Improving positivity bounds on aQGCs

Cen Zhang
Institute of High Energy Physics

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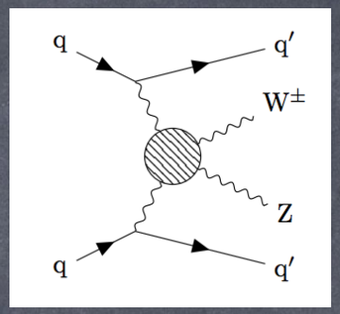
Based on ongoing project with Kimiko Yamashita and Shuang-Yong Zhou

- TGC/QGC interactions are now a core part of the LHC physics programme. QGCs mostly studied by VBS.



- Parametrization based on dim-8 SMEFT operators =>

$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,2} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,3} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,4} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi\right] \times \hat{B}^{\beta\nu} \\
O_{M,5} &= \tfrac{1}{2}\left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi\right] \times \hat{B}^{\beta\mu} + h.c. \\
O_{M,7} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu} D^\nu\Phi\right]
\end{aligned}
$$

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O_{T,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{T,2} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
O_{T,5} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,6} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
O_{T,7} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
O_{T,8} &= \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,9} &= \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},
\end{aligned}
$$

- HL/HE-LHC:
  QGC sensitivity ~ TeV scale =>

	14 TeV		27 TeV	
	$WZjj$	$W^\pm W^\pm jj$	$WZjj$	$W^\pm W^\pm jj$
f_{S_0}/Λ^4	[-8,8]	[-6,6]	[-1.5,1.5]	[-1.5,1.5]
f_{S_1}/Λ^4	[-18,18]	[-16,16]	[-3,3]	[-2.5,2.5]
f_{T_0}/Λ^4	[-0.76,0.76]	[-0.6,0.6]	[-0.04,0.04]	[-0.027,0.027]
f_{T_1}/Λ^4	[-0.50,0.50]	[-0.4,0.4]	[-0.03,0.03]	[-0.016,0.016]
f_{M_0}/Λ^4	[-3.8,3.8]	[-4.0,4.0]	[-0.5,0.5]	[-0.28,0.28]
f_{M_1}/Λ^4	[-5.0,5.0]	[-12,12]	[-0.8,0.8]	[-0.90,0.90]

(in TeV^{-4}), from HL/HE-LHC report

- Though standard, there are still things we need to understand about this parametrization. I will show that >99% of the space is redundant.

- SMEFT is meant to connect EXP data with concrete UV models. Therefore it does not make sense to study the EFT space which cannot be UV-completed (if we know in advance).
  - Particularly relevant at dim-8: it's been known that positivity bounds tell us which part of the parameter space **cannot** be UV-completed. (e.g. if some dim-8 coefficient has a wrong sign).

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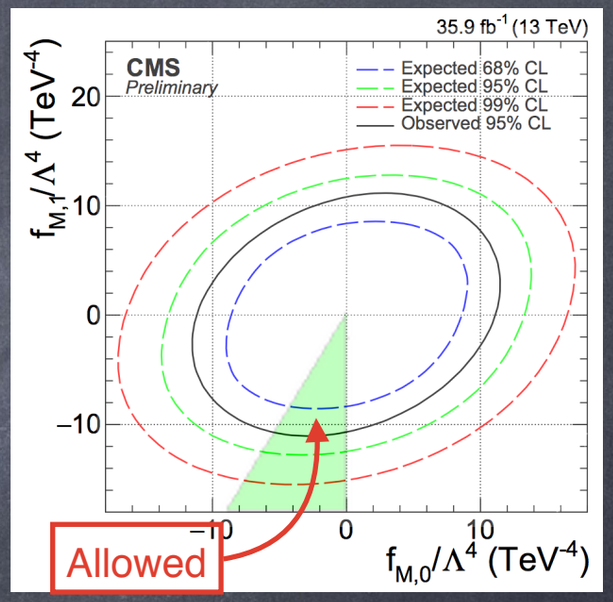
- **Currently affects the aQGC parametrization.**

[CMS-PAS-SMP-18-001]



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- **Currently affects the aQGC parametrization.**
- **However, in the future, more dim-8 effects may become accessible.**

  (e.g. new observable proposed for DY process [Alioli, Boughezal, Mereghetti, Petriello, 2003.11615], or even at future e+e-)

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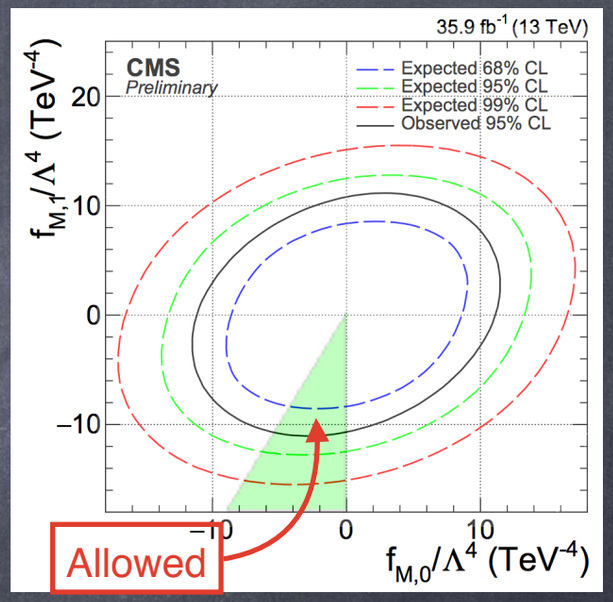

