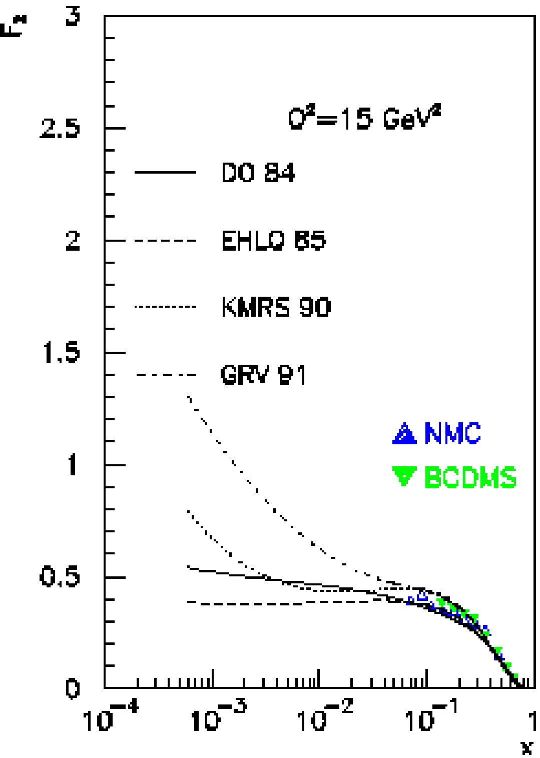
Some studies of HERA I+II combined data at low-x/low Q^2

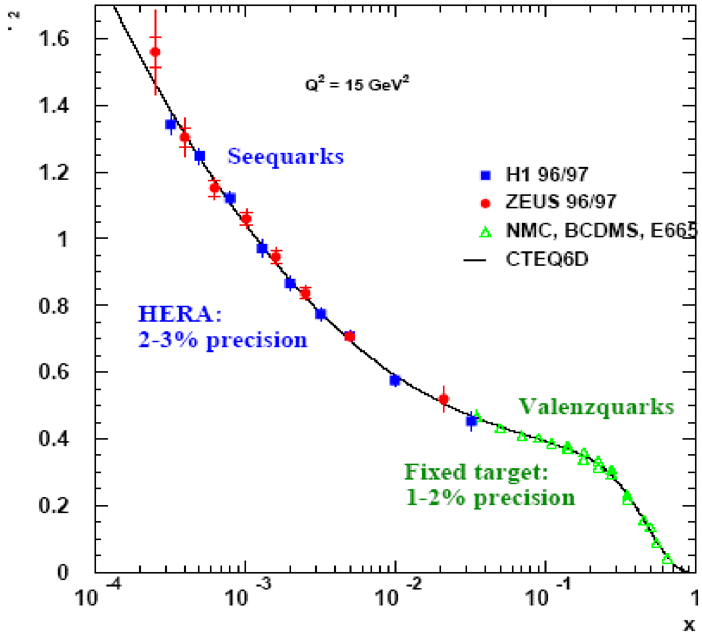
Higher twist : I Abt, A Cooper-Sarkar, B Foster, V Myronenko, K Wichmann, M Wing, arXIV:1604.02299 called HHT

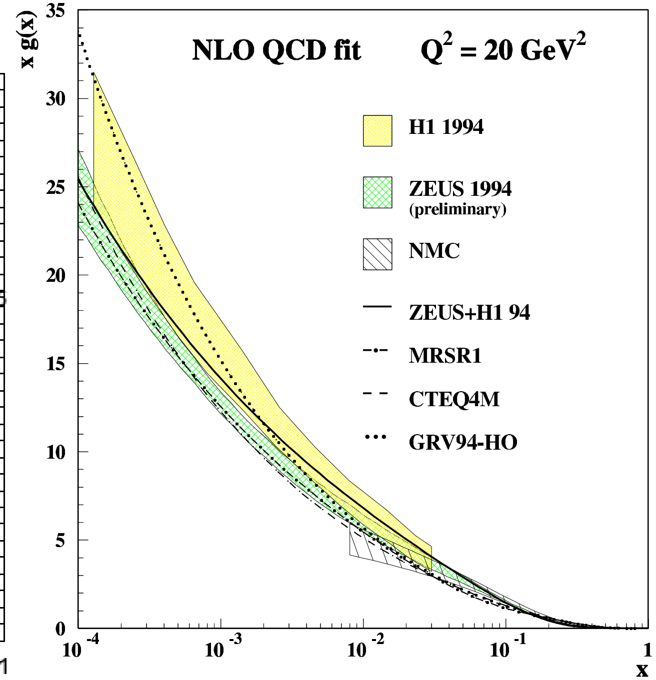
Ln(1/x) resummation: xFitter group, arXIV:1802.00064

Very low Q^2 with HERA combined data: , arXiv:1704.03187 same authors as HHT

Let us look at low-x physics at HERA







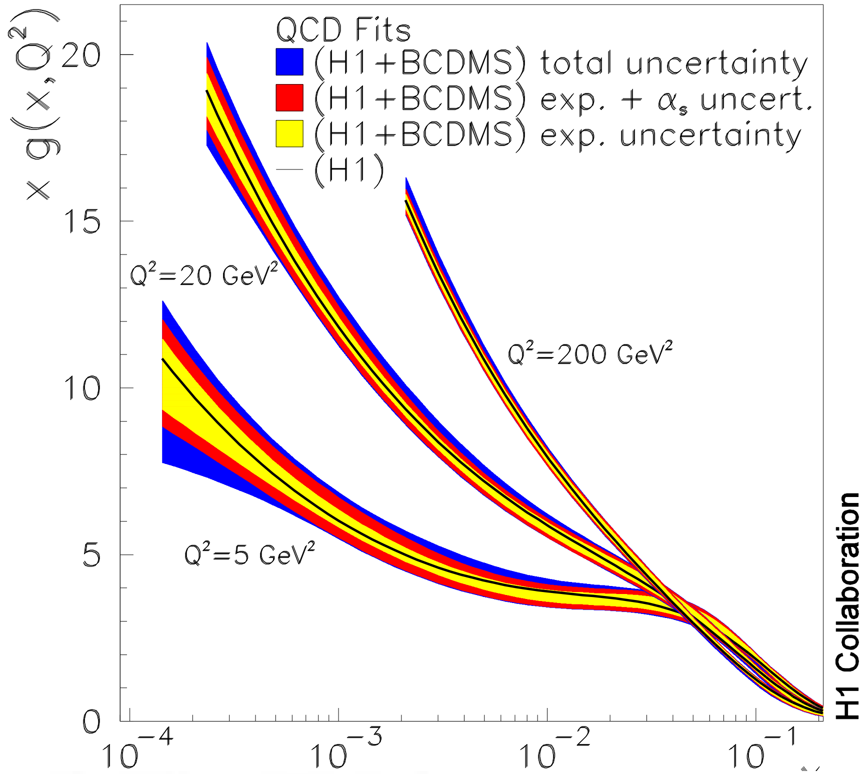
Before the HERA measurements many of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong – most theoreticians expected it to flatten out. It actually rises steeply

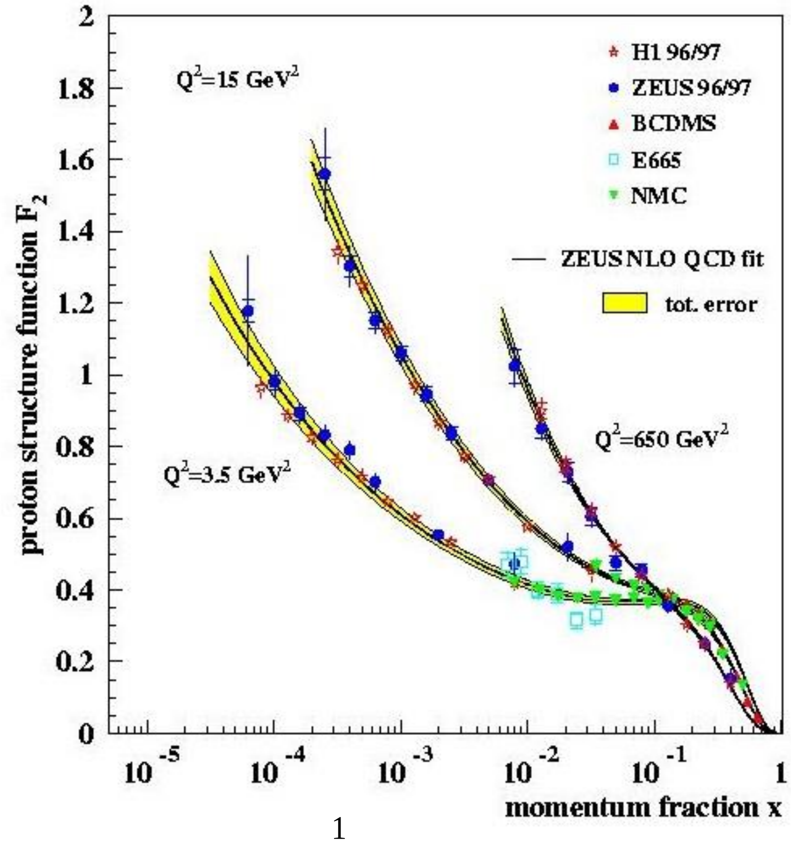
AND YET—DGLAP does predict the rise that we saw!

Now it seems that the conventional DGLAP formalism works TOO WELL at low Q^2/low-x

(we think there **should be** $\ln(^1/_x)$ corrections and/or non-linear high density corrections for $x < 5 \times 10^{-3}$)

Low-x





$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[\Sigma_q Pgq(z)q(y,Q^2) + Pgg(z)g(y,Q^2)\right]$$

At small x,

small z=x/y

$$Pgq \to \frac{C_F}{z}, \quad Pgg \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda g}$

$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0 \ln(1/x)}\right)^{\frac{1}{2}},$$

$t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

A flat gluon at low Q^2 becomes very steep **AFTER Q^2 evolution** AND F_2 becomes gluon dominated

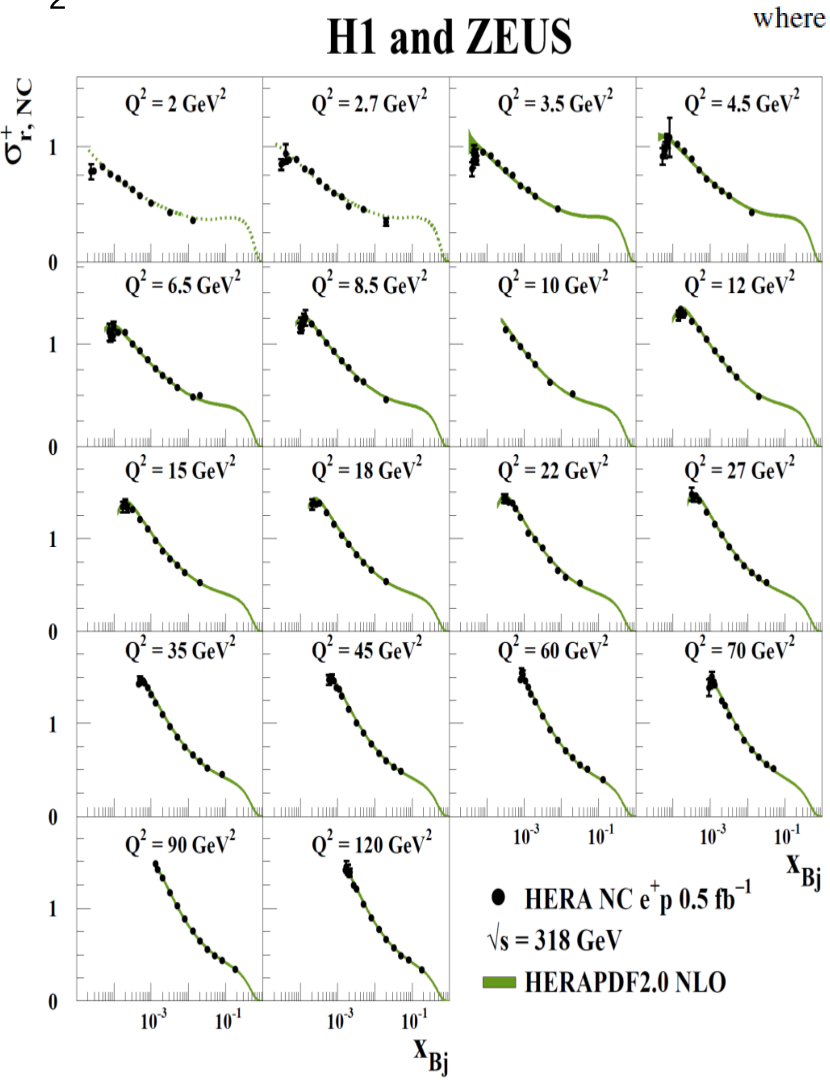
$F_2(x,Q^2) \sim x^{-\lambda s}$, $\lambda s = \lambda g - \varepsilon$

