Theory Overview:
CLFV at muon scale

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# Lepton Flavor in SM

## The Standard Model

### Lepton

<table>
<thead>
<tr>
<th>1st generation</th>
<th>2nd generation</th>
<th>3rd generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e^{L}$</td>
<td>$\nu_\mu^{L}$</td>
<td>$\nu_\tau^{L}$</td>
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<tr>
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### Quark

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### Chiral Theory
- No RH neutrino
- $L \leftrightarrow R$ nonsymmetric

- Lepton Flavor ($e$-like, $\mu$-like, $\tau$-like)
- is exact symmetry and conserved

- +Higgs boson $H$
Lepton Flavor in SM

Conserved “Charge” resulting from massless neutrinos

Electron, muon, tau number

\[ \begin{array}{ccc}
L_e & L_\mu & L_\tau \\
\hline
\bar{E} & \nu_e & \mu & \nu_\mu & \tau & \nu_\tau \\
L_e & 1 & 1 \\
L_\mu & 1 & 1 \\
L_\tau & 1 & 1 \\
\end{array} \]

Opposite (-1) for anti particles

Example of conservation

\[ \pi^- \rightarrow \mu \quad \bar{\nu}_\mu \]

\[ L_\mu \quad 0 = 1 + (-1) \]
Derivation of lepton flavor charge

Lepton Part Only

\[ L_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad e_{Ri}, \quad (i = 1, 2, 3) \]

Kinetic Part

\[ \mathcal{L}_k = \bar{L}_i i \mathcal{D}_L L_i + \bar{e}_{Ri} i \mathcal{D}_R e_{Ri} \]

\[ D_{L\mu} = \begin{pmatrix} \partial_{\mu} + \frac{i}{2} g_1 B_{\mu} - g_2 \frac{i}{2} W_{\mu}^0, \\ g_2 \frac{i}{\sqrt{2}} W^-_{\mu}, \quad \partial_{\mu} + \frac{i}{2} g_1 B_{\mu} + \frac{i}{2} g_2 W^0_{\mu} \end{pmatrix} \]

\[ D_{R\mu} = \partial_{\mu} + ig_1 B_{\mu} \]

\[ \mathcal{L}_k = \mathcal{L}_{k,\text{diag}} + \mathcal{L}_{k,W} \]

\[ \Phi_j = \{ \nu_L, e_L, e_R \} \]

Sum of 3 species of Weyl spinors

Invariant under 3 independent unitary transformation

\[ U_l \quad , \quad l = \nu_L, \quad e_L, \quad e_R \quad 3 \times 3 \quad \text{Unitary Matrix} \]

\[ l \rightarrow U_l l \quad (e_{Li} \rightarrow (U e_L e_L)_i) \quad U_l \quad \text{independent} \]
\[ \mathcal{L}_{kW} = ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma_\mu e_L + h.c. \]

To make it invariant, \( U_{\nu L} = U_{e L} \) is necessary. Reduction of symmetry

\[ 2 \text{ independent Unitary matrices} \]

Higgs Part

\[ \mathcal{L}_H = Y_{ij} \bar{L}_i e_{Rj} + h.c. \]

\((-1)^{ij} Y_{ij} \) \( 3 \times 3 \) complex :: diagonalized by 2 unitary matrices

\[ Y_{ij} \rightarrow Y_{diag} = \text{diag}\{y_e, y_\mu, y_\tau\} = U_L Y_{ij} U_R^\dagger \]

\[ L_\alpha \equiv U_{L\alpha i} L_i = \begin{pmatrix} U_{L\alpha i} \nu_L \\ U_{L\alpha i} e_L \end{pmatrix}, \quad e_{R\alpha} \equiv U_{Ri} E_{Ri}, \quad \alpha = e, \mu, \tau \]

\[ \mathcal{L}_H = Y_\alpha \bar{L}_\alpha e_{R\alpha} + h.c. \]

\[ = h^+ \left( y_e \bar{\nu}_e L e_R + y_\mu \bar{\nu}_L \mu_R + y_\tau \bar{\nu}_\tau L \tau_R \right) + h^0 \left( y_e \bar{e}_L L e_R + y_\mu \bar{\mu}_L \mu_R + y_\tau \bar{\tau}_L \tau_R \right) + h.c. \]

