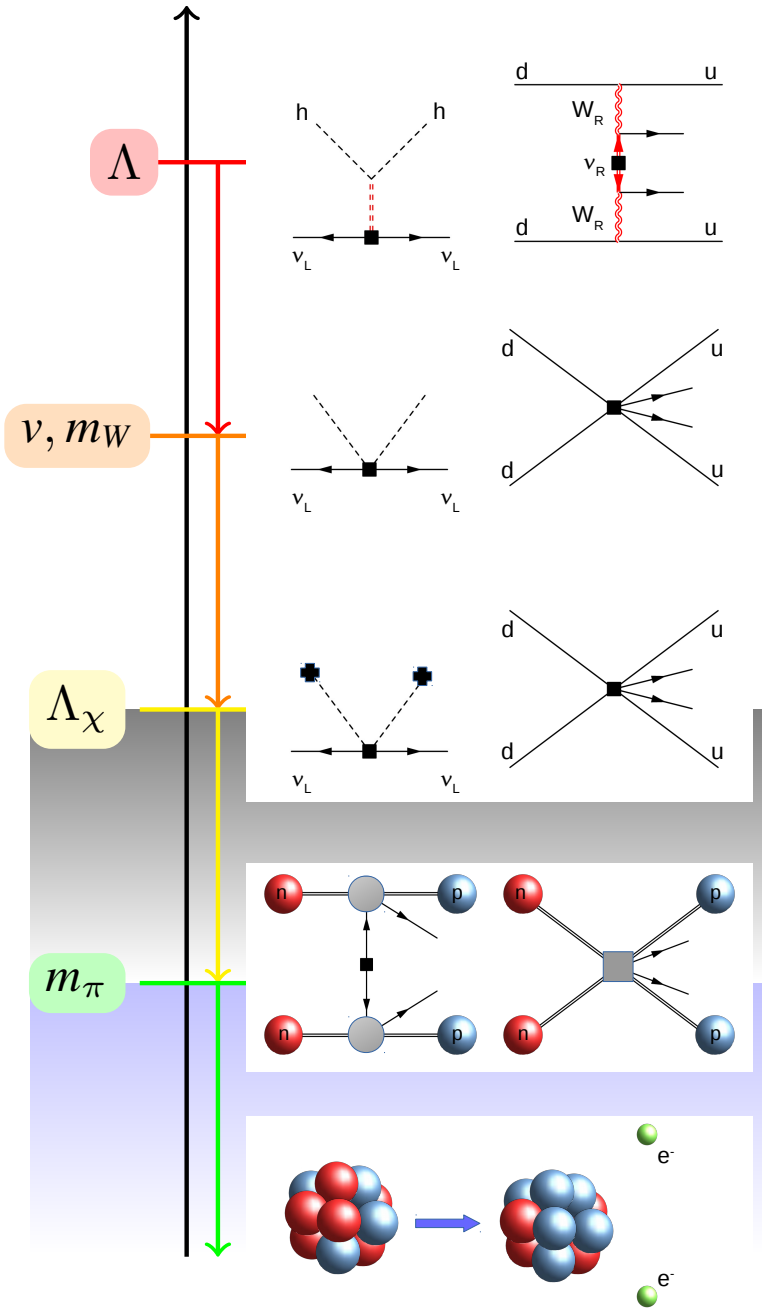
$SU(3)_c \times U(1)_{\text{em}}$ operators

non perturbative QCD

Chiral Effective Theory

Identify nucleon-level couplings
$g_A, g_S, \ldots, g_\nu^{\text{NN}}$
organize V_ν in systematic expansion

Effective Field Theories approach to LNV



new physics $\Lambda \gg v$

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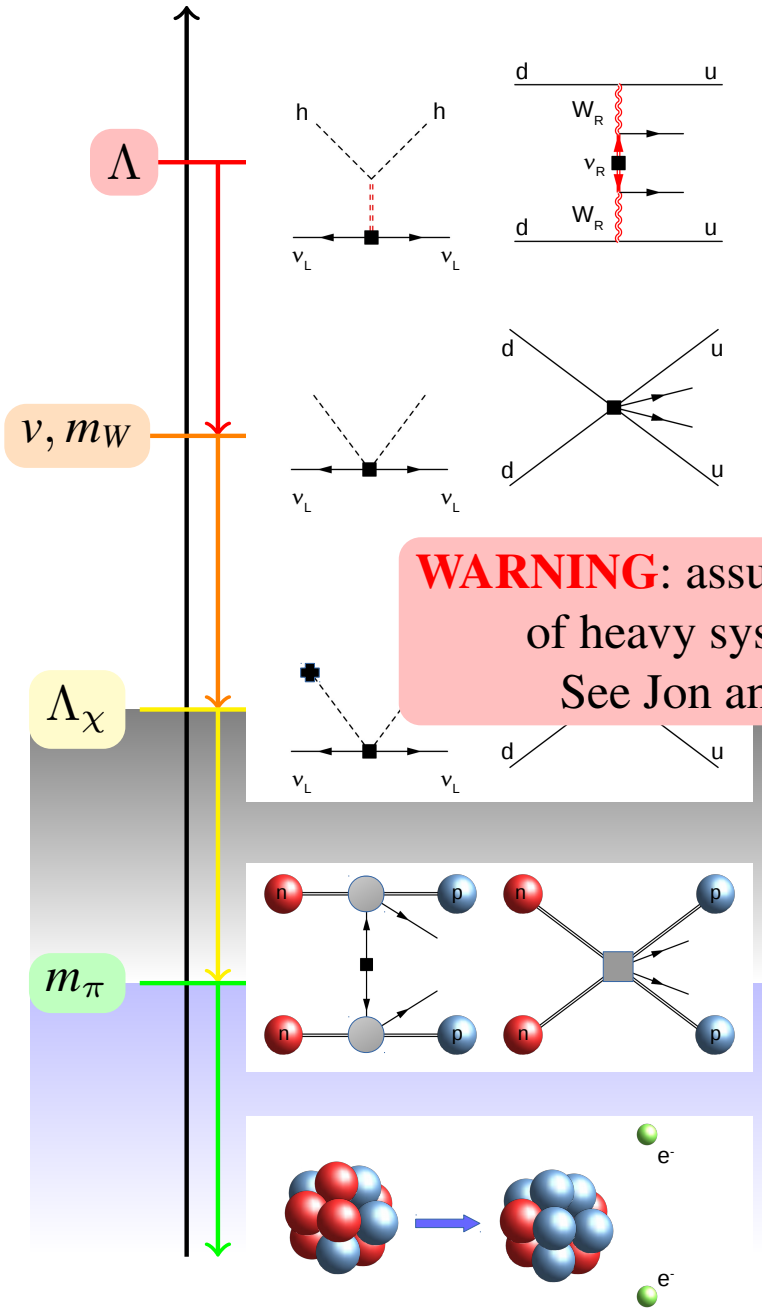
non perturbative QCD

Chiral Effective Theory

strong nuclear interactions

Many body

Effective Field Theories approach to LNV



new physics $\Lambda \gg v$

SM-EFT operators

WARNING: assume *ab initio* treatment of heavy systems is possible. See Jon and Javier's talks

$\times\ U(1)_{\mathrm{em}}$ operators

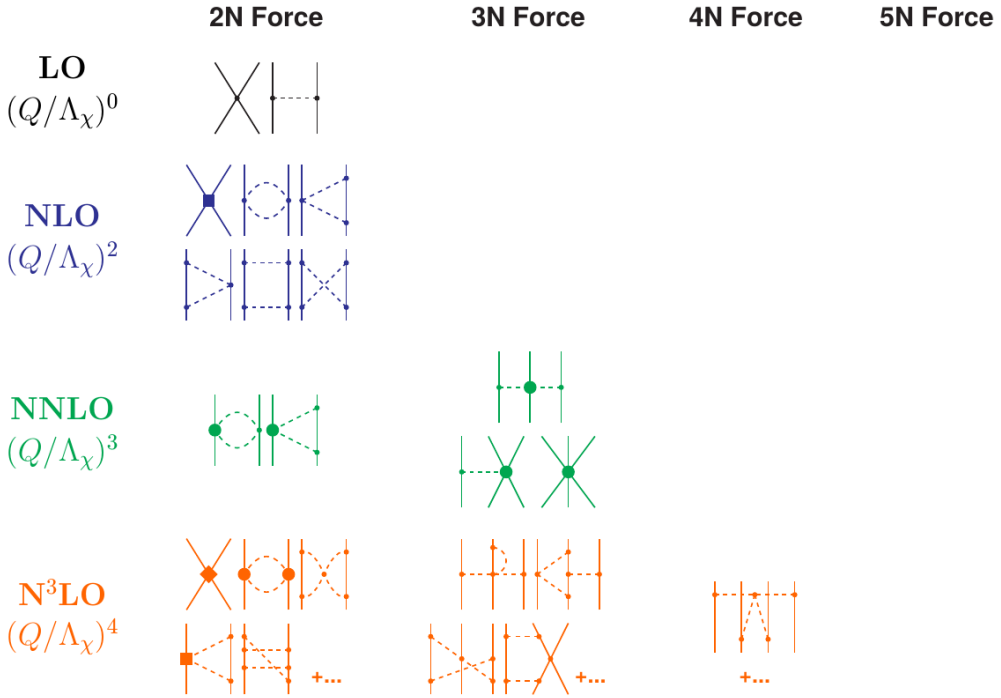
non perturbative QCD

Chiral Effective Theory

strong nuclear interactions

Many body

Chiral EFT(s)



from D. R. Entem and R. Machleidt, '17

see also:

P. Reinert, H. Krebs, E. Epelbaum, '18

M. Piarulli *et al*, '16

M. Piarulli *et al*, '14

A. Nogga, R. Timmermans, B. van Kolck, '05

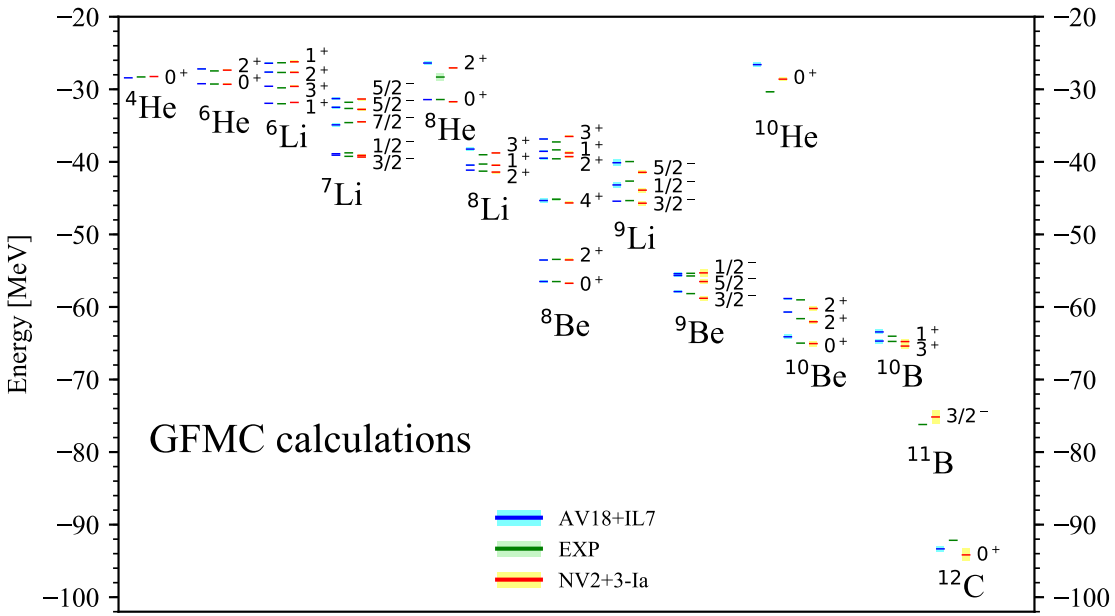
D. Kaplan, M. Savage, M. Wise, '96

Exploit QCD symmetries & scale separation in hadronic/nuclear physics

$$Q \sim m_\pi \ll \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV}$$

- expand *NN* potential and external currents in Q/Λ_χ
- LECs are fit to data in 2- and 3-nucleon systems
- and predict light-nuclear observables

Chiral EFT(s)



