Small-x Physics at EIC

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WASHINGTON, D.C. – Today, the U.S. Department of Energy (DOE) announced the selection of Brookhaven National Laboratory in Upton, NY, as the site for a planned major new nuclear physics research facility.

The Electron Ion Collider (EIC), to be designed and constructed over ten years at an estimated cost between $1.6 and $2.6 billion, will smash electrons into protons and heavier atomic nuclei in an effort to penetrate the mysteries of the “strong force” that binds the atomic nucleus together.

“The EIC promises to keep America in the forefront of nuclear physics research and particle accelerator technology, critical components of overall U.S. leadership in science,” said U.S. Secretary of Energy Dan Brouillette. “This facility will deepen our understanding of nature and is expected to be the source of insights ultimately leading to new technology and innovation.”

“America is in the golden age of innovation, and we are eager to take this next step with EIC. The EIC will not only ensure U.S. leadership in nuclear physics, but the technology developed for EIC will also support potential tremendous breakthroughs impacting human health, national competitiveness, and national security,” said Under Secretary for Science Paul Dabbar. “We look forward to our continued world-leading scientific discoveries in conjunction with our international partners.”

The EIC’s high luminosity and highly polarized beams will push the frontiers of particle accelerator science and technology and provide unprecedented insights into the building blocks and forces that hold atomic nuclei together.

Design and construction of an EIC was recommended by the National Research Council of the National Academies of Science, noting that such a facility “would maintain U.S. leadership in nuclear physics” and “help to maintain scientific leadership more broadly.” Plans for an EIC were also endorsed by the federal Nuclear Science Advisory Committee.

Secretary Brouillette approved Critical Decision-0, “Approve Mission Need,” for the EIC on December 19, 2019.
Electron-Ion Collider (EIC) White Paper

• EIC WP was finished in late 2012 + 2\textsuperscript{nd} edition in 2014

• A several-year effort by a 19-member committee + 58 co-authors

• arXiv:1212.1701 [nucl-ex]

• EIC is to be built at BNL
QCD at EIC Physics Topics

• Spin and Nucleon Structure
  – Spin of a nucleon
  – Transverse momentum distributions (TMDs)
  – Spatial imaging of quarks and gluons (GPDs)

• QCD Physics in a Nucleus
  – High gluon densities and saturation
  – Quarks and Gluons in the Nucleus
  – Connections to p+A, A+A, and cosmic ray physics
The Big Picture
Fundamental Questions in QCD

• Confinement, chiral symmetry breaking, quantitative understanding of hadron masses, structure of the proton and the nucleus.

• QCD under extreme conditions: finite-$T$ (heavy ions, Early Universe), finite-$\mu$ (neutron stars), high energy QCD asymptotics.
Fundamental Questions of QCD at EIC

• Confinement, chiral symmetry breaking, quantitative understanding of hadron masses, **structure of the proton and the nucleus**.

• QCD under extreme conditions: finite-$T$ (heavy ions, Early Universe), finite-$\mu$ (neutron stars), **high energy QCD asymptotics**.
Small-x Physics and Saturation
A. Main Concepts
Gluons at Small-$x$

- There is a large number of small-$x$ gluons (and quarks) in a proton:

\begin{center}
\begin{tabular}{c}
\text{HERAPDF1.0} \\
\text{experimental uncertainty} \\
\text{model uncertainty} \\
\text{parametrization uncertainty} \\
\end{tabular}
\end{center}
High Density of Gluons

- High number of gluons populates the transverse extent of the proton or nucleus, leading to a very dense saturated wave function:

![Diagram showing low energy and high energy proton states with a transition from few partons to many partons](diagram.png)

Low Energy

Proton \((x_0, Q^2)\)

parton

High Energy

Proton \((x, Q^2)\)

"many new smaller partons are produced"

"Color Glass Condensate"
The BFKL equation for the number of gluons $N$ reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x, Q^2) = \alpha_s K_{BFKL} \otimes N(x, Q^2)$$

Start with $N$ gluons in the proton’s wave function. As we increase the energy a new gluon can be emitted by either one of the $N$ gluons. The number of newly emitted particles is proportional to $N$. A new gluon is emitted as energy increases.

