

# Digests of recent works for extended Higgs models

- [1] Full NLO calculations for  $h(125)$  decays in non-minimal Higgs models  
[arXiv:1906.10070](#), [arXiv:1910.12769](#)
- [2] New physics effects on the production rate of  $e^+e^- \rightarrow h \gamma$   
[arXiv:1808.10268](#)
- [3] 2-loop correction to the  $hhh$  coupling in non-minimal Higgs models  
[arXiv:1903.05417](#), [arXiv:1911.11507](#)
- [4] Aligned CP violating 2HDM cancelling the EDM  
[arXiv:2004.03943](#)

Shinya Kanemura (Osaka Univ.)

with

Johannes Braathen, Mitsunori Kubota, Mariko Kikuchi,  
Kentaro Mawatari, Kodai Sakurai, Kei Yagyu

*Contributions to SNOWMASS EF-02 WG Meeting on 12 June, 2020*

# [1] H-COUP: Program to evaluate full NLO decays of $h(125)$ in non-minimal Higgs models

S. Kanemura, M. Kikuchi, K. Mawatari, K. Sakurai, K. Yagyu

Physics: [Nucl. Phys. B949 \(2019\) 114791 \(arXiv:1906.10070\)](#)

Manual: [arXiv:1910.12769](#)

# Higgs Precision Measurements

- ❑ Determination of the structure of the Higgs sector is important to determine **new physics** beyond the SM.
- ❑ Discovery of the 125 GeV Higgs boson opened a new window to narrow down the structure of the Higgs sector from its **precision measurements**.
- ❑ Precise measurements of the 125 GeV Higgs will be done at future collider experiments such as **HL-LHC and the ILC250**.
- ❑ **Precise calculations** of the Higgs properties (couplings, width, BRs, cross sections) must be important to compare future precision measurements.

H-COUP can do the job!!

# Public Tools

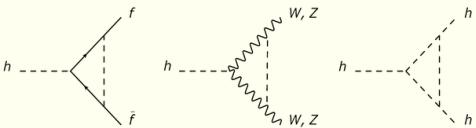
Here is a list of public tools to compute the Higgs decays for extended Higgs models.

- ▶ **(ewN)2HDECAY** : [M. Krause, M. Mühlleitner, M. Spira, 1810.00768][M. Krause, M. Mühlleitner, 1904.02103]
  - Model: 2HDMs, N2HDMs
  - Calculations of two-body-Higgs decays with full 1-loop EW and state-of-the-art QCD corrections in 17 renormalization scheme for mixing parameters
- ▶ **Prophecy4f** : [L. Altenkamp, S. Dittmaier, H. Rzehak, JHEP 1803 (2018) 110]
  - Model: SM, 2HDMs, HSM
  - $h \rightarrow WW/ZZ \rightarrow 4$  fermions with NLO QCD and NLO EW corrections
- ▶ **RECOLA2** : [A. Denner, J. N. Lang, S. Uccirati, CPC 224(2018)346 ]
  - Model: 2HDMs, HSM
  - Calculation to NLO amplitude for any process
- ▶ **2HDMC** : [D. Eriksson, J. Rathsman, O. Stal CPC. 181 (2010) 189]
  - Model: 2HDMs
  - Calculations of decays of Higgs bosons with NLO QCD
- ▶ **SHDECAY** : [R. Costa, M. Mühlleitner, M. Sampaio, R. Santos, JHEP 06 (2016) 034 ]
  - Model: HSM (real and complex)
  - Calculations of decays of Higgs bosons with NLO QCD
- ▶ **H-COUP** : Kanemura, Kikuchi, Sakurai, Yagyu, arXiv:1710.04603 (CPC)  
Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu, arXiv:1910.12769

# What is H-COUP?

Kanemura, Kikuchi, Sakurai, Yagyu, arXiv:1710.04603 (CPC)  
Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu, arXiv:1910.12769

## H-COUP



H-COUP is a set of Fortran program to calculate couplings, decay rates and BRs for  $h(125)$  in various non-minimal Higgs models.

### Model

- Higgs Singlet model
- Two Higgs doublet models  
Type I, II, X, Y
- Inert doublet model