M. Piarulli *et al*, '17

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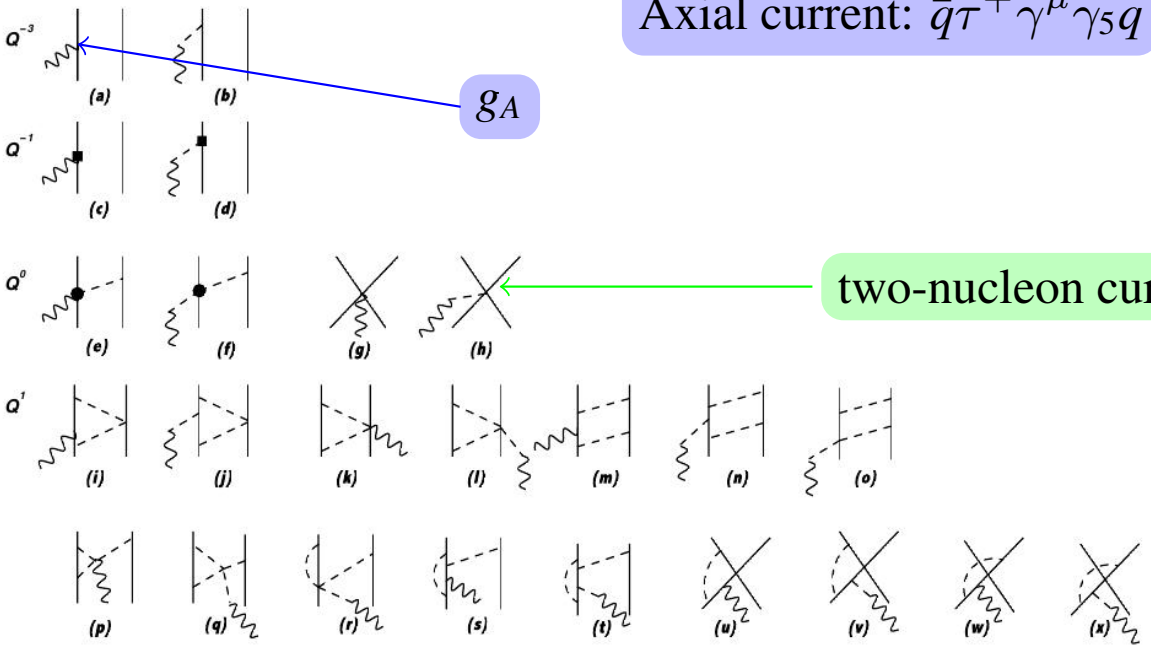
External currents in chiral EFT

Axial current: $\bar{q}\tau^{+}\gamma^{\mu}\gamma_5 q$



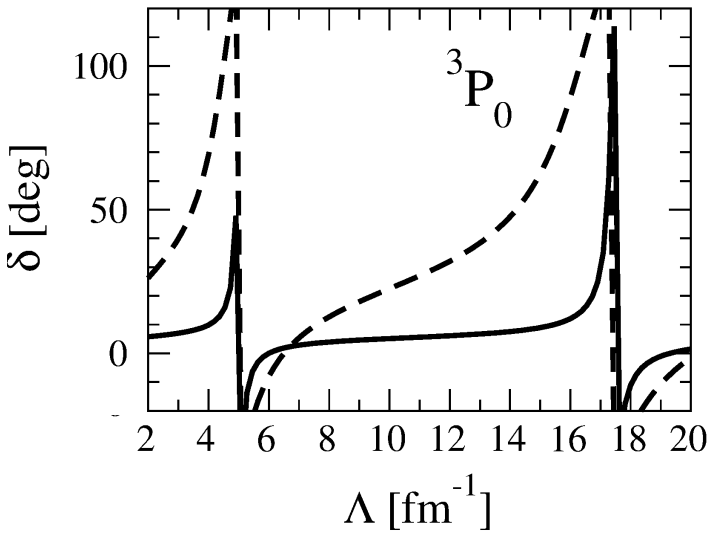
from A. Baroni *et al*, '16

- similar expansions for external currents/weak potentials

  e.g. vector, axial, scalar, pseudoscalar, tensor

- for SM operators, LECs can be fit to data
- for BSM operators, often need input from Lattice QCD (LQCD)

Power counting and renormalization



strong cut-off dependence in sol. of Schroedinger equation

A. Nogga, R. Timmermans, U. van Kolck, '05

- counting of short-range operators is based on naive dimensional analysis

Weinberg's power counting (WPC)

- WPC based on intuition from perturbation theory
- can fail because of singular chiral potentials ($\delta(\mathbf{r})$, $1/r^3$,...)

D. Kaplan, M. Savage, M. Wise, '96
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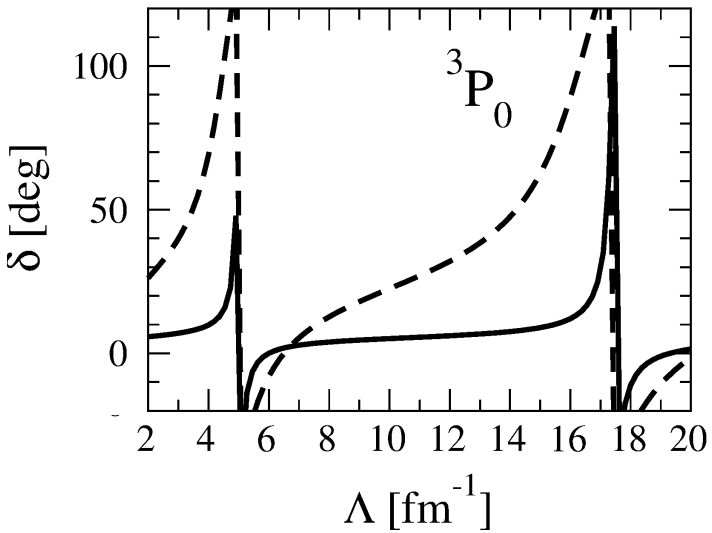
Power counting and renormalization



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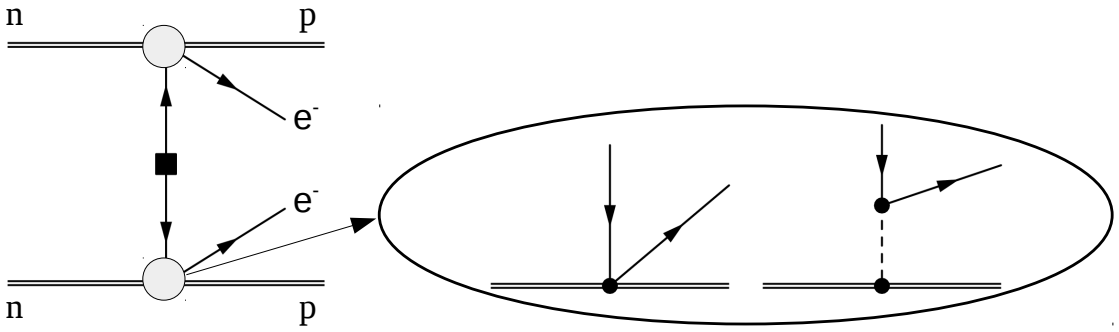
- use renormalization as **diagnostic** of chiral PC

particularly important for BSM operators
cannot fix LEC with data

Revisiting the light Majorana-ν exchange mechanism

Chiral EFT approach to light-ν exchange mechanism



- weak currents are mainly one-body

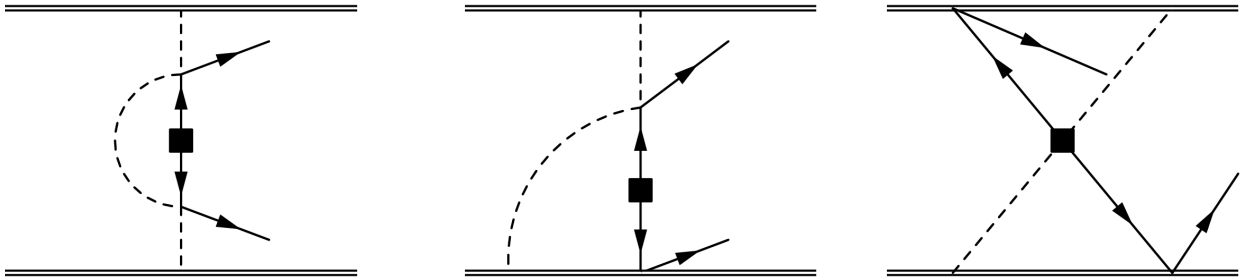
$$J_V^\mu = (g_V, \mathbf{0}) \qquad J_A^\mu = -g_A \left(0, \boldsymbol{\sigma} - \frac{\mathbf{q}}{\mathbf{q}^2 + m_\pi^2}\, \boldsymbol{\sigma}\cdot\mathbf{q}\right)$$

- $0\nu\beta\beta$ mediated by exchange of "potential" neutrinos
  $q = (q_0, \mathbf{q}) = \mathcal{O}(Q^2/m_N, Q)$

$$\begin{aligned} V_\nu &= \mathcal{A}\tau^{(1)+}\tau^{(2)+}\frac{1}{\mathbf{q}^2}\left\{\mathbf{1}^{(a)}\times\mathbf{1}^{(b)} - g_A^2\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}\left(\frac{2}{3}+\frac{1}{3}\frac{m_\pi^4}{(\mathbf{q}^2+m_\pi^2)^2}\right)+\dots\right\}. \\ \mathcal{A} &= 2G_F^2 m_{\beta\beta}\,\bar{e}_L C\,\bar{e}_L^T \end{aligned}$$

agrees with literature F. Šimkovic *et al*, '99

Standard mechanism. Higher orders



At N^{2}LO $\quad \mathcal{O}(\mathbf{q}^2/\Lambda_\chi^2)$

1. correction to the one-body currents (magnetic moment, radii, ...)

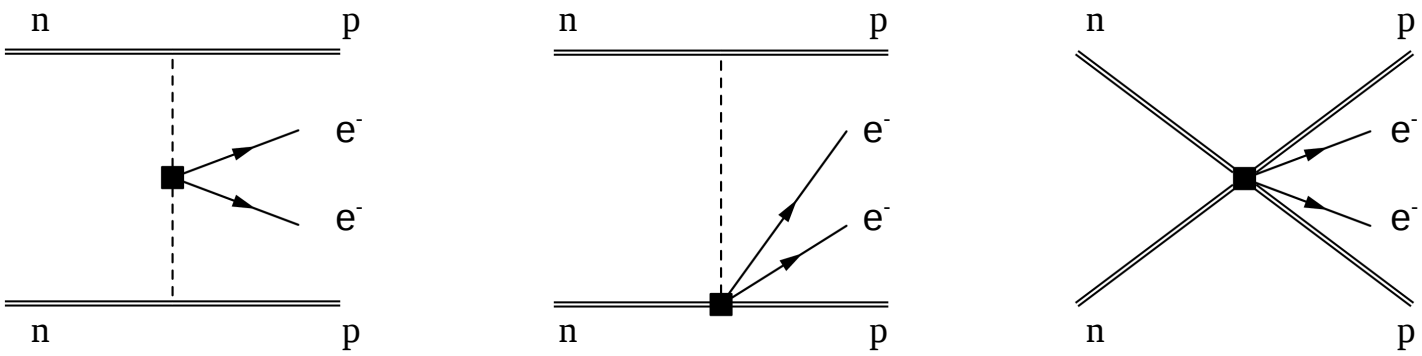
$$g_A(\mathbf{q}^2) = g_A\left(1 - r_A^2\frac{\mathbf{q}^2}{6} + \ldots\right) \qquad r_A = 0.47(7)\text{fm}$$

2. two-body corrections to V and A currents
3. pion-neutrino loops & local counterterms