Definition of lepton flavor

Flavor eigenstate = charged lepton mass (=physical) eigenstate

Since \( U_{\nu L} = U_{e L} \) kinetic term is invariant

\[ \mathcal{L}_{k,dia} = \bar{\Phi}_\alpha i \partial \Phi_\alpha \quad \Phi_\alpha = \{ \nu_\alpha L, e_{\alpha L}, e_{\alpha R} \} \]

\[ \mathcal{L}_{kW} = ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_\alpha L \gamma_\mu e_{\alpha L} + h.c. \]

Kinetic terms under flavor basis !!
Residual symmetry : : Lepton Flavor

\[ \Phi_\alpha = \{ \nu_{L\alpha}, e_{L\alpha}, e_{R\alpha} \} \quad \alpha = e, \mu, \tau \]

Grouped with “same flavor”

→ Lagrangian is invariant under phase shift of each flavor

→ Lepton flavor conservation

\[ \{ e'_L, e'_R, \nu'_{eL} \} = \exp\{-i\theta_e\}\{ e_L, e_R, \nu_{eL} \} \]

Phase transformation of electron flavor

\[ \mathcal{L}'_{k,W} = i g_2 \frac{1}{\sqrt{2}} W^+_{\mu} \bar{\nu}_e \gamma_\mu e'_L + h.c. \]

\[ = i g_2 \frac{1}{\sqrt{2}} W^+_{\mu} \bar{\nu}_e e^{i\theta} \gamma_\mu e^{-i\theta} e_L + h.c = \mathcal{L}_{k,W} \]

From Noether’s theorem Conserved current exists

In each flavor the conserved current is given by

\[ j^\mu_\alpha = \bar{\nu}_{L\alpha} \gamma^\mu \nu_{L\alpha} + \bar{e}_{L\alpha} \gamma^\mu e_{L\alpha} + \bar{e}_{R\alpha} \gamma^\mu e_{R\alpha} \]

“Charge” is expressed as follows and it conserve

\[ Q_\alpha = \int d^3x j^0_\alpha \]
For example, \( \alpha = e \), that is, electron flavor charge is given in terms of creation and annihilation operators of electrons

\[
Q_e = L_e = \int d^3 p \sum_{l=\nu_{eL}, e_L, e_R} b_l^\dagger(p) b_l(p) - d_l^\dagger(p) d_l(p)
\]

\( b^\dagger b \) number operator for particle

\( d^\dagger d \) number operator for anti-particle

Electron and electron neutrino \( L_e = +1 \)

Positron and anti-electron neutrino \( L_e = -1 \)

Similarly, muon and tau flavor charge \( L_{\mu}, L_{\tau} \) is defined.
Lepton Flavor is conserved under SM

Electron, muon, tau number

$\mathbf{L}_e \quad \mathbf{L}_\mu \quad \mathbf{L}_\tau$

$\mathbf{e}^- \quad \mathbf{\nu}_e \quad \mathbf{\mu}^- \quad \mathbf{\nu}_\mu \quad \mathbf{\tau}^- \quad \mathbf{\nu}_\tau$

$\mathbf{L}_e \quad 1 \quad 1$

$\mathbf{L}_\mu \quad 1 \quad 1$

$\mathbf{L}_\tau \quad 1 \quad 1$

Opposite\((-1)\) for anti particles

If SM is correct, in all process, these numbers are conserved

Contrapositio

If non-conserved is found, SM is not correct
With additional particles and hence additional operator in Lagrangian, in general, under the transformation

\[
\{\alpha'_L, \alpha'_R, \nu'_{\alpha L}\} = \exp\{-i\theta_\alpha\}\{\alpha_L, \alpha_R, \nu_{\alpha L}\} \quad \alpha = e, \mu, \tau
\]