### Observables

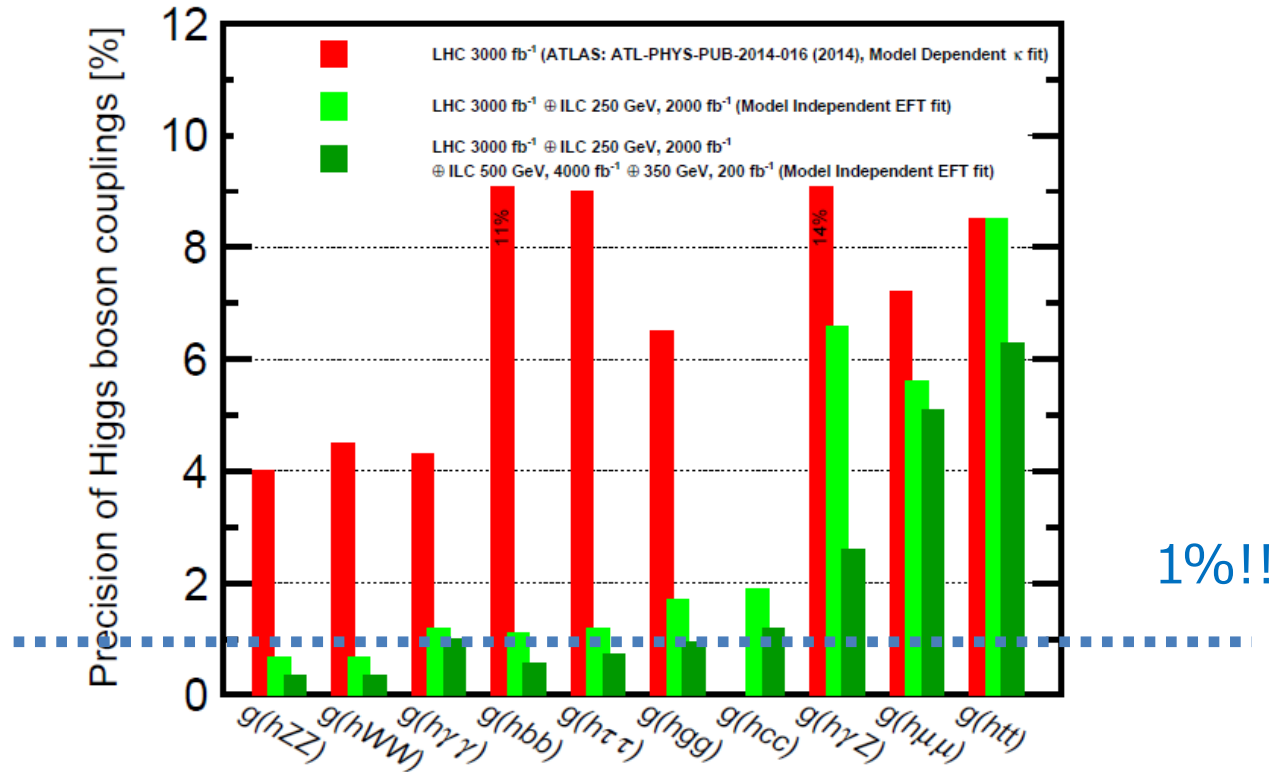
- ✓  $hff$ ,  $hVV$ ,  $hhh$  vertex functions (v1.0)  
with NLO EW
- ✓  $\text{BR}(h \rightarrow ff)$ ,  $\text{BR}(h \rightarrow VV^*)$ , (v2.0)  
 $\text{BR}(h \rightarrow \gamma\gamma)$ ,  $\text{BR}(h \rightarrow Z\gamma)$ ,  $\text{BR}(h \rightarrow gg)$   
with NNLO QCD + NLO EW

## What is New?

- ❑ All these decay rates (2 body, 3 body, loop induced processes) are calculated in the **improved EW on-shell scheme** without gauge dependence.
- ❑ These calculations are systematically applied to **various extended Higgs models**.

# Prospects of the future measurements

Fujii et al, 1710.07621 [hep-ph]



- Accuracy of the measurements can be **O(1)% level** at HL-LHC+ILC250.
- Evaluations of systematic calculations with higher-order corrections are necessary.

# Fingerprinting Higgs sectors by using H-COUP

Kanemura, Kikuchi, Mawatari, Sakurai, Yagyū, arXiv:1906.10070 (NPB)

- **Fingerprinting** of the Higgs sector can be done by using H-COUP.

$$\Delta\mu_{XX} \equiv \frac{\text{BR}(h \rightarrow XX)_{\text{NP}}}{\text{BR}(h \rightarrow XX)_{\text{SM}}} - 1$$

- Colored points show predictions in each Higgs model

HSM: Higgs singlet model

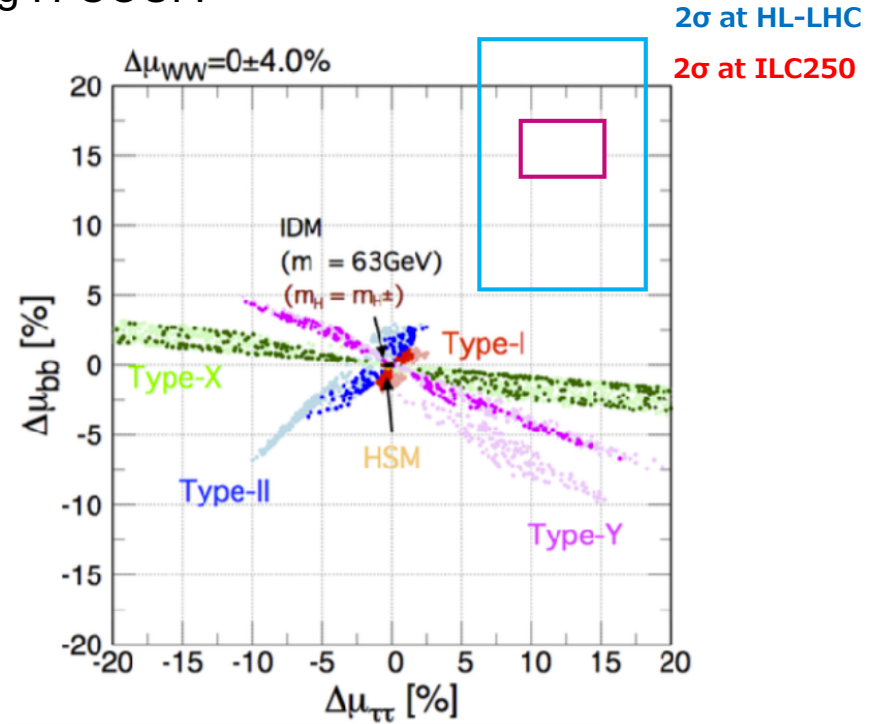
Type-I, II, X, Y: two Higgs doublet models

IDM: inert doublet models

- Lighter (Darker) points show the case for

$m_\phi > 300$  (600) GeV

( $m_\phi$  : Mass of extra Higgs bosons (degenerated))



If  $|\Delta\mu_{\tau\tau}| \gtrsim 5\%$ , 4 types of THDMs can be separated.

# Fingerprinting the Higgs sector

Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu, arXiv:1803.01865 (PLB)

□ **Fingerprinting** the Higgs sector can be done by using H-COUP.