UV divergences signal short-range sensitivity at N^{2}LO

V. Cirigliano, W. Dekens, EM, A. Walker-Loud, '17

Standard mechanism. Higher orders



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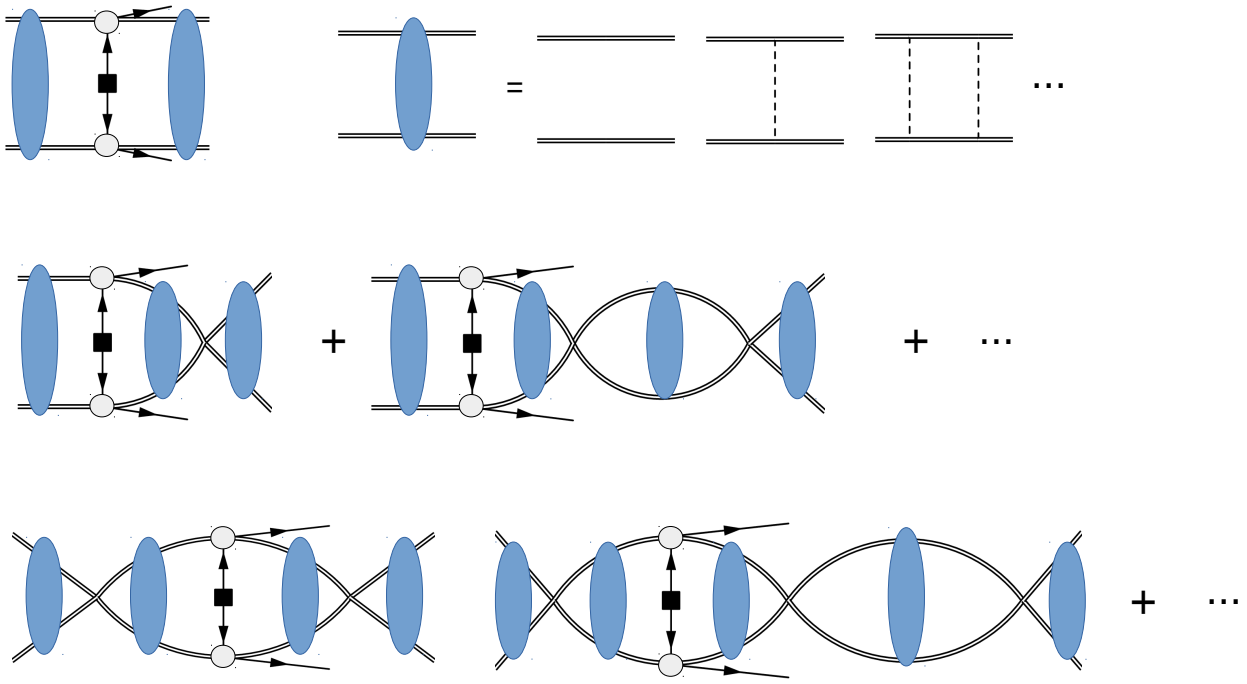
Does Weinberg's counting work for $0\nu\beta\beta$



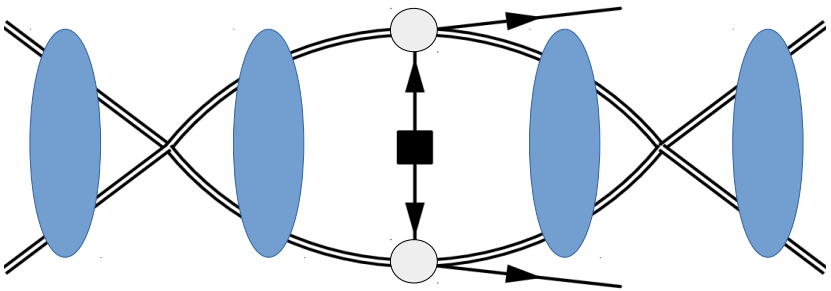
V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, S. Pastore, U. van Kolck, '18

Can address the question in a simple system $nn \to ppe^- e^-$

- solve the Schroedinger equation with LO chiral potential

$$V_{NN}^{^1S_0}(\mathbf{q}) = \tilde{C} - \frac{g_A^2}{4F_\pi^2}\frac{m_\pi^2}{\mathbf{q}^2 + m_\pi^2}$$

Does Weinberg's counting work for $0\nu\beta\beta$

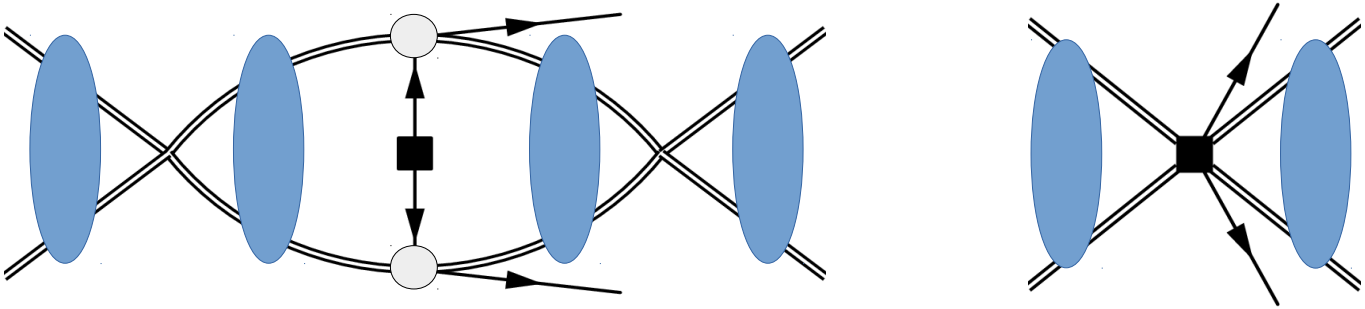


$$\frac{1}{2}(1+2g_A^2)\left(\frac{m_N\tilde{C}}{4\pi}\right)^2\left(\frac{1}{\varepsilon}+\log\mu^2\right)$$

- two-loop diagrams w. two insertions of $\tilde{C}$ have UV log divergence

need a local LNV counterterm at LO!

Does Weinberg's counting work for $0\nu\beta\beta$



- two-loop diagrams w. two insertions of $\tilde{C}$ have UV log divergence

need a local LNV counterterm at LO!

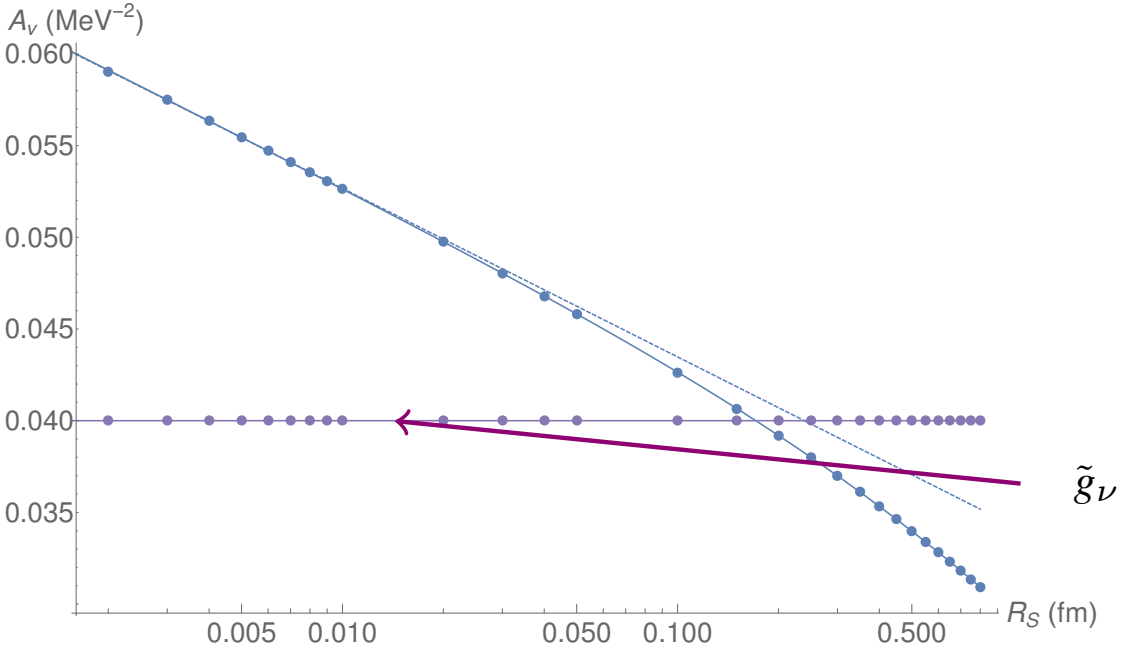
- renormalization requires to modify the LO ν potential

$$V_{\rm LNV} = V_\nu - 2g_\nu^{\rm NN}\tau^{(1)+}\tau^{(2)+}\mathcal{A}$$

- the coupling $g_\nu^{\rm NN}$ is larger than NDA

$$g_\nu^{\rm NN} \sim \frac{1}{F_\pi^2} \gg \frac{1}{(4\pi F_\pi)^2}$$

Does Weinberg's counting work for $0\nu\beta\beta$



$$g_\nu = \left(\frac{m_N \tilde{C}}{4\pi}\right)^2 \tilde{g}_\nu$$

$$\tilde{g}_\nu \sim b - \tfrac{1}{2}(1 + 2g_A^2)\log R_S$$

- divergence is not an artifact of dim. reg.

  regulate the short-range core as

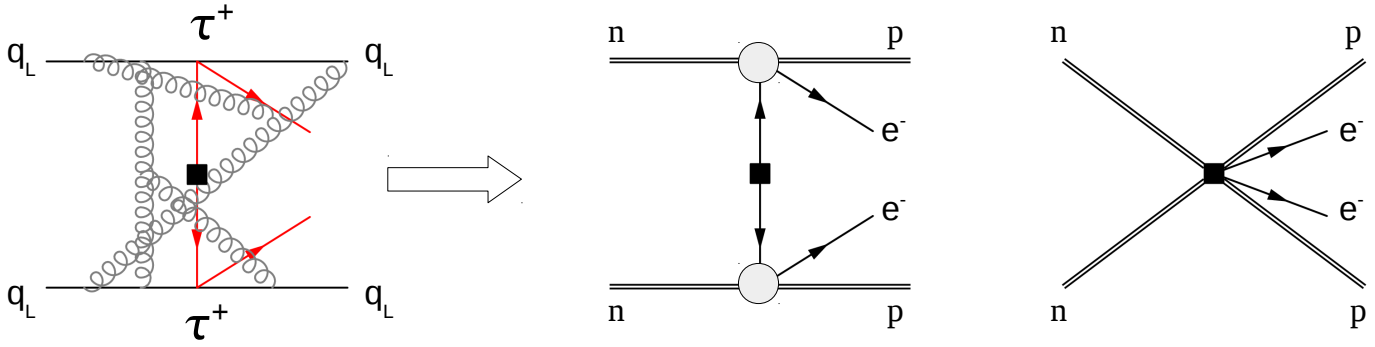
$$\delta^{(3)}(\mathbf{r}) \to \frac{1}{\pi^{3/2} R_S^3} e^{-\frac{r^2}{R_S^2}}$$

  and calculate

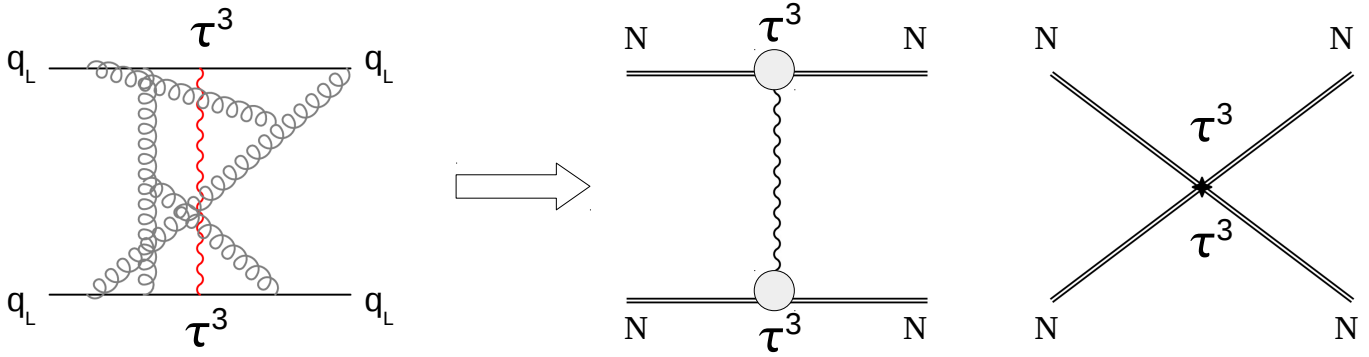
$$\mathcal{A}_\nu = \int d^3\mathbf{r}\, \psi^-_{\mathbf{p}'}(\mathbf{r}) V_\nu(\mathbf{r}) \psi^+_{\mathbf{p}}(\mathbf{r})$$

- $\mathcal{A}_\nu$ shows logarithmic dependence on R_S (+ power corrections)

Relation between $0\nu\beta\beta$ and EM isospin breaking



- the dynamics of QCD seems to imply a short-range component for V_ν
- does this happen anywhere else?
  Charge independence breaking in NN scattering



