Status and future needs for electroweak precision calculations

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- Z pole
- WW production
- Precision tests of (B)SM physics
- HZ production
- tt production

based on arXiv:1906.05379

Need for theory input

- Comparison of EWPOs / HPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs / HPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ QED/QCD, some “boring” SM effects

- “Other” eletroweak parameters (**“input” parameters**)
  $\longrightarrow$ m_{t}, α_{s}, etc. extracted from other processes

1906.05379:

“Theoretical uncertainties for electroweak and Higgs-boson measurements at FCC-ee”

Future e^+e^- colliders

	Current exp.	CEPC	FCC-ee	ILC_{250}	CLIC_{380}
M_W [MeV]	12	1	1	2.5	?
Γ_Z [MeV]	2.3	0.5	0.1	–	–
$R_\ell = \Gamma_\text{Z}^\text{had}/\Gamma_\text{Z}^\ell$ $[10^{-3}]$	25	2	1	14	38
$R_b = \Gamma_\text{Z}^b/\Gamma_\text{Z}^\text{had}$ $[10^{-5}]$	66	4.3	6	23	38
$\sin^2\theta^\ell_\text{eff}$ $[10^{-5}]$	13*	2.3	0.5	2	7.8

* naive combination of LEP/SLC/TeV/LHC

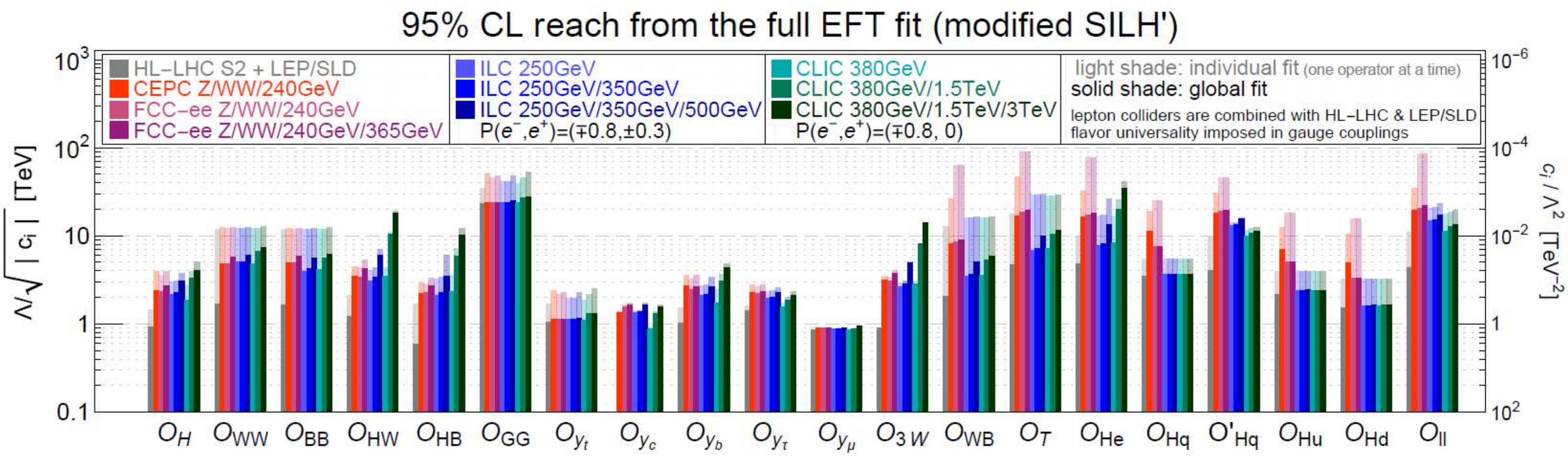
$\longrightarrow$ Improved measurements of several EWPOs necessary to improve global fit

$\longrightarrow$ Challenge for theorists (multi-loop calc. and MC tools)

New physics reach

- More operators than observables
  $\longrightarrow$ Need to make assumptions, e.g. U(3) or U(2) flavor universality

- Strong correlations between some operators
  $\longrightarrow$ All exp. inputs are important



de Blas, Durieux, Grojean, Gu, Paul ’19

Theory uncertainties

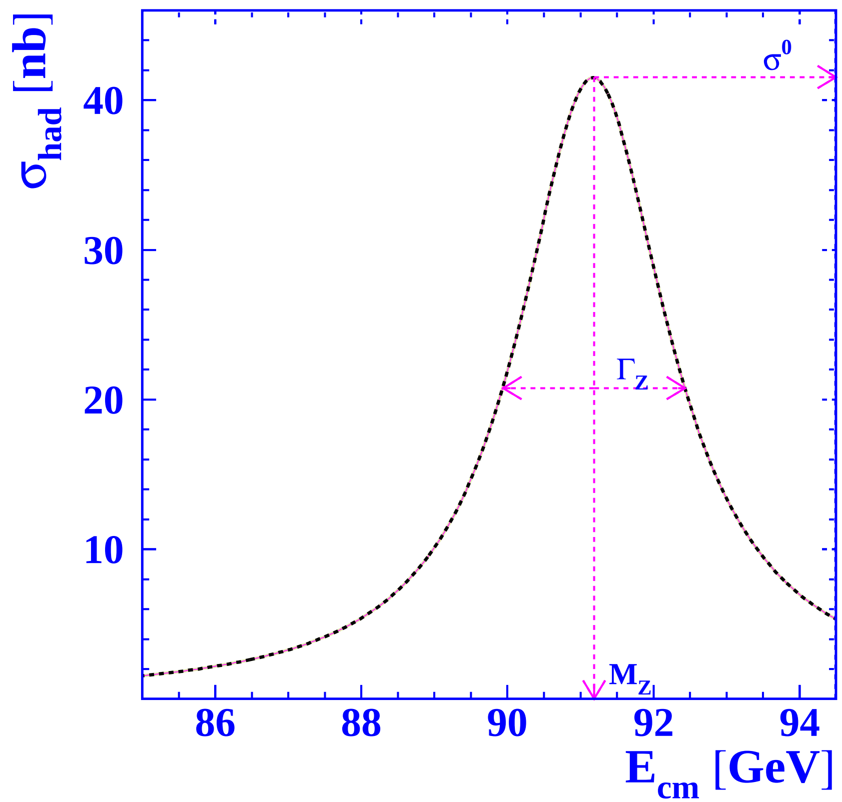
- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