□ **Scale of the second Higgs boson** can be extracted.

Colored areas show predictions in each model

HSM: Higgs Singlet Model,

Type-I, II, X, Y: Two Higgs doublet models

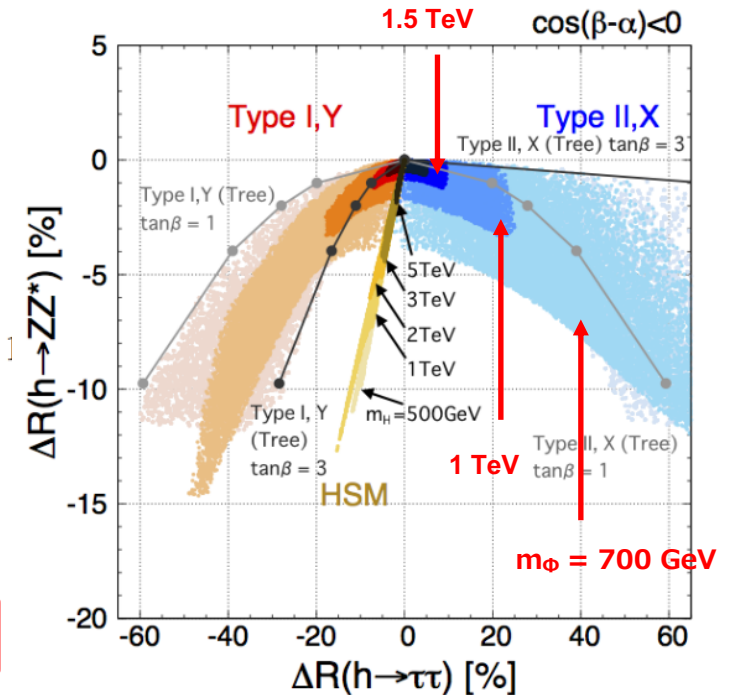
$$\Delta R(h \rightarrow XX) = \frac{\Gamma(h \rightarrow XX)}{\Gamma_{\text{SM}}(h \rightarrow XX)} - 1$$

Darker colors show larger masses of extra Higgs bosons.

( $m_\phi$  : Mass of extra Higgs bosons)

Models can be separated using direction of the deviations

If  $|\Delta R(h \rightarrow ZZ^*)| \gtrsim 4$  (2)%, the second Higgs should appear below 1 (1.5) TeV!





# Summary for the H-COUP projects

- ❑ Precise calculations of the observables of  $h(125)$  are quite important to determine the structure of the Higgs sector, which can show the direction of BSMs.
- ❑ H-COUP systematically evaluates couplings, decay rates & BRs for  $h(125)$  with **higher order corrections** in **various extended Higgs models**.
- ❑ By using H-COUP, fingerprinting of the Higgs sector is possible including NLO EW and NNLO QCD corrections.
- ❑ The scale of the second Higgs boson can be explored from the size of deviations in the decay rates of  $h(125)$ .

See also [Nucl. Phys. B949 \(2019\) 114791 \(arXiv:1906.10070\)](#) [arXiv:1910.12769](#) for details

**[2] Single Higgs production in association with  
a photon at electron-positron colliders  
in extended Higgs models**

S. Kanemura, K. Mawatari, K. Sakurai

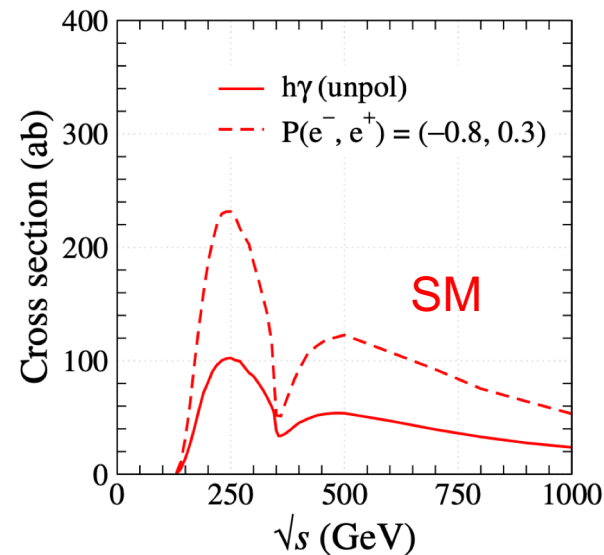
*Phys. Rev D99 (2019) 035023 (arXiv:1808.10268)*

# Introduction: $h\gamma$ production at $e^+e^-$ colliders

- The cross section peaked around  $E = 250$  GeV
- Beam polarization can enhance the cross section
- The signal is clean: a monochromatic photon

$$E_\gamma = \frac{\sqrt{s}}{2} \left(1 - \frac{m_h^2}{s}\right)$$
$$\sim 93.8 \text{ GeV} @ \sqrt{s} = 250 \text{ GeV}$$

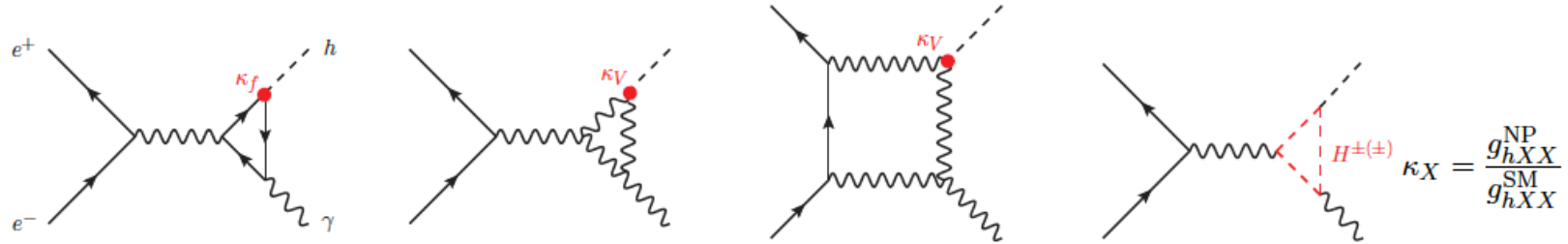
- One-loop induced  
→ Small cross section but sensitive to new physics!