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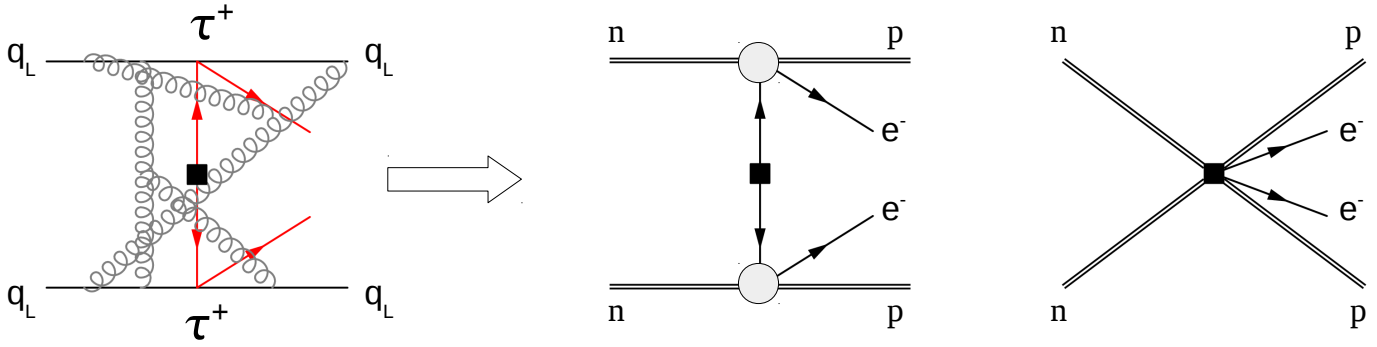

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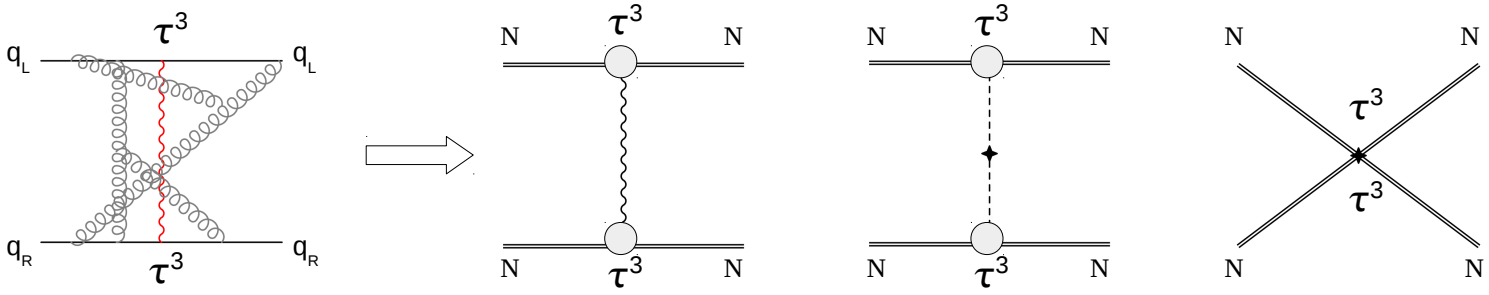
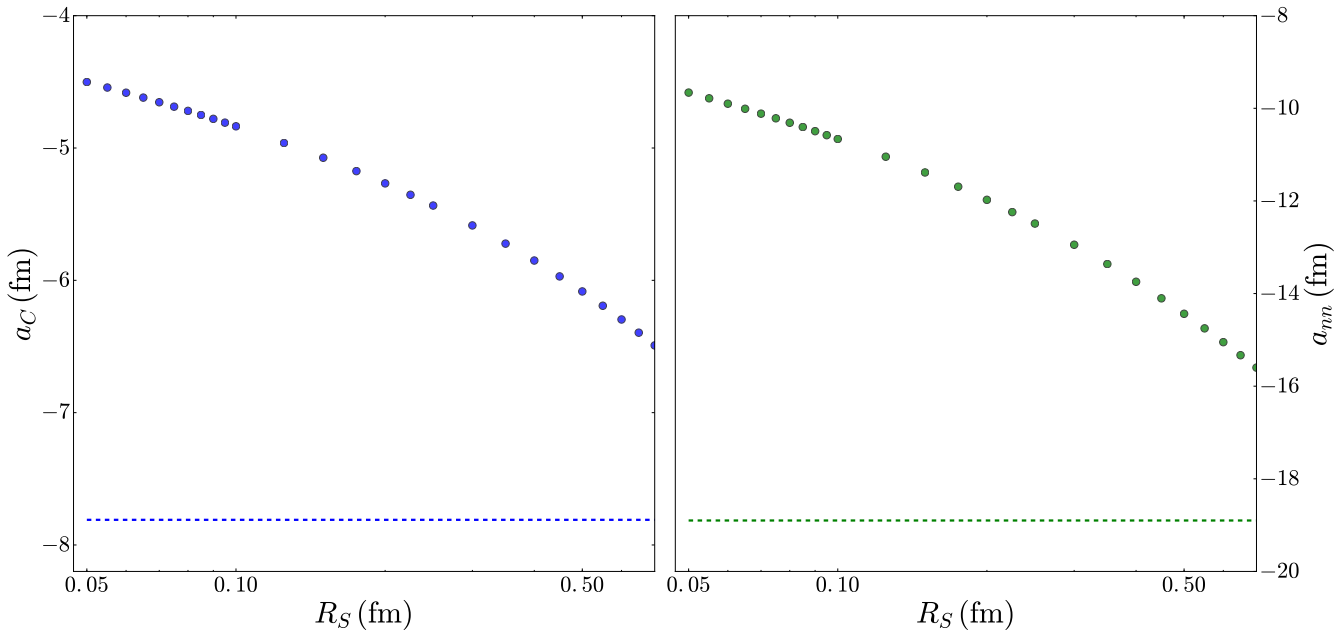


Relation between $0\nu\beta\beta$ and charge-independence breaking



Isospin breaking in the ${}^1S_0$ channel

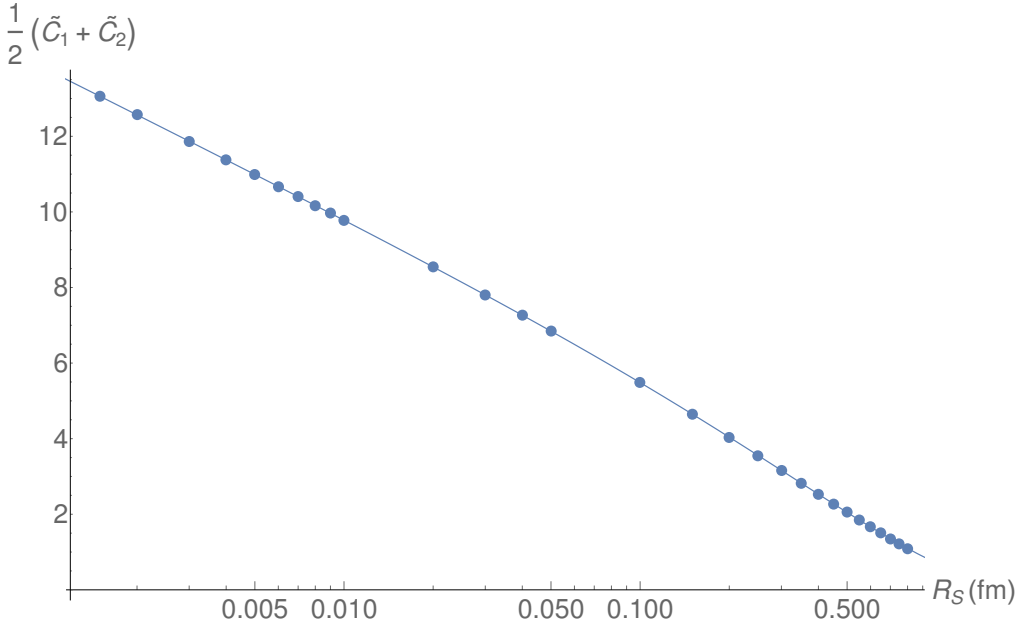
$$a_{\text{CIB}} = \frac{1}{2}\left(a_C + a_{nn} - 2a_{np}\right)$$

mostly of EM origin

- fit one charge-independent $\tilde{C}$ in np
- compute a_{nn} and a_C

log divergence! need a ct in each channel

Relation to charge-independence breaking



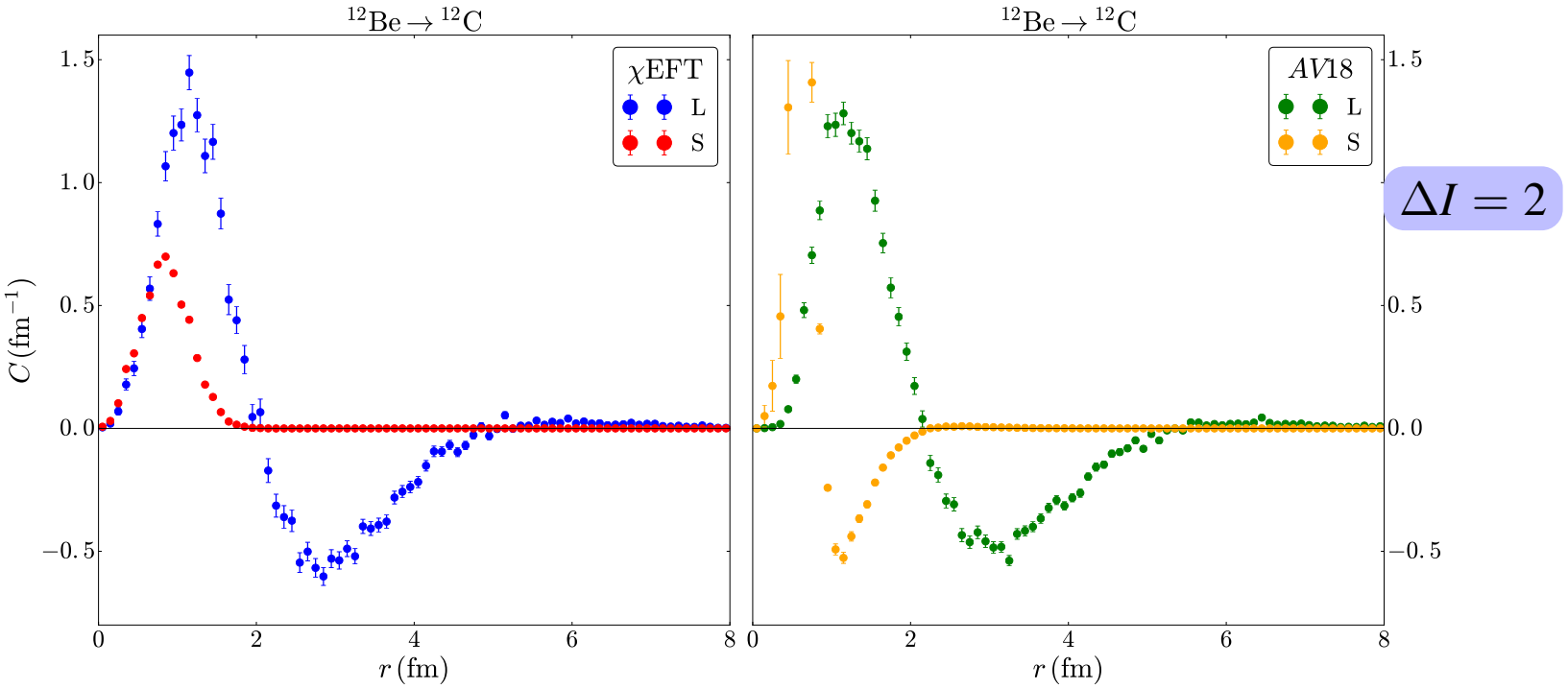
1. LO analysis of isospin breaking show log dependence

$$\frac{C_1 + C_2}{2} = \left(\frac{m_N \tilde{C}}{4\pi}\right)^2 \frac{\tilde{C}_1 + \tilde{C}_2}{2} \sim_{R_S=0.5} \frac{16}{(4\pi F_\pi)^2}$$

disagree with Weinberg's counting!