The point is that steepness should set in AFTER evolution, so at higher Q^2

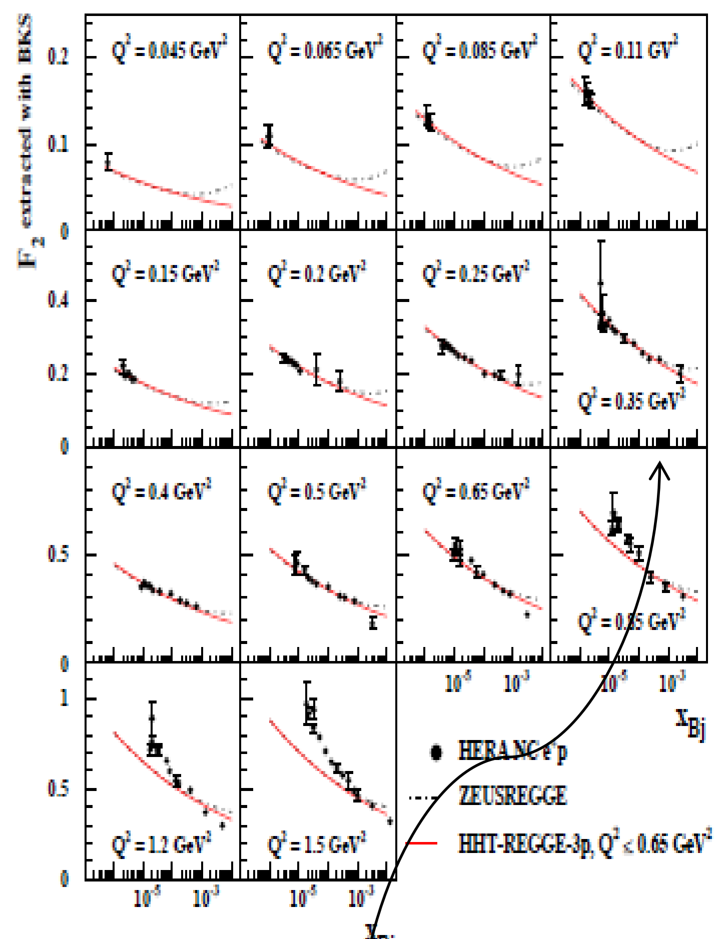
Remember σ_r is measured,
F_2 is extracted

$$\sigma_{r,\mathrm{NC}}^{e^+p} = \frac{x_{\mathrm{Bj}} Q^4}{2\pi\alpha^2} \frac{1}{Y_+} \frac{d^2\sigma(e^+p)}{dx_{\mathrm{Bj}} dQ^2} = \tilde{F}_2(x_{\mathrm{Bj}}, Q^2) - \frac{Y_-}{Y_+} x\tilde{F}_3(x_{\mathrm{Bj}}, Q^2) - \frac{y^2}{Y_+} F_{\mathrm{L}}(x_{\mathrm{Bj}}, Q^2), \quad (1)$$

where α is the fine-structure constant and $Y_\pm = 1 \pm (1-y)^2$, with the inelasticity $y = Q^2/(s\,x_{\mathrm{Bj}})$.



NEW Low Q^2 plot from 1704.03187



It was a surprise to see F$_2$ steep at small x even for low Q^2, Q^2 <~5 GeV^2 and even more of a surprise to see it steep down to Q^2 ~ 1 GeV^2

Should perturbative QCD work? α_s is becoming large - α_s at Q^2 ~ 1 GeV^2 is ~ 0.4

- Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4
- At HERA low Q2 is also low-x so ln(1/x) is becoming large

BFKL formalism (at leading order)

$$\rightarrow \quad x\, g(x, Q^2) \sim x^{-\lambda}$$

$$\lambda = \frac{\alpha_s}{\pi} C_A \ln 2 \simeq 0.5$$ for $\alpha_s \sim 0.25$ (low Q^2)

→ A singular gluon behaviour even at low-ish Q^2

→ Is this the reason for the steep behaviour of F_2 at low-x ?

However we all know that this steep behaviour was modified once NLO BFKL calculations were made. It has proved very difficult to get 'smoking gun' evidence for anything beyond DGLAP at HERA

Furthermore if the **gluon density becomes large** there maybe **non-linear effects**

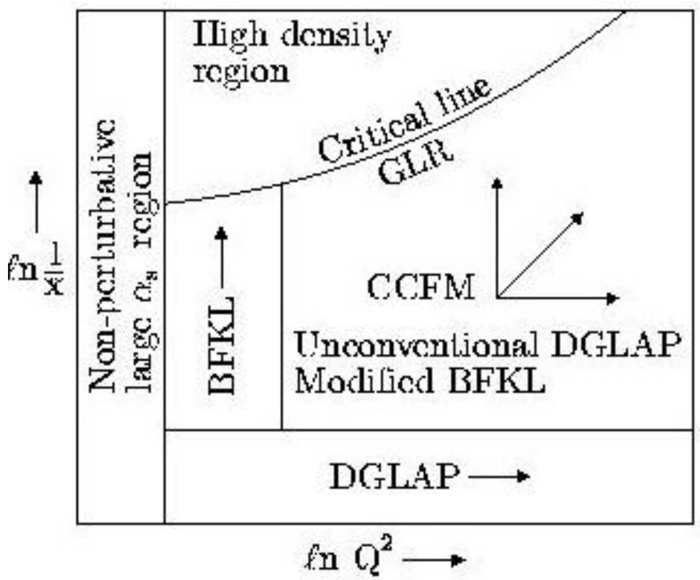
Gluon recombination g g → g

$\sigma \sim \alpha_s^2 \rho^2 / Q^2$

may compete with gluon evolution g → g g

$\sigma \sim \alpha_s \rho$

where ρ is the gluon density



Does the data *need* unconventional explanations?

One seems to be able to use DGLAP by absorbing unconventional behaviour in the boundary conditions i.e. the unknown shapes of the non-perturbative parton distributions at Q_0^2

We measure, $F_2 \sim x\,q$

$$\frac{d\,F_2}{d\ln Q^2} \sim P_{qg} \cdot x\,g$$

we can explain unusually steep $\frac{d\,F_2}{d\ln Q^2}$ by:

unusual P_{qg} → eg $ln(^1/_x)$, BFKL
OR unusual $x\,g(x,Q_0^2)$ → "valence-like" gluon ..
And indeed the gluon is weird if you push this to low Q2, and this is worse, not better at NNLO

→ need to measure other gluon sensitive quantities at low x: F_L
Unfortunately this was never done very accurately at HERA… though we will look at it

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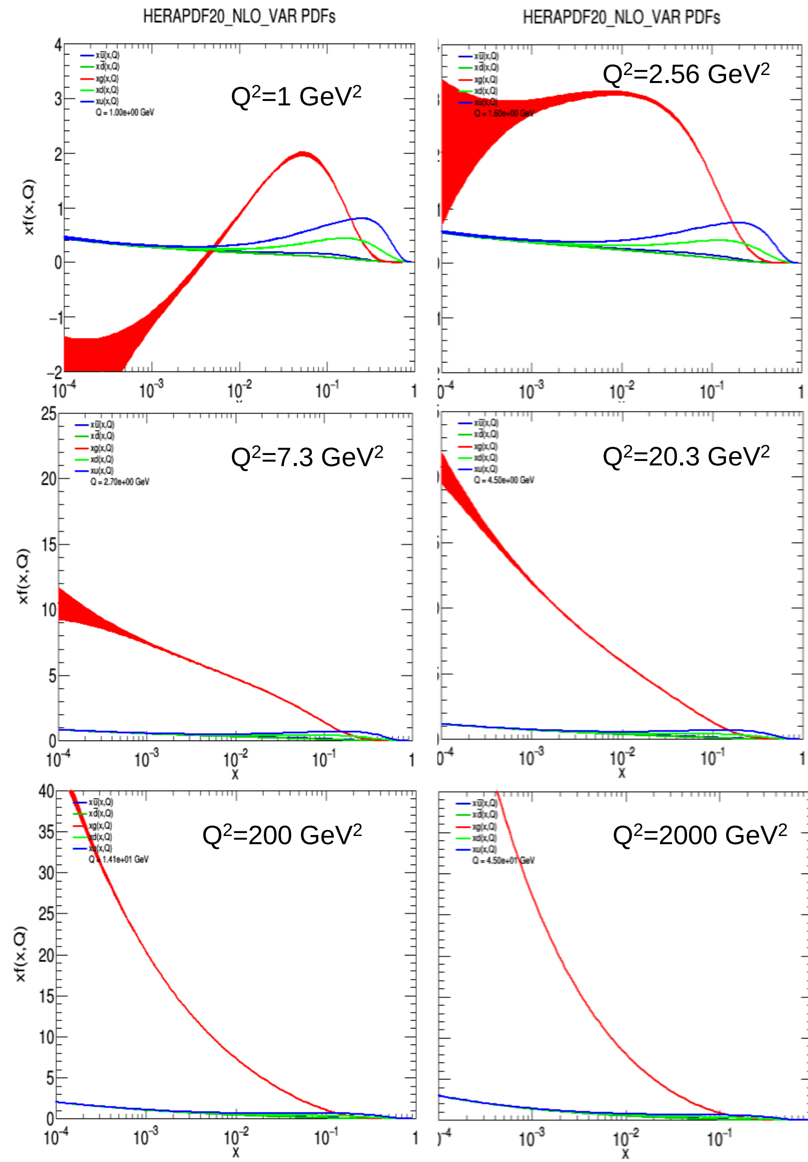
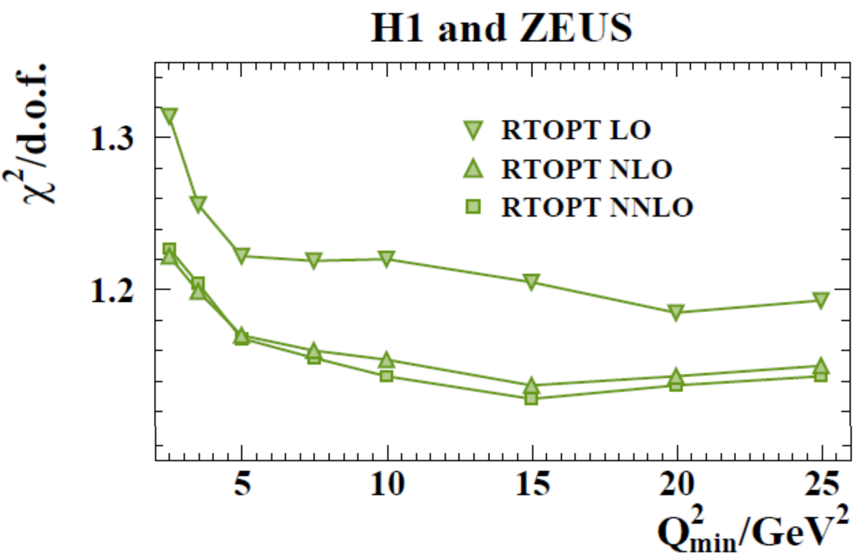