We investigate how much new physics effects can enhance the production rate using H-COUP

- 2HDMs
- Inert doublet model (IDM)
- Inert triplet model (ITM)

# 3 benchmark extended Higgs models



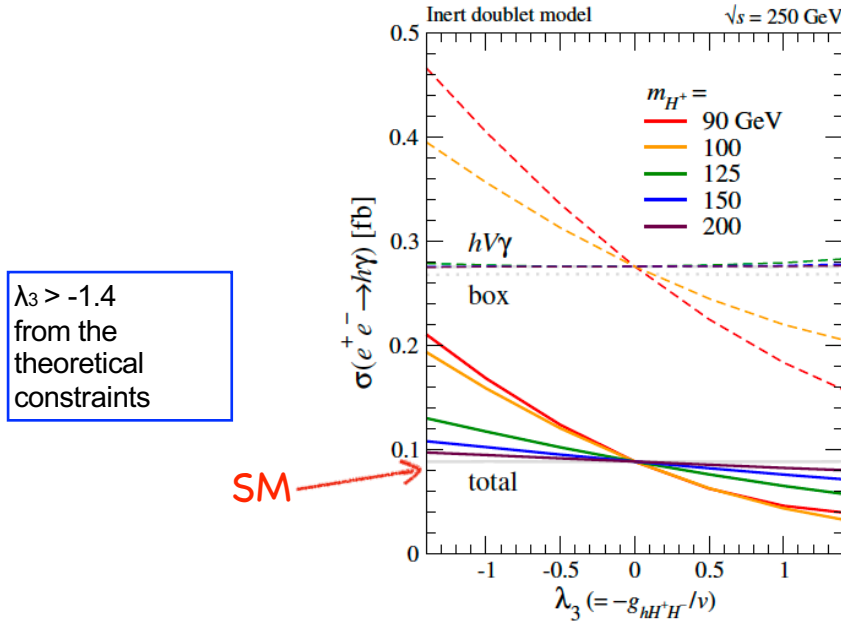
	$\mathcal{K}_f (=t)$	$\mathcal{K}_V$	$H^+$	$H^{++}$
IDM (Inert doublet model)	1	1	○	×
ITM (Y=1) (Inert triplet model)	1	1	○	○
THDM (Two Higgs doublet model)	$S\beta - \alpha - C\beta - \alpha / t\beta$	$S\beta - \alpha$	○	×

We employ H-COUP program to compute the loop amplitudes in each model

H-COUP v1: Kanemura, Kikuchi, Sakurai, Yagyu [1710.04603, CPC]  
 H-COUP v2: Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu [1910.12769]

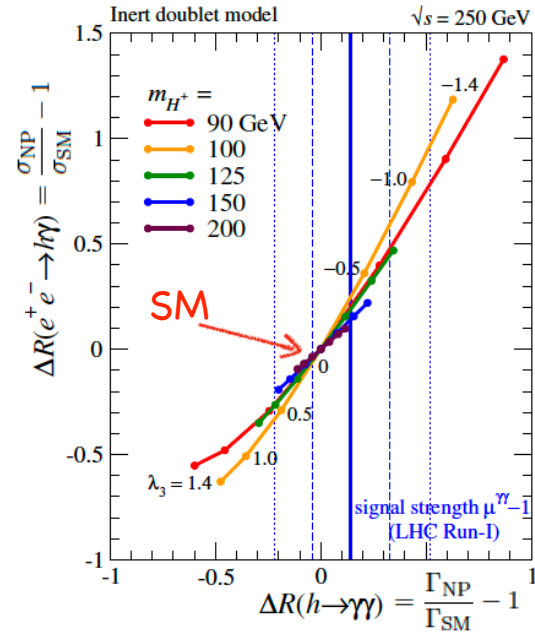
# $e^+e^- \rightarrow h \gamma$ in the IDM

$$V = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \lambda_5 \{ (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \}$$



$\lambda_3 > -1.4$   
from the  
theoretical  
constraints

Lighter  $H^+$  with negative  $\lambda_3$  can enhance the production rate

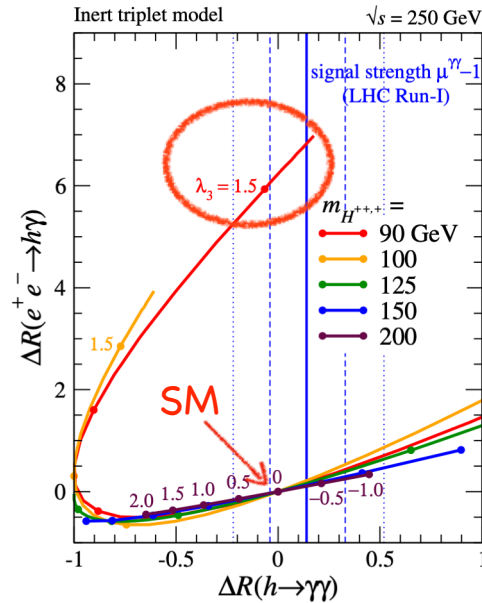


Strong positive correlation with  $h \rightarrow \gamma\gamma$   
 $\Rightarrow$  Constrained for lighter  $H^+$  by LHC