2. all high-quality chiral & pheno *NN* potentials include short-range CIB

Impact on $0\nu\beta\beta$ nuclear matrix elements



thanks to S. Pastore, M. Piarulli and B. Wiringa

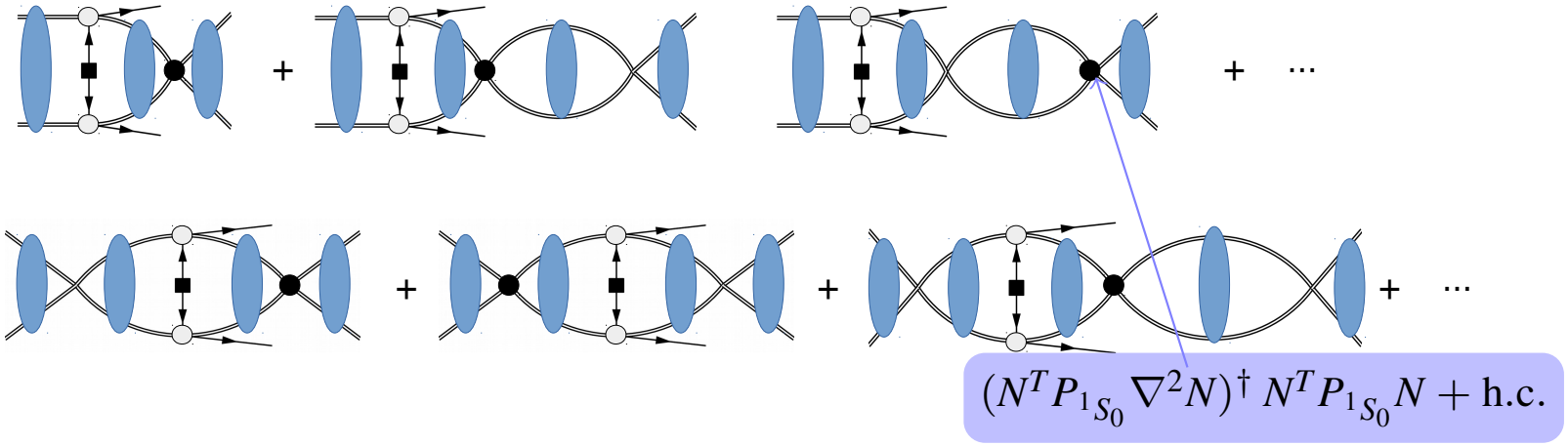
- *ab initio* calculations of ^{12}Be → ^{12}C
- large corrections to $\Delta I = 2$ transitions

AV18: $M_S/M_L = 0.8$ χEFT: $M_S/M_L = 0.7$

$> 50\%$ corrections

- ...but uncontrolled theory error from $\mathcal{C}_1 = \mathcal{C}_2$ and cancellations might be more severe in light nuclei

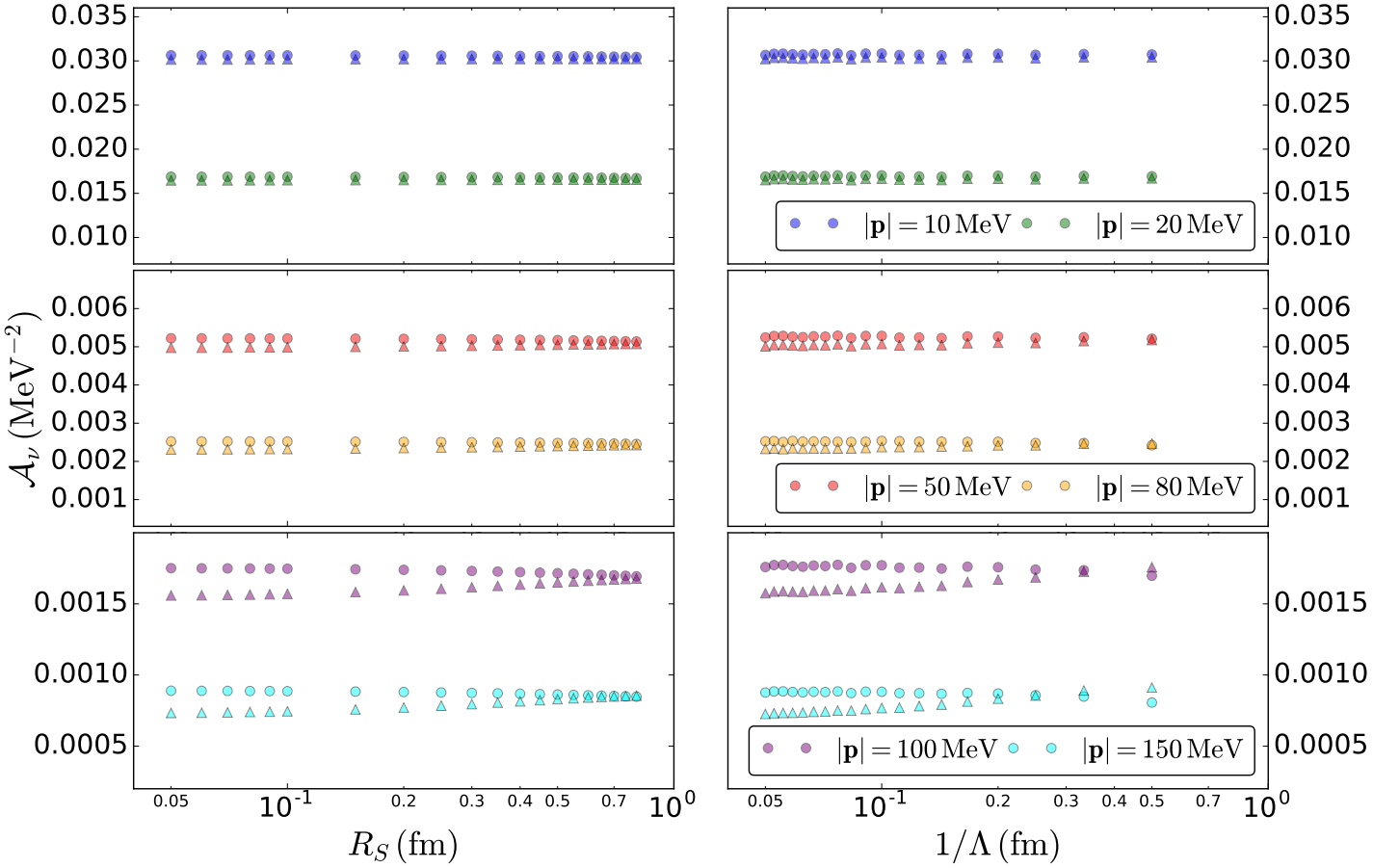
Neutrino potential at NLO



- include $^1S_0$ derivative operator in ptb. theory

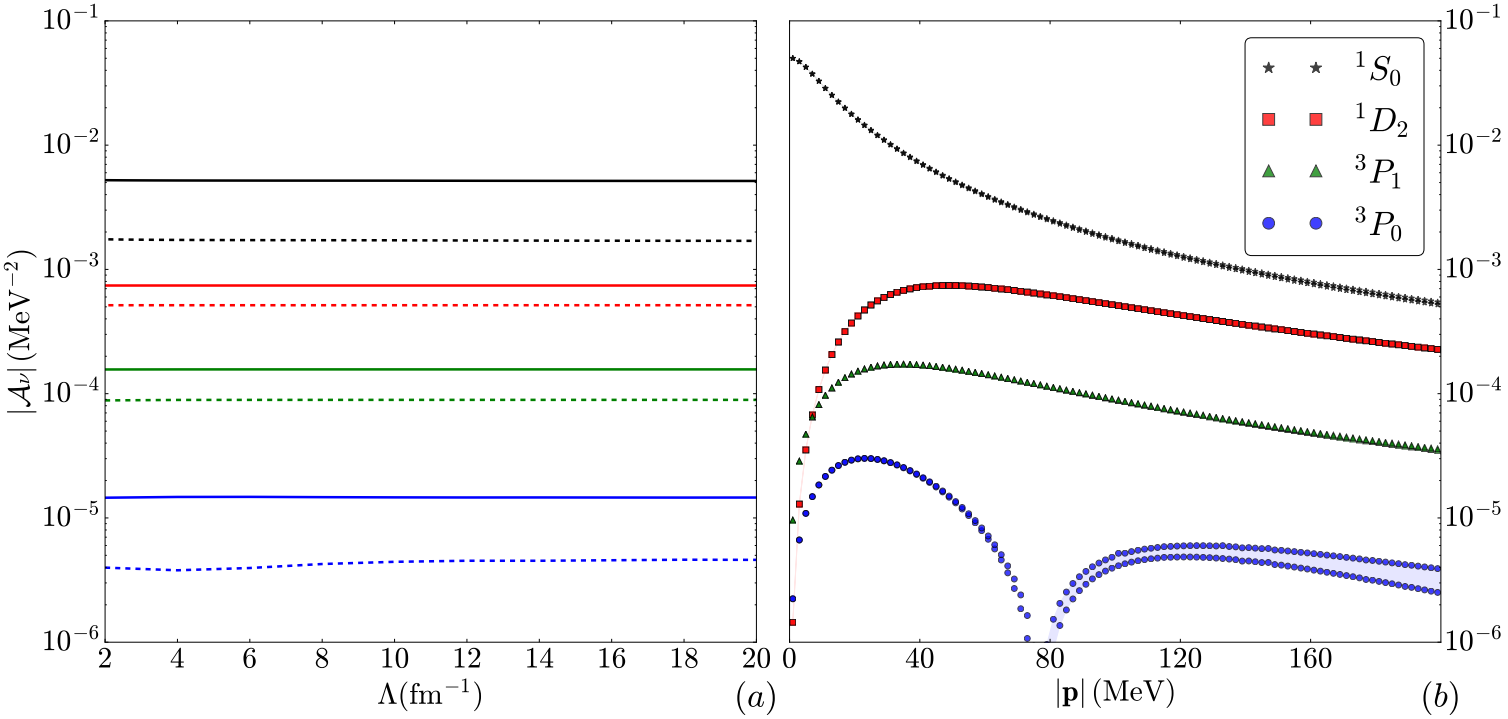
$$\mathcal{A}_\nu = -\int d^3\mathbf{r} \left(\psi_{\mathbf{p}'}^{-\,(0)\,*} + \psi_{\mathbf{p}'}^{-\,(1)\,*}\right) \left(V_\nu(\mathbf{r}) - 2g_\nu^{\mathrm{NN}} \delta_{R_S}^{(3)}(\mathbf{r})\right) \left(\psi_{\mathbf{p}}^{+\,(0)} + \psi_{\mathbf{p}}^{+\,(1)}\right),$$

Neutrino potential at NLO



- small $\mathcal{O}(Q/\Lambda)$ corrections,
  no need for derivative NLO counterterm

Higher partial waves



- Weinberg's counting leads to problems in $^3P_{0,2}$ waves

  $\Longrightarrow$ need LO counterterms in the strong interaction

- neutrino potential in P waves does not require further renormalization

The two-body neutrino potential at N^{2}LO

- one LEC at leading order

$$V_\nu^{(0)} = \tau^{(1)+}\tau^{(2)+}\left\{\frac{1}{\mathbf{q}^2}\left(1-\frac{2}{3}g_A^2\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}+\ldots\right)-2g_\nu^{\rm NN}\right\}$$

- three more LECs at N^{2}LO

$$\begin{aligned} V_\nu^{(2)} \quad = \quad & \tau^{(1)+}\tau^{(2)+}\left\{-g_{2\nu}^{\rm NN}\left(\mathbf{p}^2+\mathbf{p}'^2\right)+\frac{5}{6}\frac{g_A^2 g_\nu^{\pi\pi}}{(4\pi F_\pi)^2}\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\frac{\mathbf{q}^2}{(\mathbf{q}^2+m_\pi^2)^2}\right. \\ & \left.-\frac{g_A^2 g_\nu^{N\pi}}{(4\pi F_\pi)^2}\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\frac{1}{\mathbf{q}^2+m_\pi^2}+f(m_\pi,\mathbf{q}^2)\right\} \end{aligned}$$