Conventional NLO-DGLAP needs a valence-like gluon but a singular sea at lowQ^2
This does not get better at NNLO

Look at the DGLAP QCD fts to final HERA combined data arXiv:1506.06042

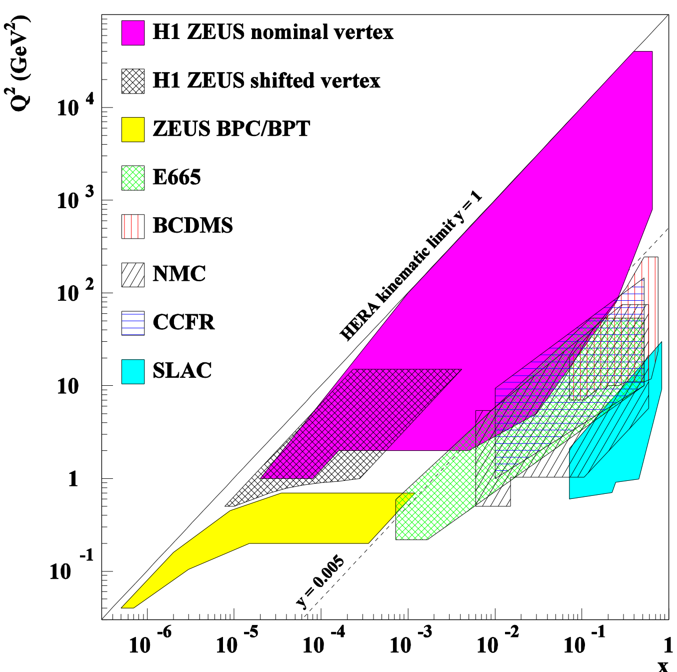


Fit χ2 deteriorates at low Q2
And NNLO is NOT better than NLO

Study two different ways of getting a better fit at low Q^2/ low-x

Adding higher twists or introducing ln(1/x) resummation

These work for 2.5 < Q^2 < 25 GeV^2..for lower Q^2 you need something else



NOTE: HERA data at low Q^2 are also at low-x

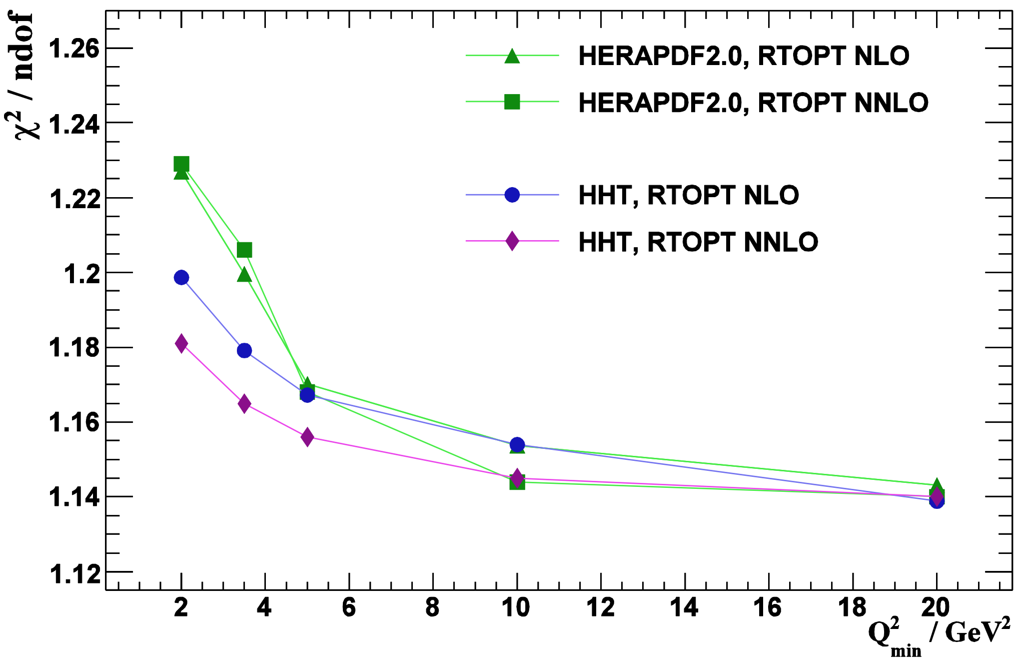
One way to improve this is to add higher twist terms - HHT analysis

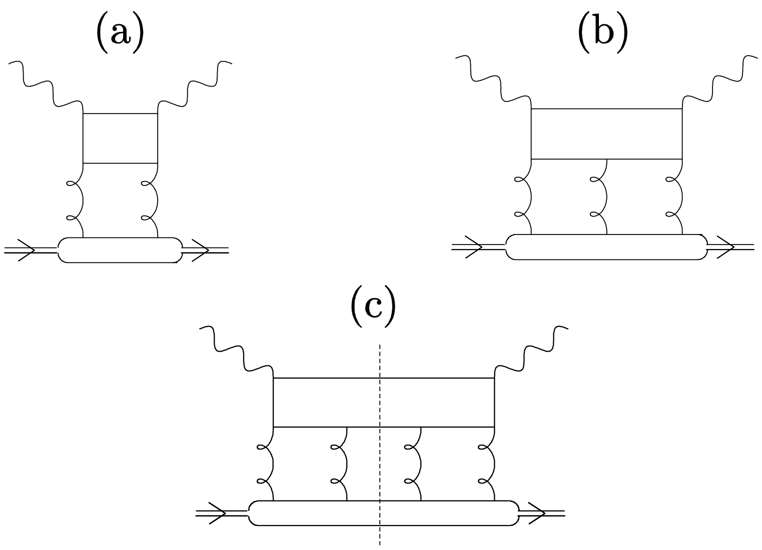
BUT NOTE- these are not the high-x, low Q^2 contributions that we usually associate with the terminology 'higher twist'

Most groups exclude those contributions by a W cut, $W^2 > 12.5$ GeV^2

ALL HERA data is at much higher $W^2 > 300$ GeV^2

What we are doing now is looking at low-x, higher twist effects





Their origin COULD be connected with the recombination of gluon ladders.

Bartels, Golec-Biernat, Kowalski suggest that such higher twist terms would cancel between σ_L and σ_T in F_2, but remain strong in F_L

Try the simplest of possible modification to the structure functions F_2 and F_L as calculated from HERAPDF2.0 formalism

$$F_{2,L} = F_{2,L}\,(1 + A_{2,L}^{HT}/Q^2)$$

We find that such a modification of F_L is favoured, whereas for F_2 it is not.

At NNLO the χ2/ndof = 1363/1131 for HERAPDF2.0

If A_2^{HT} is added this becomes 1357/1130 and $A_2^{HT} = 0.12 \pm 0.07$ GeV^2

If A_L^{HT} is added this becomes 1316/1130 and $A_L^{HT} = 5.5 \pm 0.6$ GeV^2

If both A_L^{HT} and A_2^{HT} are added the result is consistent with just adding A_L^{HT}

So now concentrating on just F_L, we call these fits HHT

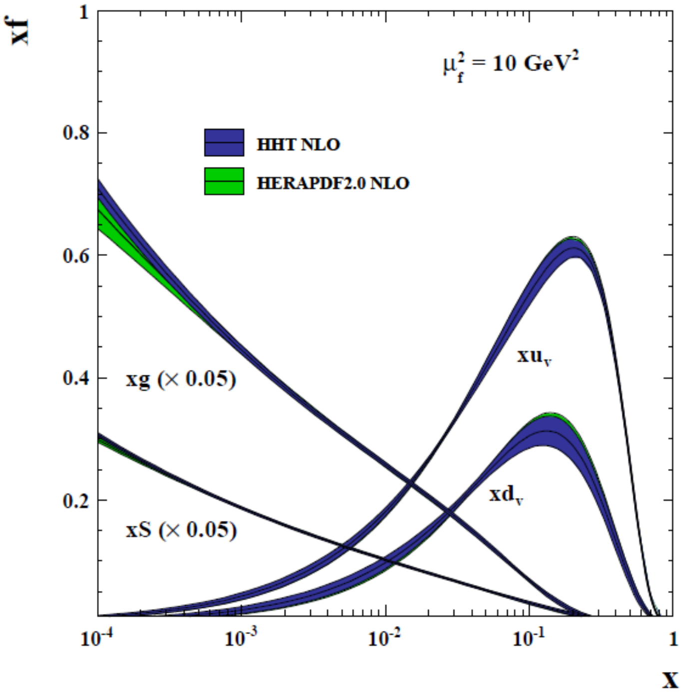
Fit at	with $Q^2_{min} = 3.5\,\mathrm{GeV}^2$	HERAPDF2.0	HHT	A_L^{HT}/GeV^2	
NNLO	χ^2/ndof	1363/1131	1316/1130	5.5±0.6	Δχ2 =-47
	χ^2/ndp for NC e^+p: $Q^2 \geq Q^2_{min}$	451/377	422/377		
	χ^2/ndp for NC e^+p: $2.0\,\mathrm{GeV}^2 \leq Q^2 < Q^2_{min}$	41/25	32/25		
NLO	χ^2/ndof	1356/1131	1329/1130	4.2±0.7	Δχ2 =-28
	χ^2/ndp for NC e^+p: $Q^2 \geq Q^2_{min}$	447/377	431/377		
	χ^2/ndp for NC e^+p: $2.0\,\mathrm{GeV}^2 \leq Q^2 < Q^2_{min}$	46/25	46/25		

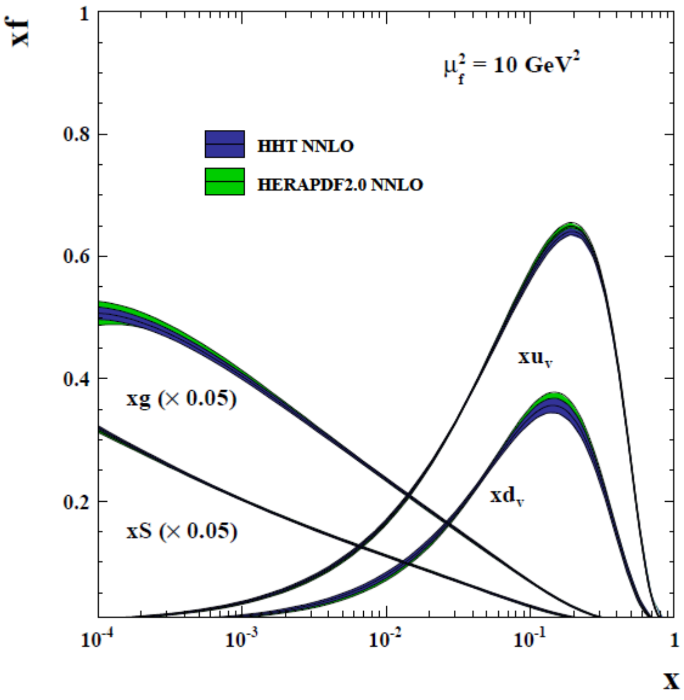
After HT is added the NNLO fit is better than the NLO fit