+ appropriate transformation for extra particles

Lagrangian is not invariant

→ Lepton flavor cannot be defined

→ Lepton flavor “charge” defined under SM Lagrangian cannot be conserved
Status of LFV with charged lepton

$$\mu^- \rightarrow e^- \gamma$$

$$L_\mu \quad 1 \quad = \quad 0 + 0$$

$$L_e \quad 0 \quad = \quad 1 + 0$$

No observation of CLFV

No doubt on SM from CLFV
τ decay

48 modes searched for, U.L.s around $\sim 10^{-8}$
2. Lepton flavor violation

\[ \{ \alpha'_L, \alpha'_R, \nu'_{\alpha L} \} = \exp \{ -i \theta_\alpha \} \{ \alpha_L, \alpha_R, \nu_{\alpha L} \} \quad \alpha = e, \mu, \tau \]

For LF to be defined, Lagrangian must be invariant under this transformation with appropriate transformation on extra particles

- If new Lagrangian is not invariant
- \(\rightarrow\) Lepton flavor cannot be defined
- \(\rightarrow\) Lepton flavor "charge" as defined under SM Lagrangian cannot be conserved

**Simplest example**

Neutrino mass term with RH neutrinos (SM singlets) \(\nu_{Ra} \) :: Dirac mass term

\[ \mathcal{L}_H^+ = \tilde{H} \tilde{Y}_{\alpha a} \tilde{L}_\alpha \nu_{Ra} + h.c \]

Flavor basis ± is fixed by charge lepton mass.

We cannot rotate lepton doublet.

If and only if nature choose \( Y_{\alpha i} \) to be diagonalized by rotating only \( \nu_{Ra} \quad \nu_{R\alpha} \equiv U_{R\alpha a} \nu_{Ra} \)

Then \( \mathcal{L}_H^+ = \tilde{H} \tilde{Y}_{\alpha} \tilde{L}_\alpha \nu_{R\alpha} \), and hence LF is defined as a result of

\[ \{ \alpha'_L, \alpha'_R, \nu'_{L\alpha}, \nu'_{R\alpha} \} = \exp \{ -i \theta_\alpha \} \{ \alpha_L, \alpha_R, \nu_{L\alpha}, \nu_{R\alpha} \} \]

Appropriate transformation ,,.., Lucky case
In general, we need by unitary transformation to get mass basis for neutrino
\[ \tilde{Y}_{\alpha a} \rightarrow \text{diag}\{\tilde{Y}_i\} = U_{NS}^{\dagger} \tilde{Y}_{\alpha a} U_R^{\dagger} \]

\[ \tilde{H} \tilde{Y}_{\alpha a} \bar{L}_\alpha \nu_{Ra} = h^0 \tilde{Y}_i \bar{\nu}_{Li} \nu_{Ri} + h^{-} \tilde{Y}_{\alpha i} \bar{\alpha}_L \nu_{Ri} \]

Diagonalized mass

In another words flavor basis do not coincide with mass basis \[ \nu_{L\alpha} \neq \nu_{Li} \]

Interaction with W boson
\[ W_\mu^+ \bar{\nu}_{\alpha L} \gamma_\mu e_{\alpha L} = W_\mu^+ \bar{\nu}_{Li} U_{NS}^{\dagger} \gamma_\mu e_{\alpha L} \]

Mass base mix with each other. ->Flavor changing processes appear seed of neutrino oscillation
Incidentally, dirac mass terms, in general, conserve

**lepton number**

\[ L_e + L_\mu + L_\tau = L \]

\[ \mu^- \rightarrow e^- \gamma \]

\[ L_1 = 1 + 0 \]

**Lepton number = A part of particle number**

\[ = (\text{particle} = 1 & \text{antiparticle} = -1) \]

c.f. Majorana mass term leads lepton number (in general particle number) violation

**neutrinoless double beta decay**

\[ \mu^- \rightarrow e^+ \]

not necessary neutrino majorana mass term
3. Neutrino oscillation

In SM, neutrino flavor is defined with paired charged lepton

Beta decay, electron is emitted (Le=1) = anti-electron neutrino emitted (Le=-1)

→ electron number 0=1+(-1)

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

\[ L_e \quad :: \quad 0 = 0 + 1 + (-1) \]

Neutrino oscillation

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]

If lepton flavor conserves,
Flavor = mass
Flavor states = particle states

creation as a whole

\[ \pi^- + p^+ \rightarrow \mu^- + \mu^+ + n \]

\[ L_\mu \quad :: \quad 0 + 0 \quad = \quad 1 + (-1) \]

detection
If neutrino is massive,

Neutrino oscillation

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\bar{\nu}_\beta = \sum \bar{\nu}_i$$