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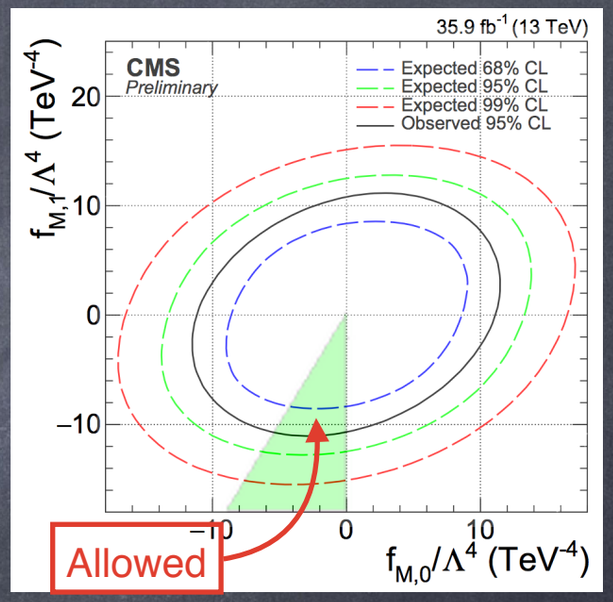

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- These bounds need to be studied, to identify the meaningful parameter space, to form a consistent interpretation of data within the SMEFT framework, and also to help focus the EXP search.

What we currently know about QGC positivity:

- Certain linear combinations of dim-8 coefficients must be positive, e.g.

$$4C_{T,0} + 4C_{T,1} + 3C_{T,2} + 12C_{T,10} \geq 0$$

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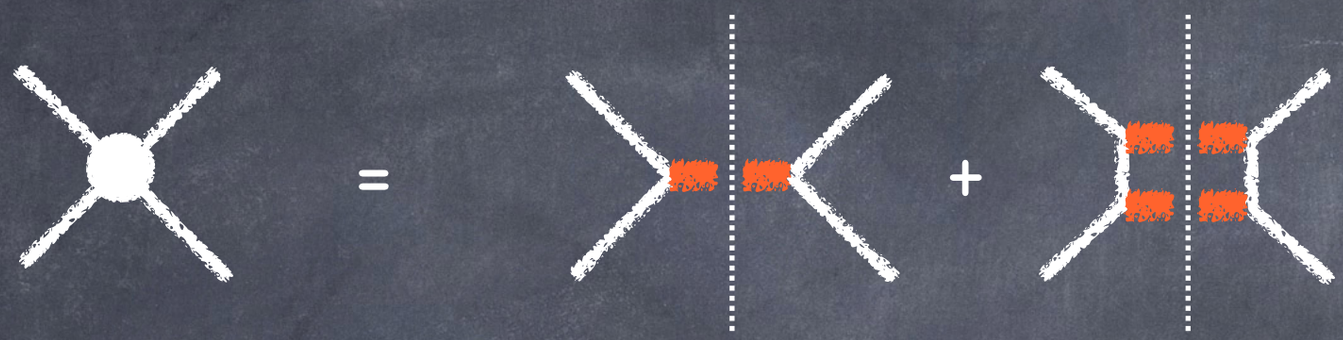
- Still not complete. Room to improve and work to be done.
- In addition, a new approach has been proposed in [**CZ**, S.-Y. Zhou, 2005.03047].
- Still need to understand the full set of bounds on all QGC operators.

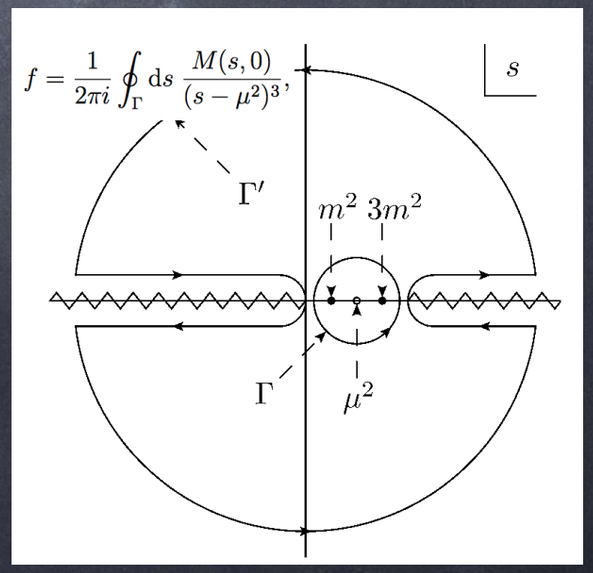
$$\frac{d^2}{ds^2}$$



+ … + s<->u crossing

$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s=\frac{1}{2}M^2, t=0\right)$$

$$= \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X}(s,\Pi_X) M^*_{kl\to X}(s,\Pi_X)}{\pi\left(s-\frac{1}{2}M^2\right)^3} + (j \leftrightarrow l)$$



[C. Cheung, G. Remmen, JHEP 16]
[de Rham, Melville, Tolley, Zhou, JHEP 17]

ijkl: particle index

$1 \leq i, j, k, l \leq n$

$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$

Forward scattering amp, at **low energy** (calculable in EFT)

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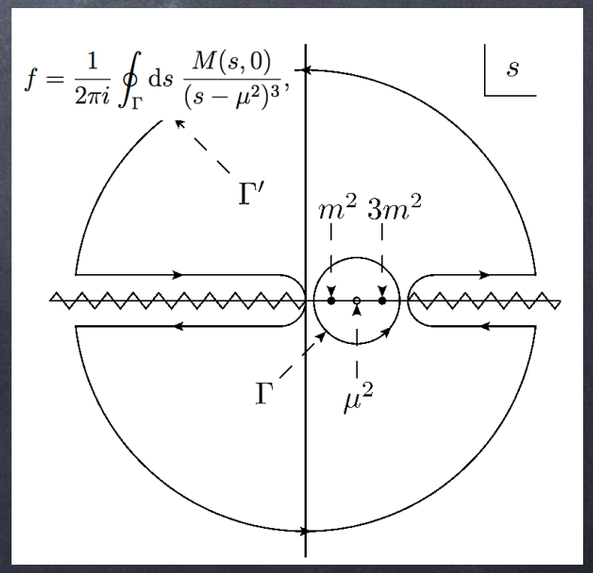