It can be emitted off of any of the $N$ gluons.

The BFKL equation for the number of gluons $N$ reads:
Nonlinear Equation

At very high energy gluon recombination becomes important. As energy (rapidity) increases, gluons not only split into more gluons, but also recombine. Recombination reduces the number of gluons in the wave function. Here $Y \sim \ln s \sim \ln 1/x$ is rapidity, $s$ is cms energy squared.

\[
\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s K_{BFKL} \otimes N(x, k_T^2) - \alpha_s [N(x, k_T^2)]^2
\]

Number of gluon pairs $\sim N^2$

I. Balitsky ’96, Yu. K. ’99 (large $N_c$) beyond large-$N_c$: JIMWLK equation
Map of High Energy QCD

Energy

Resolution, $\ln Q^2$  Number of partons

Size of gluons
Map of High Energy QCD

\[ Y = \ln \frac{1}{x} \]

Y = ln 1/x

non-perturbative region

\[ \Lambda_{\text{QCD}}^2 \]

\[ \alpha_s \sim 1 \]  
\[ \alpha_s \ll 1 \]

\[ Q_s^2(Y) \]

Saturation Scale grows with energy

\[ Q_s^2(Y) \]

BFKL, DGLAP – linear equations

BK/JIMWLK – nonlinear

energy

size of gluons

geometric scaling
Large gluon density gives a large momentum scale $Q_s$ (the saturation scale): $Q_s^2 \sim \# \text{ gluons per unit transverse area} \sim A^{1/3}$ (nuclear ooph).

For $Q_s >> \Lambda_{QCD}$, get a theory at weak coupling $\alpha_s(Q_s^2) \ll 1$ and the leading gluon field is classical.
Typical gluon “size”

Number of gluons (gluon TMD) times the phase space

Gluon “size” = $1/\text{transverse momentum}$

= $1/Q_s$
High Energy QCD: saturation physics

• The nonlinear BK/JIMWLK equations and the MV model lead to a large internal momentum scale $Q_s$ which grows with both the decreasing $x$ /increasing energy $s$ ($\lambda \approx 0.3$) and the increasing nuclear atomic number $A$

\[
Q_s^2 \sim A^{1/3} \left( \frac{1}{x} \right)^{\lambda}
\]

such that

\[
\alpha_s = \alpha_s(Q_s) \ll 1
\]

and we can calculate total cross sections, particle multiplicities, correlations, etc. , from first principles.

• Bottom line: coupling is weak, Feynman diagrams work. But: the system is dense, and physics is nonlinear!
Saturation Scale

To summarize, saturation scale is an increasing function of both energy (1/x) and A:

\[ Q_s^2 \sim \left( \frac{A}{x} \right)^{1/3} \]

Gold nucleus provides an enhancement by 197^{1/3}, which is equivalent to doing scattering on a proton at 197 times smaller-x / higher-s!
Saturation Scales at EIC

Model I = MV-inspired dipole model
Model II = running-coupling BK evolution (rcBK)

The difference in $Q_s$ values is minimal in the EIC range.
Still theoretical uncertainty remains.
References

Quantum Chromodynamics at High Energy

YURI V. KOVCHEGOV
AND EUGENE LEVIN

CAMBRIDGE MONOGRAPHS ON PARTICLE PHYSICS, NUCLEAR PHYSICS AND COSMOLOGY

Published in September 2012 by Cambridge U Press
B. Relevant Observables
Can Saturation be Discovered at EIC?

EIC has an unprecedented small-$x$ reach for DIS on large nuclear targets, allowing to seal the discovery of saturation physics and study of its properties:

\[ Q_s^2(x) \]

\[ \ln x \]

\[ \ln Q^2 \]

\[ \alpha_s \ll 1 \]

\[ \alpha_s \sim 1 \]
Can Saturation be Discovered at EIC?