If massive,

Flavor states ` particle states

$$\bar{\nu}_\gamma \neq \bar{\nu}_\beta \ e^+ \ \text{if } \gamma = e$$

creation as a whole

$$\pi^- + p^+ \rightarrow \mu^- + e^+ + n$$

$$L_e :: 0 + 0 = 0 + (-1)$$

$$L_\mu :: 0 + 0 = 1 + (0)$$

LFV!! Logically SM is incorrect!!
To insist it is due to neutrino oscillation, more information has been accumulated. If neutrino is massive,

\[
\begin{align*}
\text{Flavor eigenstate} & \neq \text{Mass eigenstate} \\
\text{Interaction state} & = \text{Particle state}
\end{align*}
\]

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

**Propagation of neutrinos**: As a particle = mass eigenstate

**Creation of neutrinos**: week interaction accompanied with partner charged lepton

Multiple propagation of neutrinos

**Quantum interference** = Neutrino Oscillation
**Reactor Neutrino Example**

**electron neutrino is emitted**

\[
\nu^f(0) = \nu_e = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

incidentally

\[
\nu_\mu = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

At a distance \( L = t \) survival probability of \( \nu_e \)

\[
P_{\nu_e \rightarrow \nu_e} = |\nu_e^\dagger \nu^f(L)|^2 = 1 - \sin^2 2\theta \sin 2\frac{\delta m^2 L}{4E}
\]

\[
\rightarrow 1 - \frac{1}{2} \sin^2 2\theta
\]

\[
= \cos^4 \theta + \sin^4 \theta
\]

Quantum interference (oscillation)

Large \( \delta m^2 \) and/or Low \( E \)

Quantum effect disappear

Merely transition

Amplitude of this transition

\[
\cos^2 \theta
\]

Similarly \( \sin^2 \theta \) for 2\textsuperscript{nd} state

incidentally

\[
P_{\nu_e \rightarrow \nu_\mu} = |(0, 1)\nu^f(x)|^2 = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}
\]

Two flavor approximation
As a result,
We have to explain neutrino masses and lepton mixings → Lepton Flavor Violation

**Neutrino**

★ Neutral (Electromagnetic and color), real under SU(2)
★ Tiny mass (also mass pattern)

Neutral :: two types of mass term

Dirac : “partner” is necessary. It is neutral (= no charge) under SM
so-called Right-Handed (RH) neutrino $\nu_R$. Higgs doublet can be reused

$$y H \bar{\nu}_R L \supset m \bar{\nu}_R \nu_L$$

Majorana: self mass term. Within renormalizable, we need to introduce SU(2) triplet $\phi$

$$y \phi L^T C L \supset \nu_L^T C \nu_L$$

If nonrenormalizable, with cutoff without new particle

$$\frac{h_{ij}}{2\Lambda} (HL_i)(HL_j)$$

though, $\Lambda$ indicates new physics = new particle
For example, $\nu_R$ majorana mass can be new scale $\Lambda$. It is singlet under SM

$$M^T \mathbf{C} \nu_R$$

is allowed.

With Dirac mass term $m \mathbf{V} \nu_L$, we have mass term for neutral particle under EM

$$\left(\nu_L, \nu_R^c\right) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

Eigenstate values are neutrino masses. Especially $m \ll M$

$$\frac{-m}{M}, M$$

Seesaw

Particle states $\nu_L, \nu_R$

Gell-mann et al, Yanagida

$\Lambda$ is RH neutrino Majorana masses. Graphically

$$m_{\nu} = \frac{(100)^2}{10^{16}} \sim 10^{-12} \text{ GeV} \sim 10^{-3} \text{ eV}$$
$m \ll M$  Is not necessary. Different type of models

Another example = loop correction  Zee model · radiative seesaw

Krauss etal

Aoki etal

Majorana mass term for left-handed neutrinos

$$\frac{h_{ij}}{2\Lambda} (H L_i)(H L_j)$$
Majorana mass term "violates"
Not only Lepton Flavor
But also Lepton Number

