Space charge effects in cooling

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- Introduction
- Application to Final Cooling
- Application to 6D Cooling
- Conclusion
- Next steps

Introduction

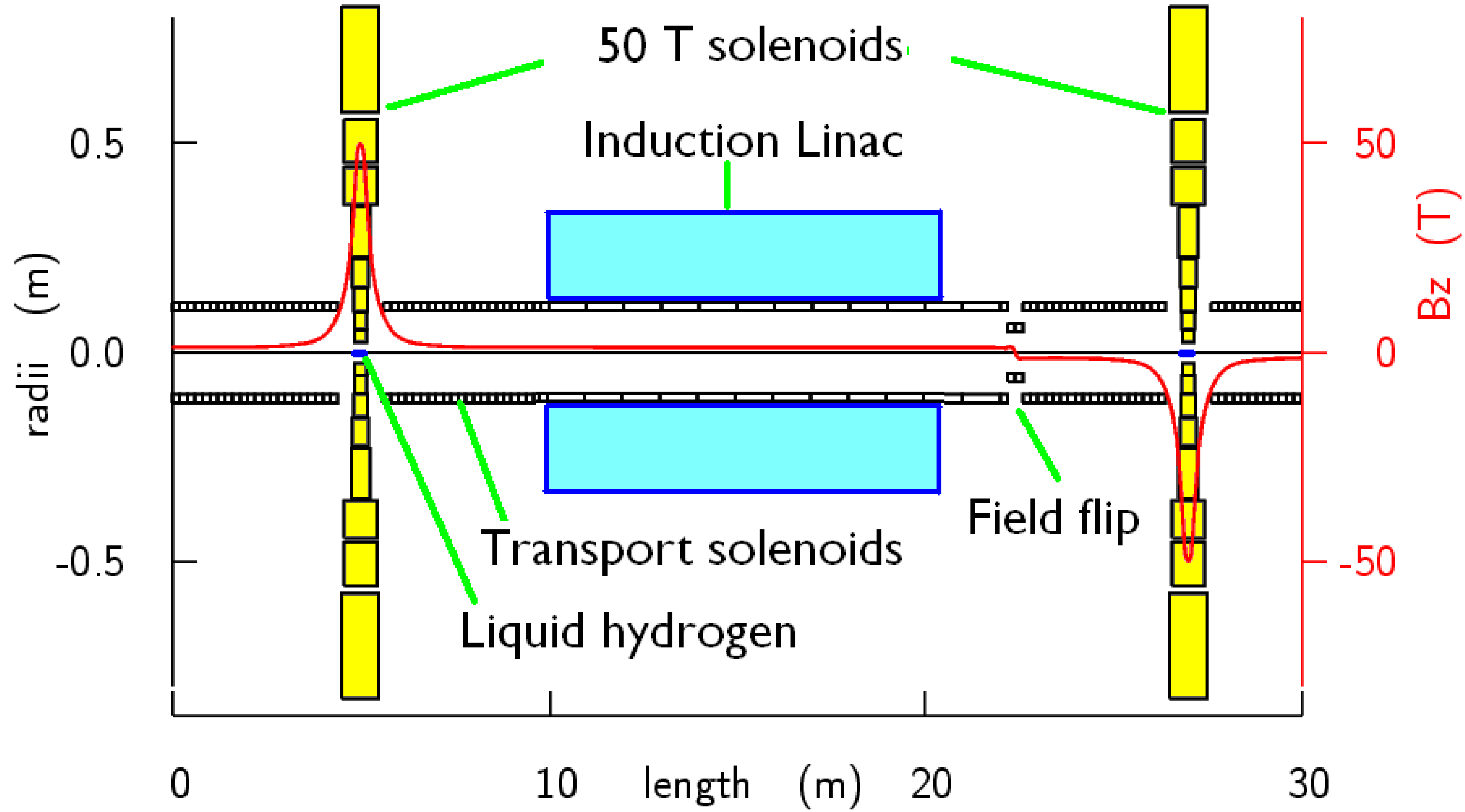
- For a luminosity of 1 $\times$ 10^{34} sec^{-1} cm^{-2} at a c of m energy of 1.5 TeV, we need bunches of 2 $\times$ 10^{12} muons.
- The transmission through acceleration = 0.867 (normal conducting) $\times$ 0.729 (super-conducting) = 0.632
- Requiring 3.16 10^{12} muons at the end of cooling,
- The bunch duration there is long: 4 m giving a relatively moderate current.

- But the rms bunch duration at the start of final cooling is only $\sigma_{ct} = 4$ cm
- The transmission through the final cooling is 0.65
- Requiring $\boxed{4.87\ 10^{12}\ \text{muons}}$ giving a peak current of

$$\frac{4.87\ 10^{12}\ 1.6\ 10^{-19}\ \ 3\ 10^{8}}{\sqrt{2\pi}\ \ 0.04} = 2.3\ \mathrm{kA} \qquad \text{at only } 135\ \mathrm{MeV/c}$$