EIC has an unprecedented small-x reach for DIS on large nuclear targets, allowing to seal the discovery of saturation physics and study of its properties:

\[
\alpha_s < 1
\]

\[
\alpha_s \approx 1
\]

\[
\ln Q^2
\]

\[
\ln x
\]

\[
s(x)
\]

\[
s(x) = 45 \text{ GeV}, 0.01 \leq y \leq 0.95
\]

\[
s(x) = 90 \text{ GeV}, 0.01 \leq y \leq 0.95
\]

Measurements with \( A \geq 56 \) (Fe):
- eA/μA DIS (E-139, E-665, EMC, NMC)
- νA DIS (CCFR, CDHSW, CHORUS, NuTeV)
- DY (E772, E866)
(i) Nuclear Structure Functions
Nuclear structure functions $F_2$ and $F_L$ which could be measured at EIC (values = EPS09+PYTHIA). Shaded area = $(x, Q^2)$ range of the world $e+A$ data.
Nuclear Shadowing

- Saturation effects may explain nuclear shadowing: reduction of the number of gluons per nucleon with decreasing $x$ and/or increasing $A$:

\[ R_2 = \frac{A F_2^P}{A F_2}, \]

\[ R_L = \frac{F_L A}{A F_L^P}, \]

But: as DGLAP does not predict the $x$- and $A$-dependences, it needs to be constrained by the data.

Note that including heavy flavors (charm) for $F_2$ and $F_L$ should help distinguish between the saturation versus non-saturation predictions.
Nuclear Shadowing for Charm

may help distinguish saturation vs DGLAP-based prediction
(ii) Di-Hadron Correlations
De-correlation

- Small-x evolution ↔ multiple emissions
- Multiple emissions → de-correlation.

- B2B jets may get de-correlated in $p_T$ with the spread of the order of $Q_S$
Di-hadron Correlations

Depletion of di-hadron correlations is predicted for e+A as compared to e+p. (Dominguez et al ‘11; Zheng et al ‘14). This is a signal of saturation.
(iii) Diffraction
Diffraction in optics

Diffraction pattern contains information about the size $R$ of the obstacle and about the optical “blackness” of the obstacle.

In optics, diffraction pattern is studied as a function of the angle $\theta$. In high energy scattering the diffractive cross sections are plotted as a function of the Mandelstam variable $t = k \sin \theta$. 
Optical Analogy

Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:

Coherent: target stays intact;
Incoherent: target nucleus breaks up, but nucleons are intact.
Diffraction terminology

\(\gamma^*\) hadron or nucleus

\(p^+\) \(x_P\) \(\beta\) \(p^+\)

\(M_X\)

rapidity gap

rapidity gap

(a)

(b)
Diffraction on a black disk

• For low $Q^2$ (large dipole sizes) the black disk limit is reached with $N=1$

• Diffraction (elastic scattering) becomes a half of the total cross section

\[
\frac{\sigma_{q\bar{q}A}^{el}}{\sigma_{q\bar{q}A}^{tot}} = \frac{\int d^2b \ N^2}{2 \int d^2b \ N} \rightarrow \frac{1}{2}
\]

• Large fraction of diffractive events in DIS is a signature of reaching the black disk limit!