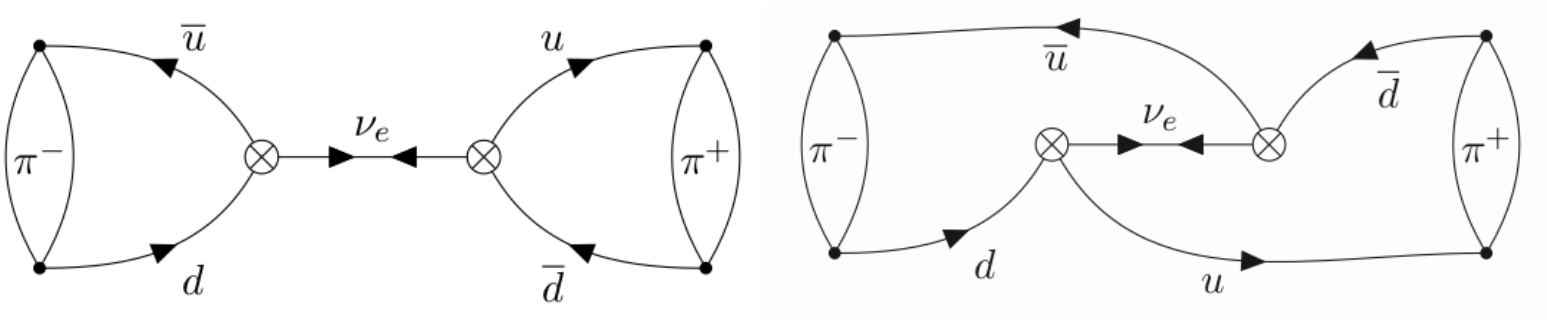
- three-body contributions at N^{2}LO

J. Menendez, D. Gazit, A. Schwenk, '11,
L.-J. Wang, J. Engel, J. M. Yao, '18

- "closure corrections" can also be incorporated in the formalism

V. Cirigliano, W. Dekens, EM, A. Walker-Loud, '17

Evaluation of the LECs



W. Detmold and D. Murphy, ‘20

- data driven extraction might not be possible
  - no LNV data
  - chiral symmetry constraints to CIB in light nuclei to be explored
- LQCD offers the most direct avenue
- long distance contributions to π $0\nu\beta\beta$ already computed

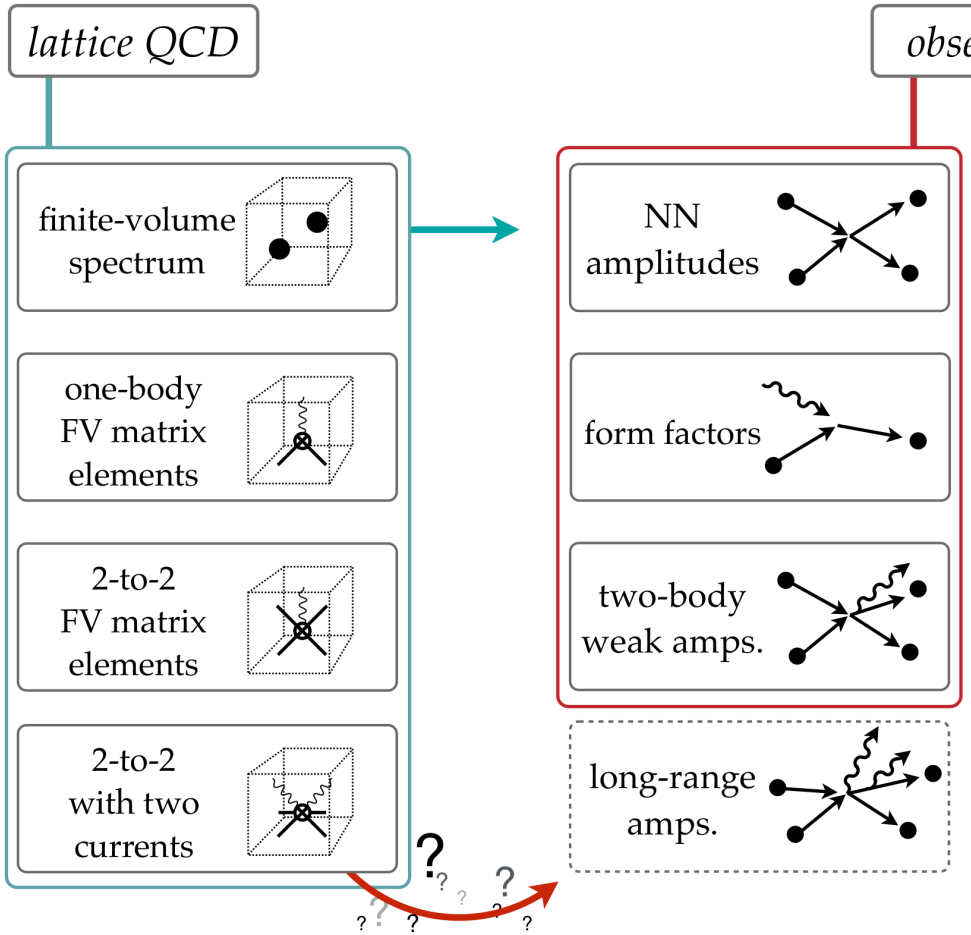
$$|g_\nu^{\pi\pi}(\mu)|_{\mu=m_\rho} = -10.89 \pm 0.79$$ X.-Y. Tuo, X. Feng and L.-C. Jin, ‘19

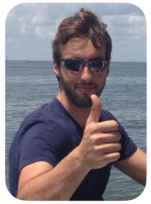
$$|g_\nu^{\pi\pi}(\mu)|_{\mu=m_\rho} = -10.78 \pm 0.52$$ W. Detmold and D. Murphy, ‘20

- extension to *NN* amplitudes in progress

Need for energy-dependent amplitudes

In order to reliably study $nn \to pp + ee$, we will need to determine the amplitude for every single subprocess for a range of kinematic points:





Alessandro Baroni
LANL Postdoc

Zohreh Davoudi
Maryland

Max Hansen
CERN

Matthias Schindler
South Carolina

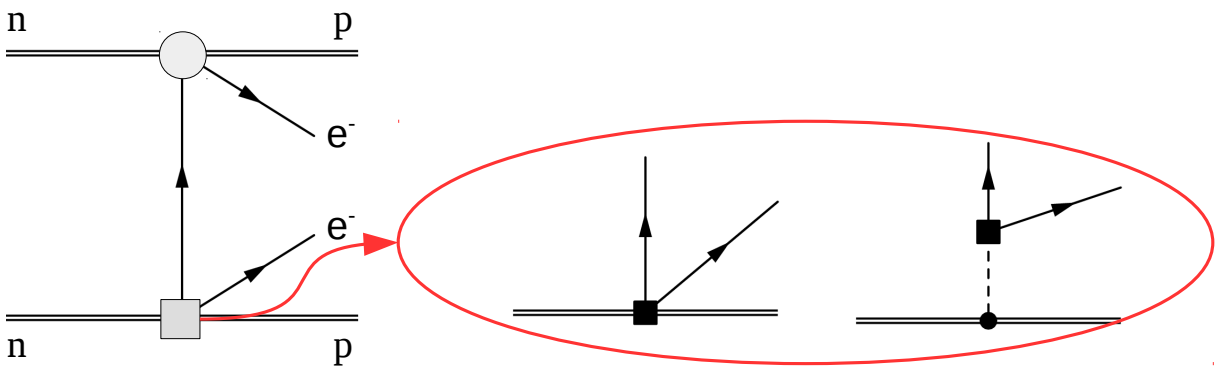
RB, Davoudi, Hansen, Schindler, Baroni (2019)

INT program "Beyond the Standard Model Physics with Nucleons and Nuclei"

Talks by Raul Briceno, Xu Feng, Zohreh Davoudi

Chiral EFT for non-standard long-range mechanisms

Long-range corrections to $0\nu\beta\beta$



$$\mathcal{L}^{(6)}_{\Delta L=2} = \frac{2G_F}{\sqrt{2}} \Big\{ C^{(6)}_{\mathrm{VL},ij}\, \bar{u}_L \gamma^\mu d_L\, \bar{e}_{R,i}\, \gamma_\mu\, C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{VR},ij}\, \bar{u}_R \gamma^\mu d_R\, \bar{e}_{R,i}\, \gamma_\mu\, C\bar{\nu}^T_{L,j}$$
$$+ C^{(6)}_{\mathrm{SR},ij}\, \bar{u}_L d_R\, \bar{e}_{L,i}\, C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{SL},ij}\, \bar{u}_R d_L\, \bar{e}_{L,i}\, C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{T},ij}\, \bar{u}_L \sigma^{\mu\nu} d_R\, \bar{e}_{L,i} \sigma_{\mu\nu}\, C\bar{\nu}^T_{L,j} \Big\}$$

- induced from SMEFT dim-7 operators
- need axial, vector, scalar, pseudoscalar and tensor one-body currents
- nucleon matrix elements are well determined on the lattice

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, EM, ‘17 and ‘18,
H. Pas, M. Hirsch, S. G. Kovalenko, H. V. Klapdor-Kleingrothaus, ‘99

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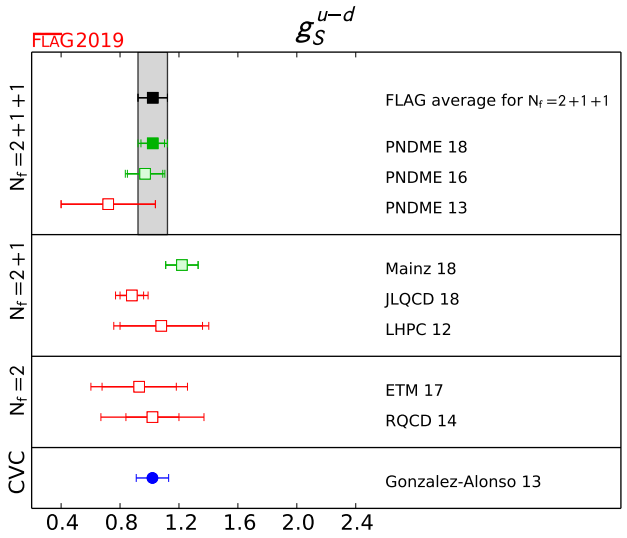

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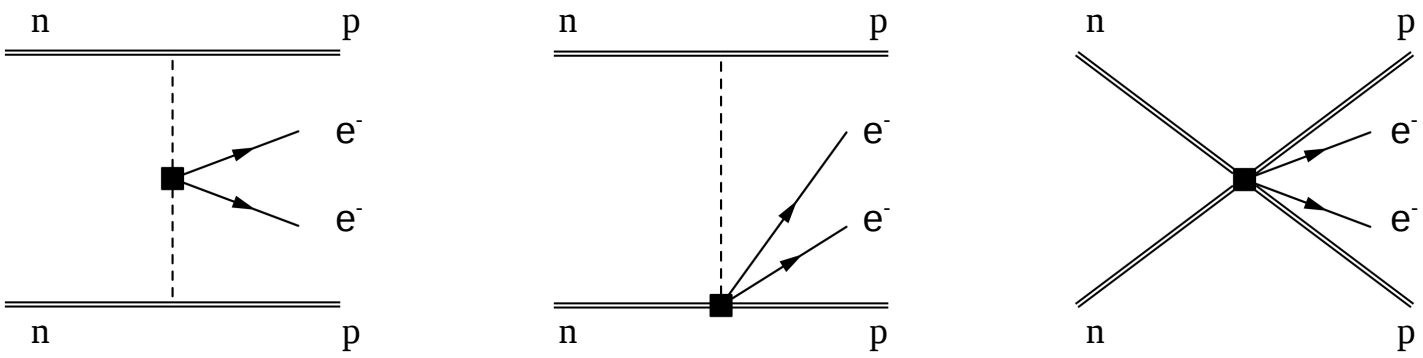
Chiral EFT for non-standard short-range mechanisms

Dim. 9 operators



$$\mathcal{L}^{(9)} = \frac{1}{v^5} \sum_i \mathcal{O}_i \left(C^{(9)}_{i\mathrm{R}} \bar{e}_R C \bar{e}_R^T + C^{(9)}_{i\mathrm{L}} \bar{e}_L C \bar{e}_L^T \right) + \mathcal{L}_{\text{vector}}$$