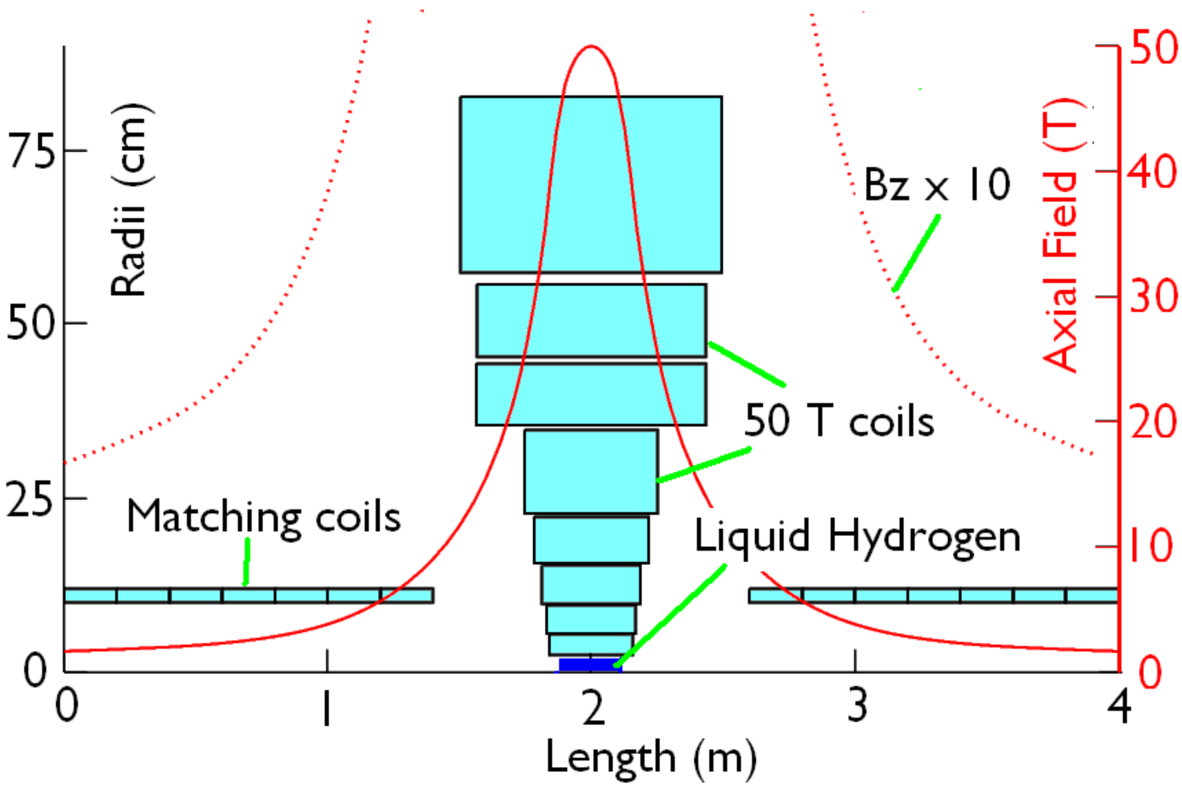
- This is a large current, at a low mom, so coherent effects can be significant
- This talk only addresses space charge effects
- Impedance effects also need study

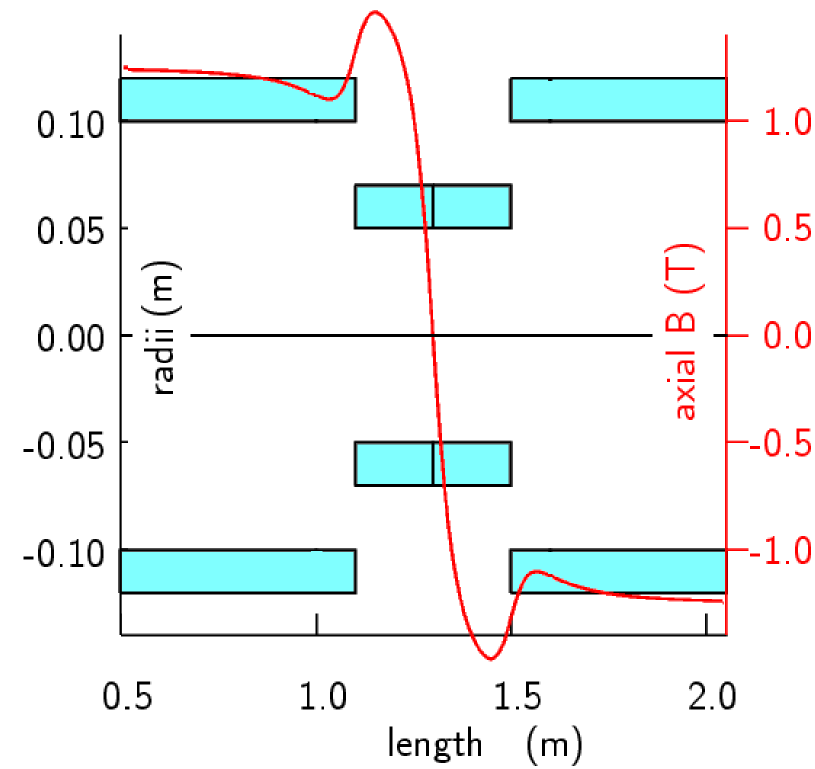
Part I: Application to Final Cooling



- This example that used 50 T. With 40 T would be similar
- It shows the last and penultimate stages, with Induction Acc of long bunches
- The transport solenoid field is 1.2 T, with field flip before the following solenoid

Details of matching to transport solenoid & field flip





- Adiabatic match from 50 T $\rightarrow$ 1.25 T
- Rapid field flip with beta matching

Transverse Space Charge Tune Shift

From S Y Lee (p110), for an arbitrary transverse shape the maximum space charge tune shift per unit length is

$$\frac{\Delta\nu_{\rm space}}{L} = \left(\frac{dN_\mu}{dz}\right) \frac{F_B\ r_\mu}{2\pi\ \epsilon_N\ \beta_v\gamma^2}$$

where ϵ_N is the normalized 95% transverse emittance, r_μ is the classical muon radius ($\approx\ 1.35\ 10^{-17}$ m), and F_B depends on the transverse bunch distribution. For a Gaussian, S. Y. Lee gives it a value of 3.8, but Ankenbrandt has pointed out that it should be 3.0.

For a Gaussian longitudinal charge distribution with rms length of σ_z

$$\text{The maximum} \quad \frac{dN_\mu}{dz} = \frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}$$

so

$$\frac{\Delta\nu_{\rm space}}{L} = \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right) \frac{r_\mu}{4\pi\ \epsilon_\perp\ \beta_v\gamma^2}$$

where $\epsilon_\perp\ =\ \epsilon_N/6$ is the rms normalized emittance

Relative tune shifts: $\Delta\nu/\nu$

This tune shift $\Delta\nu$ over length L should be compared with the normal tune ($\nu = \phi/2\pi$ where ϕ is the phase advance) over the same length L.

$$\text{We can define an average} \quad < \beta_\perp > = \frac{L}{2\pi\ \nu}$$

so:

$$\nu \ = \ \frac{L}{2\ \pi\ \ < \beta_\perp >}$$

Then

$$\frac{\Delta\nu_{\rm space}}{\nu} \ = \ \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right)\ \frac{r_\mu\ \ < \beta_\perp >}{2\ \epsilon_\perp\ \beta_v \gamma^2} \tag{1}$$

In a periodic system L will be the length of one cell, and $\nu = \phi/2\pi$ will be the tune of that cell, where ϕ is the phase advance.