A substantial part of the improvement comes from the NCe^+p 920 data

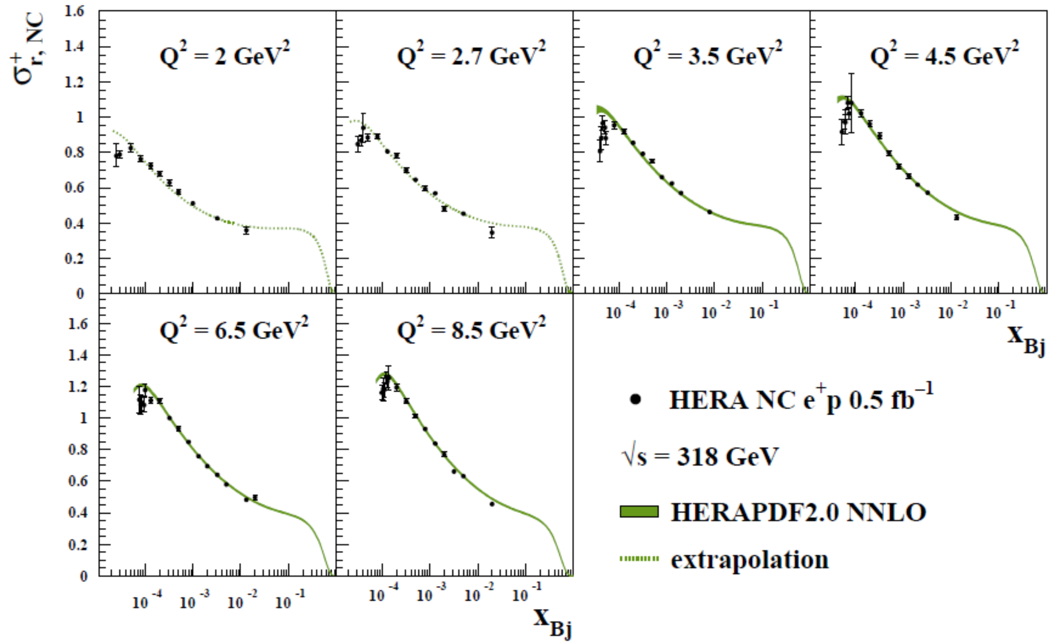
This persists even below the usual cut-off $Q^2_{min} = 3.5$ GeV^2

NOTE: the HHT PDFs themselves barely change from HERAPDF2.0 – the higher twist modification does not affect high-scale LHC physics





On the other hand the peculiar behaviour of the gluon wrt the Sea at low-x/Q^2 remains

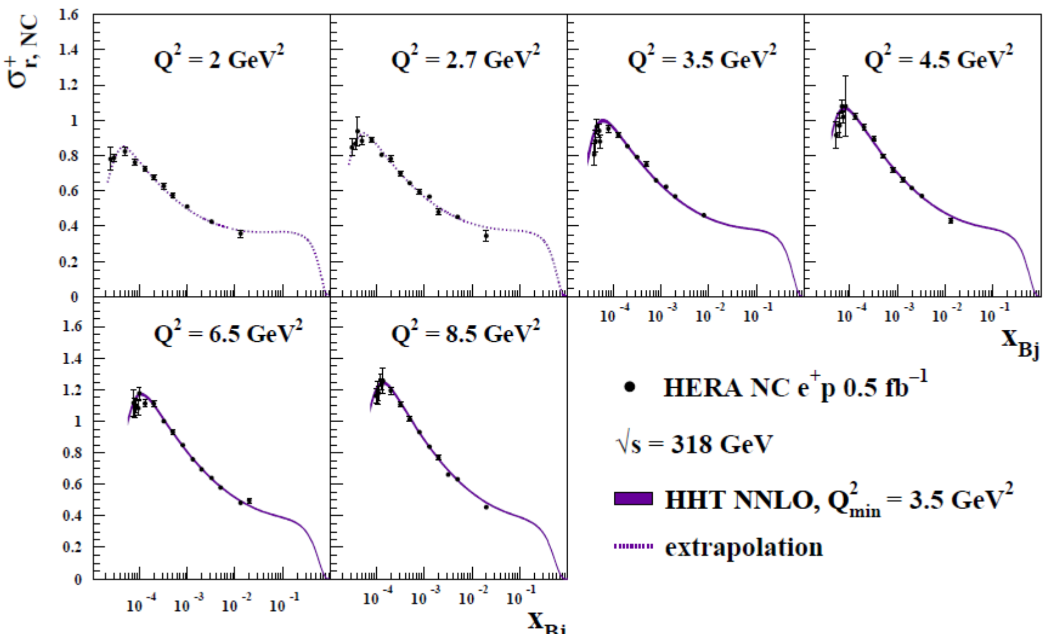


So now let's look at why the HHT fits do so well

It is because they describe the turn over of the cross section at low x, Q2 much better

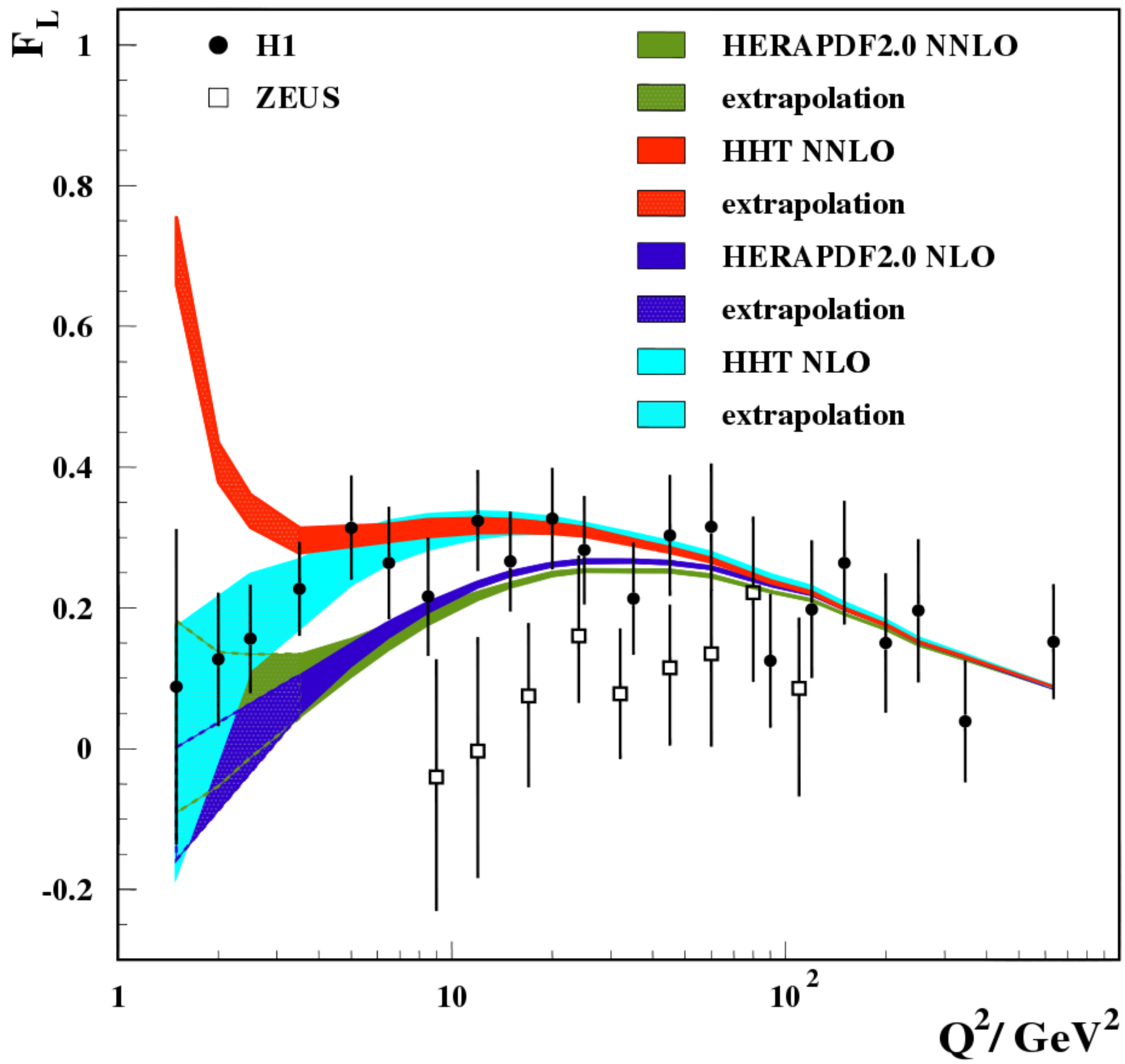
$$\sigma_{red} = F_2 - y^2/Y_+ F_L$$

The data clearly wants a larger F_L and this is what the higher twist term provides



The fit is also better for low-x values above the turn over

The HHT fits give a larger F_L at low Q^2 for both NLO and NNLO



You might think that -since F_L is related to the gluon -

$$xG(x,Q^2) \approx \frac{3}{5}\, 5.9 \left[\frac{3\pi}{4\alpha_s} F_L(0.4x,Q^2) - \tfrac{1}{2} F_2(0.8x,Q^2)\right]$$

Simple LO relationship gives the idea

- an easier way to obtain larger F_L would be to drop the negative term in the gluon PDF parametrisation.

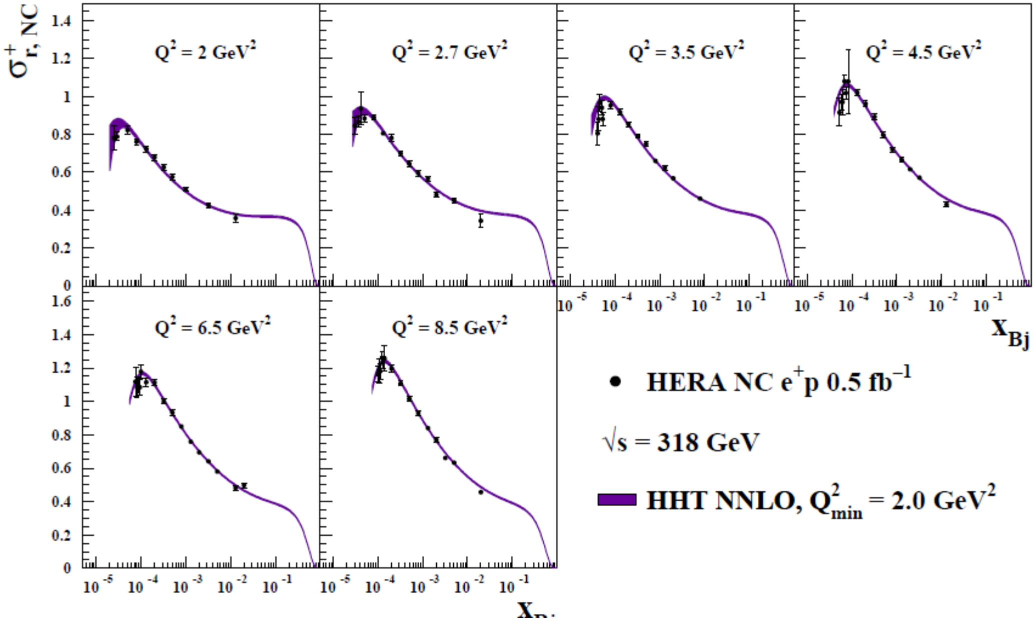
$$xg(x) = A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g},$$

So we did- we call this the alternative gluon (AG) parametrisation

This makes almost no difference for the NLO fits
Whereas it is strongly disfavoured for the NNLO fits.
At NNLO the fit wants a negative term in the gluon parametrization AND a higher twist term in F_{L}.