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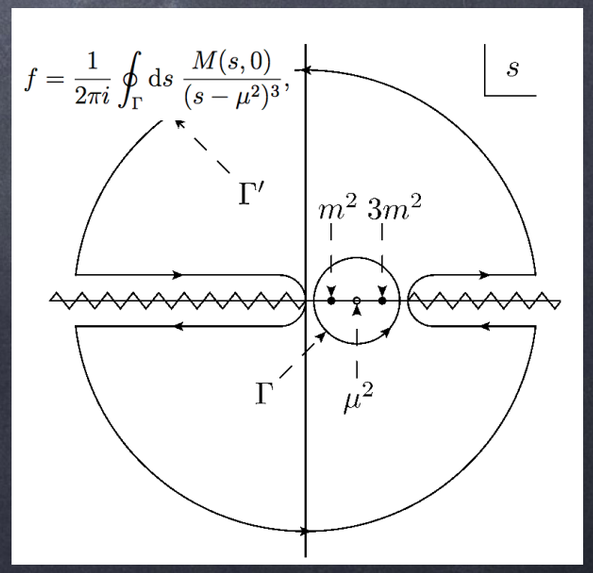
$\epsilon < 1$

s<->u crossing

X = BSM states summation & PS integration

Amplitude of SM -> X



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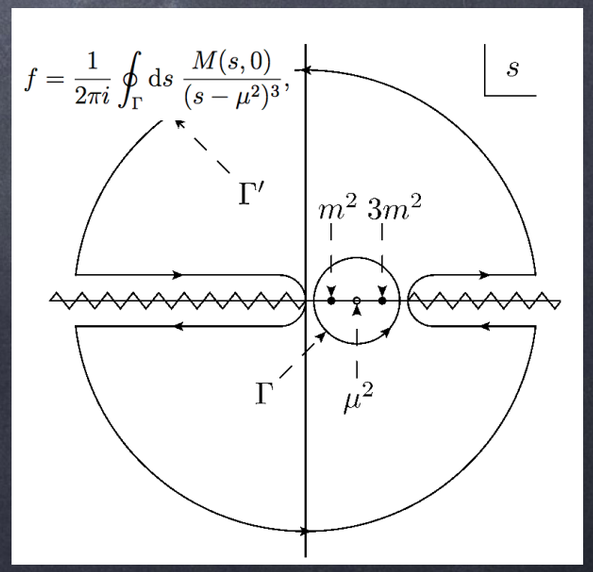

- L.H.S : calculable in EFT
  - At tree level, simply <u>linear combination of $C^{(8)}$</u>
- R.H.S : integration of BSM contribution
- Might think of this as a matching formula.

[C. Cheung, G. Remmen, JHEP 16]
[de Rham, Melville, Tolley, Zhou, JHEP 17]

The traditional approach: elastic scattering

With $M^{ijkl} \equiv \frac{d^2}{ds^2} M_{ij\to kl}\left(\frac{1}{2}M^2\right), \quad m_X^{ij} \equiv M_{ij\to X}(\mu, \Pi_X)$

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

- When i=k, j=l, RHS -> complete squares >0, i.e. a discrete set of inequalities:

$$M^{ijij} \geq 0$$

M (or C8) must stay inside

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- More generally, consider $u^i v^j u^{*k} v^{*l} \cdot M^{ijkl}, \quad u, v \in \mathbb{C}^n$
  RHS -> $|u \cdot m_X \cdot v|^2 + |u \cdot m_X \cdot v^*|^2 \geq 0$
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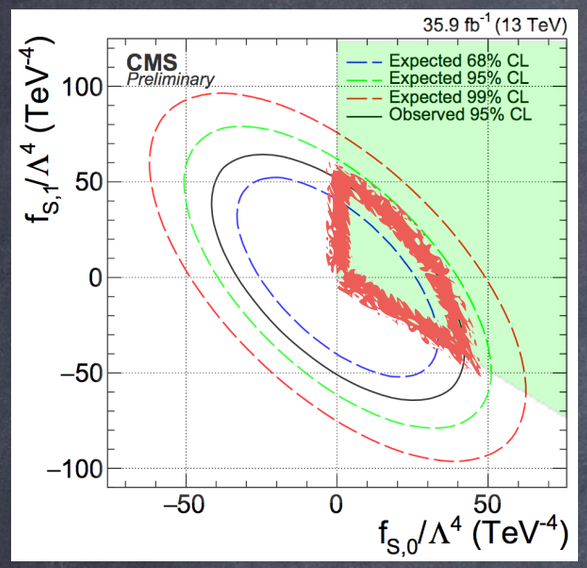
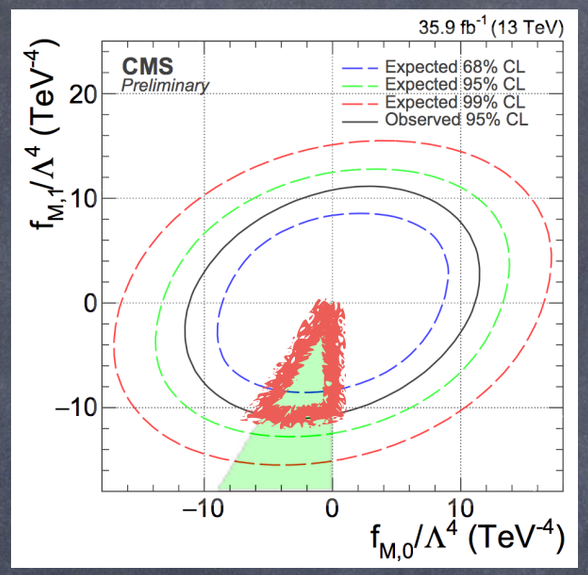
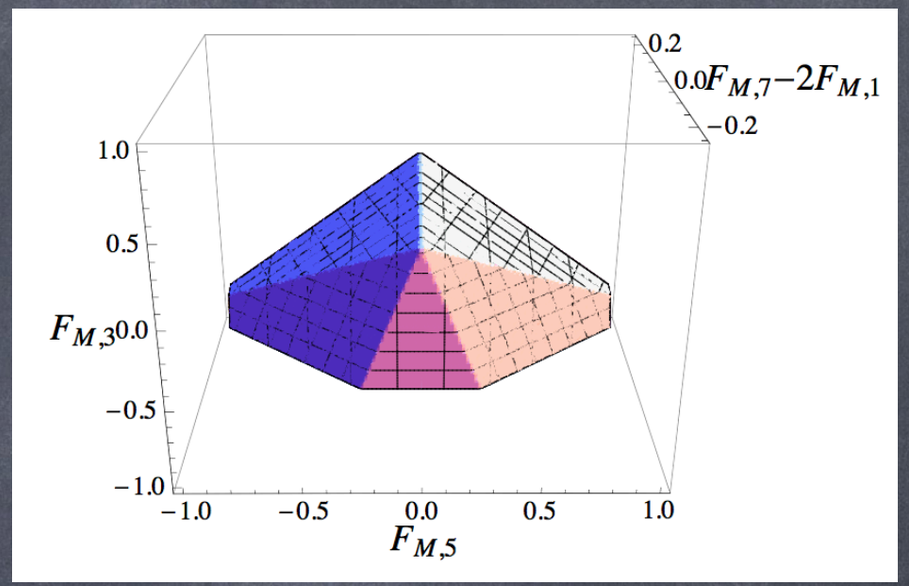
- This is the elastic scattering between two superposed states (by the u,v vectors). Vary u,v to get the full set of bounds.

The traditional approach: elastic scattering

- [Q. Bi, CZ, S.-Y. Zhou, JHEP 19] Mass eigenstates with superposed helicity states:







- [Remmen, Rodd, JHEP 19] Gauge eigenstates, all superposition of Goldstones:

$$c_{S,0} + c_{S,1} + c_{S,2} > 0$$
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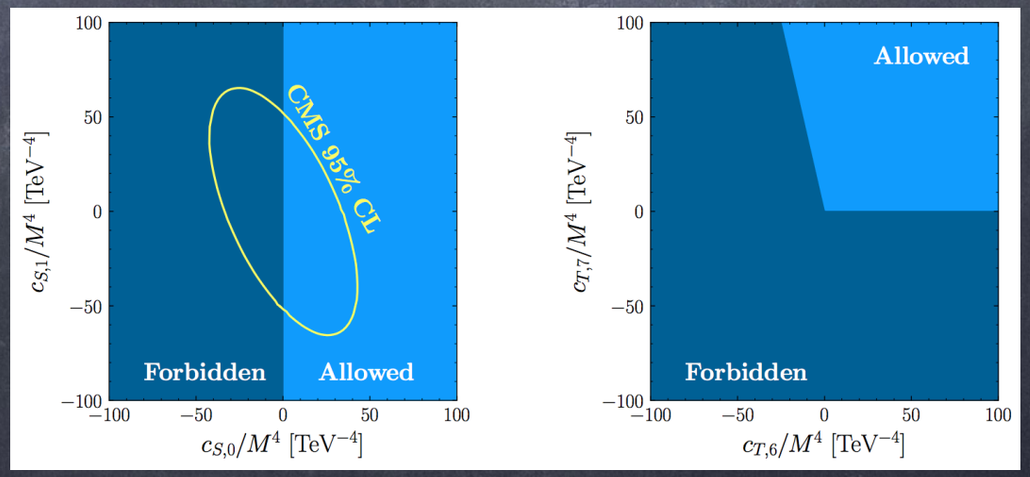


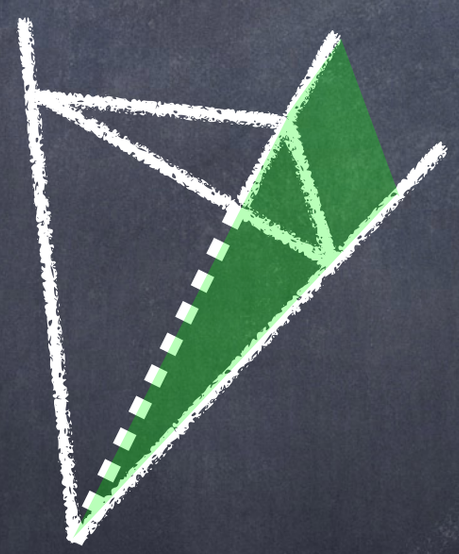
- A lot of room to improve: superposition of gauge components of W/B, together with Goldstones, etc.; still, best bounds not guaranteed.

[CZ and S.-Y. Zhou 2005.03047]

The new approach

(Previous approach)
Inequality representation:

- A convex cone is the set of points satisfying a number of linear inequalities, each representing a <u>facet</u>.