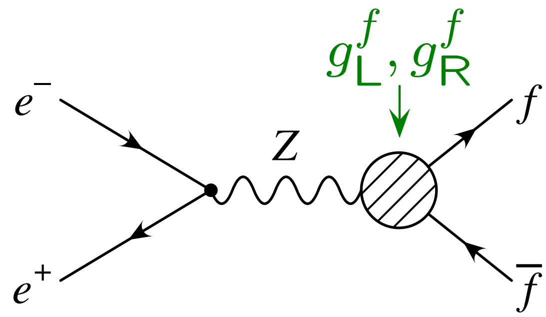
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\mathrm{CM}} \sim M_{\mathrm{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\mathrm{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\mathrm{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\mathrm{Z}}^2)^2+M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2} = \frac{12\pi}{M_{\mathrm{Z}}^2}R_eR_f$

$$\Gamma_{ff} = C\left[(g_{\mathrm{L}}^f)^2 + (g_{\mathrm{R}}^f)^2\right]$$



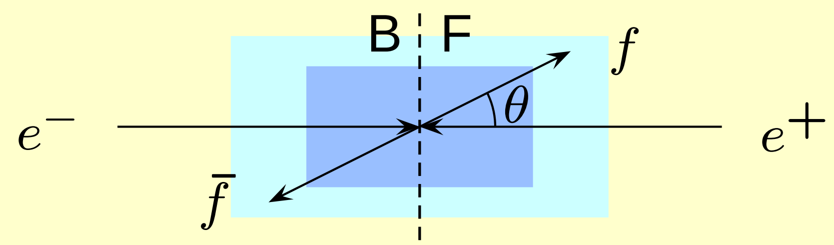


Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$



Other asymmetries:

- Average τ pol. in $e^+e^- \to \tau^+\tau^-$, $\qquad \langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$
- Left-right asymmetry (for polarized e^-): $\qquad A_{\mathrm{LR}} = \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = -\mathcal{A}_e$

Z lineshape

Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

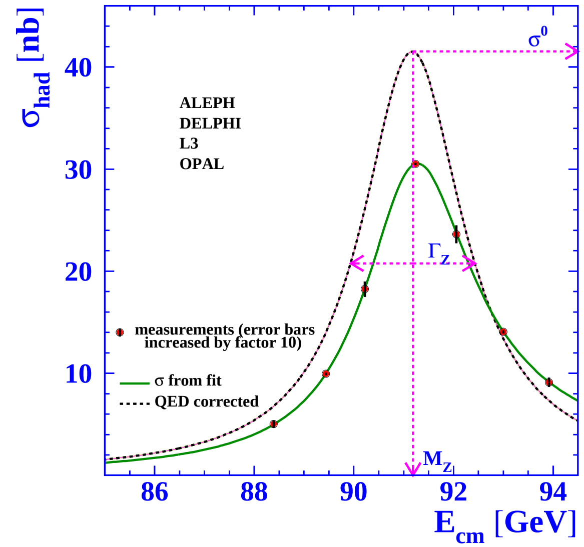
Soft photons (resummed) + collinear photons

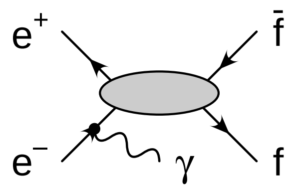
$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

Universal ($m{=}n$) logs known to $n = 6$,
also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

LEP EWWG '05

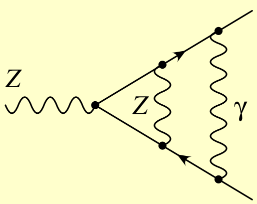




Z decay

Factorization of massive and QED/QCD FSR:

$$\Gamma_f \approx \frac{N_c M_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma_Z'}\right]_{s=M_Z^2}$$



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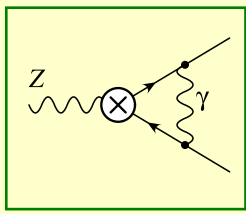
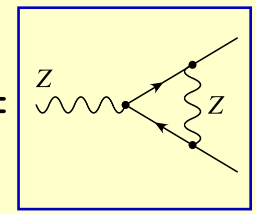


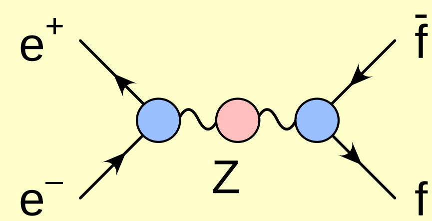
+ finite, with $\otimes$ =



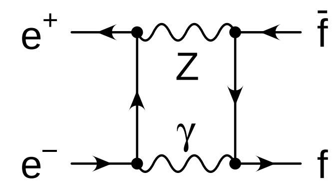
$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

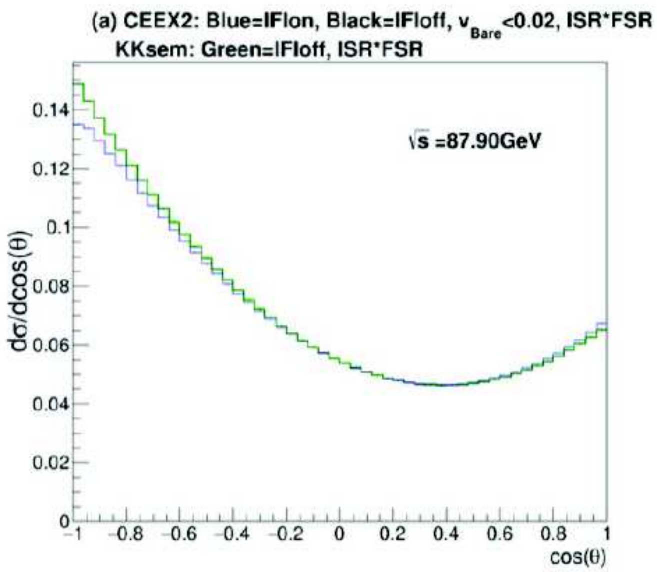
g_V^f, g_A^f, Σ_Z': Electroweak corrections