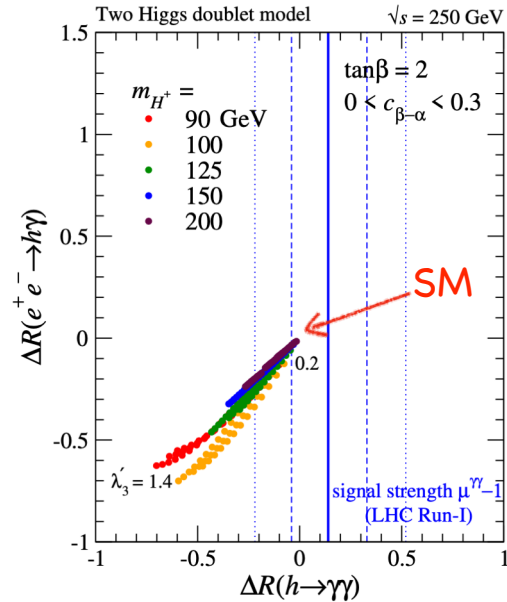
# $e^+e^- \rightarrow h\gamma$ in the ITM and the THDM

$$V = \mu_1^2 |\Phi|^2 + \mu_2^2 \text{Tr}[\Delta^\dagger \Delta] + \frac{1}{2} \lambda_1 |\Phi|^4 + \frac{1}{2} \lambda_2 (\text{Tr}[\Delta^\dagger \Delta])^2 + \lambda_3 |\Phi|^2 \text{Tr}[\Delta^\dagger \Delta] + \frac{1}{2} \lambda_4 \text{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

$$V = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \{ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \}$$



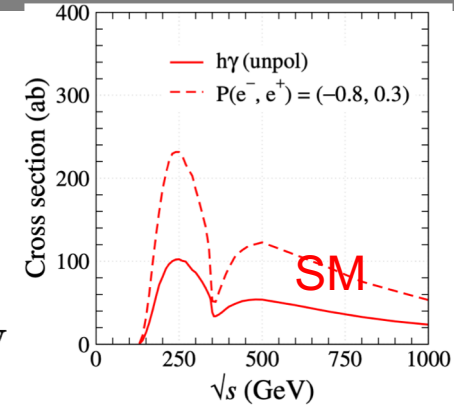
We can find a particular parameter region where the  $h\gamma$  production significantly, but the  $h \rightarrow \gamma\gamma$  decay still remains at in the SM



Qualitative behaviours are similar to the case with the IDM but the enhanced parameter region is excluded by the theory constraints

# Summary for $h\gamma$ production at $e^+e^-$ colliders

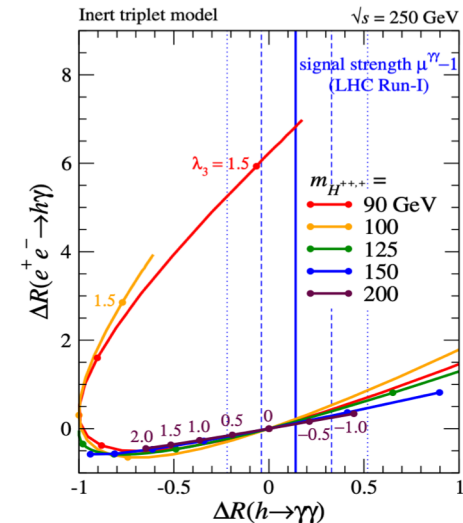
- In SM, the cross section peaked around  $E = 250$  GeV
- Beam polarization can enhance the cross section
- The signal is clean: a monochromatic photon  $E_\gamma = \frac{\sqrt{s}}{2} \left(1 - \frac{m_h^2}{s}\right) \sim 93.8 \text{ GeV} @ \sqrt{s} = 250 \text{ GeV}$
- One-loop induced  
→ Small cross section but **sensitive to new physics!**



We studied **IDM, ITM, 2HDM** as bench mark models using H-COUP

Light charged scalars can enhance the event rates by a factor 2 under current theoretical and experimental constraints

For ITM, thanks to doubly charged scalars, a particular parameter region is found where the  $h\gamma$  production significantly increases, but  $h \rightarrow \gamma\gamma$  does not.



See also [Kanemura, Mawatari, Sakurai Phys. Rev. D99 \(2019\) 03523](#) for details

**[3] Leading two-loop corrections to the  $hhh$  couplings in non-minimal Higgs model**

Johannes Braathen and Shinya Kanemura

*PLB 796 (2019) 38-46 & EPJC 80 (2020) 3, 227*



# Intro: Why investigate the Higgs trilinear coupling $\lambda_{hhh}$ ?

- **To probe the shape of Higgs potential and nature of EWPT:**

- The Higgs is responsible for EWSB, but we don't know the shape of its potential away from the EW vacuum  $\rightarrow$  this is determined by  $\lambda_{hhh}$
- In turn,  $\lambda_{hhh}$  determines the strength of the EWPT (crucial e.g. for EW Baryogenesis scenario)

- **To distinguish aligned scenarios with or without decoupling:**

2HDM scenarios with alignment (i.e. SM-like tree-level Higgs couplings) seem strongly favoured by experimental results  $\rightarrow$  why?