But in a long solenoid L is an arbitrary length along the solenoid, and the average $\beta_\perp$ is its value everywhere.

In a long solenoid (as in final cooling)

The length is arbitrary:

$$\beta_\perp = \frac{2\,P_\mu}{B\,c} \qquad \text{and} \qquad \frac{\Delta\nu_{\text{space}}}{\nu} = \left(\frac{N_\mu}{\sqrt{2\pi}\,\sigma_{ct}}\right)\frac{r_\mu\,M_\mu}{\epsilon_\perp\,B\,\beta_v\gamma\,c} \qquad (2)$$

where P_μ and M_μ are in electron Volts/c, c is the velocity of light, and B is the axial field in Tesla.

At the end of the last stage of final cooling:

$E=$	5	MeV
$N_\mu=$	3.16	10^{12}
$\sigma_{ct}=$	4	m
$\epsilon_\perp=$	25	μm

- In the 40 T solenoid: $\Delta\nu_{40}/\nu =$ 0.004 which is negligible
- In the following 1.2 T transport: $\boxed{\Delta\nu_1/\nu = 0.19}$ which is significant,

but the matching is adiabatic and insensitive to changes in focus strength, providing the sum $(\nu\ -\ \Delta\nu)$ remains significantly positive, e.g. if

$$\Delta\nu_1/\nu\ < 0.5$$

which is clearly satisfied for this stage.

Effects on Field Flip

The places where a tune shift could have significant consequences, are at the field flips. These are not adiabatic and have limited momentum, or other source of focus strength variation.

In the flip prior to the last stage of final cooling:

$E=$	7.3	MeV
$N_\mu=$	3.15	10^{12}
$\sigma_{ct}=$	4	m
$\epsilon_\perp=$	28	μm

With the transport field of 1.2 T: $\boxed{\Delta\nu_2/\nu = \ 0.12}$ giving a maximum tune spread of $\pm$ 6%.

This can be compared to the momentum acceptance of of $\pm$ 15% acceptance of the flip used in the above last stage cooling simulations. Since the actual rms momentum spread is only 2% little effect would be expected from the additional space charge of $\pm$ 6%.

But the effect of tune shift on a non-adiabatic lattice is not the same as a corresponding change in momentum, so this needs checking

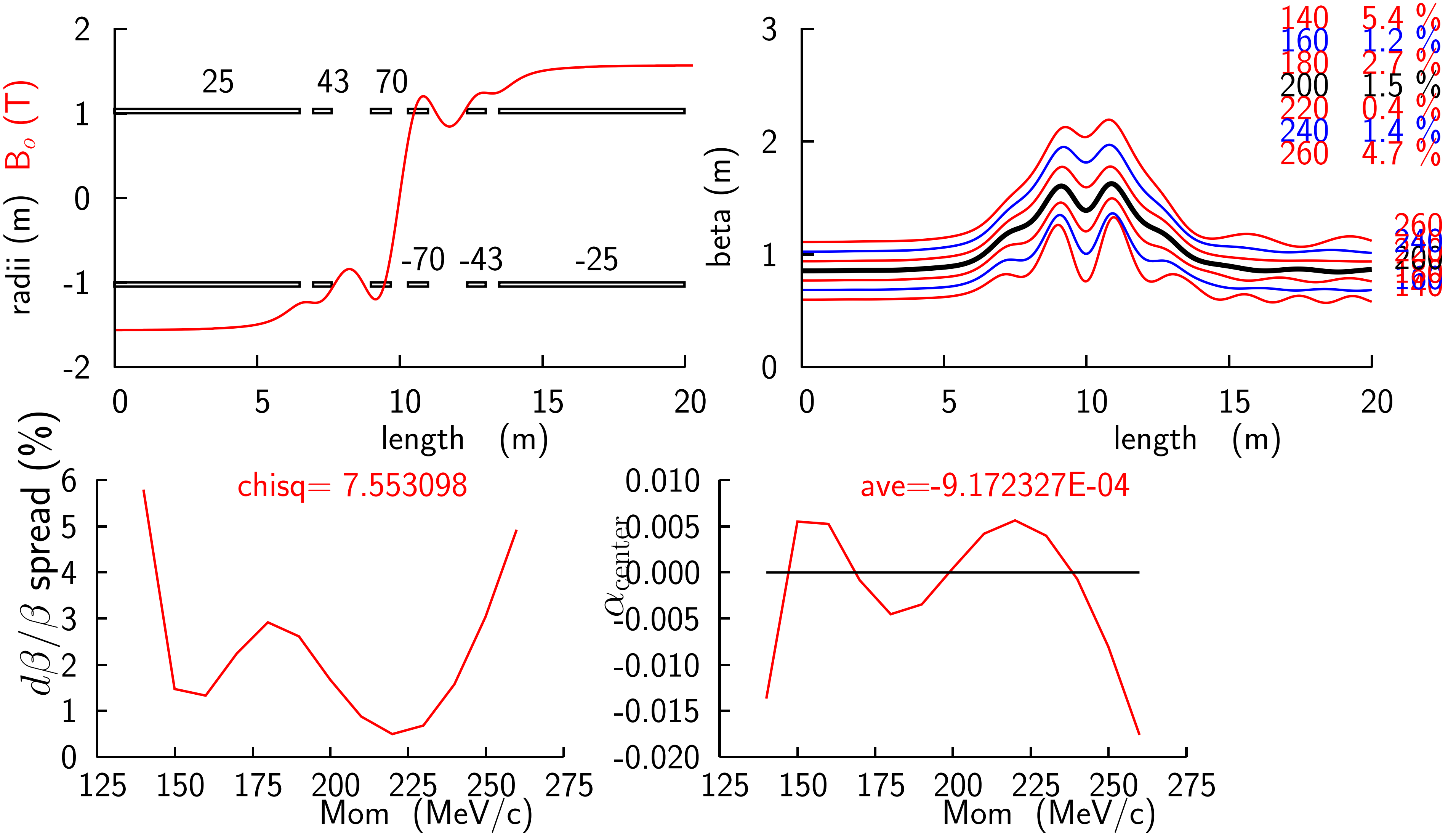
Tune shifts for other stages of final cooling