Lepton number changing process

Example

\[(A, Z) \rightarrow (A, Z + 2) + 2e^-\]
\[(0\nu2\beta)\] decay

\[m_{ee} = U_{eh}^2 m_h = c_{13}^2 (m_1 c_{12}^2 + m_2 s_{12}^2 e^{2i\alpha}) + m_3 s_{13}^2 e^{2i\beta'}\]

Also \[\mu^- \rightarrow e^+\] in muonic atom
prediction from neutrino oscillation and constraint from Neutrinoless double beta decay

Two kinds of prediction from neutrino oscillation
Normal hierarchy (NH) and inverted hierarchy (IH)

Note
“Majorana mass for left-handed neutrino → neutrinoless double beta decay”
Always holds but conversion is not true!!

Example from SUSY

Lepton flavor violation and particle number violation has different origin !!!

Majorana mass term ` neutral
Majorana mass term = real representation, can be charged
Mohapatra 1986
4. Charged Lepton Flavor violation

Lepton Flavor is exact symmetry in SM as long as neutrinos are massless.

Charged Lepton Flavor Violation (cLFV) through Lepton Mixing in the neutrino oscillation.

But ...

\[ \text{BR}(\mu \rightarrow e\gamma) \sim \left( \frac{\delta m_{\mu}^2}{m_W^2} \right)^2 < 10^{-54} \]

Invisible, eternally

Strong suppression of FCNC by GIM.

Detection of the LFV signal: Clear evidence for beyond SM.
Indeed, in physics beyond SM,

**Large FCNC is expected**

Particularly Combining with neutrino oscillation

Large FCNC in charged lepton is expected

*(must appear ??)*

**e.g. a supersymmetric model**

Enhancement of LFV through the slepton mixing

Detectable at future experiments

Search for LFV with charged lepton is inevitable
cLFV from muon decay

Upper limit on Br

\[ \mu^+ \rightarrow e^+ \gamma < 4.2 \times 10^{-13} \]
\[ \mu^+ \rightarrow e^+ e^+ e^- < 1.0 \times 10^{-12} \]
\[ \mu^- \text{Ti} \rightarrow e^- \text{Ti} < 6.1 \times 10^{-13} \]
\[ \mu^- \text{Au} \rightarrow e^- \text{Au} < 7 \times 10^{-13} \]
\[ \mu^+ e^- \rightarrow \mu^- e^+ < 8.3 \times 10^{-11} \]

-W. J. Marciano, T. Mori, and J. M. Roney

Long history
cLFV from tau decay

Upper bound \(\sim \frac{1}{\# \text{ of taus}}\)
Effective operators for (muon)CLFV

\[ \mathcal{L}_{\mu^- e^- \rightarrow e^- e^- e^+} = \]
\[ - \frac{4G_F}{\sqrt{2}} \left[ m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} \mu^+ \rightarrow e^+ \gamma \right. \]
\[ + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R) (\bar{e}_L e_R) \]
\[ + g_3 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_R \gamma_\mu e_R) + g_4 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_L \gamma_\mu e_L) \]
\[ + g_5 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_L \gamma_\mu e_L) + g_6 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_R \gamma_\mu e_R) \]
\[ + (H.c.) \right], \]

\[ \mu^+ \rightarrow e^+ e^- e^+ \]
\[ \mu^- N \rightarrow e^- N \]
Effective operators for CLFV

A) Loop vs Tree

\[ \mu^+ \rightarrow e^+\gamma \quad \text{:: Loop only, dipole} \]

Gauge Symmetry forbids tree contribution

\[ \mu^+ \rightarrow e^+e^-e^+ \quad \text{:: Loop and Tree} \]

\[ \mu^- N \rightarrow e^- N \]

e.g. Loop = dipole + quark bilinear = \[ \mu^- N \rightarrow e^- N \]

\[ \sim \alpha \text{ smaller than } \mu \rightarrow e\gamma\]

Tree :: singlet particle is necessary for conversion!