These facets are positivity bounds. (which may or may not correspond to a pair of u,v)

(New approach)
Generator representation:

- A convex cone is the set of points positively generated from a number of rays, each representing an <u>edge</u>.

What are the generators, physically?

[CZ and S.-Y. Zhou 2005.03047]

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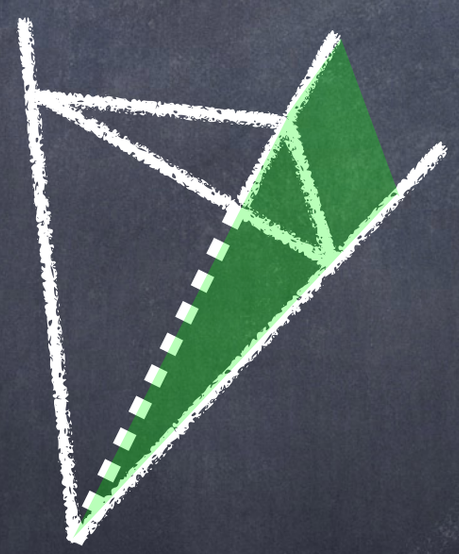

"Vertex enumeration"

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[CZ and S.-Y. Zhou 2005.03047]

The new approach

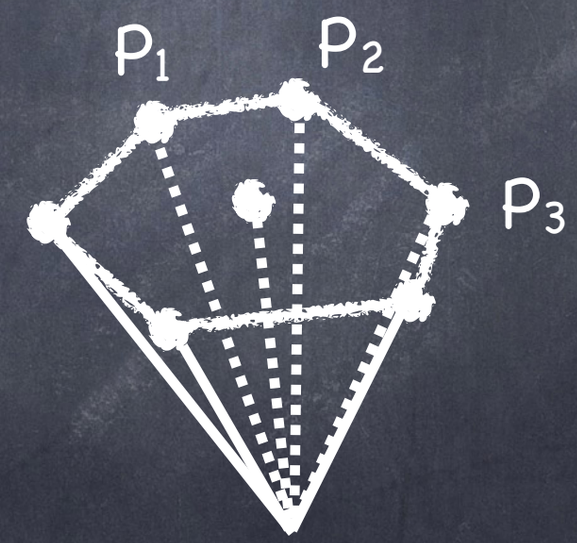
- Based on symmetries of the problem
- Dispersion relation:

$$M^{ijkl} = \sum_{X\in r}{}' \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \frac{|< X|M|r >|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_r^{i(j|k|l)}$$

Dynamics

Symmetry

- P_r^{ijkl} is the projective operator of an irrep r, obtained by CG coefficients.
- The generators are simply (subset of) $P_r^{i(j|k|l)}$
- Computing all P's, and applying vertex enumeration leads to, in principle, the best constraints.
- Difficulty arises if an infinite number of P's are present...



M (or C8) must stay inside

Toy example: WW->WW

- Traditional approach (elastic scattering) gives:

bounds	channel ($\|1>+\|2> \to \|1>+\|2>$)
$F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_y^2$
$4F_{T,1} + F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_x^2$
$F_{T,2} + 8F_{T,10} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_y^1 - W_x^2$
$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_x^1 + W_y^2$

(Other channels are not independent)

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bounds	channel ($\|1>+\|2> \to \|1>+\|2>$)
$F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_y^2$
$4F_{T,1} + F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_x^2$
$F_{T,2} + 8F_{T,10} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_y^1 - W_x^2$
$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_x^1 + W_y^2$

(Other channels are not independent)

- While the new approach gives better bounds

$$
\begin{aligned}
&F_{T,2} \geq 0,\\
&4F_{T,1} + F_{T,2} \geq 0,\\
&F_{T,2} + 8F_{T,10} \geq 0,\\
&8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,\\
&12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,\\
&4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.
\end{aligned}
$$

Toy example: WW->WW

- Traditional approach (elastic scattering) gives:

bounds	channel ($\|1>+\|2> \to \|1>+\|2>$)
$F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_y^2$
$4F_{T,1} + F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_x^2$
$F_{T,2} + 8F_{T,10} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_y^1 - W_x^2$
$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_x^1 + W_y^2$

(Other channels are not independent)

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$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.$$

Cannot be obtained from any elastic channel

Still open questions:

- Bounds: what are the best set of bounds on all QGC, available from the dispersion relation?

- General approaches: establish more concrete and systematic algorithms? which may apply to other operators/channels...

  - Elastic approach: how to determine $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$ w.r.t. all u,v vectors? The determination of a degree-4 polynomial >0 is NP hard.

  - New approach: in case continuous generators show up (like circular cones), how to get bounds?

  - Numerical alternatives?

First step: bounds on transversal QGCs

Transversal operators:

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \quad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \quad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_{T,5} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \quad O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

Note $O_{T,10}$ and $O_{T,11}$ have been missed in standard QGC parameterization.

Pointed out by [Remmen, Rodd, JHEP 12 (2019) 032]

See also dim-8 basis: [C. Murphy 2005.00059], [H.-L. Li 2005.00008]

Preliminary results: the traditional (elastic approach)

Consider scattering two superposition of 8 SM modes: $W_x^1, W_y^1, W_x^2, W_y^2, W_x^3, W_y^3, B_x, B_y$

Positive-definiteness of rank-4 polynomial with 32 variables.

-> We assume the superposition can be factorized in gauge/helicity space.