- One (boring) explanation is **decoupling**, i.e. BSM states are out of reach.
- Instead if there is **alignment without decoupling**, Higgs boson properties, e.g.  $\lambda_{hhh}$ , might **deviate significantly** from SM predictions because of large **non-decoupling effects** (NDE) in BSM loops!
- Large NDE are known to appear in 2HDM at one loop [Kanemura, Okada, Senaha, Yuan '04]  $\rightarrow$  **what happens at two loops?** Does pert. theory break down? Are huge new effects possible?
- Currently there is still room for large deviations in  $\lambda_{hhh}$ :  $-3.7 < \lambda_{hhh} / (\lambda_{hhh})^{\text{SM}} < 11.5$  (95% CL) [ATLAS-CONF-2019-049]  $\rightarrow$  but potential deviations can be **probed at future colliders!**

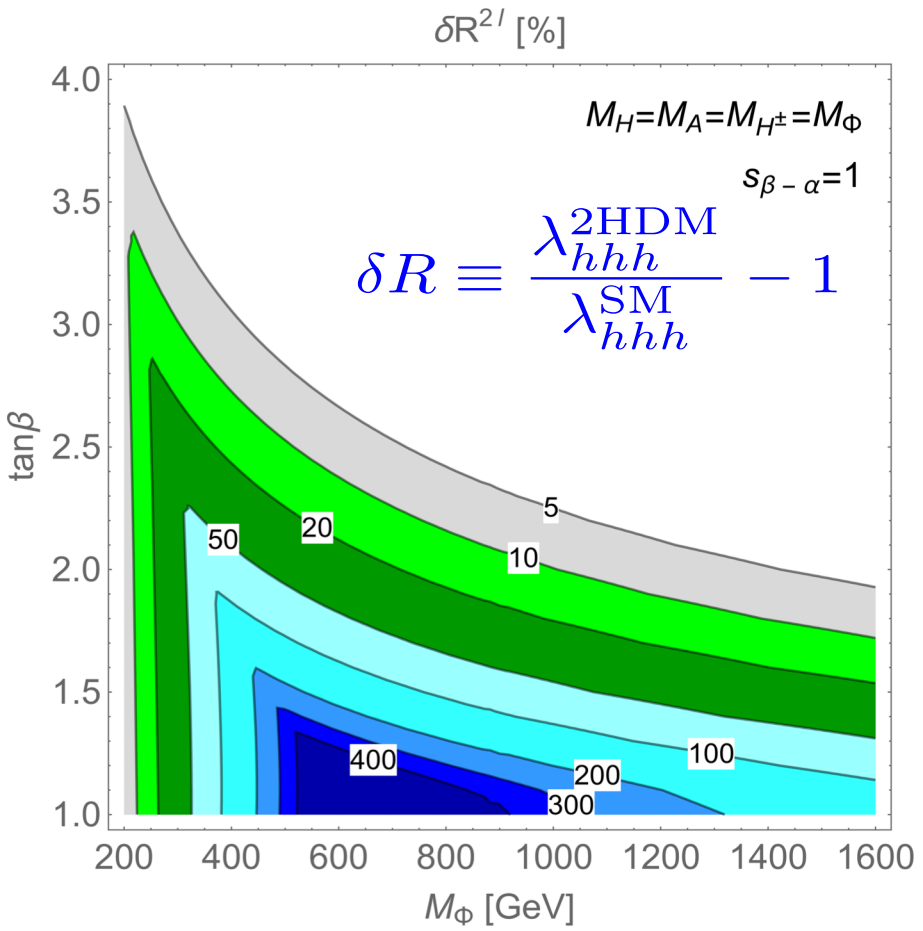
# Setup of our two-loop calculation for the 2HDM

- We consider a CP-conserving 2HDM, with (softly broken)  $Z_2$  symmetry to avoid tree-level FCNCs
- 5 physical Higgs mass eigenstates:  $h, H$ : CP-even;  $A$ : CP-odd;  $H^\pm$ : charged
- BSM scalars ( $\Phi = H, A, H^\pm$ ) in 2HDM have **2 sources of mass**:
  - (1) Higgs VEV  $v$
  - (2) soft  $Z_2$  breaking scale  $M$
$$\left. \begin{array}{l} (1) \text{ Higgs VEV } v \\ (2) \text{ soft } Z_2 \text{ breaking scale } M \end{array} \right\} \Rightarrow M_\Phi^2 = M^2 + \lambda_\Phi v^2$$

( $\lambda_\Phi$ : some combination of quartic couplings)
- We compute the **dominant two-loop corrections**, from top quark and additional BSM scalars ( $\Phi=H, A, H^\pm$ )
  - Effective potential approx.  $\rightarrow$  neglect sub-leading external momenta effects
  - Neglect sub-leading effects from light fermions, gauge bosons, and light scalars
  - Alignment limit ( $\sin(\beta-\alpha)=1$ )  $\rightarrow$  evade/relax experimental constraints

# Example of results: maximal possible BSM corrections in the 2HDM

- We plot the maximal deviation  $\delta R$  at (1+2) loops, allowed under criterion of tree-level unitarity
- Max. deviations for low  $\tan\beta$  and  $M_\phi \sim 600-800$  GeV  $\rightarrow$  BSM scalars acquire all their mass from the Higgs VEV and become heavy
  - 1 loop: up to  $\sim 300\%$  deviation at most
  - 2 loops: extra 100% (for same points)
- For increasing  $\tan\beta$ , unitarity constraints become more stringent  $\rightarrow$  smaller effects
  - **Blue region:** probed already at HL-LHC (which can reach 50% accuracy on  $\lambda_{hhh}$ )
  - **Green region:** probed at ILC (50% acc. at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)



# Summary for the hhh coupling at 2 loop

- **First two-loop calculation of  $\lambda_{hhh}$  in 2HDM**, in a scenario with alignment
- Two-loop corrections are typically 10-20% of one-loop contributions (max. 30%)
  - Non-decoupling effects found at one loop are not drastically changed.
  - Possible BSM deviations in  $\lambda_{hhh}$  are **accessible at future colliders**: HL-LHC (50% accuracy) and ILC (down to 10% at 1 TeV).
  - In the future perspective of precise measurements of  $\lambda_{hhh}$ , computing corrections beyond one loop will be necessary!
- Precise calculation of Higgs couplings (e.g.  $\lambda_{hhh}$ ) can allow **distinguishing aligned BSM scenarios with or without decoupling**, by accessing non-decoupling effects!