- The 40T tune shifts are greatest at stage 3, and fall thereafter
- The transport solenoid fields were adjusted to keep the $\Delta\nu_1/\nu \le 0.5$
- The maximum transport field was then 2.7 T
- The maximum $\Delta\nu_2/\nu$ at the field flips is now 0.36, giving spreads of $\pm$ 18%, which is greater than the simulated flip's momentum acceptance

	field T	emit μm	E1 MeV	E2 MeV	$\Delta\nu_{40}/\nu$ %	$\Delta\nu_1/\nu$	$\Delta\nu_2/nu$
1	1.5	281.0	34.6	66.6	1.1	0.48	0.30
2	1.9	202.1	34.8	66.9	1.7	0.48	0.36
3	2.7	150.3	36.0	67.1	1.6	0.49	0.23
4	2.4	116.2	36.0	54.5	1.7	0.49	0.28
5	2.4	93.0	30.6	41.3	1.7	0.49	0.29
6	2.3	76.7	24.9	32.4	1.6	0.49	0.29
7	2.0	64.7	20.7	25.7	1.5	0.50	0.30
8	1.8	55.8	17.4	20.0	1.2	0.47	0.26
9	1.4	48.2	13.6	15.0	0.9	0.47	0.26
10	1.2	41.6	10.3	10.7	0.7	0.42	0.22
11	1.2	36.0	7.5	7.2	0.5	0.31	0.17
12	1.2	31.2	5.1	7.0	0.4	0.23	0.14
13	1.2	27.2	5.1	7.4	0.4	0.19	0.12

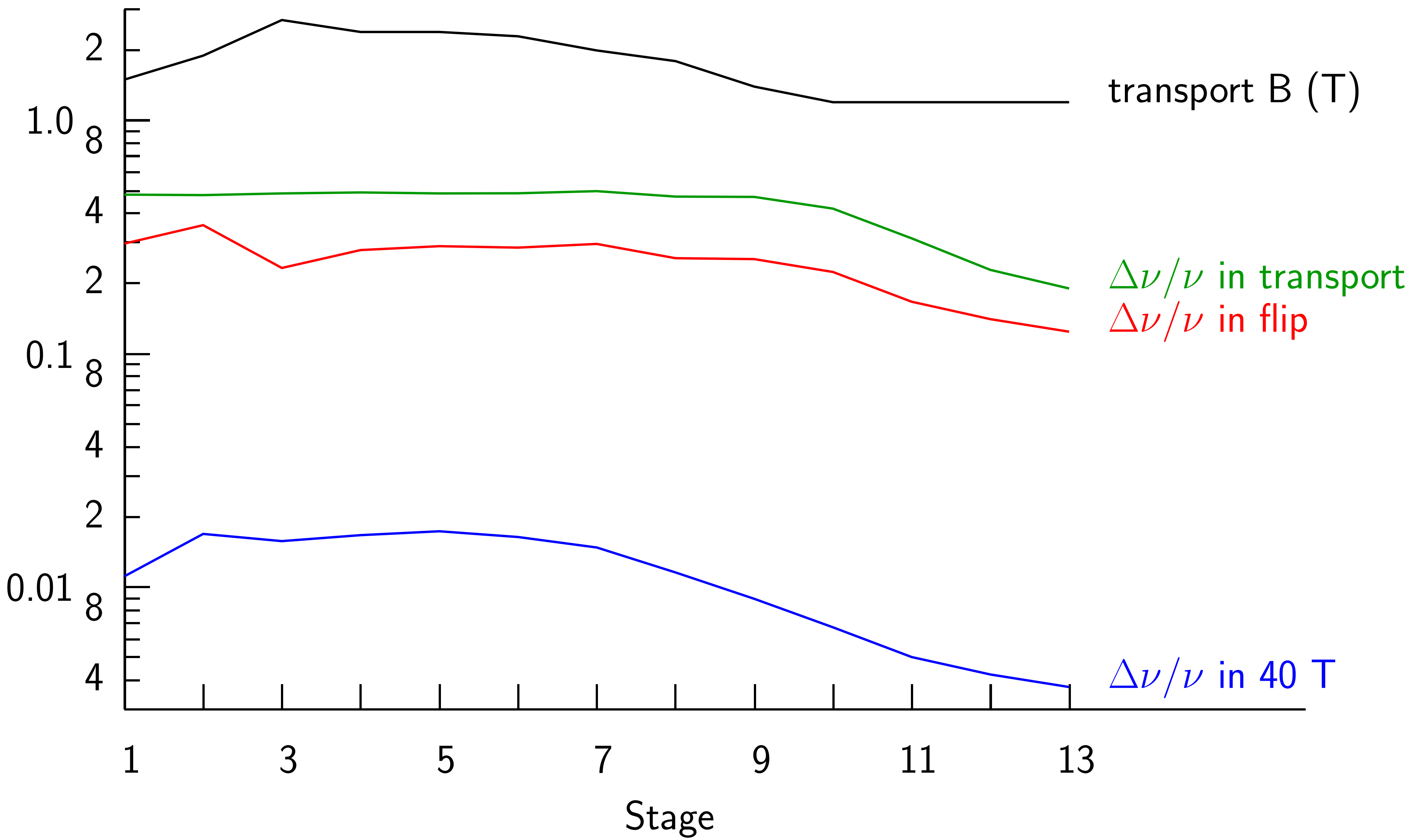
But we can design field flips with far greater acceptance (e.g. $\pm$ 27%) if 4 matching coils are used (see next slide)