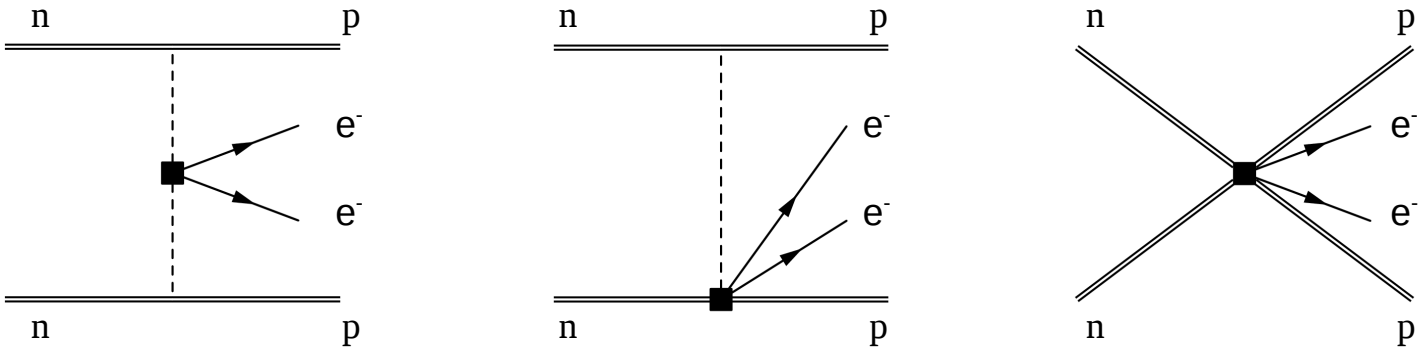
1. LL LL : $\mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_L \, \gamma_\mu d_L$
2. LR LR : $\mathcal{O}_2 = \bar{u}_L d_R \, \bar{u}_L \, d_R, \qquad \mathcal{O}_3 = \bar{u}_L^\alpha d_R^\beta \, \bar{u}_L^\beta \, d_R^\alpha$
3. LL RR : $\mathcal{O}_4 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \, \gamma_\mu d_R, \quad \mathcal{O}_5 = \bar{u}_L^\alpha \gamma^\mu d_L^\beta \, \bar{u}_R^\beta \, \gamma_\mu d_R^\alpha$

- several unjustified assumptions in the literature . . .

  e.g. $\langle pp | \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \, \gamma_\mu d_R | nn \rangle = \langle p | \bar{u}_L \gamma^\mu d_L | n \rangle \, \langle p | \bar{u}_R \, \gamma_\mu d_R | n \rangle = (1 - 3 g_A^2)$

  inconsistent with QCD, miss chiral dynamics

LNV interactions from dim. 9 operators



- $\pi\pi$ couplings

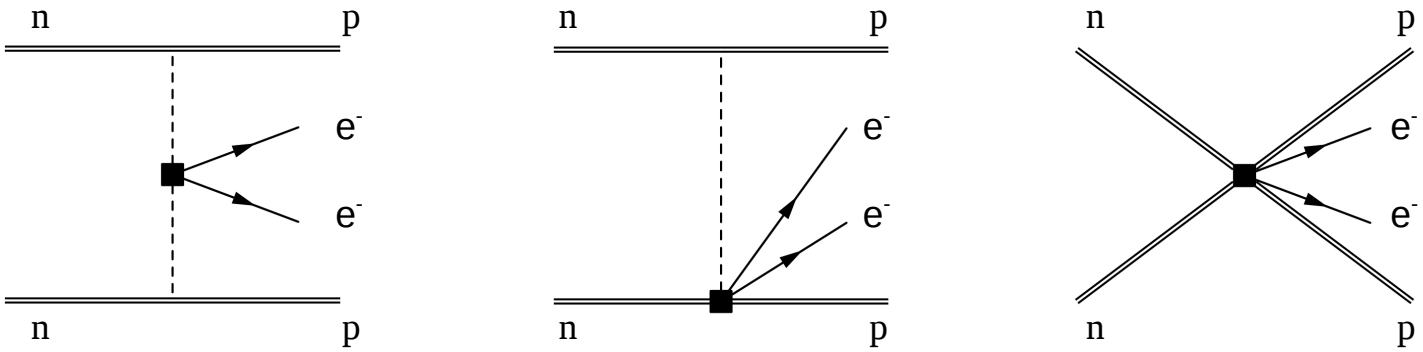
$$\mathcal{L}_\pi = \frac{F_0^2}{2}\left[\frac{5}{3} g_1^{\pi\pi} C_{1\mathrm{L}}^{(9)}\, \partial_\mu \pi^- \partial^\mu \pi^- + \left(g_4^{\pi\pi} C_{4\mathrm{L}}^{(9)} + g_5^{\pi\pi} C_{5\mathrm{L}}^{(9)} - g_2^{\pi\pi} C_{2\mathrm{L}}^{(9)} - g_3^{\pi\pi} C_{3\mathrm{L}}^{(9)}\right) \pi^- \pi^-\right] \times \frac{\bar{e}_L C \bar{e}_L^T}{v^5} + (L \leftrightarrow R) + \ldots$$

- size depends on chiral properties of $\mathcal{O}_{1,\ldots,5}$

$$g_1^{\pi\pi} \sim \mathcal{O}(1), \qquad g_{2,3,4,5}^{\pi\pi} \sim \mathcal{O}(\Lambda_\chi^2)$$

G. Prezeau, M. Ramsey-Musolf, P. Vogel, ‘03

LNV interactions from dim. 9 operators



- πN couplings, only important for $\mathcal{O}_1$
- NN couplings

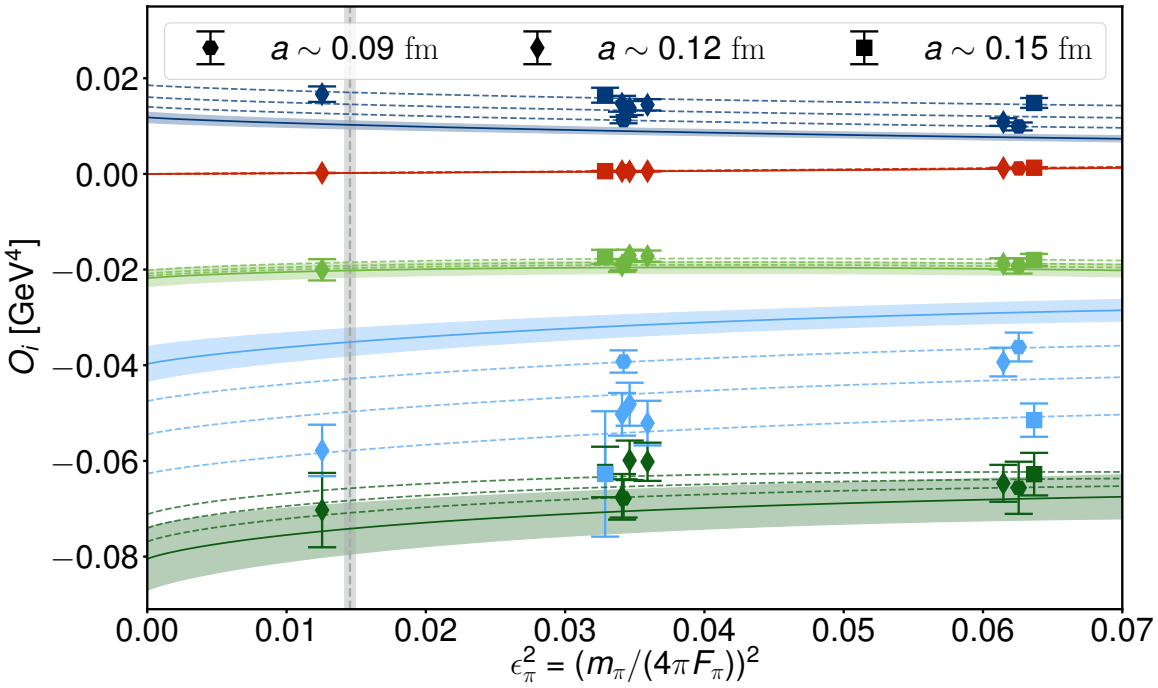
$$\mathcal{L}_{NN} = \left(g_1^{NN} C_{1\mathrm{L}}^{(9)} + g_2^{NN} C_{2\mathrm{L}}^{(9)} + g_3^{NN} C_{3\mathrm{L}}^{(9)} + g_4^{NN} C_{4\mathrm{L}}^{(9)} + g_5^{NN} C_{5\mathrm{L}}^{(9)}\right) (\bar{p}n)\ (\bar{p}n)\ \frac{\bar{e}_L C \bar{e}_L^T}{v^5}$$

- size depends on chiral properties of $\mathcal{O}_{1,\ldots,5}$

$$g_1^{NN} \sim \mathcal{O}(1), \qquad g_{2,3,4,5}^{NN} \sim \mathcal{O}\left(\frac{\Lambda_\chi^2}{F_\pi^2}\right)$$

enhanced w.r.t NDA!

$\pi\pi$ matrix elements



$g_1^{\pi\pi} = +0.4$

$g_2^{\pi\pi} = -(1.8\,\mathrm{GeV})^2$

$g_3^{\pi\pi} = +(1.0\,\mathrm{GeV})^2$

$g_4^{\pi\pi} = -(1.7\,\mathrm{GeV})^2$

$g_5^{\pi\pi} = -(3.6\,\mathrm{GeV})^2$

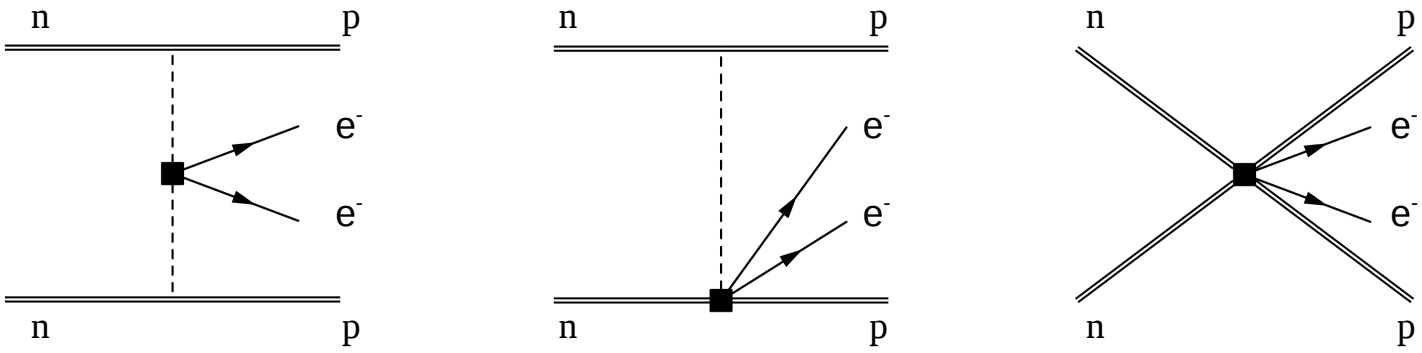
A. Nicholson *et al.*, CalLat collaboration, '18

- $\pi\pi$ matrix elements well determined in LQCD

good agreement with NDA

- $nn \to pp$ will allow to determine g_i^{NN}
  and test the chiral EFT power counting

$0\nu\beta\beta$ potential



- NME differ dramatically from factorization
  e.g $C_4^{(9)}$

$$M = -\frac{g_4^{\pi\pi} C_4^{(9)}}{2m_N^2}\left(\frac{1}{2}M_{AP,sd}^{GT} + M_{PP,sd}^{GT}\right) \sim -0.60 C_4^{(9)}$$

$$M_{\text{fact}} = -\frac{3g_A^2 - 1}{2g_A^2}\frac{m_\pi^2}{m_N^2} C_4^{(9)} M_{F,sd} \sim -0.04 C_4^{(9)}$$

bigger error than from NMEs ...