See also *Braathen and Kanemura, PLB 796 (2019) 38-46 & EPJC 80 (2020) 3, 227 for details*

# **[4] Aligned CP-violating 2HDM canceling the electric dipole moment**

Shinya Kanemura, Mitsunori Kubota and Kei Yagyu

*arXiv:2004.03943*

# Introduction to aligned CPV 2HDM

Baryogenesis requires the Sakharov's conditions:

1. B violation
2. C and CP violation
3. Departure from equilibrium.

Sakharov (1967); Kuzmin, Rubakov, Shaposhnikov (1985); Huet and Sather (1995); Kajantie, Laine, Rummukainen, Shaposhnikov (1996)

SM cannot satisfy them:

Kobayashi-Maskawa phase is numerically not sufficient.

Phase transition is not first-order getting out of equilibrium.

Extended Higgs model with extra CP-phases and modifications to Higgs potential can realize electroweak baryogenesis.

However, such a CP-phase is normally strongly constrained by the current data for the electric dipole moment (EDM).

We investigate a scenario with  $O(1)$  CP-phases in 2HDM but avoiding the EDM constraint

# Aligned CPV 2HDM and EDM

- general 2HDM

$$\mathcal{V} = -\mu_1^2 |\Phi_1|^2 - \mu_2^2 |\Phi_2|^2 - \left[ \mu_3^2 (\Phi_1^\dagger \Phi_2) + h.c. \right] + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4$$

$$+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \left\{ \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2) + \lambda_6 |\Phi_1|^2 + \lambda_7 |\Phi_2|^2 \right] (\Phi_1^\dagger \Phi_2) + h.c. \right\}$$

$$\mathcal{L}_Y = -\bar{Q}_L \left( \sqrt{2} \frac{M_d}{v} \Phi_1 + \rho_d \Phi_2 \right) d_R + \dots$$

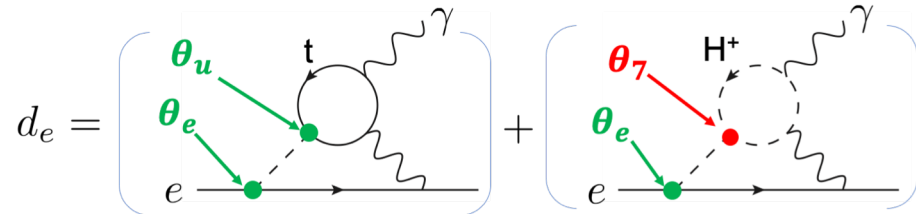
- We assume two kinds of alignment:

- **No mixing among the neutral Higgs bosons**, such that the lightest one is treated as  $h(125)$ . It leaves only one physical CPV-phase  $\theta_7 = \arg[\lambda_7] - \arg[\lambda_5]/2$  in the potential.
- **No off-diagonal element of Yukawa interaction** avoiding dangerous flavor changing neutral currents. It is realized by the following equation. Pich and Tuzon, PRD (2009)

$$\rho_d = \frac{\sqrt{2}}{v} M_d \zeta_d, \text{ where } \zeta_d = |\zeta_d| e^{i\theta_d}$$

$\rho_u$  and  $\rho_e$  are in a similar way.

- Remaining CP-phases,  $\theta_f$  ( $f=u, d, e$ ) and  $\theta_7$ , give the contributions to the electron EDM ( $d_e$ ) They can be destructive.



Barr and Zee, PRL (1990)

# Numerical Results for EDM

- We calculate the contour plot of the electron EDM  $d_e$  as a function of  $\theta_u$  and  $\theta_7$ .

- Input:

$$m_{H^+} = 230[\text{GeV}]$$

$$|\zeta_u| = 0.01, |\zeta_d| = 0.1, \quad m_{H_2^0} = 280[\text{GeV}]$$

$$|\zeta_e| = 0.5, |\lambda_7| = 0.3 \quad m_{H_3^0} = 230[\text{GeV}]$$

- Output:

— :  $d_e = 0$

- - -: Latest EDM bound

- Area between the green dashed lines is allowed.

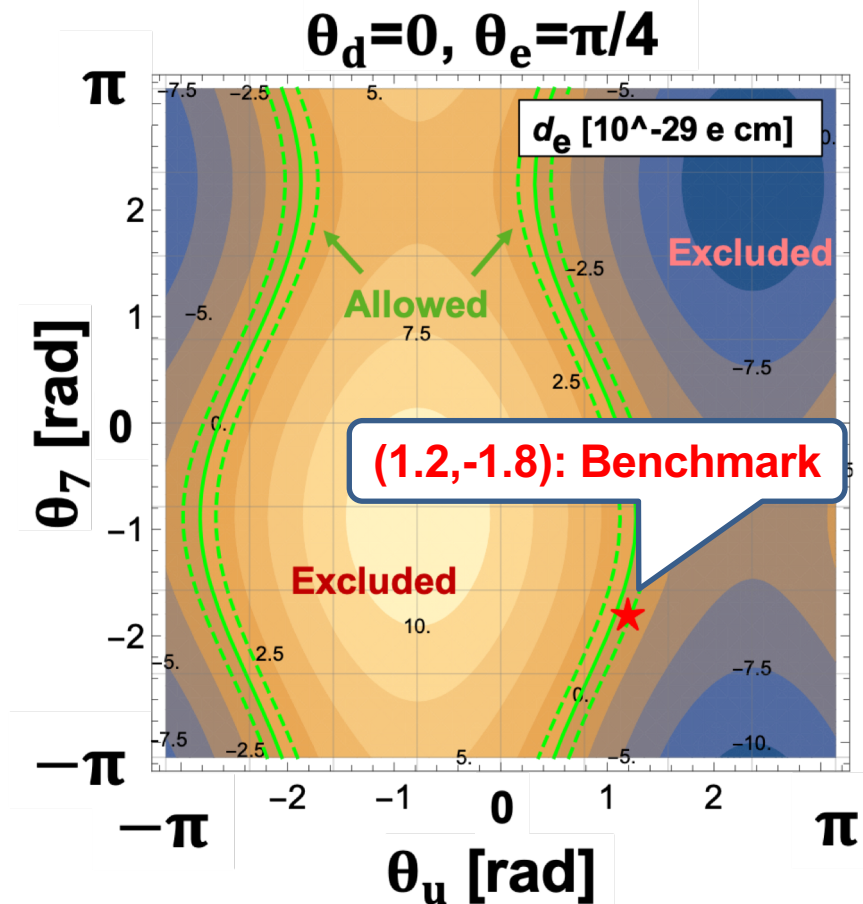
- At the Benchmark point (1.2, -1.8):