• HERA: $\sim15\%$ (unexpected!) ; EIC: $\sim25\%$ expected from saturation
Diffractive over total cross sections

- Here’s an early EIC measurement which may distinguish saturation from non-saturation approaches:

\[
\begin{align*}
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{tot}}}{dM_x^2} (\text{GeV}^{-2})
\end{align*}
\]

\[
\begin{align*}
\beta
\end{align*}
\]

\[
\begin{align*}
Q^2 = 1 \text{ GeV}^2 \\
x = 1 \times 10^{-3}
\end{align*}
\]

15 GeV on 100 GeV 
\[
\int L dt = 1 \text{ fb}^{-1}/A
\]

sat = Kowalski et al ‘08, plots generated by Marquet
no-sat = Leading Twist Shadowing (LTS), Frankfurt, Guzey, Strikman ‘04, plots by Guzey
Exclusive Vector Meson Production

• An important diffractive process which can be measured at EIC is exclusive vector meson production:

\[ q \gamma^* x_N \rho, \omega, \phi, J/\psi, \ldots \]
Exclusive VM Production as a Probe of Saturation

\[ \int L \, dt = 10 \, \text{fb}^{-1}/A \]
\[ 1 < Q^2 < 10 \, \text{GeV}^2 \]
\[ x < 0.01 \]
\[ |\eta_{(e\text{decay})}| < 4 \]
\[ p(e_{\text{decay}}) > 1 \, \text{GeV}/c \]
\[ \delta t/t = 5\% \]

$J/\psi$ is smaller, less sensitive to saturation effects
$\phi$ meson is larger, more sensitive to saturation effects

Plots by T. Toll and T. Ullrich using the Sartre event generator ($b$-Sat ($=\text{GBW}+b$-dep+$\text{DGLAP}$) + WS + MC).

- $J/\psi$ is smaller, less sensitive to saturation effects
- $\phi$ meson is larger, more sensitive to saturation effects
Connections to p+A and A+A collisions
Connections to p+A

- In the saturation framework particle production in p+A is described by the dipole amplitude, just like structure functions in DIS. **(Universality!)**
- Correlations in both processes are described by other Wilson line operators like quadrupoles. **(Universality!)**
- Some evidence of saturation has been seen in d+Au collisions at RHIC:

$$R_{pA} = \frac{1}{N_{coll}} \frac{dN^{pA}/d^2p_T\,dy}{dN^{pp}/d^2p_T\,dy}$$

![Graph showing connections to p+A](image)
Di-hadron back-to-back azimuthal correlation function decorrelates for central d+Au collisions in agreement with saturation predictions (cf. e+A):
Connections to Heavy Ion Physics

- CGC Physics also plays important role in the early-time dynamics of heavy ion collisions
- By exploring it at EIC we would get a better handle on formation of QGP and on fluctuations, including multiplicity and azimuthal harmonic flow coefficients $v_n$. 

![Diagram of heavy ion collision with CGC framework](image)

![Graph showing 2dNch/dy vs N_part](image)
Connections to Cosmic Rays

- There is a known problem in Auger data indicating that cosmic rays behave like protons at lower energies and like nuclei at higher energies, according to the existing QCD Monte-Carlos.

- $X_{\text{max}}$ = atmospheric depth of the cosmic ray shower maximum

- It could be that the problem is with our understanding of QCD at this super-high energies.

- Perhaps saturation physics, with input from EIC, could help improve our understanding of the Auger data.
Proton Spin
Our understanding of nucleon spin structure has evolved:

• In the 1980’s the proton spin was thought of as a sum of constituent quark spins (left panel)

• Currently we believe that the proton spin is a sum of the spins of valence and sea quarks and of gluons, along with the orbital angular momenta of quarks and gluons (right panel)
Proton Helicity Sum Rule

• Helicity sum rule (Jaffe-Manohar form):

\[
\frac{1}{2} = S_q + L_q + S_g + L_g
\]

with the net quark and gluon spin

\[
S_q(Q^2) = \frac{1}{2} \int_0^1 dx \, \Delta \Sigma(x, Q^2) \quad \quad S_g(Q^2) = \int_0^1 dx \, \Delta G(x, Q^2)
\]

• \(L_q\) and \(L_g\) are the quark and gluon orbital angular momenta
Proton Spin Puzzle

- The spin puzzle began when the EMC collaboration measured the proton $g_1$ structure function ca 1988. Their data resulted in
  \[ \Delta \Sigma \approx 0.1 \div 0.2 \]

- It appeared quarks do not carry all of the proton spin (which would have corresponded to $\Delta \Sigma = 1$).