Conclusion: χEFTs, LQCD & $0\nu\beta\beta$

- systematic organization of ν potentials in powers of Q/Λ_χ
- power counting (and matrix elements) can be tested in simpler systems
- LQCD can provide the required nonperturbative QCD input
- one step towards *ab initio* calculations of $0\nu\beta\beta$

Standard mechanism:

- short-range $0\nu\beta\beta$ operator @ LO

additional $\mathcal{O}(1)$ contribution

- g_ν^{NN} can be guessed from isospin breaking in *NN* scattering

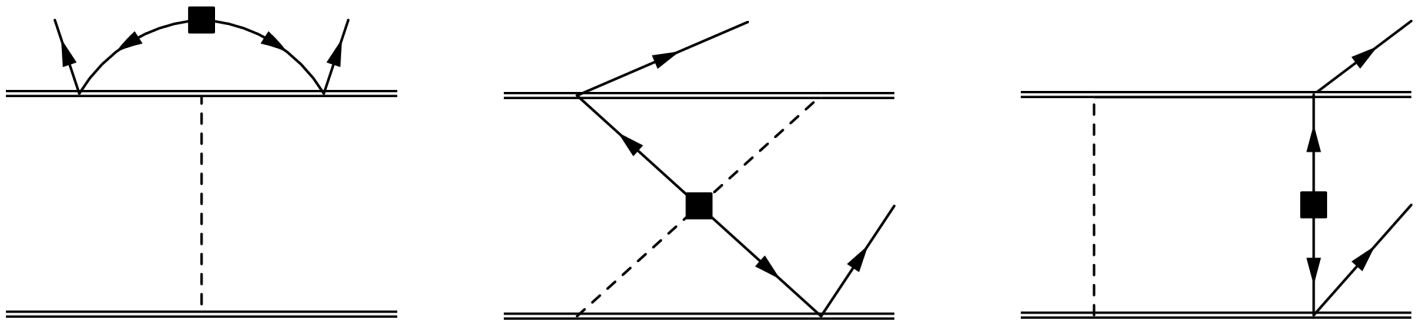
need LQCD calculations of *NN* amplitudes

Non Standard mechanisms:

- all $\pi\pi$ couplings known from LQCD
- more work to do in *NN* sector

Backup

Loop corrections to the standard mechanism



axial-axial component

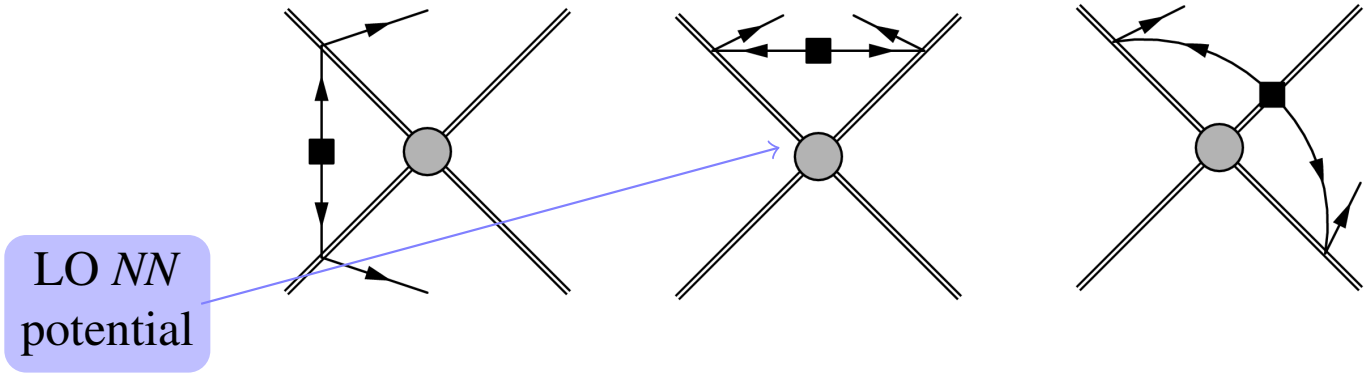
$$\mathcal{V}_{AA}^{a,b} = \frac{g_A^4}{(4\pi F_\pi)^2} \frac{2}{\mathbf{q}^2 + m_\pi^2} \left\{ \boldsymbol{\sigma}^{(a)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q} \log \frac{m_\pi^2}{m_\nu^2} + \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}\, \mathbf{q}^2 \log \frac{m_\pi^2}{m_\nu^2} \right) + \ldots$$

- the infrared dependence does not drop out!

what's going on?

- ultrasoft neutrinos are still propagating!

Loop corrections to the standard mechanism



- to define V_ν, need to subtract usoft neutrinos

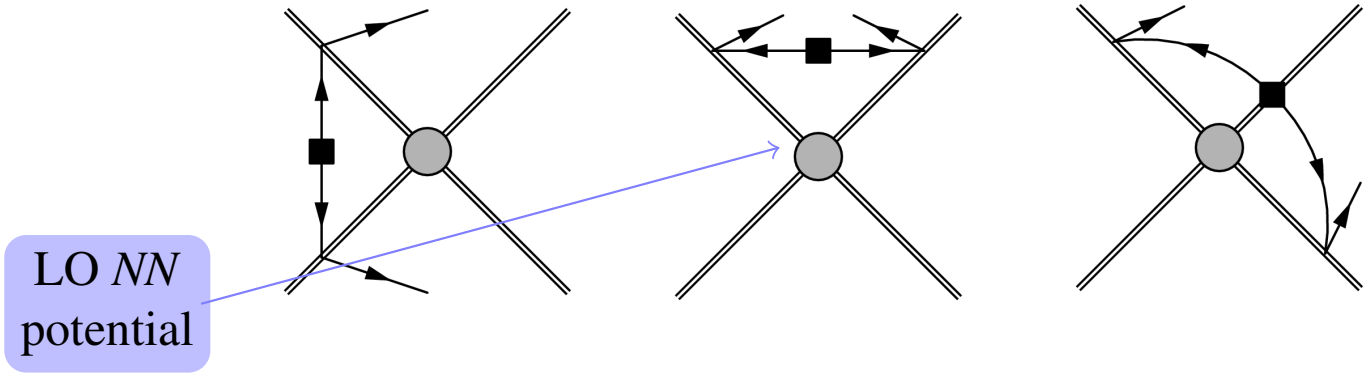
$$V_{\nu,2}^{(a,b)} = \tau^{(a)+}\tau^{(b)+}\left(\mathcal{V}_{VV}^{(a,b)} + \mathcal{V}_{AA}^{(a,b)} + \tilde{\mathcal{V}}_{AA}^{(a,b)}\,\log\frac{m_\pi^2}{\mu_{\rm us}^2} + \mathcal{V}_{CT}^{(a,b)}\right).$$

- the $\mu_{\rm usoft}$ dependent piece

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = 2\,\frac{g_A^4}{(4\pi F_\pi)^2}\,\frac{\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q} + \mathbf{q}^2\,\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}{\mathbf{q}^2+m_\pi^2} + \frac{g_A^2}{(4\pi)^2}\,16C_T\,\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}$$

- $\mathcal{V}_{AA}^{(a,b)}$ is some ugly function of $|\mathbf{q}|/m_\pi$

Loop corrections to the standard mechanism



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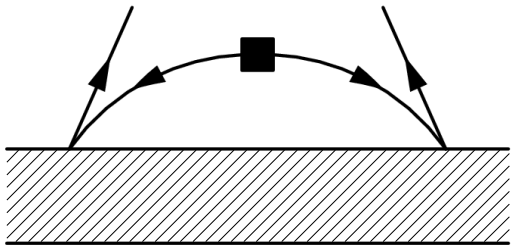
Usoft contribution to the amplitude



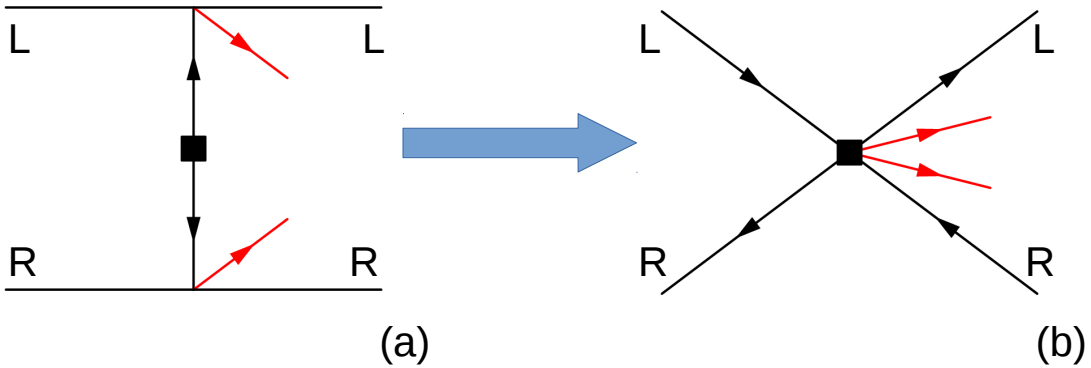
overlap $\langle n|J_\mu|i\rangle$
same as in $2\nu\beta\beta$!

- usoft neutrinos couple to the nuclear bound states

$$T_{\text{usoft}}(\mu_{\text{us}}) = \frac{T_{\text{lept}}}{8\pi^2} \sum_n \langle f|J_\mu|n\rangle\langle n|J^\mu|i\rangle \left\{ (E_2+E_n-E_i)\left(\log\frac{\mu_{\text{us}}}{2(E_2+E_n-E_i)}+1\right)+1 \leftrightarrow 2 \right\},$$

- $|i\rangle$, $|f\rangle$, $|n\rangle$ eigenvector of the LO strong Hamiltonian
- μ_{usoft} dependence cancel with $V^{(a,b)}_{\nu,2}$
- contrib. suppressed by $E/(4\pi k_F)$

Neutrino mass dependence and consistency checks



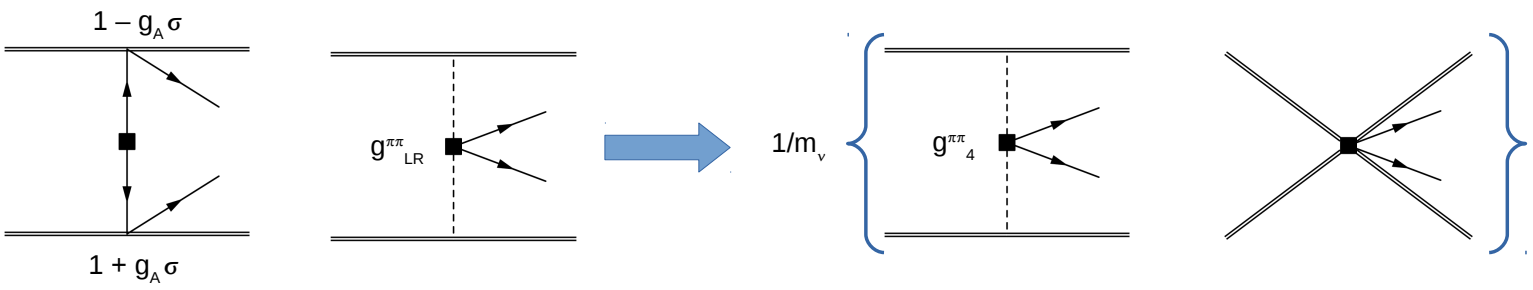
Can we understand the transition from long-range to short-range ME?

- use massive neutrino (application to sterile neutrinos contributions to $0\nu\beta\beta$)
- need to include all structures are low and high mass for correct matching e.g. LO $\pi\pi$ coupling for LR interactions
- from dispersion relations

$$g_{\rm LR}^{\pi\pi}(m_\nu) \xrightarrow{m_\nu \to \infty} \frac{\Lambda_\chi^2}{m_\nu^2} F_\pi^2$$

correct behavior to match $g_4^{\pi\pi}$

Neutrino mass dependence and consistency checks



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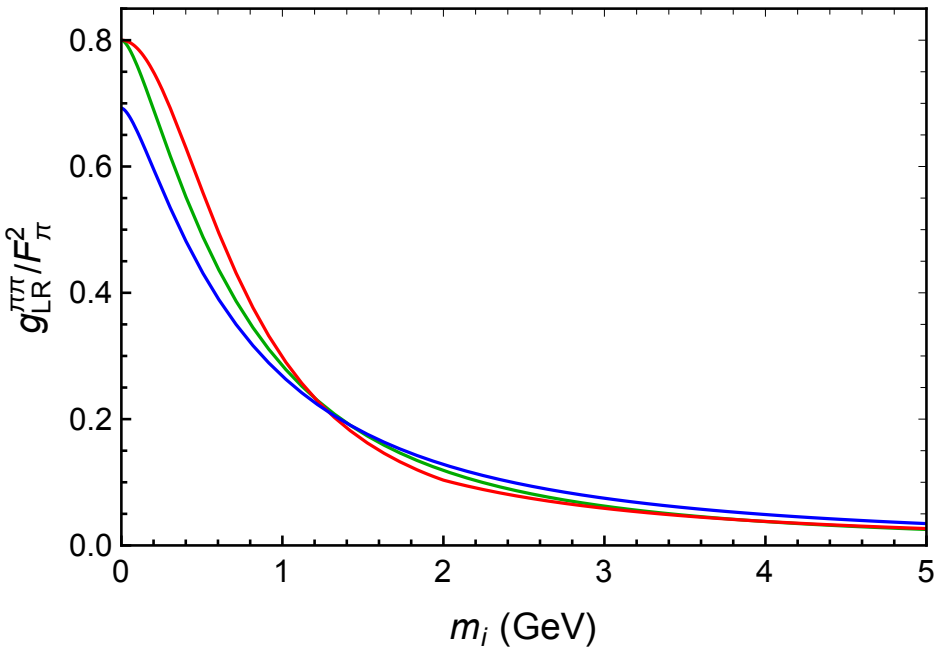

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