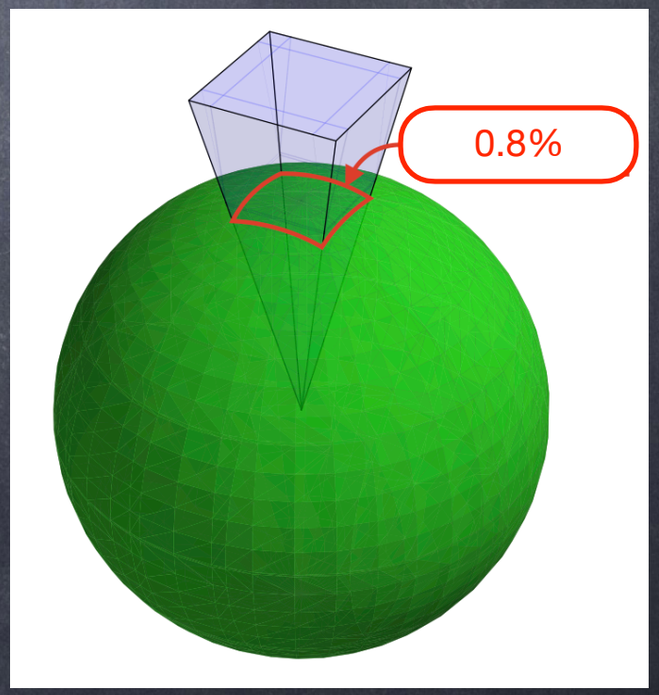
Linear bounds:

$$2F_{T,0} + 2F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} \geq 0$$
$$2F_{T,0} + F_{T,1} + F_{T,2} + 2F_{T,10} \geq 0$$
$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$
$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$

Quadratic bounds:

$$4\sqrt{[2(F_{T,0} + F_{T,1}) + F_{T,2}](2F_{T,8} + F_{T,9})} \geq \max(0, -2(2F_{T,5} + 2F_{T,6} + F_{T,7}), 4F_{T,5} + F_{T,7})$$
$$2\sqrt{F_{T,9}(F_{T,2} + 4F_{T,10})} \geq \max(0, -(2F_{T,11} + F_{T,7}), 2F_{T,11})$$
$$2\sqrt{[4F_{T,10} + 4(F_{T,0} + F_{T,1}) + 3F_{T,2}](4F_{T,8} + 3F_{T,9})} \geq |2F_{T,11} + 4F_{T.5} + F_{T,7}|$$

+ some cubic bounds...



- The parameter space is constrained to 0.8% of the total (in terms of solid angle)
  - Compared with previous result, 2.1% for S+M+T operators...

Preliminary results: the new (generator) approach

The new approach: need to consider an infinite set of projectors (which are potentially generators), continuously parametrized by r:

$$\vec{E}_1 = (1,0,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_2 = (0,1,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_3 = (0,0,1,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_4 = (0,0,0,1,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_5 = \left(-\frac{1}{6},\frac{1}{6},0,0,-\frac{5}{3},0,0,\frac{5}{3},0,0,\frac{5}{6},0,0\right)$$
$$\vec{E}_6 = \left(0,0,-1,1,0,-\frac{3}{4},0,0,\frac{3}{4},0,0,0,1\right)$$
$$\vec{E}_7(r) = \left(0,0,0,0,1,r,r^2,0,0,0,0,0,0\right)$$
$$\vec{E}_8(r) = \left(0,0,0,0,0,0,0,1,r,r^2,0,0,0\right)$$
$$\vec{E}_9(r) = \left(0,0,0,0,0,0,0,0,0,0,1,r,r^2\right)$$
$$\vec{E}_{10}(r) = \left(-\frac{1}{3},\frac{1}{3},-\frac{4r}{3},\frac{4r}{3},-\frac{1}{3},0,-r^2,\frac{1}{3},0,r^2,-\frac{1}{3},0,-\frac{4r}{3}\right)$$
$$\vec{E}_{11}(r) = \left(\frac{1}{2},\frac{1}{2},\frac{r^2}{2},\frac{r^2}{2},-1,-\frac{3r^2}{8},0,-1,-\frac{3r^2}{8},0,-\frac{1}{2},r,-\frac{r^2}{2}\right)$$
$$\vec{E}_{12}(r) = \left(1,0,r^2,0,-2,-\frac{3r^2}{4},0,0,0,0,1,-2r,r^2\right)$$

Question: what is the cone spanned by all these vectors?

How to identify its boundary?

Analytically: a tower of linear, quadratic, cubic, ... inequalities.

- So far only able to obtain the first two levels

Linear:

$$F_{T,2} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$
$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$
$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$

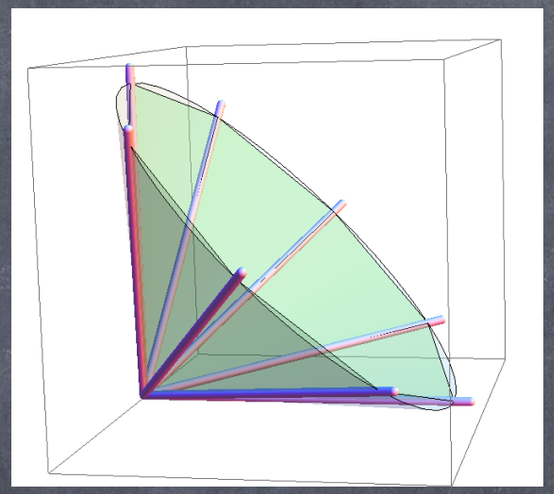
- The parameter space is constrained to 0.687% of the total. (Conservative)