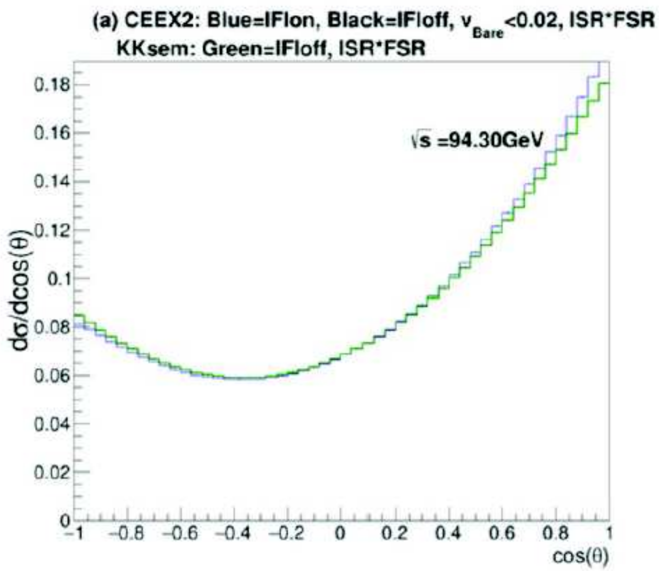


- Interference between ISR and FSR suppressed by Γ_Z/M_Z on Z resonance



- Still relevant for high precision an off-resonance





Jadach, Yost ’18

- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost ’18

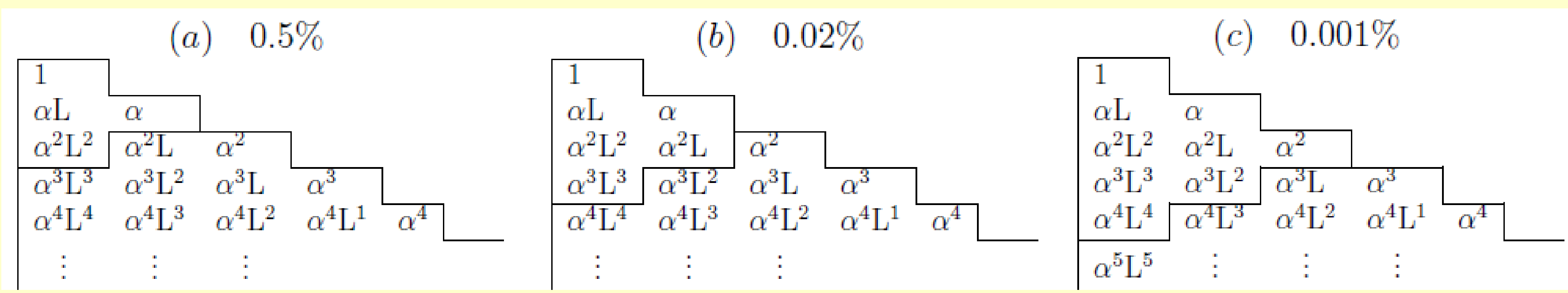
Greco, Pancheri-Srivastava, Srivastava ’75

Monte-Carlo methods for QED effects

- Implementation in MC program to evaluate exp. efficiency and particle ID
- Current state of art: e.g. `KORALZ, KKMC` — Jadach, Ward, ...
  $\longrightarrow \mathcal{O}(\alpha^2 L)$ accuracy $[L = \ln(s/m_e^2)]$
- One to two orders improvement needed:

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

Jadach, Skrzypek '19

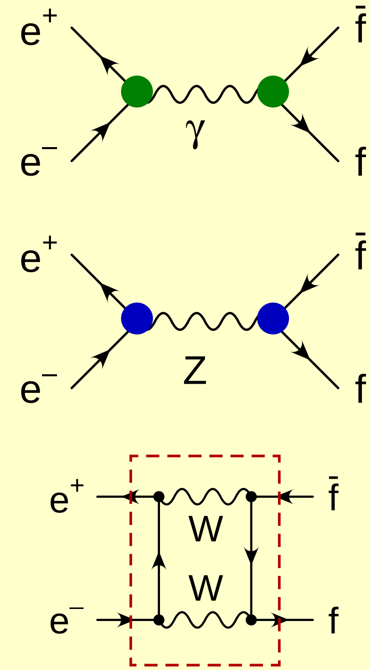


$\longrightarrow$ Need matching of h.o. matrix elements with QED parton shower (exclusive in all fs particles)

“Hard” matrix element

Consistent (gauge-invariant) theory setup:

Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:



$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings

At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Current state of art: full one-loop for S, T

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

see, e.g., Bardin, Grünewald, Passarino ’99

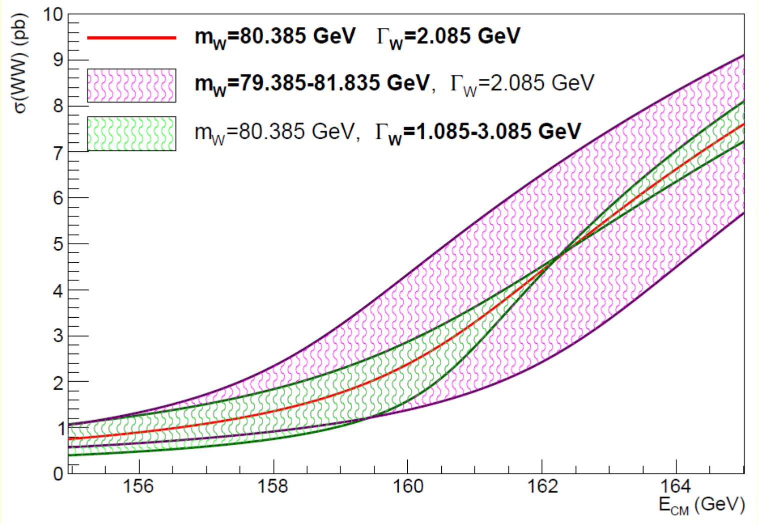
$\longrightarrow$ Sensitivity to some NP beyond EWPO

WW threshold

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

$$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05

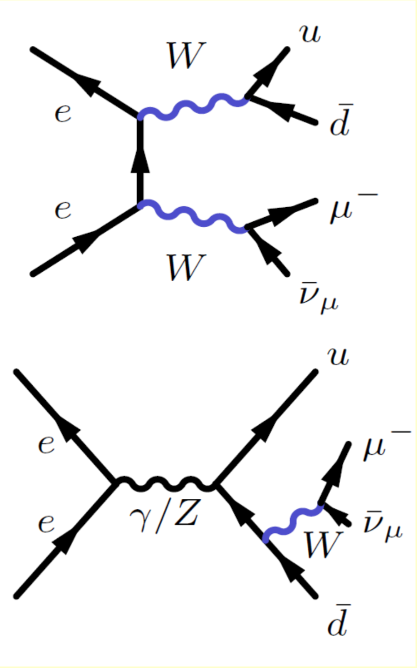
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

  Beneke, Falgari, Schwinn, Signer, Zanderighi '07

  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\text{th}}M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08

  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\text{th}}M_W \lesssim 0.6$ MeV



Precision test of (B)SM physics

	Current exp.	Current th.	CEPC	FCC-ee
M_{W} [MeV]	12	4	1	1
Γ_Z [MeV]	2.3	0.4	0.5	0.1
$R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^\ell$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\mathrm{had}}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta_{\mathrm{eff}}^\ell$ $[10^{-5}]$	13	4.5	2.3	0.5

$\longrightarrow$ To probe NP, need to compare with SM theory predictions ($\longrightarrow$ theory error)

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not CEPC/FCC-ee!