Charge 2 is OK for \( \mu \rightarrow 3e \)
Leptoquark is OK for \( \mu \rightarrow e \)
We can parameterize the relative strength

$$\mathcal{L} = \frac{1}{1 + \kappa} \frac{m_\mu}{\Lambda^2} \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + \frac{\kappa}{1 + \kappa} \frac{1}{\Lambda^2} \left( \bar{\mu}_L \gamma^\mu e_L \right) \left( \bar{q}_L \gamma_\mu q_L \right)$$

$$\mathcal{K}_\mu \sim \alpha \ : \ dipole \ type, \ say \ SUSY \ with \ R \ parity$$

In general, Model Dependent
B) Vector vs Scalar

cLFV is mediated by new particle(s)

Vector Boson ::

Boson with broken gauge

So-called Z’ Model, Extra U(1) from SO(10) GUT

Kaluza-Klein mode of gauge

Higher dimensional models have massive modes of gauge bosons

Scalar Boson ::

From symmetry = SUSY

Extension of Higgs :: more 2plet, 3plet for nu mass

Explanation for new physics
Example from Vector type interaction

If Vector boson has no charge

\[ \mu^+ \rightarrow e^+e^-e^+ \quad \text{and} \quad \mu^- N \rightarrow e^- N \]

can occur at tree level

in a wide sense $Z'$ model

\[ \mu^+ \rightarrow e^+\gamma \quad \leftrightarrow \quad \mu^- N \rightarrow e^- N \]

irrelevant
cLFV Interaction

\[
\frac{gZ'}{\sin \theta_W} \bar{l_i} Q_{ij} \gamma^\mu l_j Z'_\mu
\]

Different Q’s!!

Table 1: The cLFV experiments and the corresponding $Z'$ charges probed at lowest order pro-

Figure 2: $\mu^+ \rightarrow e^+ \gamma$ in the $Z'$ models.

Figure 3: Non-photonic diagram of $\mu^- - e^-$ conversion in the $Z'$ models.
Figure 4: Constraint of $Z'$ by the current search for $\mu^+ \rightarrow e^+\gamma$.

$$B(\mu \rightarrow e\gamma) = 1.3 \times 10^{-13} \left( \frac{g_x}{g_Y} \right)^4 \left( \frac{Q_{13}Q_{23}}{10^{-5}} \right)^2 \left( \frac{1\text{TeV}}{m_{Z'}} \right)^4$$
10TeV

Figure 5: Constraint of $Z'$ by the current search for $\mu^- - e^-$ conversion.

$$B(\mu N \rightarrow e N) = 3.1 \times 10^{-11} \left( \frac{g_{Z'}}{g_Y} \right)^4 \left( \frac{Q_{12}}{10^{-5}} \right)^2 \left( \frac{1 \text{ TeV}}{m_{Z'}} \right)^4$$
Direct Search at LHC, excluded $< 3\text{TeV}$
Example from Vector type interaction

If R parity is broken,

\[ W_{RPV} = \frac{\lambda_{ijk}}{2} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i D_j D_k + \mu'_i L_i H_u \]

Tree contribution may dominate for $\mu - e$ conversion

**Leptoquark**

While $\mu \rightarrow e + \gamma$

Induced by loop

**distinction of models**

Andre` de Gouve^a, Smaragda Lola, and Kazuhiro Tobe

<table>
<thead>
<tr>
<th>( \lambda'<em>{121} \lambda'</em>{221} )</th>
<th>( \text{MSSM with } \nu_R )</th>
<th>( \frac{\text{Br}(\mu \rightarrow e \gamma)}{\text{Br}(\mu \rightarrow 3e)} )</th>
<th>( \frac{\text{R}(\mu \rightarrow e \text{ in } T_i)}{\text{Br}(\mu \rightarrow 3e)} )</th>
<th>( A_p )</th>
<th>( A_{p_1} )</th>
<th>( A_{p_2} )</th>
<th>( A_{p_1} / A_{p_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.6 \times 10^2</td>
<td>2 \times 10^5</td>
<td>0.92</td>
<td>-100%</td>
<td>-26%</td>
<td>-5%</td>
<td>5.6</td>
</tr>
</tbody>
</table>
Orthodox scenario