For HERAPDF2.0 AG the χ2/ndof = 1389/1131 cf 1363/1130 for the standard fit
For HHT AG the χ2/ndof = 1350/1130 cf 1316/1130 for the standard fit

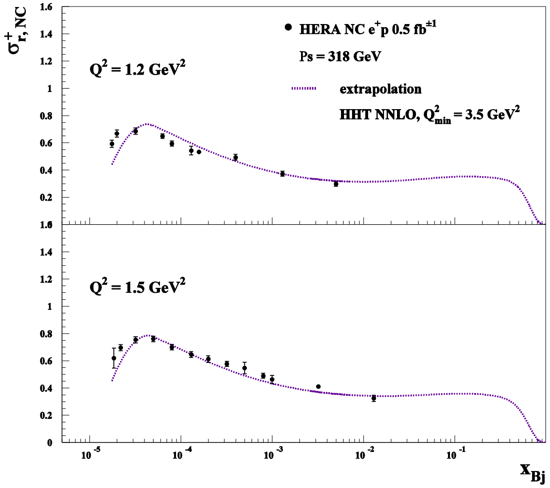
These two contributions clearly affect the fit in different ways



Looking at the extrapolations of our fits below Q^2_{min} =3.5 GeV^2 made us bold enough to extend the fit down to Q^2_{min}=2.0 GeV^2

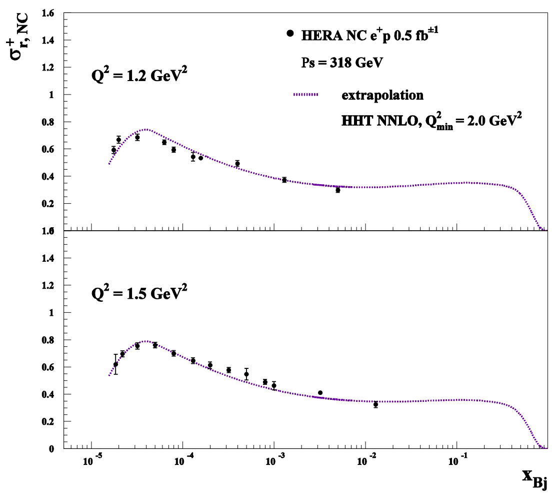
Fit at	with $Q^2_{min} = 2.0\,\text{GeV}^2$	HERAPDF2.0	HHT	A_L^{HT}/GeV^2
NNLO	χ^2/ndof	1437/1171	1381/1170	5.2±0.7
	χ^2/ndp for NC e^+p: $Q^2 \geq Q^2_{min}$	486/402	457/402	
	χ^2/ndp NC e^+p: $Q^2_{min} \leq Q^2 < 3.5\,\text{GeV}^2$	31/25	26/25	
NLO	χ^2/ndof	1433/1171	1398/1170	4.0±0.6
	χ^2/ndp for NC e^+p: $Q^2 \geq Q^2_{min}$	487/402	466/402	
	χ^2/ndp NC e^+p: $Q^2_{min} \leq Q^2 < 3.5\,\text{GeV}^2$	40/25	31/25	

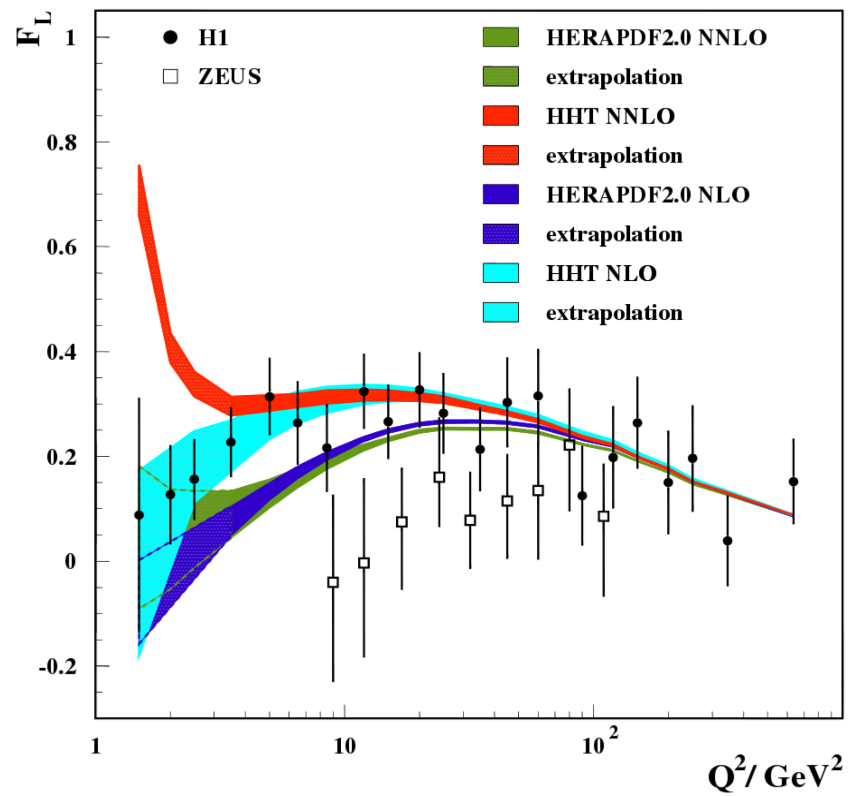
Not much changes for the NNLO fit and the NLO fit improves a little



So we got even bolder and looked at lower Q^2- by backward evolution

But beware…is this actually reasonable? What does FL itself look like?





NNLO HHT F_L prediction is becoming untamed at low Q^2– **this approach cannot be pushed below Q^2 ~2.5 GeV²**
This comes from NNLO coefficient functions and the $1/Q^2$ term just makes it worse

Now examine the alternative approach of ln(1/x) resummation adding terms to DGLAP splitting functions and coefficient functions – see talk of Bertone

The programme to do these High Energy Leading log resummation (HELL) has been implemented in xFitter

1. Here we explore consequences for a HERAPDF style fit

HELL implements resummation corrections to the fixed order splitting functions and coefficient functions up to NLL accuracy in ln(1/x), denoted as NLLx. The fixed order quantities are calculated by APFEL within the FONLL variable flavour number scheme.

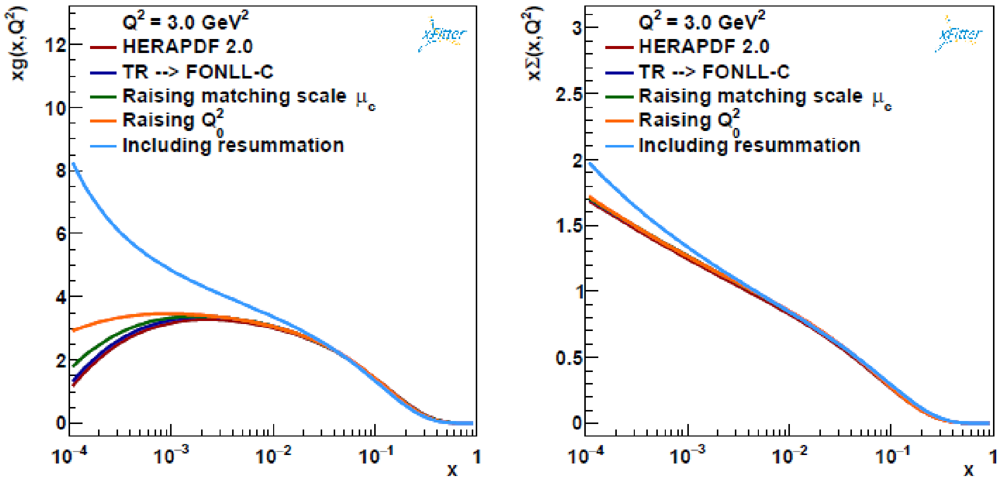
2.Thus we must use FONLL for the HERAPDF fit

4. The computation of ln(1/x) resummation is unreliable at low scales due to the large value of α_S thus the starting scale is raised to Q^2_0=2.56GeV^2 rather than the usual HERAPDF value of Q^2_0=1.9GeV^2.

3. Consequently the charm quark threshold, μ_c, must be displaced above Q_0 while keeping the charm mass, m_c, fixed. (see 1707.05343)

5. Finally NLLx resummation can be applied

	Step-1	Step-2	Step-3	Step-4	Step-5
	HERAPDF2.0 NNLO	TR→FONLL-C	raise the charm matching scale μ_c	raise the initial scale Q_0	include NLLx resummation
HERA χ^2/d.o.f.	1363/1131	1387/1131	1390/1131	1388/1131	1316/1131



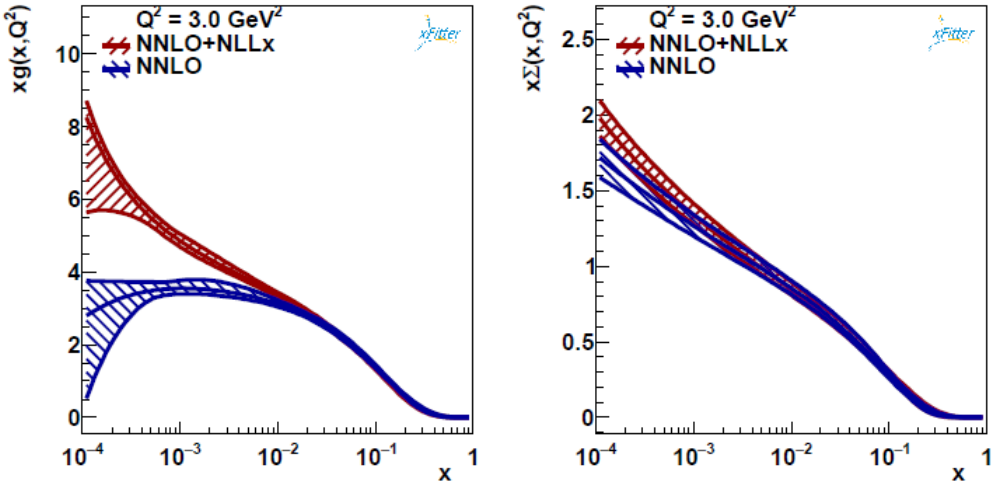
2.The increase in χ2 for FONLLC is well known and relates to the treatment of FL; terms up to $\mathcal{O}(\alpha_S^3)$ are included for RTOPT, but terms up to $\mathcal{O}(\alpha_S^2)$ are included for FONLLC. The gluon does not change shape

3. Raising the charm matching scale makes very little difference to χ2 or to gluon shape

4. Raising the initial scale has no effect on χ2, but does marginally change the shape of the gluon—this is a model variation which will be accounted for

5. The χ2 for the NNLO fit improves dramatically at the final step
The shape of the gluon is also changed dramatically from flattening/turning over at low-x to singular at low-x

After these adjustments – and adding PDF uncertainties we have



A decrease in χ2 of 74 using ln(1/x) resummation

Largely due to the NC e+p 920 data

But also less need for shifts of systematic uncertainties

$$\chi^2 = \sum_i \frac{\left[D_i - T_i\left(1 - \sum_j \gamma^i_j b_j\right)\right]^2}{\delta^2_{i,\mathrm{unc}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i} + \sum_j b_j^2 + \sum_i \ln \frac{\delta^2_{i,\mathrm{unc}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i}{\delta^2_{i,\mathrm{unc}} D_i^2 + \delta^2_{i,\mathrm{stat}} D_i^2},$$

	NNLO fit with new settings	NNLO+NLLx fit with new settings
Total $\chi^2 (= \tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{d.o.f.}$	$1468\,(1327 + 119 + 22)/1207$	$1394\,(1305 + 91 - 2)/1207$
dataset inclusive $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(1264 + 103 + 21)/1145$	$(1239 + 78 - 4)/1145$
- subset NC 920 $\tilde{\chi}^2/\mathrm{n.d.p.}$	$447/377$	$413/377$
- subset NC 820 $\tilde{\chi}^2/\mathrm{n.d.p.}$	$67/70$	$65/70$
dataset charm $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(47 + 12 - 1)/47$	$(50 + 11 - 1)/47$
dataset beauty $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(16 + 2 + 3)/29$	$(16 + 2 + 3)/29$

What is included in PDF uncertainties?

