CompF6: Tensor networks for HEP

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LOI with R. Somma, B. Şahinoğlu, and G. Vidal + LOI from J. Unmuth-Yockey

Snowmass Computational Frontier

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Quantum computations/simulations for HEP theory?

Problems in HEP where perturbation theory and classical sampling (Monte Carlo) are challenged:

- Real-time evolution for QCD
- Jet Physics (crucial for the LHC program)
- Near conformal systems (BSM, needs very large lattices)
- Early cosmology
- Finite density QCD (sign problem)
- Strong gravity

Tensor/RG tools from QI/Condensed matter

- DMRG (density matrix renormalization group $\sim$1990): takes into account the entanglement between coarse-grained blocks in computations of ground states of lattice Hamiltonians in 1D.
- MPS (matrix product state $\lesssim$ 2006): used to simulate dynamics of 1D quantum systems under the constraint of low entanglement; PEPS (projected entangled pair states): MPS for $D \geq 2$.
- MERA (multi-scale entanglement renormalization ansatz $\sim$2006): renormalization scheme keeping track of local entanglement.
- TRG (tensor renormalization group, $\sim$2006): real space RG technique for classical (Lagrangian) lattice models models.
- TNR (tensor network renormalization $\sim$2015): insertion of optimized unitary and isometric tensors to remove short-range entanglement at each coarse-graining step.
- .....

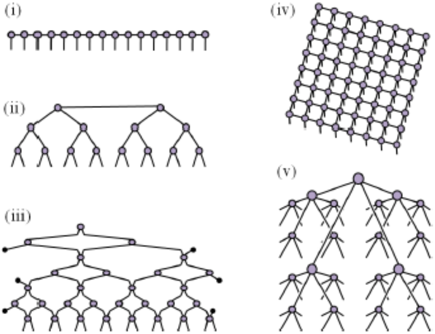


FIG. 7: Examples of tensor network states for 1D systems: (*i*) matrix product state (MPS), (*ii*) tree tensor network (TTN), (*iii*) multi-scale entanglement renormalization ansatz (MERA). Examples of tensor network states for 2D systems: (*iv*) projected entangled-pair state PEPS, (*v*) 2D TTN. (2D MERA not depicted).

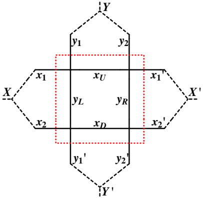
From S. Singh, R. Pfeifer, and G. Vidal Phys. Rev. A 82, 050301(R), 2010.

Tensor Networks and AdS/CFT

- MERA: the removal of local entanglement is essential for defining a proper real space renormalization group transformation for quantum states.
- The discrete geometry that appears at the critical point is a discrete version of anti de Sitter space (AdS).
- Finite temperature quantum states include black hole-like objects.
- For the full holographic perspective, see B. Swingle, Phys. Rev. D 86, 065007.
- cMERA, MPO, error corrections, quantum chaos ....
- For recent developments see Daniel Harlow at 2:46 today.

The Tensor Renormalization Group (TRG) method

- Exact blocking



Unique feature: the blocking separates the degrees of freedom inside the block (integrated over), from those kept to communicate with the neighboring blocks. The only approximation is the truncation in the number of states kept.

- Applies to lattice models with compact fields: Ising model, $O(2)$ model, $O(3)$ model, $SU(2)$ principal chiral model, Abelian and $SU(2)$ gauge theories, in arbitrary dimensions.
- Schwinger model with Wilson and staggered fermions.
- No sign problems: complex temperature and chemical potential.
- Fixed points and critical exponents (improved with TNR).
- Transfer matrix: connects smoothly to the Hamiltonian picture
- Used to design quantum circuits and quantum simulators.
- Comparison with MPS and quantum links needed.

From compact to discrete: pure gauge $U(1)$ example

$$Z_{PG} = \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_{x,\mu}}{2\pi} e^{\beta \sum_{x,\mu<\nu} \cos(A_{x,\mu}+A_{x+\hat{\mu},\nu}-A_{x+\hat{\nu},\mu}-A_{x,\nu})}.$$

We can do the “hard integrals" exactly (Bessel functions)

$$\mathrm{e}^{\beta \cos(A_{x,\mu}+A_{x+\hat{\mu},\nu}-A_{x+\hat{\nu},\mu}-A_{x,\nu})} = \sum_{m_{x,\mu,\nu}=-\infty}^{+\infty} \mathrm{e}^{im_{x,\mu,\nu}(A_{x,\mu}+A_{x+\hat{\mu},\nu}-A_{x+\hat{\nu},\mu}-A_{x,\nu})} I_{m_{x,\mu,\nu}}(\beta).$$

Integration over $A_{x,\mu}$ yields the selection rule (discrete Maxwell's eqs.):

$$\sum_{\nu>\mu}[m_{x,\mu,\nu} - m_{x-\hat{\nu},\mu,\nu}] + \sum_{\nu<\mu}[-m_{x,\nu,\mu} + m_{x-\hat{\nu},\nu,\mu}] = 0.$$

The partition function with PBC can now be written using tensors:

$$Z = (I_0(\beta))^{VD(D-1)/2} \times \mathrm{Tr} \prod_{links} A^{(links)}_{m_1,\ldots m_{2(D-1)}} \prod_{pl.} B^{(pl.)}_{m_1 m_2 m_3 m_4}.$$

Graphical representation of the partition function

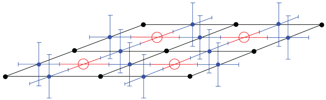
B-tensors are attached to a plaquette (x, μ, ν) and carry one index m

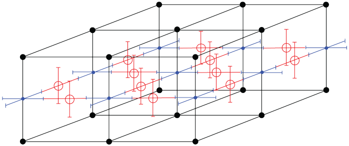
$$B^{(x,\mu,\nu)}_{m_1 m_2 m_3 m_4} = \begin{cases} I_{m_1}(\beta)/I_0(\beta), & \text{if all } m_i \text{ are the same} \\ 0, & \text{otherwise.} \end{cases}$$

These are assembled (traced) together with *A*-tensors attached to links with $2(D-1)$ legs orthogonal to the link.

$$A^{(x,\mu)}_{m_1 \ldots m_{2(D-1)}} = \delta_{\sum_{\nu>\mu}[m_{x,\mu,\nu} - m_{x-\hat{\nu},\mu,\nu}], \sum_{\nu<\mu}[m_{x,\nu,\mu} - m_{x-\hat{\nu},\nu,\mu}]}$$

Transfer matrix="lasagna" with magnetic layer (left) x electric layer (right). You can "see" the Hilbert space in 2+1D on the electric layer (red *B*-tensors, right).





Recent TRG developments and perspectives

- Non-compact ϕ^4 with Gaussian quadratures.
- Supersymmetry.
- Truncations are compatible with symmetries (= selection rules).
- Noise-robust implementations of Abelian Gauss' s law.
- Judah Unmuth-Yockey LOI
  - Importance of Anisotropic TRG and Triad TRG algorithms for practical calculations in 2+1 and 3+1 dimensions.
  - Symmetries are crucial for sparse linear algebra.
  - TRG is ready for high-performance parallel computing.
- Many steps to go on the "Kogut ladder" (RMP 1979) towards QCD.
- A lot to learn from the QI community (TNR, CDL, CTMRG, ...).
- Hybrid quantum/classical methods, comparisons of truncations ...
- Quantum computations and quantum simulation experiments (John Preskill's talk at 2:22)
- Apologies for US/HEP/lattice bias and lack of references.
- Thanks for listening!