Field flip with 4 matching coils



- Alpha crosses zero, for perfect matches, at 4 momenta
- Match is now good from 145 to 250 MeV/c ($\pm$ 26%)
- But again, this may not handle the space charge tune shifts

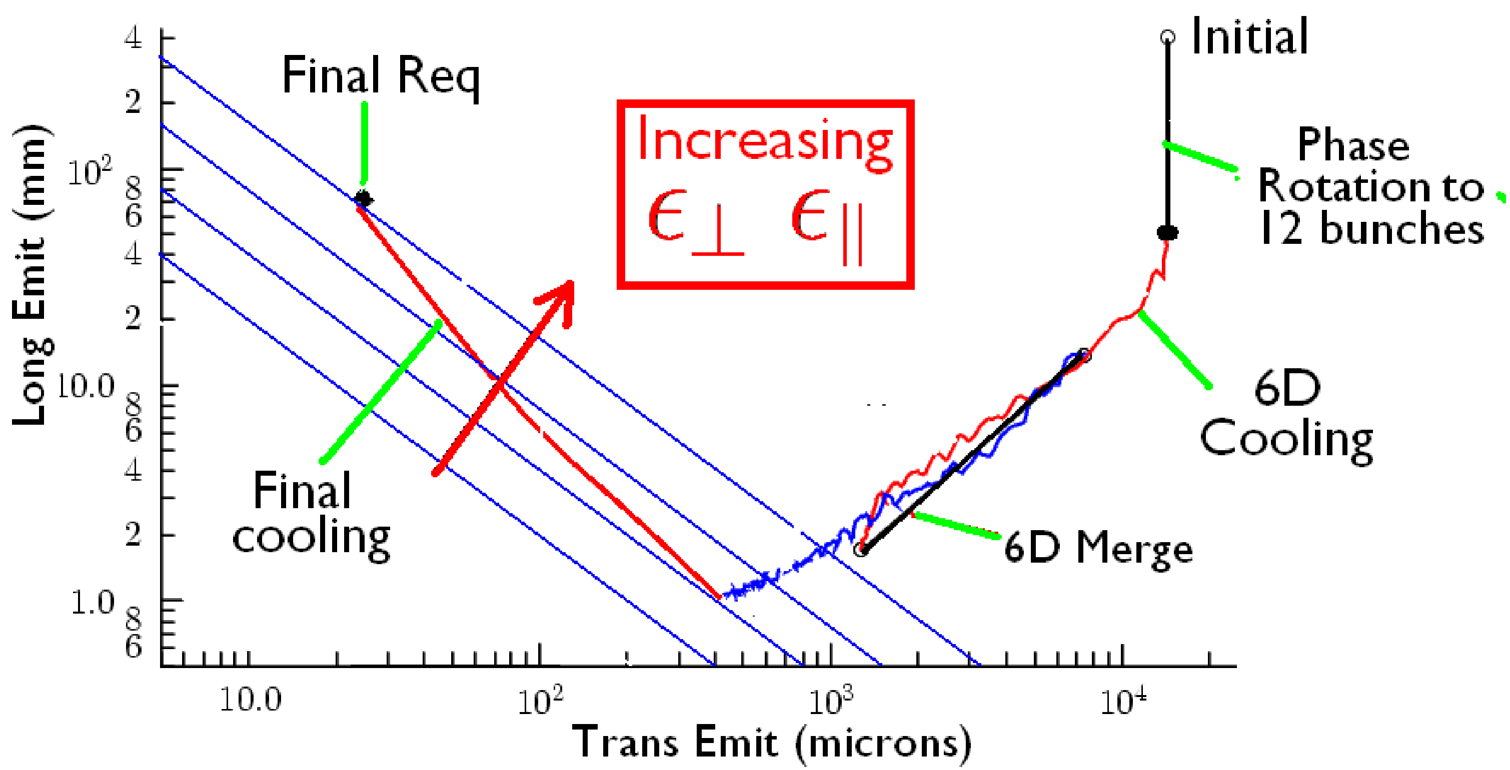
Plot tune shifts vs stage



Why is tune shift getting less even as $\epsilon_\perp$ is falling ?

From 2 $$\frac{\Delta\nu_{\text{space}}}{\nu} = \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_{ct}}\right) \frac{r_\mu\ m_\mu/c}{\epsilon_\perp\ B\ \beta_v\gamma}$$

and $$\epsilon_\parallel = \beta_v\gamma\ \sigma_z\ \frac{dp}{p}$$ so $$\frac{\Delta\nu_{\text{space}}}{\nu} = \left(\frac{N_\mu\ r_\nu\ m_\mu/c}{B\sqrt{2\pi}}\right) \frac{\beta_v\ dp/p}{\epsilon_\perp\ \epsilon_\parallel}$$



$\epsilon_\parallel$ is rising faster than $\epsilon_\perp$ is falling, so $\epsilon_\perp\epsilon_\parallel$ is rising, causing $\Delta\nu/\nu$ to fall

Longitudinal Space Charge

With the outrageous* assumption that the bunch is very long compared with its radial width, and ignoring pipe impedance effects ($L = 0, \mathcal{E}_w = 0$), then, from S.Y.Lee p341 eq. 3.346:

$$\mathcal{E}_{\rm sc} = \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\,\frac{d\lambda}{dz}$$

where $g_o = [1 + 2\ \ln(b/a)]$, $\lambda = dN_\mu/dz$ is the line beam density, $\epsilon_o = 8.8\ 10^{-12}$, and $e = 1.6\ 10^{-19}$