- Missing spin can be
  - Carried by gluons
  - In the orbital angular momenta of quarks and gluons
  - At small $x$ (both helicity and OAM):
    \[ S_q(Q^2) = \frac{1}{2} \int_0^1 dx \Delta \Sigma(x, Q^2) \quad S_g(Q^2) = \int_0^1 dx \Delta G(x, Q^2) \]
    Can’t integrate down to zero, use $x_{\text{min}}$ instead!
  - Or all of the above!
EIC & Spin Puzzle

- Parton helicity distributions are sensitive to low-x physics.
- EIC would have an unprecedented low-x reach for a polarized DIS experiment, allowing to pinpoint the values of quark and gluon contributions to proton’s spin:

\[\Delta G \text{ and } \Delta \Sigma \text{ are integrated over } x \text{ in the } 0.001 < x < 1 \text{ interval.}\]
Recent Small-x Results for Spin


\[ \Delta q(x, Q^2) \sim \left( \frac{1}{x} \right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \]

\[ \Delta G(x, Q^2) \sim \left( \frac{1}{x} \right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \]

\[ L_{q+\bar{q}}(x, Q^2) = -\Delta\Sigma(x, Q^2) \sim \left( \frac{1}{x} \right)^{\frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}}} \]

\[ L_G(x, Q^2) \sim \Delta G(x, Q^2) \sim \left( \frac{1}{x} \right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}}} \]

• At large $N_c$ & $N_f$ we have obtained (Yu. K., Y. Tawabutr, arXiv:2005.07285 [hep-ph])

\[ \Delta\Sigma(x, Q^2) \bigg|_{\text{large-}N_c\&N_f} \sim \left( \frac{1}{x} \right)^{\alpha_h^q} \cos \left[ \omega_q \ln \frac{1}{x} + \varphi_q \right] \]

with

\[ \omega_q \approx \frac{0.22 N_f}{1 + 0.1265 N_f} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \]
Conclusions

• EIC would allow to map out the spin structure of the proton, helping resolve the spin crisis.

• EIC would measure quark and gluon distributions (PDFs and TMDs) both as functions of $x$ and $k_T$, for nucleons and nuclei.

• EIC would help us understand spatial distribution of gluons and quarks (GPDs) in the nucleons and nuclei.

• EIC is a unique opportunity to complete the discovery of saturation/CGC physics and to study its properties. By discovering saturation, we would make a significant progress in understanding high-energy QCD, answering one of the fundamental questions in the field and paving the way for better understanding of strong interactions at the future accelerators.
Backup Slides
Big Questions EIC Would Address

• How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

• Where does the saturation of gluon densities set it? What is the dynamics? Is it universal?

• How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?
Dipole picture of DIS

• In the dipole picture of DIS the virtual photon splits into a quark-antiquark pair, which then interacts with the target.

• The total DIS cross section and structure functions are calculated via:
Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude $N$:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2x_\perp}{2\pi} d^2b_\perp \int_0^1 \frac{dz}{z(1-z)} \left| \overline{\Psi} \gamma^* q \bar{q} (\mathbf{x}_\perp, z) \right|^2 N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)$$

with rapidity $Y=\ln(1/x)$
Dipole Amplitude

The dipole-nucleus amplitude as a function of the dipole size is

\[ \frac{1}{Q} \]

Color transparency

Black disk limit,

\[ \sigma_{tot} < 2\pi R^2 \]

\[ \alpha_s \ll 1 \]

\[ x_{\perp} \]

\[ \frac{1}{Q_s} \]

\[ \frac{1}{\Lambda} \]
Nonlinear evolution at large $N_c$

As $N=1-S$ we write

$$\frac{\partial}{\partial Y} N_{x_0,x_1}(Y) = \frac{\alpha_s N_c}{2 \pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \left[ N_{x_0,x_2}(Y) + N_{x_2,x_1}(Y) - N_{x_0,x_1}(Y) - N_{x_0,x_2}(Y) N_{x_2,x_1}(Y) \right]$$