Source of LFV

Slepton mixing

CMSSM + RH neutrino

Most exhaustedly studied

\[ W = f_{\nu}^i \beta \bar{N}_i L_{\beta} H_u \]

\[ (m_{L}^2)_{\alpha}^\beta \simeq \frac{(6 + a_0^2)m_0^2}{16\pi^2} (f_{\nu}^i f_{\nu})_{\alpha} \beta \log \frac{M_G}{M_R} \]

\[ \simeq \frac{(6 + a_0^2)m_0^2}{16\pi^2} U^\text{Dirac}_{\alpha k} (U^\text{Dirac}^*)_{\beta k} |f_{\nu_k}|^2 \log \frac{M_G}{M_R} \]

\[ \tilde{\chi}_A^0 \]

\[ l_\beta \rightarrow \tilde{\ell}_X \rightarrow \tilde{\ell}_X \rightarrow l_\alpha \]

Dipole dominant
1207.7227 Calibbi et al
5. Summary

Lepton Flavor

- Exact Symmetry in the Standard Model
  - If SM is correct then LF conserves
  - $\leftrightarrow$
  - LFV then SM is not correct

Neutrino Oscillation

- Manifestation of Lepton Flavor Violation
  - $\rightarrow$
  - SM must be extended so that neutrinos are massive

Neutrino masses

- Dirac or Majorana? Tree or Induced?
- If Majorana $\rightarrow$ Lepton Number is also violated
  - neutrinoless double beta decay, $\mu^- \rightarrow e^+$ in muonic atom
Charged Lepton Flavor Violation

SU(2) connection indicates LFV in Charged lepton
Clean signal for Physics beyond the Standard Model
Not observed yet though many searches have been done
Muon decay, Tau decay, LFV in final state (decay product)

Classification of new physics
Tree vs Loop : $\mu^+ \rightarrow e^+\gamma$ : always loop effect
Scalar vs Vector

Model dependence

Most precise measurements with muon
$\mu^+ \rightarrow e^+\gamma$ and $\mu^- \rightarrow e^-$

Which one will be observed first?
Example of model dependent analysis with other signals
Back Up
Connection among CLFVs
an Example

J.S & M. Yamanaka
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MEGII experiment updates/discovers(?)
\[ \mu^+ \rightarrow e^+ \gamma \]
COMET/DeeMe/Mu2E will discover(?)
\[ \mu^- \rightarrow e^- \]
In near future

Sensitivity is same.
If COMET find CLFV first then ...?
μ-β conversion and then?

If μ-β conversion is found, while other cLFV processes will never be found.

E.g.
R-parity violating SUSY gives such a situation.

Tree contribution for CLFV
Scalar/Vector with LFV
Direct coupling with qq and μ-β

☑ No correlations among cLFV
☑ How to confirm the scenario?
Aim of this work

To find out distinctive signals to discriminate the scenario and other new physics models

To show the feasibility to determine the parameters in the RPV scenario through observing the signals

How to confirm a model?
R-parity violating SUSY

- Candidate of new physics: R-parity violating SUSY

- Consistent with experimental/theoretical status
  - New physics is required to cancel Higgs quadratic divergence
  - TeV scale SUSY predicts grand unification of interactions
  - So far no typical SUSY signals have been observed

- RPV terms in superpotential in SUSY

\[ \mathcal{W}_R = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \]

Offers LFV Scalar

Omit the term to avoid proton decay
Framework of our scenario

- Naturally realized by RG evolution with universal masses @ GUT scale

- Slepton contribution to RPV: only 3rd generation

- Different generation of left- and right-handed leptons
  \[ \lambda_{ijk} \ (i \neq k \text{ and } j \neq k) \]

- Assumption to realize the interesting situation

- RPV terms in superpotential in SUSY

\[ \mathcal{W}_R = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \]
Framework of our scenario

- For quarks, flavor diagonal components are much larger than off-diagonal components

\[ \lambda'_{ijj} \gg \lambda'_{ijk} \quad (j \neq k) \]