$$d_e = -0.96 [10^{-29} \text{ e cm}]$$

$$|d_e^{\text{exp.}}| < 1.1 [10^{-29} \text{ e cm}]$$

[ACME collaboration, Nature (2018)]

- CP-phases can be O(1) without large electron EDM.





# Summary for CPV 2HDM cancelling EDM

We discussed CPV 2HDM cancelling the EDM

EDM data can be satisfied by the destructive interference between the contributions from multiple  $O(1)$  CP-phases in the Yukawa sector and in the Higgs sector.

Such  $O(1)$  CPV phases can be the source of the baryon number asymmetry of the Universe.

Note: In our paper [arXive: 2004.03943], we also obtain the following conclusions:

CP-phases can be tested by the angular distributions of decays of extra Higgs bosons.

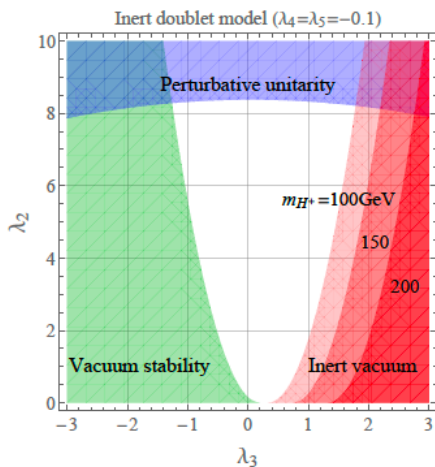
Destructive interference can be conserved until the high scales at least  $O(10^7)$  GeV, so stable.

Buck-up slides

# Supplement: Theoretical constraints

## IDM

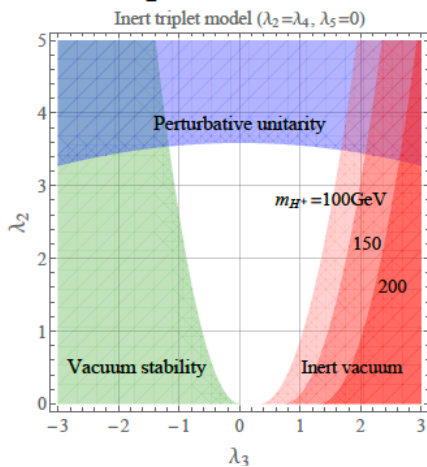
$$\begin{aligned}
 V &= \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 \\
 &+ \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 \\
 &+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\
 &+ \frac{1}{2} \lambda_5 \{ (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \}
 \end{aligned}$$



$$\begin{aligned}
 g_{hH^+H^-} &= -\frac{2}{v} (m_{H^+}^2 - \mu_2^2) \\
 &= -v \lambda_3
 \end{aligned}$$

## ITM

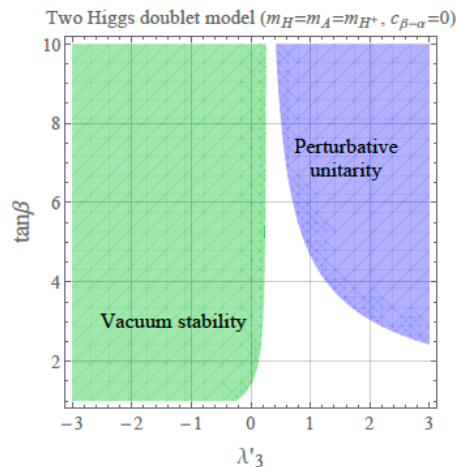
$$\begin{aligned}
 V &= m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 \\
 &- m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 &+ \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 \\
 &+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\
 &+ \frac{1}{2} \lambda_5 \{ (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \}
 \end{aligned}$$



$$\begin{aligned}
 g_{hH^+H^-} &= -\frac{2}{v} (m_{H^+}^2 - \mu_2^2) = -v \lambda_3 \\
 g_{hH^+H^-} &= -\frac{2}{v} \left\{ \frac{1}{2} (m_{H^+}^2 + m_H^2) - \mu_2^2 \right\} \\
 &= -v \left( \lambda_3 + \frac{1}{2} \lambda_5 \right)
 \end{aligned}$$

## 2HDM

$$\begin{aligned}
 V &= m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 \\
 &- m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 &+ \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 \\
 &+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\
 &+ \frac{1}{2} \lambda_5 \{ (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \}
 \end{aligned}$$



$$\begin{aligned}
 g_{hH^+H^-} &= -\frac{2}{v} \left\{ (m_{H^+}^2 - M^2 + \frac{1}{2} m_h^2) s_{\beta-\alpha} \right. \\
 &\quad \left. - (M^2 - m_h^2) \cot 2\beta c_{\beta-\alpha} \right\} \\
 &\equiv -v \lambda_3'
 \end{aligned}$$