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_{W}, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '13,14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhanced by y_{t} and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

Projected future theory uncertainties

	CEPC	FCC-ee	Current th.	perturb. error with 3-loop†
M_W [MeV]	1	1	4	1
Γ_Z [MeV]	0.5	0.1	0.4	0.15
R_ℓ [10^{-3}]	2	1	5	1.5
R_b [10^{-5}]	4.3	6	10	5
$\sin^2\theta^\ell_\mathrm{eff}$ [10^{-5}]	2.3	0.5	4.5	1.5

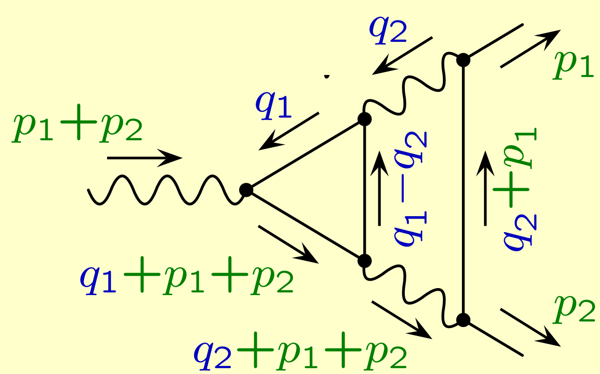
† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$

(N_f^n = at least n closed fermion loops)

Also "parametric" uncertainties from inputs for m_t, α_s, $\Delta\alpha$

Theory calculations

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(1000) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods are more general, but computing intensive
- Special numerical techniques can balance precision and evaluation time

Analytic calculations

- Mostly used for diagrams with up to two scales
  ($e.g.$ M_W & m_t or M_W & M_Z)
- Reduce to **master integrals** with integration-by-parts and other identities
  Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...

  Public programs:

	`Reduze`	von Manteuffel, Studerus '12
	`FIRE`	Smirnov '13,14
	`LiteRed`	Lee '13
	`KIRA`	Maierhoefer, Usovitsch, Uwer '17

- Evaluate master integrals with differential equations or Mellin-Barnes rep.
  Kotikov '91; Remiddi '97; Smirnov '00,01; Henn '13; ...

  $\longrightarrow$ Result in terms of Goncharov polylogs / multiple polylogs

  $\longrightarrow$ Some problems need iterated elliptic integrals / elliptic multiple polylogs
  Broedel, Duhr, Dulat, Trancredi '17,18
  Ablinger er al. '17

  $\longrightarrow$ Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios,
  $e.g.\ M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

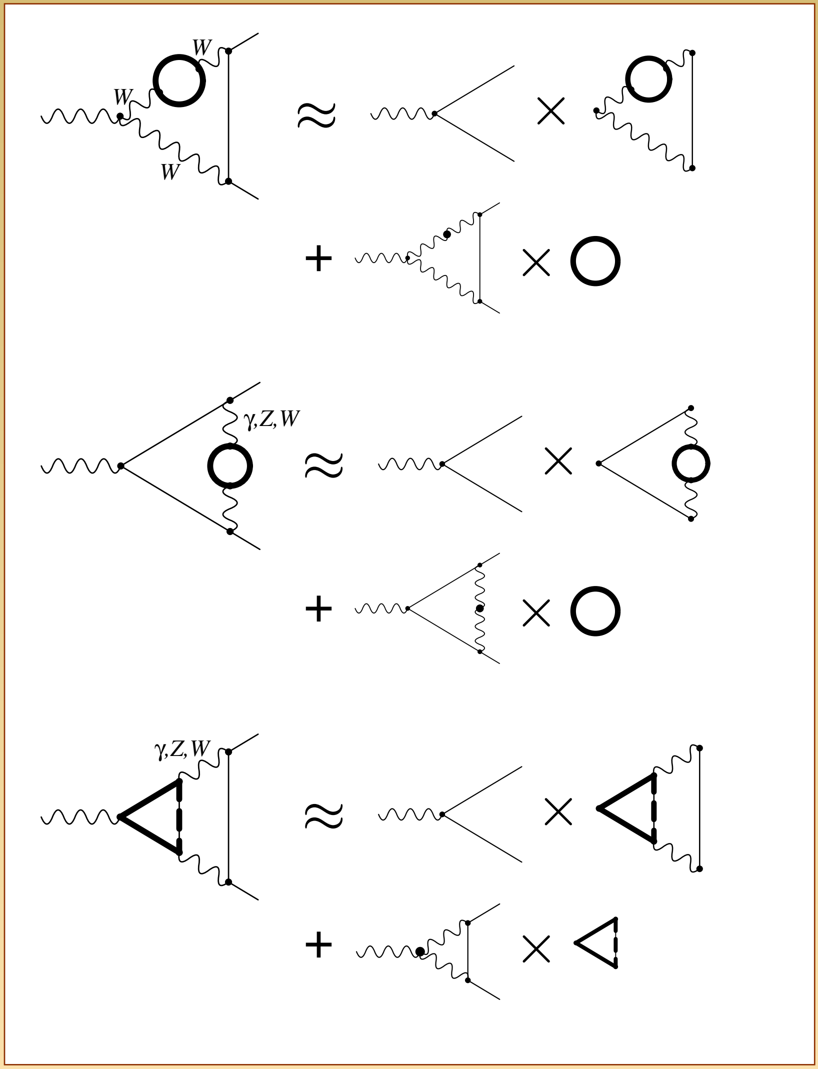
$\longrightarrow$ Used in some 2/3-scale problems

$\longrightarrow$ Public programs:

exp Harlander, Seidensticker, Steinhauser '97
asy Pak, Smirnov '10

$\longrightarrow$ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Direct numerical integration

Two general approches:

⟶ Automated treatment of UV/IR divergencies

⟶ No restriction on number of loops or legs

- **Sector decomposition:**

Public programs:	`SecDec`	Carter, Heinrich '10; Borowka et al. '12,15,17
	`FIESTA`	Smirnov, Tentyukov '08; Smirnov '13,15

- **Mellin-Barnes representations:**

Public programs:	`MB/MBresolve`	Czakon '06; Smirnov, Smirnov '09
	`AMBRE/MBnumerics`	Gluza, Kajda, Riemann '07 Dubovyk, Gluza, Riemann '15 Usovitsch, Dubovyk, Riemann '18

- Diagrams with internal thresholds can cause numerical instabilities
- Specialized techniques (for some type of diagrams) often improve computing time, robustness, precision (but not automated)

Summary

- **Electroweak & Higgs precision tests** at future e^+e^- colliders require 1–2 orders improvement in SM theory calculations and tools
  - Z-pole: 3-loop & leading 4-loop EW + multi-loop/leg merging for QED MC
  - off Z-pole / backgrounds: ($\geq$2)-loop EW
  - WW & Higgs: 2-loop EW for 2$\rightarrow$2 processes (+ 4-loop QCD)
    ($\geq$1)-loop for backgr. and non-resonant terms

- Improvements needed both for **fixed-order loop corrections** as well as **MC tools**

- Development of **new calculational techniques** (numerical/semi-numerical) may be crucial

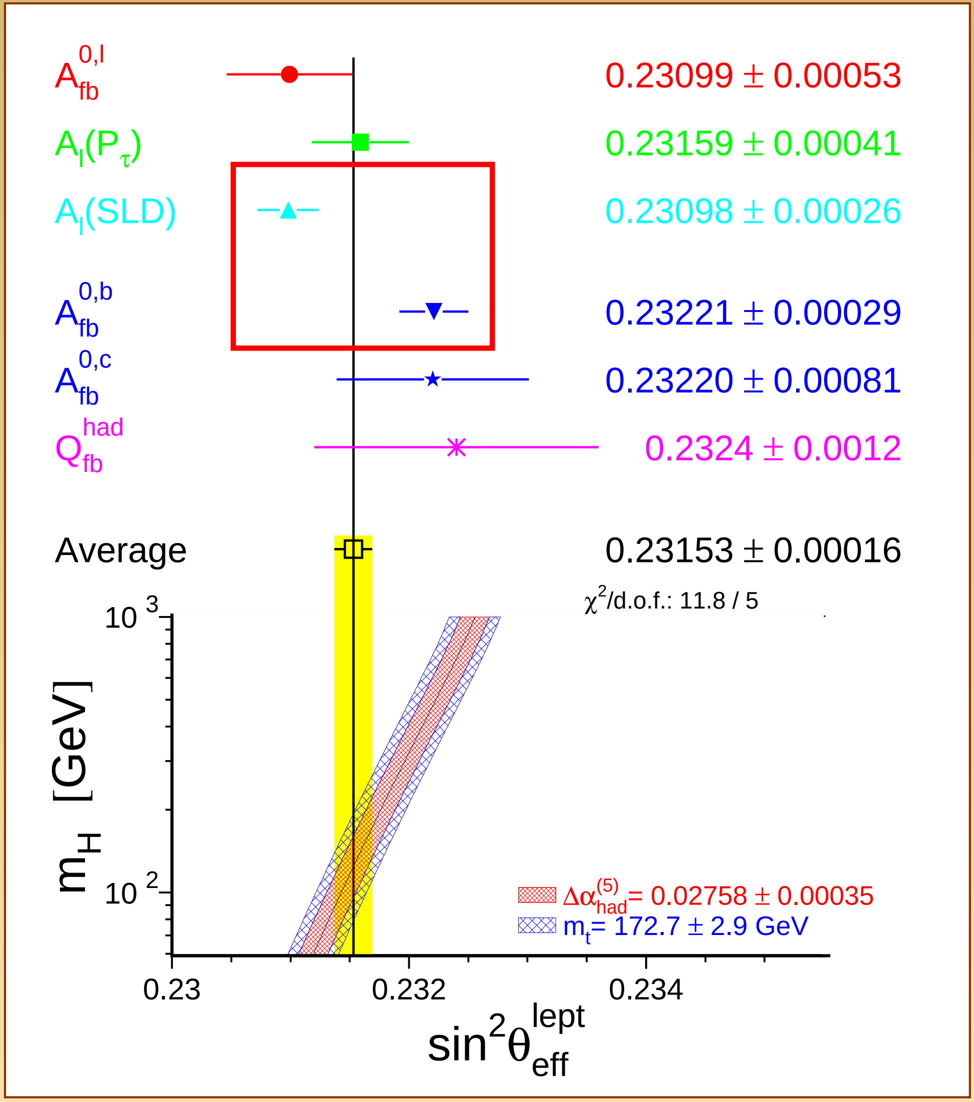
  $\longrightarrow$ Results can only be shared as numerical grid or fit formula

- Besides massive community effort of theorists, no obvious show-stopper in achieving the goals

Backup slides

Present status of $\sin^2\theta^\ell_{\rm eff}$

- Most precise determination from $A_{\rm LR}$ at SLD and $A^b_{\rm FB}$ at LEP
- Disagreement by $\sim 4\sigma$
  $\longrightarrow$ Underestimated systematics?
- Default at CEPC, FCC-ee: $\sin^2\theta^\ell_{\rm eff}$ from $A^{\mu\mu}_{\rm FB}$



LEP EWWG '05

Z-pole asymmetries

Left-right asymmetry: (using polarization e^- beams)

$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e + \Delta A_{\gamma Z} + \Delta A_{\gamma}$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

Limited by systematic uncertainty of P_{e^-}

0.5% at SLD, 0.1% possible in future

Karl, List '17

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4}$ $\Rightarrow$ $\delta \sin^2 \theta^{\ell}_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_{W}	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha_{\mathrm{s}}^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_{\mathrm{s}}$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^\ell_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{\text{bos}}^2) \sim \mathcal{O}(\alpha_{\text{bos}})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{\text{bos}}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

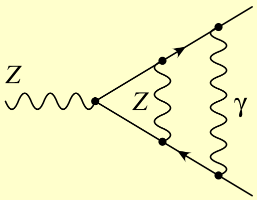
$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{\text{lq}}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

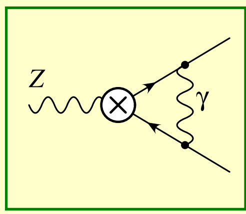
Total: $\delta\Gamma_Z \approx 0.5$ MeV

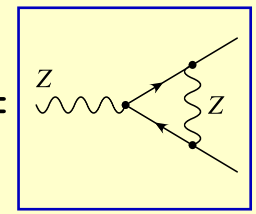
Z decay

Factorization of massive and QED/QCD FSR:

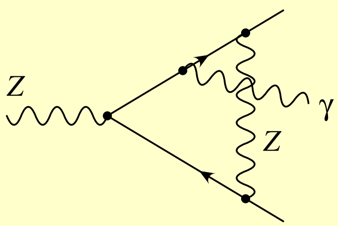
$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma_Z'}\right]_{s=\overline{M}_Z^2}$$



$=$


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

$\longrightarrow$ How to account for in MC simulations?