Experimental uncertainties for sure, but also model and parametrisation uncertainties according to the usual HERAPDF procedure

$\Delta m_c = \pm 0.05$, $\Delta m_b = 0.25$, $\Delta\alpha_s = 0.001$ around 0.118

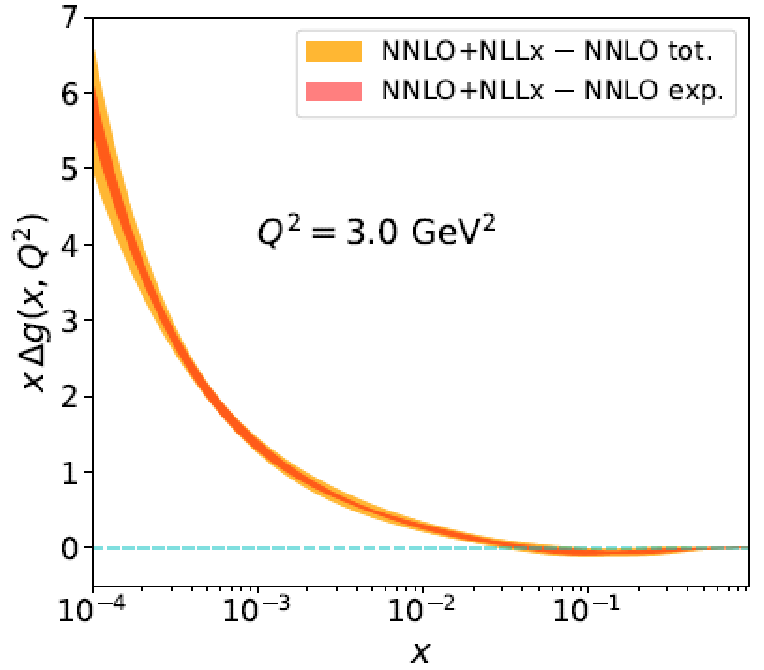
$Q^2_0 = 2.88$ rather than 2.56 GeV^2

$Q^2_{min} = 2.5, 5.0$ rather than 3.5 GeV^2

The largest difference comes from changing the Q^2_{min} to 5 GeV^2

Parametrisation uncertainties are evaluate by adding extra terms D,E,F to the polynomials $P_i(x) = (1 + D_i x + E_i x^2)e^{F_i x}$. which describe the PDFs $xq_i(x) = A_i x^{B_i}(1-x)^{C_i} P_i(x)$,

This can give different shapes to the PDFs even when the χ2 of the fit is barely different. The largest difference comes from a D_{uv} term.



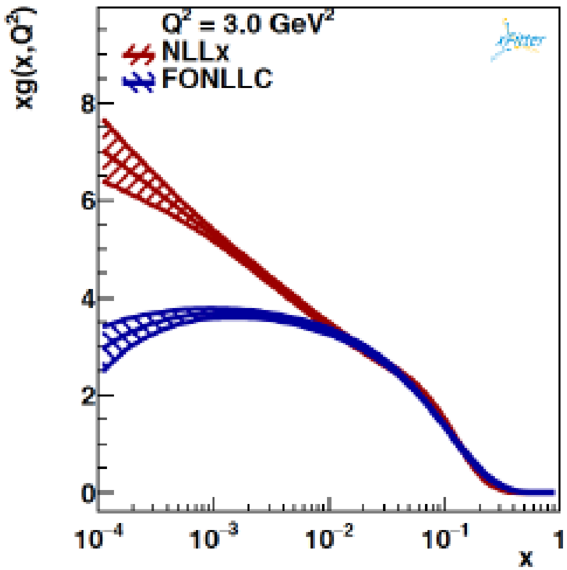
Clearly since the data used in the NNLO and NNLO+NLLx fits are the same the uncertainties on the fits are highly correlated. Thus to evaluate the really difference in the gluon shapes we must account for correlations.

Uncertainties are evaluated by MC replicas of the data using the same random number sequence for both fits to evaluate the spread of the synchronised differences- and this is shown above as $x\Delta g(x,Q^2)$

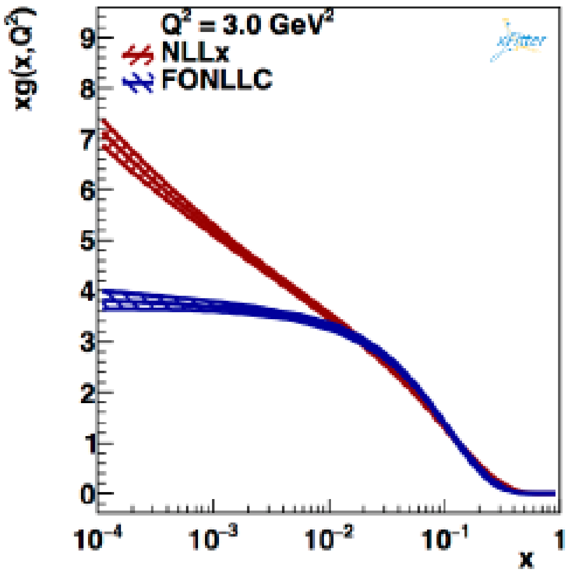
Adding the negative gluon term

Do we really need the negative term of gluon? → We produced a version of the **final NNLO+NLLx and NNLO fits without the negative term** just to check this

NNLO+NLLx (standard)

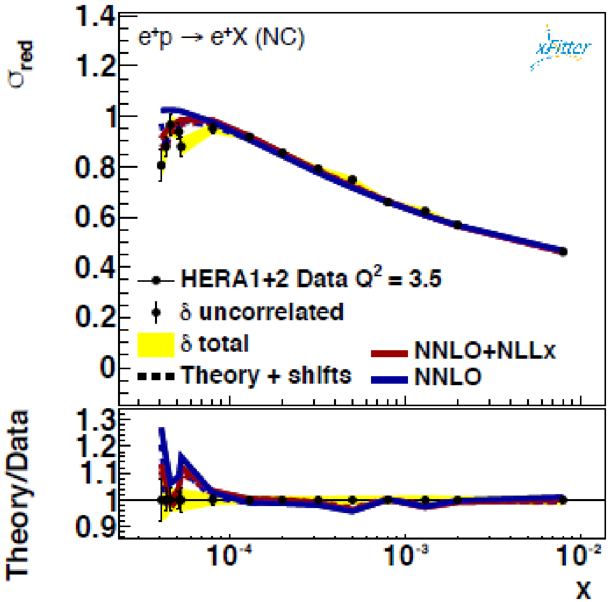


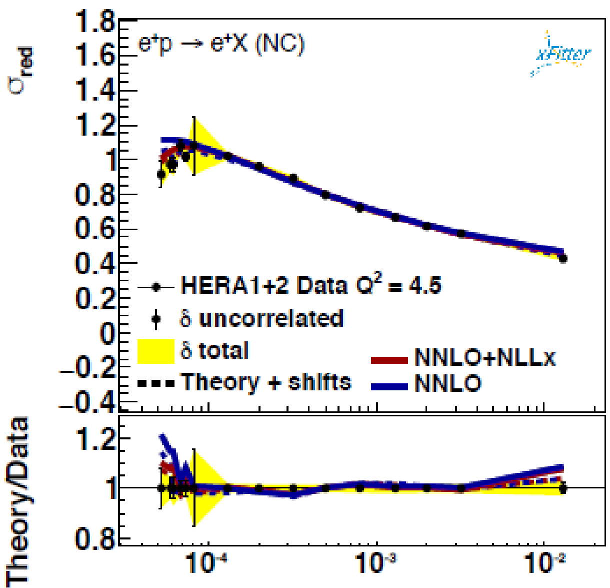
NNLO+NLLx (w/o neg term gluon)



The point is that even without the negative term the gluon for NLLO likes to take a flattish shape at low-x, whereas for NNLO+NLLx it takes a singular shape

Comparison to data

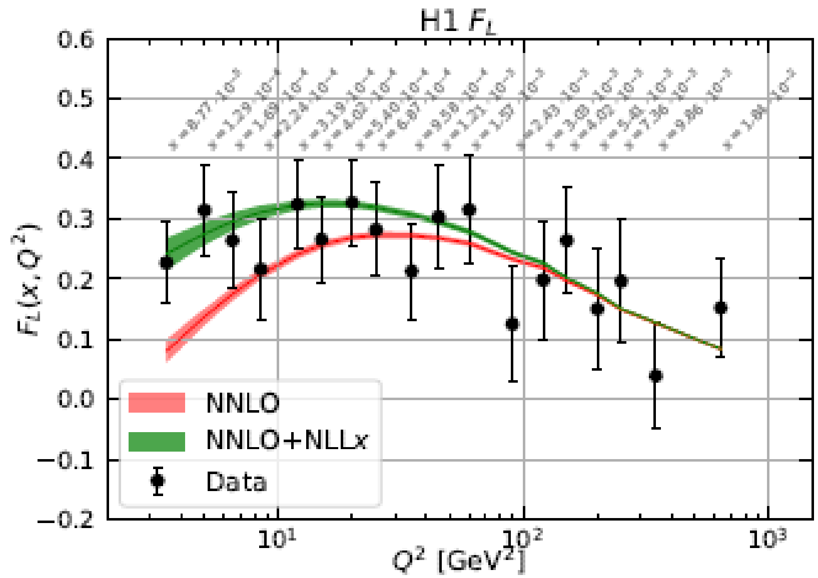




Comparison to data in the lowest Q^2 bins shows that the fit with low x resummation is much better able to follow the turn over of the data that happens at low-x, low Q^2, high-y due to the F_L term in the reduced cross section

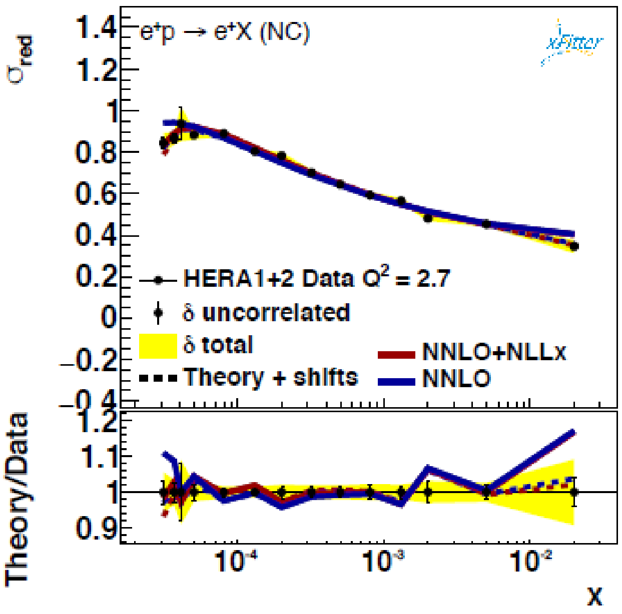
$$\sigma_{\rm red} = F_2 - \frac{y^2}{Y_+} F_L$$