- RPV terms in superpotential in SUSY

\[ \mathcal{W}_R = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k - \lambda''_{ijk} U^c_i D^c_j D^c_k \]
Exotic processes in the scenario

\[ \mathcal{L}_{\text{RPV}} = 2 \left\{ \lambda_{312} \tilde{\nu}_\tau \tilde{\mu}_R e_L + \lambda_{321} \tilde{\nu}_\tau \tilde{e}_R \mu_L + \lambda_{132} \tilde{\tau}_L \tilde{\mu}_R \nu_e + \lambda_{231} \tilde{\tau}_L \tilde{e}_R \nu_\mu \right\} \\
+ \left\{ \lambda'_{311} (\tilde{\nu}_\tau \tilde{d}_R d_L - \tilde{\tau}_L \tilde{d}_R u_L) + \lambda'_{322} (\tilde{\nu}_\tau \tilde{s}_R s_L - \tilde{\tau}_L \tilde{s}_R c_L) \\
+ \lambda'_{333} (\tilde{\nu}_\tau \tilde{b}_R b_L - \tilde{\tau}_L \tilde{b}_R t_L) \right\} + \text{h.c.} \]

- \( \mu\text{-}\epsilon \) conversion@tree level
- Negligible rates of other cLFV processes
Current bound for the scalar with LFV
Correlations of distinctive signals

Contour plot of

- $\text{BR}(\mu^- + N \to e^- + N)$
- $\sigma(pp \to \mu\bar{e})$
- $\sigma(pp \to jj)$

- sneutrino mass $m_{\tilde{\nu}_\tau} = 1\,\text{TeV}$
- collision energy $\sqrt{s} = 14\,\text{TeV}$

☑ $\mu$-$\bar{e}$ conversion search is a strong tool for exploring RPV

☑ PRISM explores all parameter space wherein LHC can survey
Correlations of distinctive signals

Contour plot of

- $\text{BR}(\mu^- + N \rightarrow e^- + N)$
- $\sigma(pp \rightarrow \mu \bar{e})$
- $\sigma(pp \rightarrow jj)$

- sneutrino mass $m_{\tilde{\nu}_\tau} = 1\text{TeV}$
- collision energy $\sqrt{s} = 14\text{TeV}$

- COMET/DeeMe found m–e conversion white band
Correlations of distinctive signals

Contour plot of

\[ \text{BR}(\mu^- + N \rightarrow e^- + N) \]
\[ \sigma(pp \rightarrow \mu\bar{e}) \]
\[ \sigma(pp \rightarrow jj) \]

- sneutrino mass \( m_{\tilde{\nu}_\tau} = 1 \text{TeV} \)
- collision energy \( \sqrt{s} = 14 \text{TeV} \)

- COMET/DeeMe found m–e conversion white band
- Dijet resonance is found with 10 fb\(^{-2}\) white small region
Correlations of distinctive signals

Contour plot of

- BR($\mu^- + N \rightarrow e^- + N$)
- $\sigma(pp \rightarrow \mu\bar{e})$
- $\sigma(pp \rightarrow jj)$

- sneutrino mass $m_{\tilde{\nu}_\tau} = 1$ TeV
- collision energy $\sqrt{s} = 14$ TeV

- $\mu\bar{e}$ resonance is found with 10 fb$^{-4}$ blue star point
- J–PARC and LHC precisely determine the RPV parameters!
More on coupling discrimination

- **Non Standard Interaction**

Pion decay in scalar channel – chiral enhancement

Exotic decay

\[ \pi^+ \rightarrow \mu^+ \nu_e \]

\[ \varepsilon^S_{\mu e} = \sqrt{2} \frac{m^2_\pi}{m_\mu m_e} \frac{\lambda^*_3 \lambda_1}{G_F m^2_\tau} \]

312 : LH electron only

- **ILC with polarization**

LHC signal is same for 312(LH e) and 321 (RH e)

Can you distinguish them?
Scalar type

SUSY :: Still main target!?

2< doublet higgs :: SUSY is restricted version

Higgs triplet :: doubly charged

Radiative generation of neutrino masses

Krauss et al

sometimes doubly charged

\[ \mu^+ \rightarrow e^+ e^- e^+ \]

Is more relevant
**SUSY**

Neutral scalar : Heavy neutral higgs , sneutrino

**With R-Parity**

Scalar (Higgs) can contribute at tree level

Naïve 2< doublets, this coupling can be large, though...

In SUSY , slepton mixing must be contributed , that is, the couplings has same or less magnitude as dipole

Furthermore, these higgsses are probably very heavy