Quadratic:

$$F_{T,9}\left(F_{T,2} + 4F_{T,10}\right) \geq F_{T,11}^2$$
$$16\left(2\left(F_{T,0} + F_{T,1}\right) + F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right) \geq \left(4F_{T,5} + F_{T,7}\right)^2$$
$$32\left(2F_{T,8} + F_{T,9}\right)\left(3F_{T,0} + F_{T,1} + 2F_{T,2} + 4F_{T,10}\right) \geq 3\left(4F_{T,5} + F_{T,7}\right)^2$$
$$2\sqrt{2}\sqrt{F_{T,9}\left(F_{T,2} + 8F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{\left(8F_{T,0} + 4F_{T,1} + 3F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right)} \geq \max\left(-8F_{T,5} - F_{T,7}, 8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)$$
$$4\sqrt{F_{T,9}\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{6}\sqrt{\left(2F_{T,8} + F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left[-3\left(8F_{T,5} + F_{T,7}\right), 3\left(8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)\right]$$
$$\sqrt{6}\sqrt{\left(4F_{T,8} + 3F_{T,9}\right)\left(6F_{T,0} + 2F_{T,1} + 3F_{T,2} + 6F_{T,10}\right)} \geq \max\left[-3\left(2F_{T,5} + F_{T,11}\right), 3\left(2F_{T,5} + F_{T,7} + F_{T,11}\right)\right]$$
$$2\sqrt{\left(12F_{T,8} + 7F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(-12F_{T,5} - F_{T,7} - 2F_{T,11}, -12F_{T,5} + 4F_{T,6} - F_{T,7} - 2F_{T,11}, -12F_{T,5} - F_{T,7} + 2F_{T,11}, 12F_{T,5} + 4F_{T,6} + 5F_{T,7} + 2F_{T,11}\right)$$

Numerically: might as well directly determine if a given point is included in the convex hull of all generators (convex inclusion)

- Infinite number of (potential) generators, but numerically, we sample them with a large number (N of order 100~1000) of discrete ones, i.e. polyhedral cone inscribed to a "circular" cone =>



- Determination of inclusion can be turned in to a linear programming problem.

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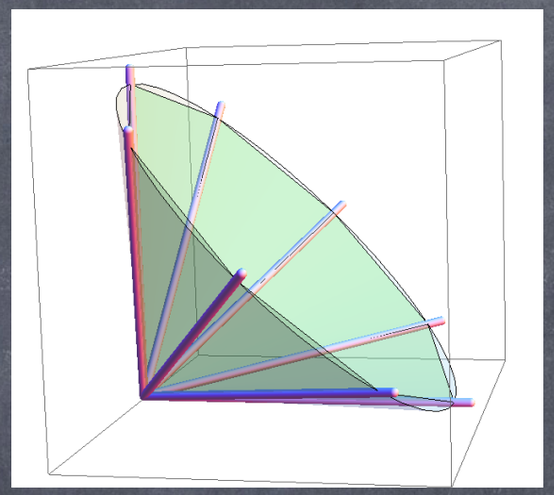
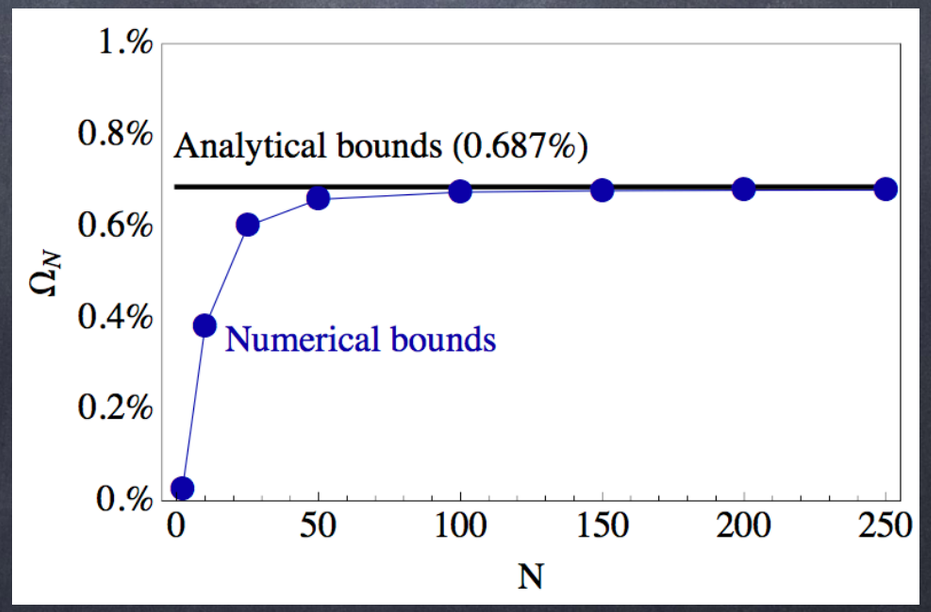


- Determination of inclusion can be turned in to a linear programming problem.

- Volume: ~0.681% (1 - 79.3/N^2)
- **<u>The true volume seems to be 0.681%.</u>**
- Analytical bounds of the first two orders seem sufficient (~1% relaxed).



Summary and to-do list:

- For studying VBS, multi-boson, and QGCs: should keep in mind that, at least, 99.32% of (transversal) QGC parameter space is redundant (because of no UV physics)
- Will work towards full set of QGC bounds.
- Things to think about:
  - Techniques:
    - Further investigation of analytical/numerical approaches.
    - Impacts of double insertion of dim-6, SM loops, etc.
  - Physics:
    - Implication on EXP strategy? On EFT global fits?
    - Apart from VBS, other opportunities to directly test positivity nature on dim-8 operators (e.g. at a future ee collider)?

Thank you

Backups

Preliminary results: the traditional (elastic approach)

The traditional (elastic) approach: consider the scattering of two superposition of 8 SM modes: $W_x^1, W_y^1, W_x^2, W_y^2, W_x^3, W_y^3, B_x, B_y$

- This is the determination of the positive-definiteness of a 4th-order polynomial with 32 variables. Too hard...