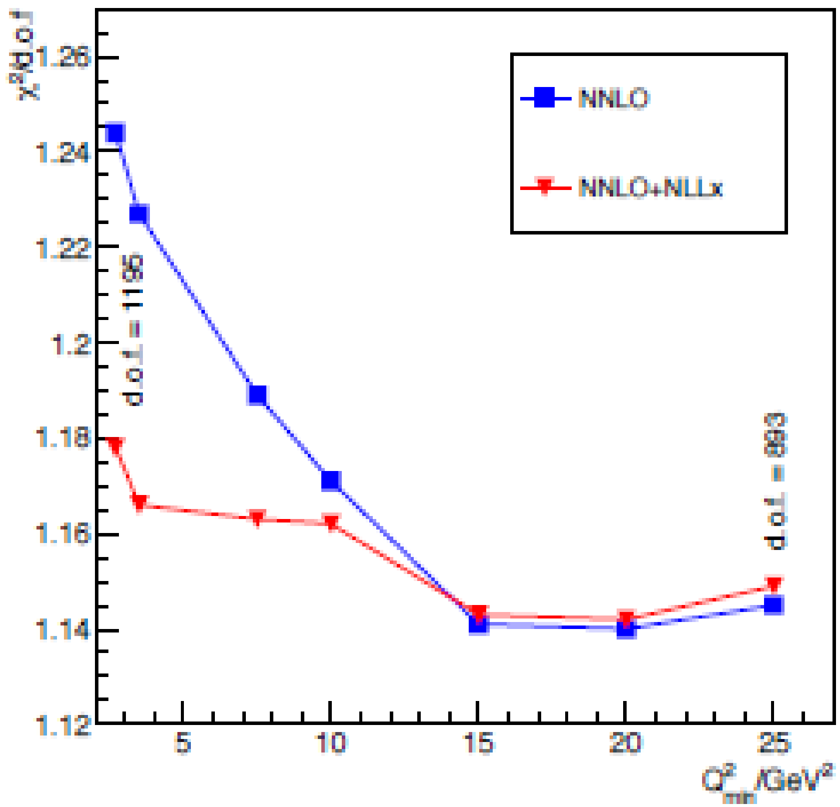
Looking at H1 F_L data directly shows that FL is larger at low Q^2/x for the NLLx fit



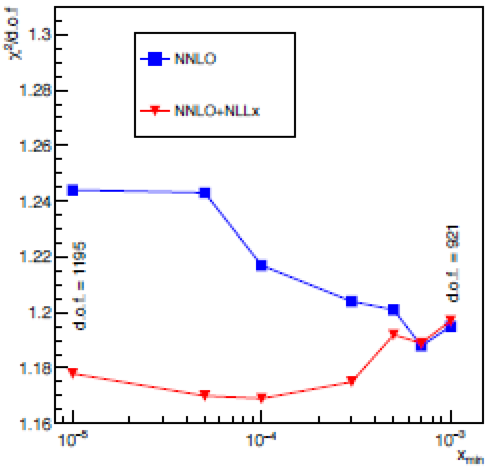
Theory +shifts means shifts due to experimental systematics- the term Σγb in the χ2

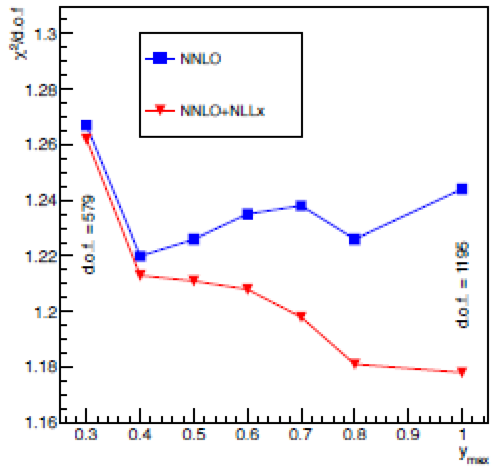
We also extend below the usual low Q2 cut –off for the data, down to Q2=2.7GeV2.
The NNLO+NLLx fit including these data gives very similar PDFs to the standard fit





To dileneate the kinematic region for which NLLX resummation improves the fits we perform a χ2 scan in Q^2_{min} for NNLO and NNLO+NLLx fits
This shows an improvement at low Q^2, Q^2_{min} < 15 GeV^2





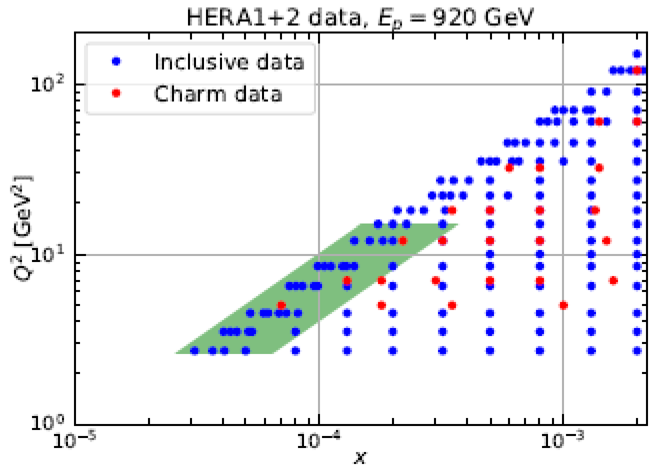
We also scan vs x_{min} seeing improvement for x_{min} < 5.10 $^{-4}$

And against y^{max} seeing improvement for y_{max}>0.4

This emphasizes the importance of low x resummation at high-y for the DIS data because of the role of the FL term

$$\sigma_{\text{red}} = F_2 - \frac{y^2}{Y_+} F_L$$

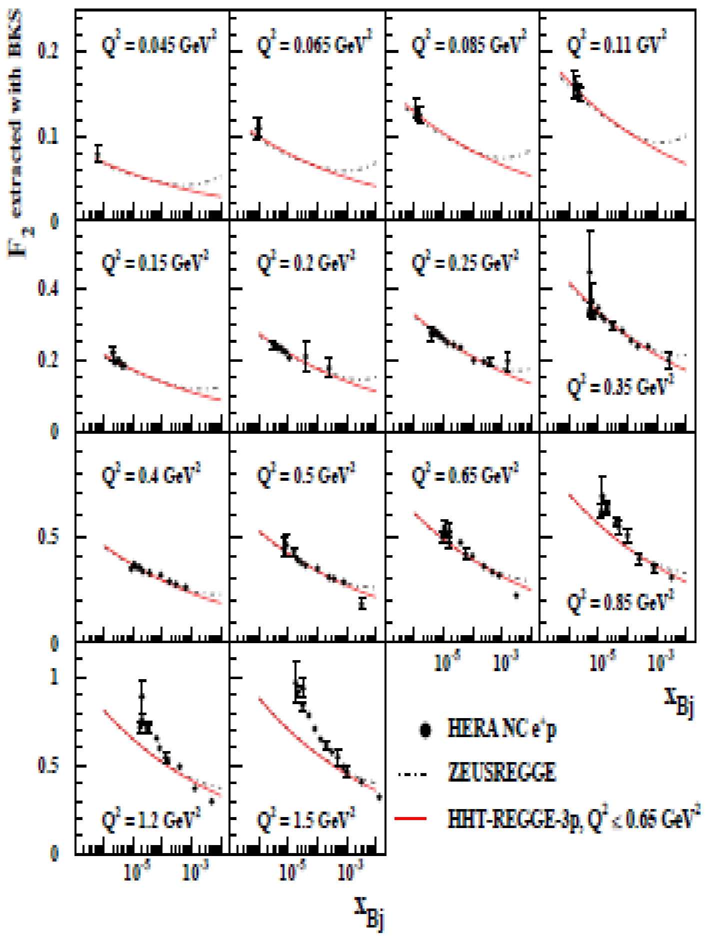
The scans shown here were done refitting the PDFs at each step—thus they dileneate a region where the fixed order calculation is poor –even with refitting –as illustrated on the x, Q^2 plane here.



However note that the calculation of the NLLx modifications cannot be pushed below Q^2~ 2.5 GeV^2

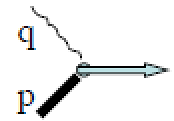
But there are other approaches to looking for effects beyond DGLAP, consider the transition to the non-perturbative regime

Linear DGLAP evolution doesn't work for $Q^2 < 1$ GeV2, WHAT does? – REGGE ideas?



arXiv:1704.03187

Small x is high W^2, $x = Q^2/2p.q \;\; Q^2/W^2$



$\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ – Regge prediction for high energy cross-sections

α is the intercept of the Regge trajectory
α=1.08 for the SOFT POMERON

Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including
$\sigma(\gamma p) \sim (W^2)^{0.08}$
for real photon- proton scattering

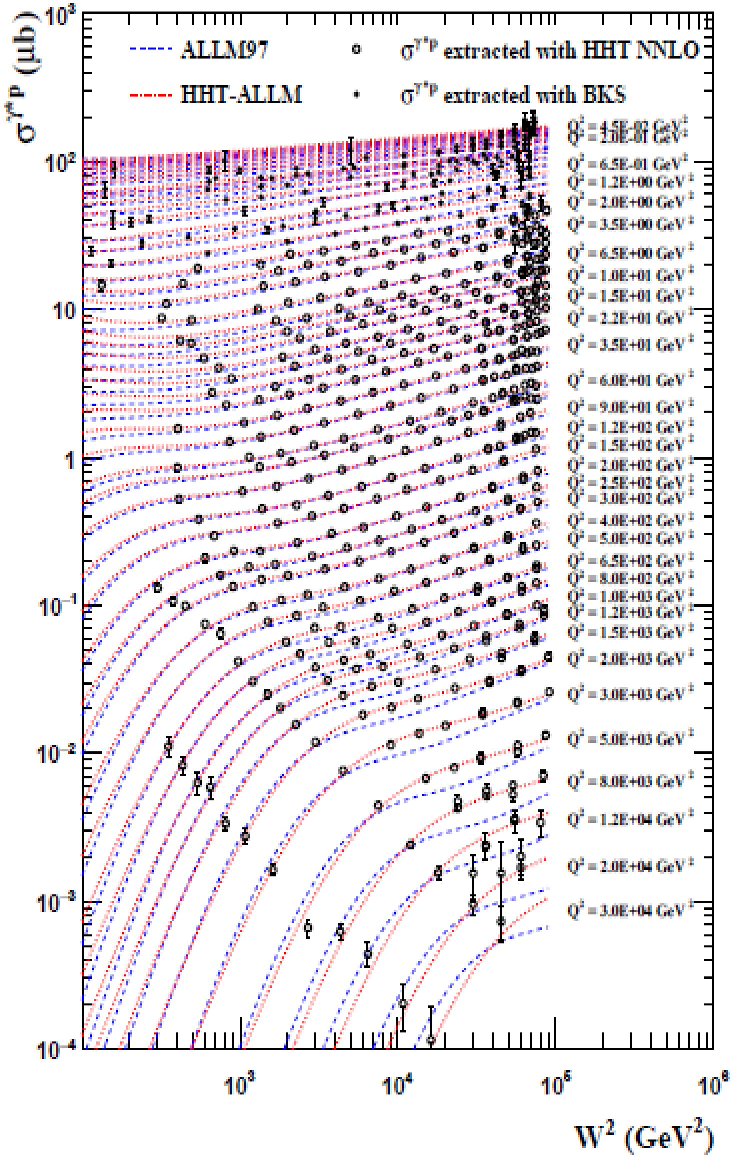
For virtual photons, at small x

$$\sigma(\gamma^* p) = \frac{4\pi^2\alpha \;\; F_2}{Q^2}$$

$\rightarrow \quad \sigma \sim (W^2)^{\alpha-1} \rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$

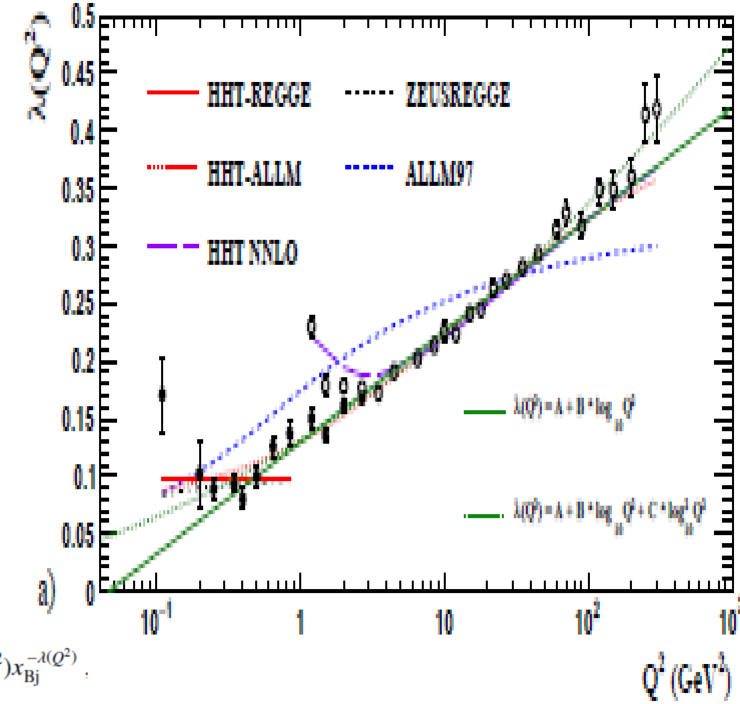
so a SOFT POMERON would imply
$\lambda = 0.08$ gives only a very gentle rise of F_2 at small x

For $Q^2 > 1$ GeV^2 we have observed a much stronger rise…..



gentle rise

much steeper rise



$F_2 = C(Q^2) x_{Bj}^{-\lambda(Q^2)}$.