Other electroweak precision parameters

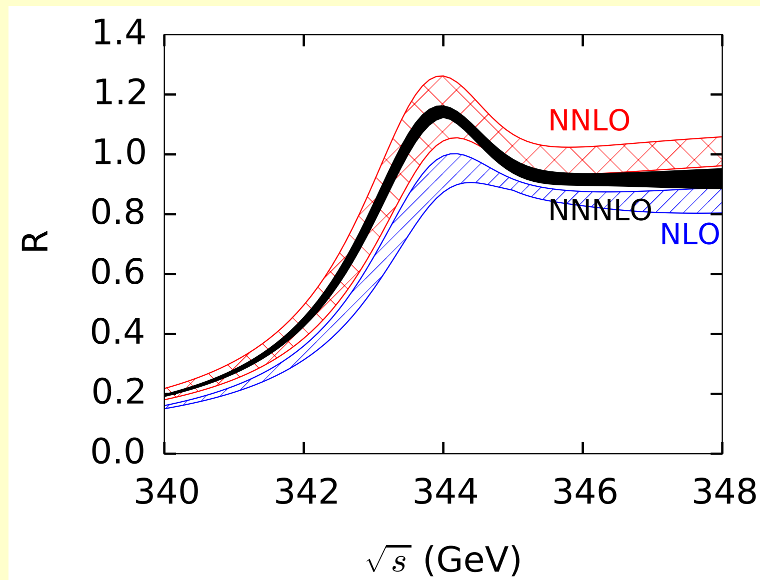
- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ $\delta M_Z, \delta\Gamma_Z \sim 0.1$ MeV could be achievable

- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16
  Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:

$$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp} \oplus [50\ {\rm MeV}]_{\rm QCD} \oplus [10\ {\rm MeV}]_{\rm mass\ def.} \oplus [70\ {\rm MeV}]_{\alpha_s} > 100\ {\rm MeV}$$



Beneke et al. '15

Other electroweak precision parameters

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From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp}$	$[20\ \text{MeV}]_{\rm exp}$	
$\oplus\ [50\ \text{MeV}]_{\rm QCD}$	$\oplus\ [30\ \text{MeV}]_{\rm QCD}$	(h.o. resummation)
$\oplus\ [10\ \text{MeV}]_{\text{mass def.}}$	$\oplus\ [10\ \text{MeV}]_{\text{mass def.}}$	
$\oplus\ [70\ \text{MeV}]_{\alpha_s}$	$\oplus\ [15\ \text{MeV}]_{\alpha_s}$	$(\delta\alpha_s \lesssim 0.0002)$
> 100 MeV	$\lesssim 50$ MeV	

Other electroweak precision parameters

- $m_b,\ m_c$: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\mathrm{MS}}} \sim 30$ MeV, $\delta m_b^{\overline{\mathrm{MS}}} \sim 25$ MeV LHC HXSWG '16

  $\longrightarrow$ estimated improvements $\delta m_b^{\overline{\mathrm{MS}}} \sim 13$ MeV, $\delta m_b^{\overline{\mathrm{MS}}} \sim 7$ MeV
  Lepage, Mackenzie, Peskin '14

- M_{H}: from kinematic constraint fits $HZ(\ell\ell)$, $H(b\bar{b})Z$
  $\longrightarrow$ $\delta M_{\mathrm{H}} \sim 10...20$ MeV
  $\longrightarrow$ theory errors subdominant

Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

    $\longrightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$

    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

    $\longrightarrow$ Subject to sizeable non-pertubative power corrections

    $\longrightarrow$ Systematic uncertainties in power corrections?

  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^\ell$):

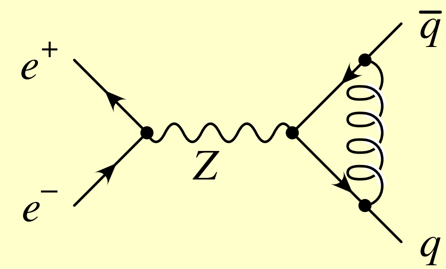
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    $\longrightarrow$ No (negligible) non-perturbative QCD effects

    FCC: $\delta R_\ell \sim 0.001$

    $\Rightarrow \quad \delta\alpha_s < 0.0002$ (subj. to theory error)

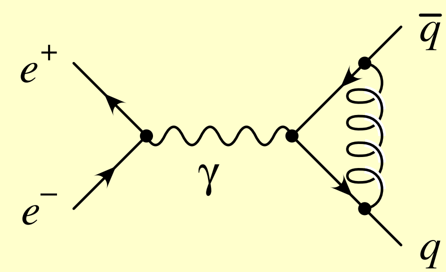
    Caviat: R_ℓ could be affected by new physics



  - $R = \frac{\sigma[ee \to \text{had.}]}{\sigma[ee \to \mu\mu]}$ at lower $\sqrt{s}$

    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$

    Kühn, Steinhauser, Teubner '07



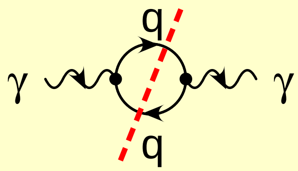
    $\longrightarrow$ dominated by s-channel photon, less room for new physics

    $\longrightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Shift of finestructure constant

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- Hadronic effects from $e^+e^- \to$ had. data