- One solution: assuming the superposition can be factorized in gauge/helicity space,

$$u^i = x^a \alpha^b, \; v^i = y^a \beta^b, \; i = (a, b)$$

  with polarization index a and gauge index b.

- The result is conservative, but it converts the problem into "quadratically constrained quadratic programming" problems, can be solved analytically. E.g.

$$\begin{array}{ll} \text{minimize} & 2b_1 M_{b1} + b_3 M_{b3} + 2b_4 M_{b5} + b_6 M_{b6} + b_7 M_{b7} \\ \text{subject to} & b_7 \geq 0, 0 \leq b_1 \leq b_3, |b_4| \leq 2\sqrt{b_1 b_7}, b_6 \geq 2\sqrt{b_3 b_7} \end{array}$$

W-boson polyhedral cone

- Consider $T^{ijkl} = T^{ilkj} \in \mathbb{R}^{(6^4)},\ T^{(ij),(kl)} \succeq 0$ is 36x36 matrix

$$T^{ijkl} m^{ij} m^{kl} = T^{ijkl} m^{il} m^{kj} \geq 0 \quad \Rightarrow \quad T^{ijkl} M^{ijkl} \geq 0$$

- If $T^{ijkl} \neq \sum_\alpha \rho_\alpha u^i_\alpha v^j_\alpha u^k_\alpha v^l_\alpha\,,\ \rho_\alpha \geq 0$

  then we have a new bound not covered by $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

- The ER approach gives two such matrices =>

- Eigenvalues are 15,10,10,10,6,6,6,6,6,6,6,6,6,6,5,2,2,2,2,2 plus 16 0's.

- Same T's apply to other theories.

$$\begin{pmatrix}
\frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{7}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \\
0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 \\
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0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
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\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
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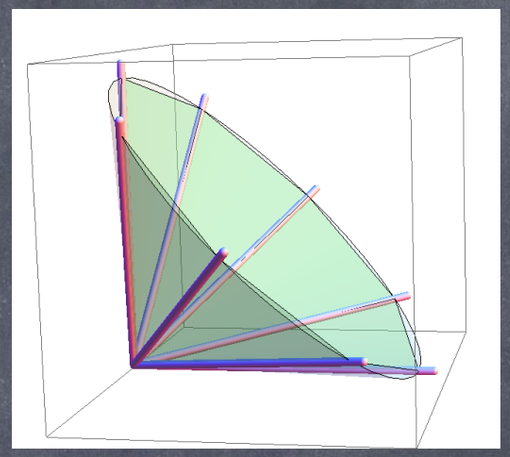
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 \\
0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 \\
\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

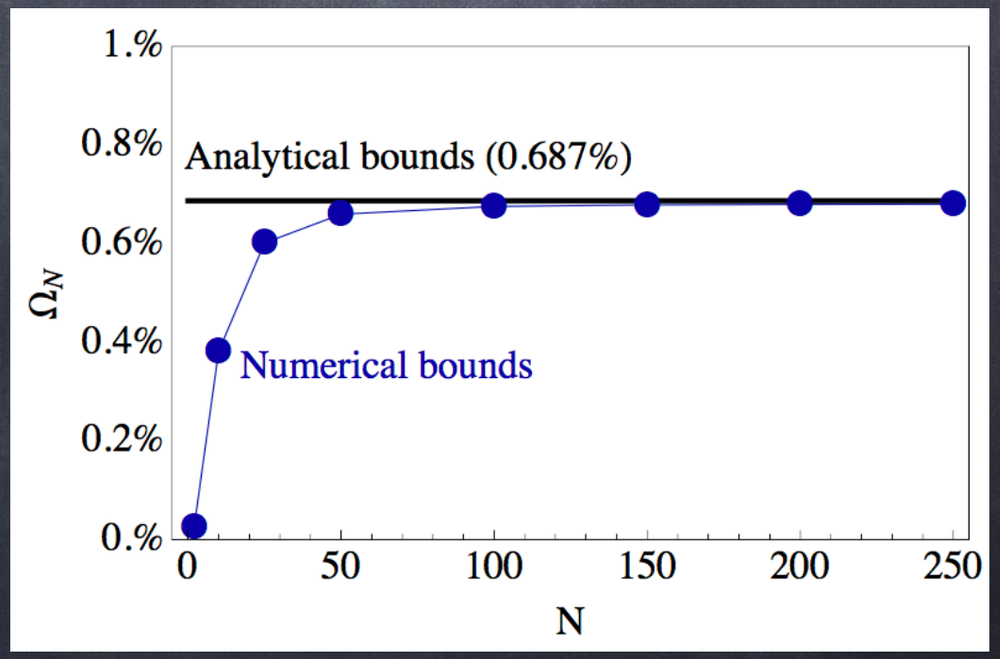
Numerically: might as well directly determine if a given point is included in the convex hull of all ERs (convex inclusion)

- Infinite number of (potential) ERs, but numerically, we sample them with a large number (N of order 100~1000) of discrete ERs, i.e. polyhedral cone inscribed to a "circular" cone.



- The inclusion determination is equivalent to a linear programming:

$$\begin{array}{ll} \text{minimize} & 0 \\ \text{subject to} & \sum_i w_i \vec{e}_{N,i} = \vec{f},\ w_i \geq 0 \end{array}$$

where e's are the ERs, f is the given point, w's are real numbers

- Can be done efficiently with classic programming algorithms.
- Volume: ~0.681% (1 - 79.3/N^2)
- **<u>The true volume seems to be 0.681%.</u>**