For a Gaussian bunch: $$\lambda = \frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\ \exp\left(-\frac{z^2}{2\ \sigma_z^2}\right)$$

$$\frac{d\lambda}{dz} = \frac{N_\mu\ z}{\sqrt{2\pi}\ \sigma_z^3}\ \exp\left(-\frac{z^2}{2\ \sigma_z^2}\right)$$

$$\frac{d_2\lambda}{dz^2}(\text{max at z} = 0) = \frac{N_\mu}{\sqrt{2\pi}\ \sigma_z^3}$$

$$\frac{d\mathcal{E}_{\rm sc}}{dz}(\text{max}) = \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\,\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z^3}$$

* e.g. in transport at start of Final: $\sigma_{x,y}$ =1.3 cm, σ_z =4 cm

rf Requirement

To stop such a bunch from growing we need a slope of rf gradient vs. time $d\mathcal{E}_{\rm rf}/dz$ that is greater than the maximum slope of space charge gradients:

$$\frac{d\mathcal{E}_{\rm rf}}{dz} \ > \ \frac{d\mathcal{E}_{\rm sc}}{dz}(\max)$$

Assuming the bunch length small compared with the rf wavelength, and no phase rotation (ϕ=0 without space charge), we require:

$$\omega \ < \mathcal{E}_{\rm rf} > \ \ \sin(\phi) \ > \ \frac{d\mathcal{E}_{\rm sc}}{dz}$$

where, in the absence of phase rotation, ϕ is the rf phase with respect to the zero crossing, ω is the rf frequency, and $< \mathcal{E} >_{\rm rf}$ is the average rf gradient. i.e. we require:

$$\omega \ < \mathcal{E}_{\rm rf} > \ \ \sin(\phi) \ > \ \frac{e \ g_o}{4\pi \ \epsilon_o \ \gamma^2} \frac{N_\mu \ \beta_v \ c}{\sqrt{2\pi} \ \sigma_z^3}$$

$$\sin(\phi) \ > \ \xi \ = \ \frac{e \ g_o}{4\pi \ \epsilon_o \ \gamma^2} \frac{N_\mu \ \beta_v \ c}{\sqrt{2\pi} \ \sigma_z^3 \ \omega \ \ < \mathcal{E}_{\rm rf} >} \qquad (3)$$

In reality, there will be need for phase rotation, but ξ remains a useful measure of the severity of the longitudinal space charge.

At the start of final cooling

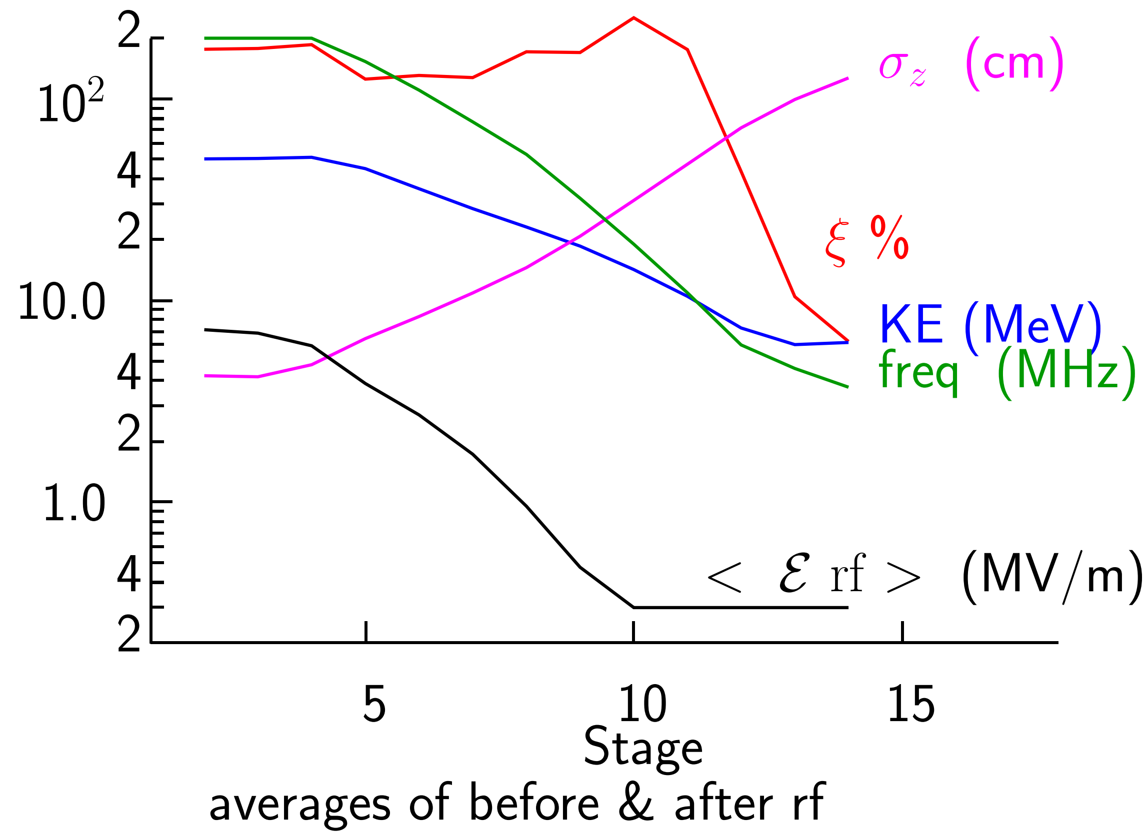
Assuming the beam pipe over beam rms size of a factor of 5, then $g_o = [1 + 2\ln(a/b)] = 4.2$ and then where η is fraction len with rf:

	N_μ 10^{12}	g_o	E MeV	σ_{ct} cm	freq MHz	Grad MV/m	η	ξ %
before rf	4.80	4.9	34.6	4.07	201	15.5	0.46	217
after rf	4.74	4.9	66.6	4.41	201	15.5	0.46	129