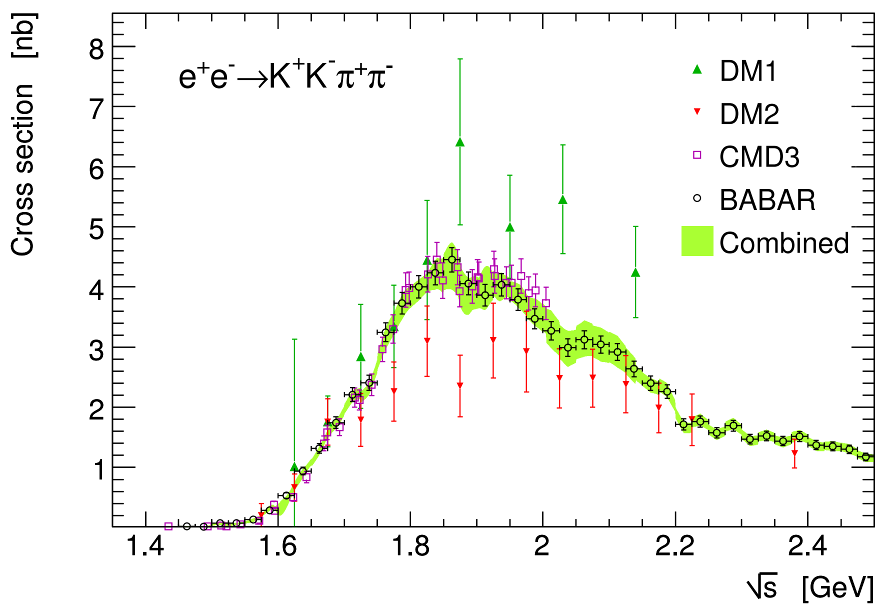


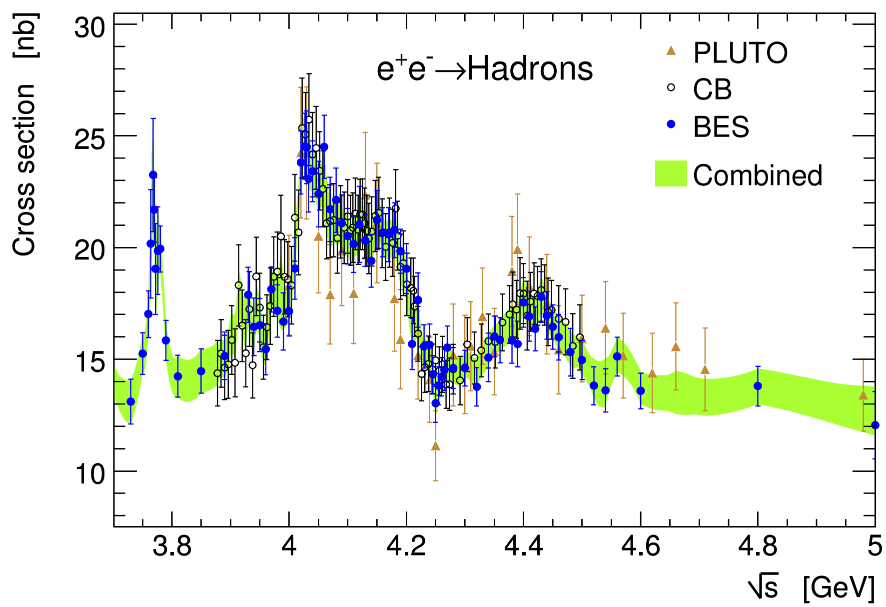
- Last 5 years: new data from BaBar, VEPP, BES
  $\longrightarrow$ Robust precision $\sim 10^{-4}$

Davier et al. '17,19; Jegerlehner '17
Keshavarzi, Nomura, Teubner '18

- With future data from BES, VEPP, Belle and improvements in QCD:
  $\delta(\Delta\alpha) \sim 5 \times 10^{-5}$

Jegerlehner '19





Davier et al. '17

Shift of finestructure constant

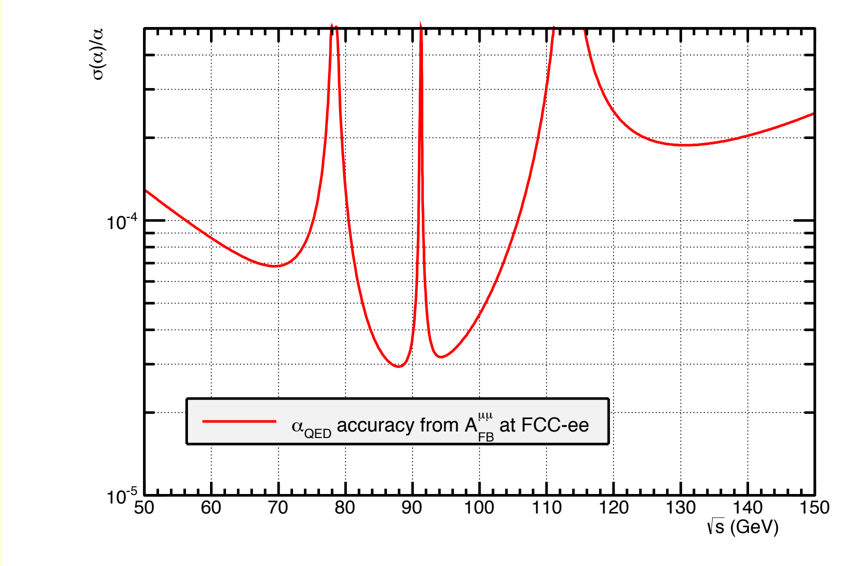
- $\Delta\alpha_{\text{had}}$: Could be limiting factor

  a) From $e^+e^- \to$ had. using dispersion relation
  Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
  Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely

  b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
  (i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
  $\to \delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$

  Janot '15

  Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions



Theory predictions for Higgs precision observables

Measurable properties of $h(125)$:

- **Spin, CP:** already underway at LHC
- **Mass:** direct LHC measurement more precision than SM/MSSM prediction
- **BRs, couplings:** currently $\mathcal{O}(20\%)$, improvement will greatly enhance sensitivity to higher new-physics scales Englert et al. ’14

Target precision of future e^+e^- colliders:

	CEPC	FCC-ee
hbb	1.0%	0.4%
hcc	1.9%	0.7%
$h\tau\tau$	1.2%	0.5%
$h\mu\mu$	5%	6%
hWW	1.1%	0.2%
hZZ	0.25%	0.15%
$h\gamma\gamma$	1.6%	1.5%
hgg	1.2%	0.8%

SM predictions for Higgs decays

Review: Lepage, Mackenzie, Peskin '14, see also LHC HXSWG '13

hbb:

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
- $\mathcal{O}(\alpha)$ QED+EW — Dabelstein, Hollik '92; Kniehl '92
- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large m_t — Kwiatkowski, Steinhauser '94; Butenschoen, Fugel, Kniehl '07