Balitsky ‘96, Yu.K. ‘99
Solution of BK equation

numerical solution by J. Albacete ’03
(earlier solutions were found numerically by
Golec-Biernat, Motyka, Stasto,
by Braun and by Lublinsky et al in ‘01)

BK solution preserves the black disk limit, \( N<1 \) always
(like the linear BFKL equation)

\[
\frac{1}{Q_s} x_\perp (\text{GeV}^{-1})
\]

\[
N(x_{\perp}, Y)
\]

\[
\alpha S Y = 0, 1.2, 2.4, 3.6, 4.8
\]

\[
\sigma^{q\bar{q}A} = 2 \int d^2 b \; N(x_{\perp}, b_{\perp}, Y)
\]
Dipole Amplitude as a Probe of Spatial Gluon Distribution

- Dipole amplitude is related to gluon distribution.
- It is related to the Wigner distribution for low-x gluons:

\[ N(\vec{x}_\perp, \vec{b}_\perp, Y = \ln 1/x_{Bj}) \leftarrow (\text{Fourier transform}) \Rightarrow W(\vec{k}_\perp, \vec{b}_\perp, x_{Bj}) \]

- Just like for the Wigner distribution, one can extract the gluon transverse momentum distribution (TMD) out of it:

\[ \int d^2 b_\perp N(\vec{x}_\perp, \vec{b}_\perp, Y = \ln 1/x_{Bj}) \leftarrow (\text{Fourier transform}) \Rightarrow f(\vec{k}_\perp, x_{Bj}) \]

- Dipole amplitude gives us information about the spatial distribution of small-x gluons.
Quasi-elastic DIS

Consider the case when nothing but the quark-antiquark pair (pions) is produced:

\[
\sigma_{el} A = \int \frac{d^2 x_\perp}{4 \pi} \int_0^1 \frac{dz}{z (1 - z)} \left| \Psi^{\gamma^* \rightarrow q\bar{q} (\vec{x}_\perp, z)} \right|^2 N^2 (\vec{x}_\perp, \vec{b}_\perp, Y)
\]

The quasi-elastic cross section is then

Buchmuller et al ‘97, McLerran and Yu.K. ‘99
Exclusive VM Production:
Probe of Spatial Gluon Distribution

- Differential exclusive VM production cross section is

\[
\frac{d\sigma^{\gamma^*+A\rightarrow V+A}}{dt} = \frac{1}{4\pi} \left| \int d^2b \, e^{-i \vec{q}_\perp \cdot \vec{b}_\perp} \, T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) \right|^2
\]

- the T-matrix is related to the dipole amplitude N:

\[
T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) = i \int \frac{d^2x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \, \Psi^{* \rightarrow q\bar{q}}(\vec{x}_\perp, z) \, N(\vec{x}_\perp, \vec{b}_\perp, Y) \, \Psi^V(\vec{x}_\perp, z)^*
\]

Brodsky et al ‘94, Ryskin ‘93

- Can study t-dependence of the d\sigma/dt and look at different mesons to find the dipole amplitude N(x,b,Y) (Munier, Stasto, Mueller ’01).

- Learn about the gluon distribution in space.
Dipole Amplitude and Other Operators

• Dipole scattering amplitude is a universal degree of freedom in saturation physics.
• It describes the total DIS cross section and structure functions:

\[ q \gamma^* \]

\[ x_\perp \]

\[ x_\perp \]

• It also describes single inclusive quark and gluon production cross sections in DIS and in p+A collisions. <- **Universality**!
• Works for diffraction in DIS and p+A. <- **Universality**!
• For correlations need also quadrupoles (J.Jalilian-Marian, Yu.K. ’04; Dominguez et al ‘11) and other Wilson line operators.