This is a problem

And later

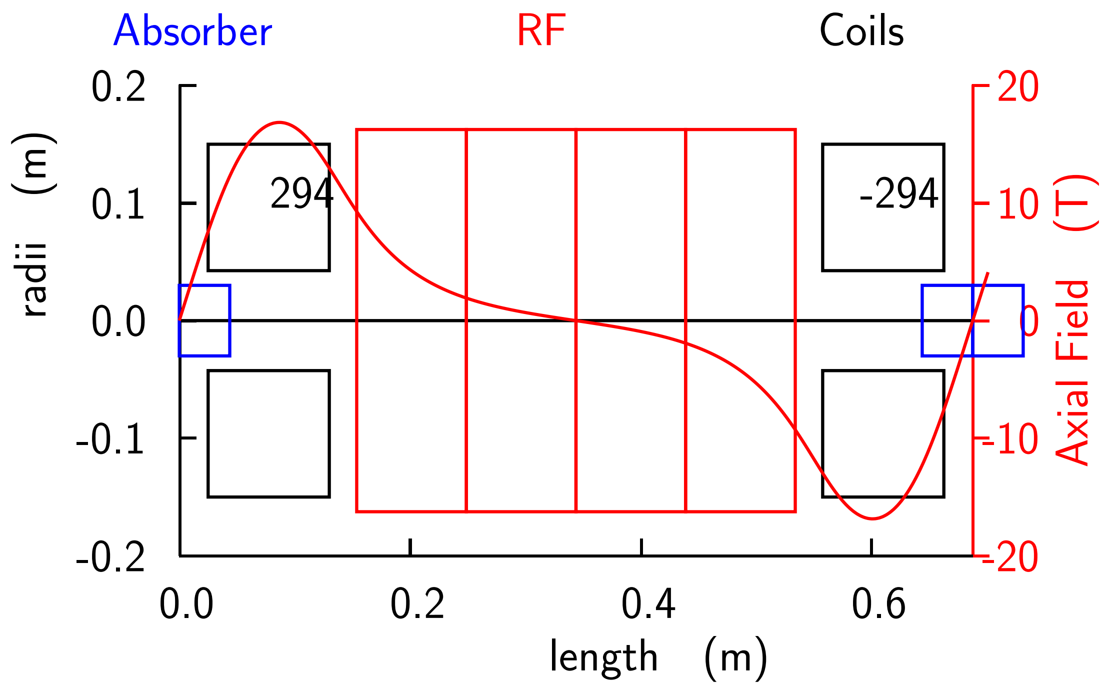
- Improves first, but has 2nd maximum in stage 10 of 242%
- Needs fuller simulation
- Study higher freq, smaller dp/p $\rightarrow$ longer σ_z, higher $\mathcal{E}_{\mathrm{rf}}$
- Maybe soluble (see 6D example later)

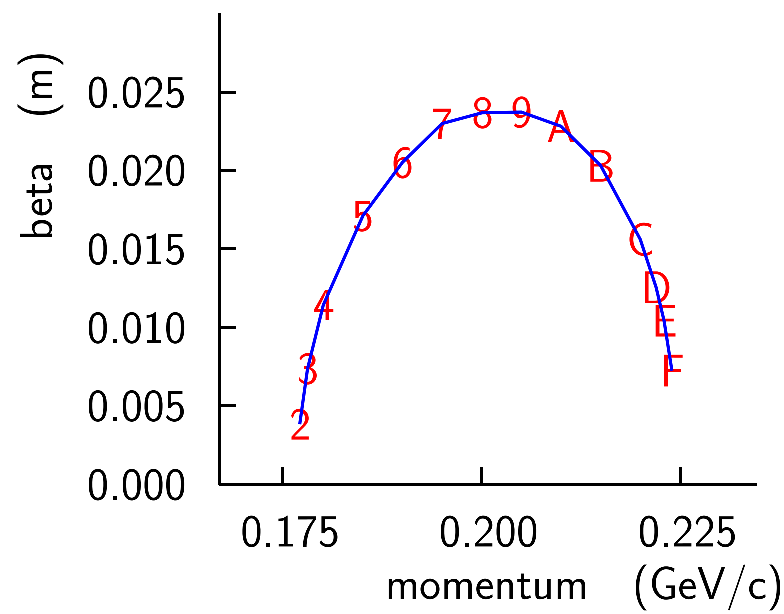


Part II Tune shifts in 6D cooling Lattices

e.g.
The final 6D RFOFO cooling lattice

- Bending field by tilted solenoids or weak dipole
- Maximum axial field 17 T
- Mean momentum 200 MeV/c
- Mom acceptance $\pm$ 11.7 %
- Max $\beta_\perp = 2.37$ cm





Transverse Space-Charge Tune shifts in 6D cooling

from eq. 2 $$\frac{\Delta\nu_{\rm space}}{\nu} = \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right)\ \frac{r_\mu\ <\beta_\perp>}{2\ \epsilon_\perp\ \beta_v\gamma^2}$$

The phase advance per cell, as a function of momentum, in an RFOFO lattice (as in Guggenheim cooling) covers the range from π to 2π. So the average phase advance per cell is 1.5 π, and $<\beta> = L/1.5\pi$, where L is the cell length.

For the last cells in the Guggenheim 6D cooling after the merge, the cell lengths are 0.6875 m long, so