  $\longrightarrow$ Use for error estimate

Current theory error: $\Delta_{th} < 0.4\%$

With full 2-loop: $\Delta_{th} \sim 0.2\%$

Parametric error:

$$\left.\begin{array}{l} \delta m_b = 0.030 \text{ GeV} \\ \delta\alpha_s = 0.001 \end{array}\right\} \rightarrow \Delta_{par} \approx 0.8\%$$

$$\left.\begin{array}{l} \delta m_b = 0.005 \text{ GeV} \\ \delta\alpha_s = 0.0001 \end{array}\right\} \rightarrow \Delta_{par} \approx 0.3\%$$

SM predictions for Higgs decays

$h\tau\tau$:

With full 2-loop (no QCD): $\Delta_{\rm th} < 0.1\%$

Parametric error negligible

hWW^*/hZZ^*:

- complete $\mathcal{O}(\alpha) + \mathcal{O}(\alpha_{\rm s})$ for $h \to 4f$ Bredenstein, Denner, Dittmaier, Weber '06
- leading $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_{\rm s})$ and $\mathcal{O}(\alpha\alpha_{\rm s}^2)$ for large $m_{\rm t}$ Djouadi, Gambino, Kniehl '97

Kniehl, Spira '95; Kniehl, Steinhauser '95

$\longrightarrow$ Small (0.2%) effect Kniehl, Veretin '12

Theory error: $\Delta_{\rm th,EW} < 0.3\%$, $\Delta_{\rm th,QCD} < 0.5\%$

With NNLO final-state QCD corrections: $\Delta_{\rm th,QCD} < 0.1\%$

Parametric error:

$\delta M_{\rm H} \sim 10$ MeV $\longrightarrow \Delta_{\rm par} \approx 0.1\%$

Note: Distributions affected by corrections $\longrightarrow$ implementation into MC tools

SM predictions for Higgs decays

hgg:

- $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ (in large m_t-limit) QCD corrections — Baikov, Chetyrkin '06; Schreck, Steinhauser '07
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04

Theory error (dominated by QCD): $\Delta_{th} \approx 3\%$
With $\mathcal{O}(\alpha_s^4)$ in large m_t-limit (4-loop massless QCD diags.): $\Delta_{th} \approx 1\%$

Parametric error: $\delta\alpha_s = 0.001 \rightarrow \Delta_{par} \approx 3\%$
$\delta\alpha_s = 0.0001 \rightarrow \Delta_{par} \approx 0.3\%$

h$\gamma\gamma$:

- $\mathcal{O}(\alpha_s^2)$ QCD corrections — Zheng, Wu '90; Djouadi, Spira, v.d.Bij, Zerwas '91; Dawson, Kauffman '93; Maierhöfer, Marquard '12
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04; Actis, Passarino, Sturm, Uccirati '08

Theory error: $\Delta_{th} < 1\%$

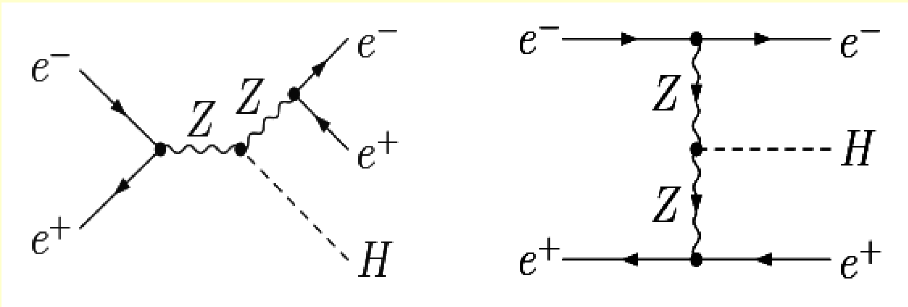
Parametric error negligible

SM predictions for Higgs production

hZ production:

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl '92; Denner, Küblbeck, Mertig, Böhm '92; Consoli, Lo Presti, Maiani '83; Jegerlehner '86; Akhundov, Bardin, Riemann '86
- Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available — Boudjema et al. '04
- Can be combined with h.o. ISR QED radiation — Greco et al. '17
- $\mathcal{O}(\alpha\alpha_s)$ corrections — Gong et al. '16; Chen, Feng, Jia, Sang '18

Theory error: $\Delta_{th} \sim O(1\%)$



With full 2-loop corrections for $ee \to HZ$:
$\Delta_{th} \lesssim O(0.3\%)$

Parametric error: negligible if $\delta M_H < 100$ MeV

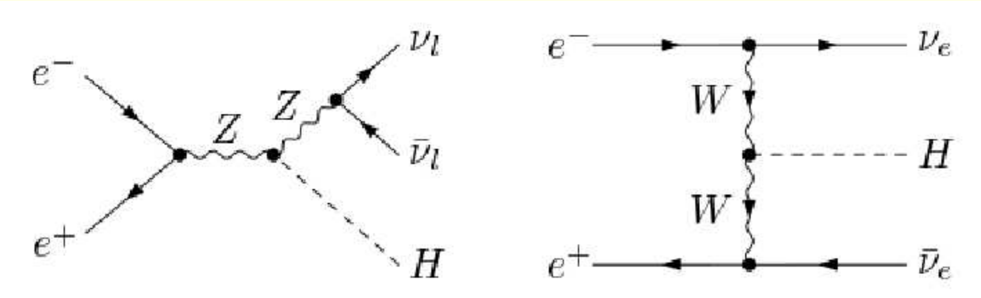
SM predictions for Higgs production

WW fusion:

- $\mathcal{O}(\alpha)$ corrections with h.o. ISR — Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03

Theory error: $\Delta_{\text{th}} \sim O(1\%)$?

Parametric error: negligible



Full $\mathcal{O}(\alpha^2)$ calculation for 2$\rightarrow$3 process is very challenging

$\longrightarrow$ Contributions with closed fermion loops maybe feasible

Main challenges

For Higgs and WW physics:

- Full $\mathcal{O}(\alpha^2)$ for $2 \rightarrow 2$ processes
- $\mathcal{O}(\alpha_s^4)$ QCD corrections
- Also need $O(\alpha)$ (or better?) corrections for backgrounds: $e^+e^-b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, etc.
  $\longrightarrow$ Technology exists, but work needed   Denner, Dittmaier, Roth, Wieders '05

For Z pole:

- 3-loop EW and mixed EW-QCD corrections for Zff vertices
- Leading 4-loop effects
- Initial-final QED effects / mergering multi-loop and Monte-Carlo