$$\beta_{\rm ave} = \frac{0.6875}{1.5\ \pi} = 0.146\ \ {\rm m}$$

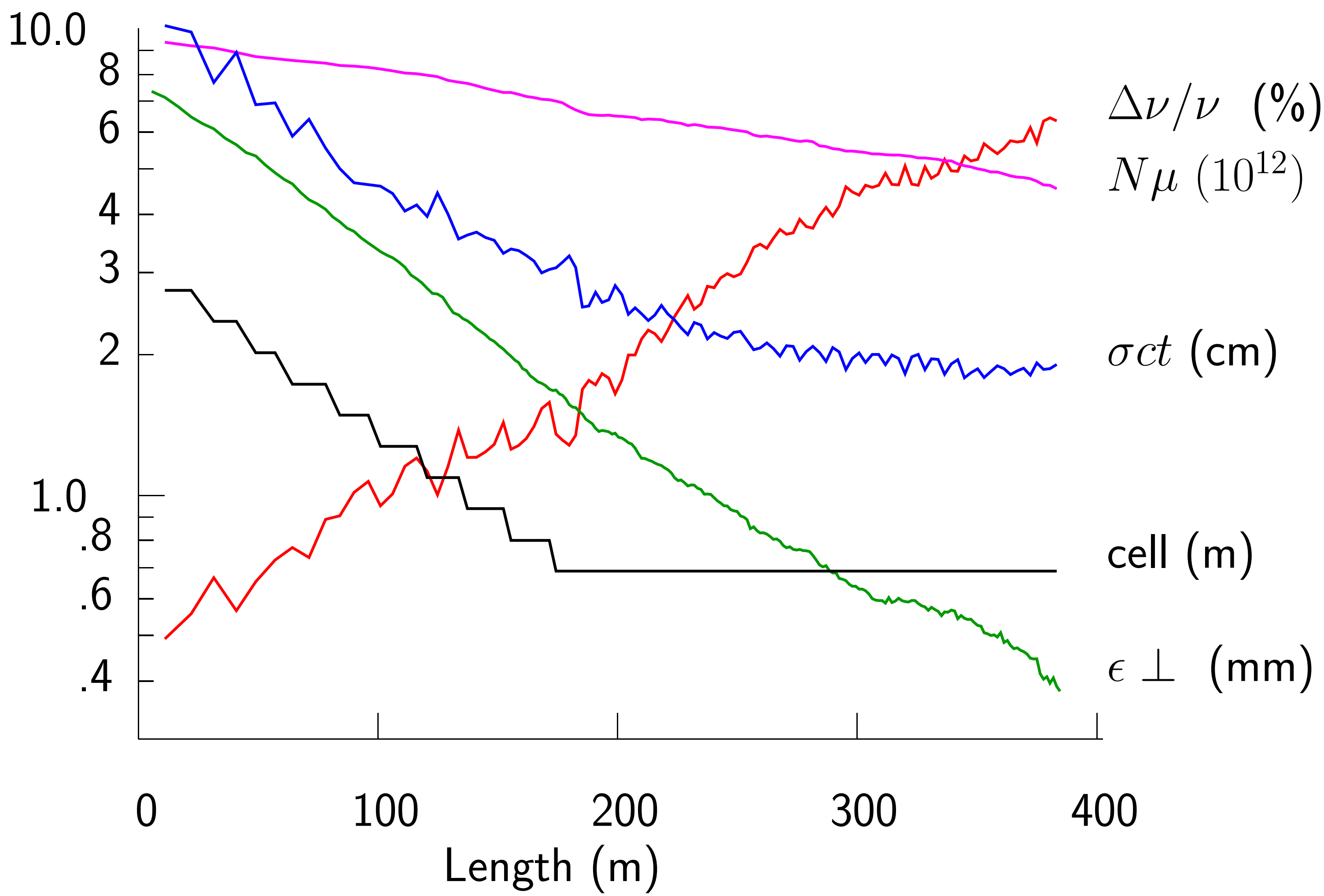
With

$P=$	207	MeV/c
$N_\mu=$	4.87	10^{12}
$\sigma_{ct}=$	1.87	cm
$\epsilon_\perp=$	400	μm

Gives $\Delta\nu/\nu\ =\ 0.068$

which is a small fraction of the acceptance of $\pm$ 0.25/0.75 $=$ 0.33

Tune shifts earlier in the 6D cooling



- Earlier, in the 6D cooling, the number of muons is greater
- But both emittances ($\epsilon_\perp$ & $\epsilon_\parallel$) are larger
- Giving progressively smaller tune shifts

Longitudinal Space-Charge Tune shifts in 6D cooling

from eq. 3
$$\xi \;=\; \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\ \frac{N_\mu\ \beta_v\ c}{\sqrt{2\pi}\ \sigma_z^3\ \omega\ \mathcal{E}_{\rm rf}\ \eta}$$

	N_μ 10^{12}	g_o	mom MV/m	σ_z cm	freq MHz	$\mathcal{E}_{\rm rf}$ MV/m	η	ξ %
Current	4.81	4.2	135	1.66	805	15.5	0.5	361

This is a problem

Can it be Fixed ?

- An Example
  - Pipe diam could be 7 $\rightarrow$ 5 sigma
  - Frequency .805 $\rightarrow$ 1.3 GHz & Grad 15.5 $\rightarrow$ 20 MV/m
  - Mom 207 $\rightarrow$ 135 MeV/c For the same $\epsilon_\parallel$ $\sigma_z \rightarrow 2.54$ cm

	N_μ 10^{12}	g_o	mom MV/m	σ_z cm	freq MHz	$\mathcal{E}_{\rm rf}$ MV/m	η	ξ
Current	4.81	4.2	135	1.66	805	15.5	0.5	361
New ?	4.81	4.2	207	2.54	1300	20	0.5	0.78

This is NOT a simulated case, but shows what might be possible

Conclusion

- Transverse tune shifts in final cooling
  - Large tune shifts require increased transport focus fields 1.2 $\longrightarrow$ 2.7 T
  - But effects remain large (50 %)
  - Require fuller simulation
- Longitudinal space charge in final cooling
  - Effects appear disastrous (242%)
  - Need to re-optimization sequence
- Transverse effects in 6D cooling
  - Negligible effects in transverse ($\pm$ 3.4 %)
- Longitudinal effects in 6D
  - Even worse (361 %)
  - Require fuller simulation
  - With un-simulated parameters $\xi \rightarrow 78$ %
- Need to check these calculations

Next steps

- Modify space charges for our short bunches
- Debug ICOOL's transverse space charge calculations
  - Simulate field flips with space charge
  - Simulate RFOFO lattice with space charge

- Re-optimize final cooling to lower long space charge
- Re-design later 6D cooling to reduce long space charge

- Design final cooling's first 'match and re-acceleration'
- Debug ICOOL's longitudinal space charge
  - Simulate matching and re-acceleration with long space charge
  - Simulate RFOFO lattice with long space charge

- Study RF and pipe Impedance and wake field effects
- If needed: full wake/space charge simulations