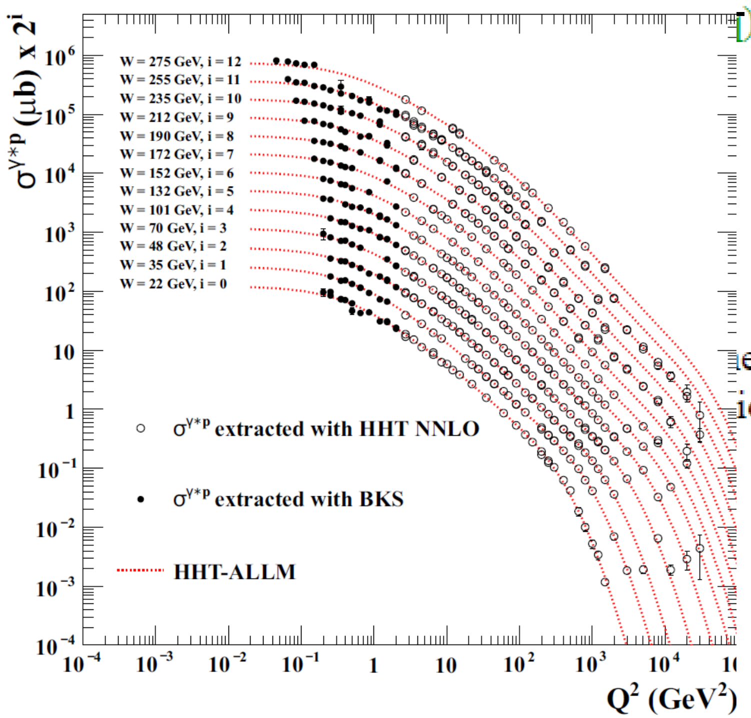
The slope of F_2 at small x , $F_2 \sim x^{-\lambda}$, is equivalent to a rise of $\sigma(\gamma^* p) \sim (W^2)^\lambda$ which is only gentle for $Q^2 < 1$ GeV^2

As well as the soft Pomeron, $\alpha - 1 = \lambda = 0.08$ (REGGE) should we consider

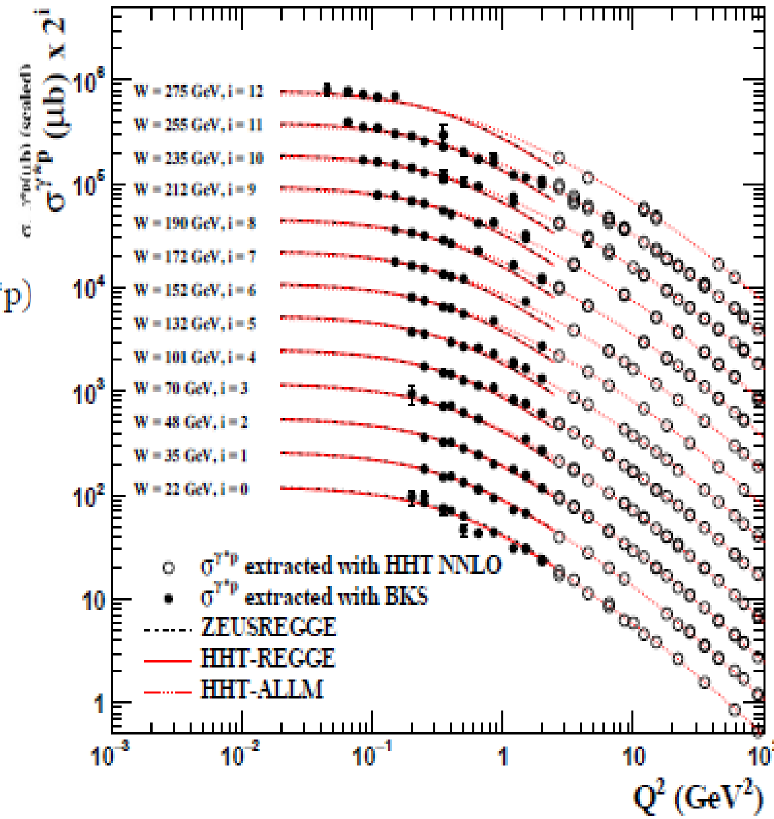
- a QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$- (NNLO-DGLAP)
- a BFKL POMERON, $\alpha - 1 = \lambda \sim 0.5$
- a mixture of HARD and SOFT Pomerons to explain the transition $Q^2 = 0$ to high Q^2? (Donnachie and Landshoff mark2, or ALLM)

What about the Froissart bound ? – the rise MUST be tamed – non-linear effects?

Dipole models provide another way to model the transition $Q^2=0$ to high Q^2



σ(γ*p)



$$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2)),\ R_0(x)^2 \sim (x/x_0)^\lambda \sim 1/xg(x)$$

r/R_0 small → large Q^2, x $\sigma \sim r^2 \sim 1/Q^2$, F2 flat Bjorken scaling	r/R_0 large → small Q2, x $\sigma \sim \sigma_0$ → saturation of the dipole cross-section

But $\sigma(\gamma^*p) = \frac{4\pi\alpha^2}{Q^2} F_2$ is general (at small x)

σ(γp) is finite for real photons, $Q^2=0$. At high Q^2, F_2 ~flat (weak $\ln Q^2$ breaking) and $\sigma(\gamma^*p) \sim 1/Q^2$

GBW dipole model

Summary

Higher twist term:1604.02299

- Improves the description of HERA data at low-x, low-Q^2, high-y, one extra parameter
- Gluon remains the same as before the twist term is added, still has valence-like shape when the sea is rising
- Description fails for $Q^2 < \sim 2.5$ Gev^2

Low-x resummation:1802.00064

- Improves the description of HERA data at low-x, low-Q2, high-y, without need for further parameters
- Results in a rising low-x gluon, which is always larger than the total Sea
- Description cannot be used for $Q^2 < \sim 2.5$ GeV^2

VERY low Q2: 1704.03187

Modern HERA combined data is available for the extracted quantities F_2, σ (γ*p), d lnF_2/d ln(1/x), d F_2/d ln(Q^2)

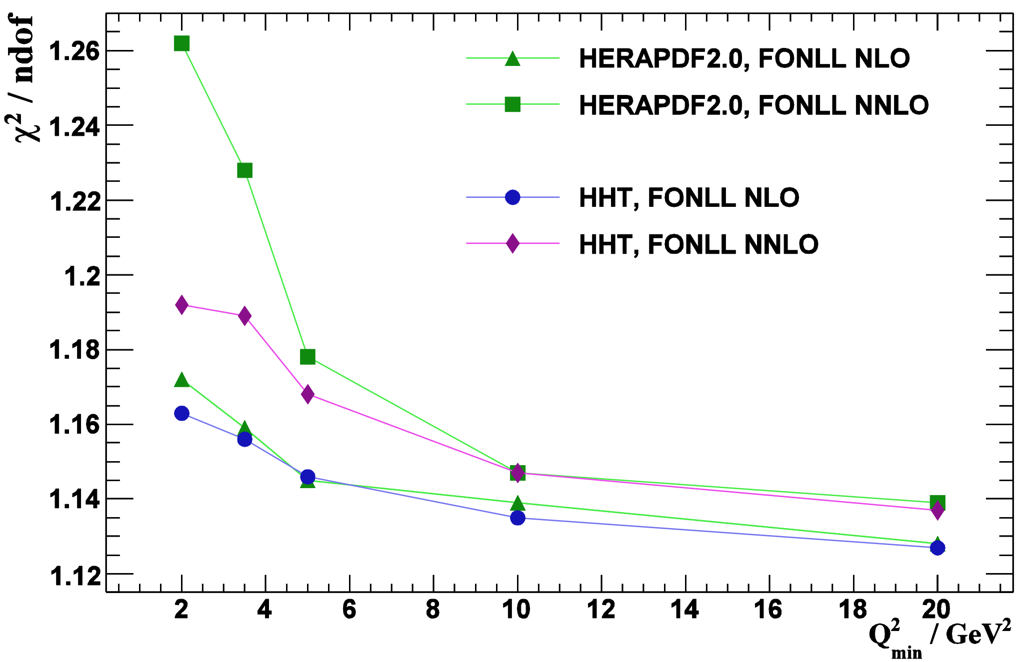
Back-up

Higher twist effects

Another consideration is that we know that the rate of decrease $\chi2$/ndof with increasing Q^2_{min} differs with the heavy flavour scheme used AND with the order in α_S to which F_L is evaluated

So let's take a look at FONLL

For FONLL-C at NNLO a higher twist term in F_L brings a substantial decrease in the $\chi2$/ndof with a similar value of A_L^{HT}=6.0 ± 0.7 GeV^2 to that for the RTOPT scheme. For FONLL-B at NLO a higher twist term in F_L brings almost no decrease in $\chi2$/ndof . This is probably related to the order in α_S to which F_L is evaluated



For FONLL-C/RTOPT at NNLO, F_L is evaluated to $O(\alpha_S^2)$/ $O(\alpha_S^3)$

For FONLL-B/RTOPT at NLO, F_L is evaluated to $O(\alpha_S)$/ $O(\alpha_S^2)$

The value of F_L at $O(\alpha_S)$ is relatively large in any scheme and thus there is little need for higher twist.

However as soon as F_L is evaluated to $O(\alpha_S^2)$ or higher the need for higher twist appears

HT study- AMCS only

What do we get if we make it $FL=FL(1+ Ax^b/Q^2)$?
In the NNLO Q2>2 fit where chisq =1381.
We get chisq= 1376 with A=0.31 and x=-0.313– this has strong x dependence
It gives you Ax^b = 5.6, 2.7,1.3 for $x=10^{-4,3,2}$
Seems to indicate we were dominated by the lower x end
AND it is very saturation like in form x^-0.3/Q^2

Further considerations:

Since we have change the heavy quark scheme the charm and beauty masses used may not be optimal for the new scheme. Thus charm and beauty data from HERA are included in the fit and charm and beauty mass scans are performed to determine new values m_c=1.46 and m_b=4.5GeV. Only m_c differs from that of the HERAPDF and the charm threshold is move to μ_c=1.64 correspondingly.
We include these heavy flavour data in the fits from now on since they are potentially sensitive to low x resummation.

Since we have a very different shape of the gluon PDF a parametrisation scan is performed at NNLO+NLLx to determine if the HERAPDF parametrisation is adequate. The form of the parametrisation is confirmed, however the negative term in the gluon is now small ~3σ from zero. In fact this is also the case for the NNLO fit due to the raised starting scale Q^2_0=2.56GeV^2
Nevertheless the resulting gluon shapes are very different.
The form of the common parametrisation used for both NNLO and NLLO+NLLx is

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}}x\right), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

Note it is not the negative term which makes the gluon turn over at low-x for NNLO, the main term can also have a valence like shape if Bg is positive- and at NNLO it DOES

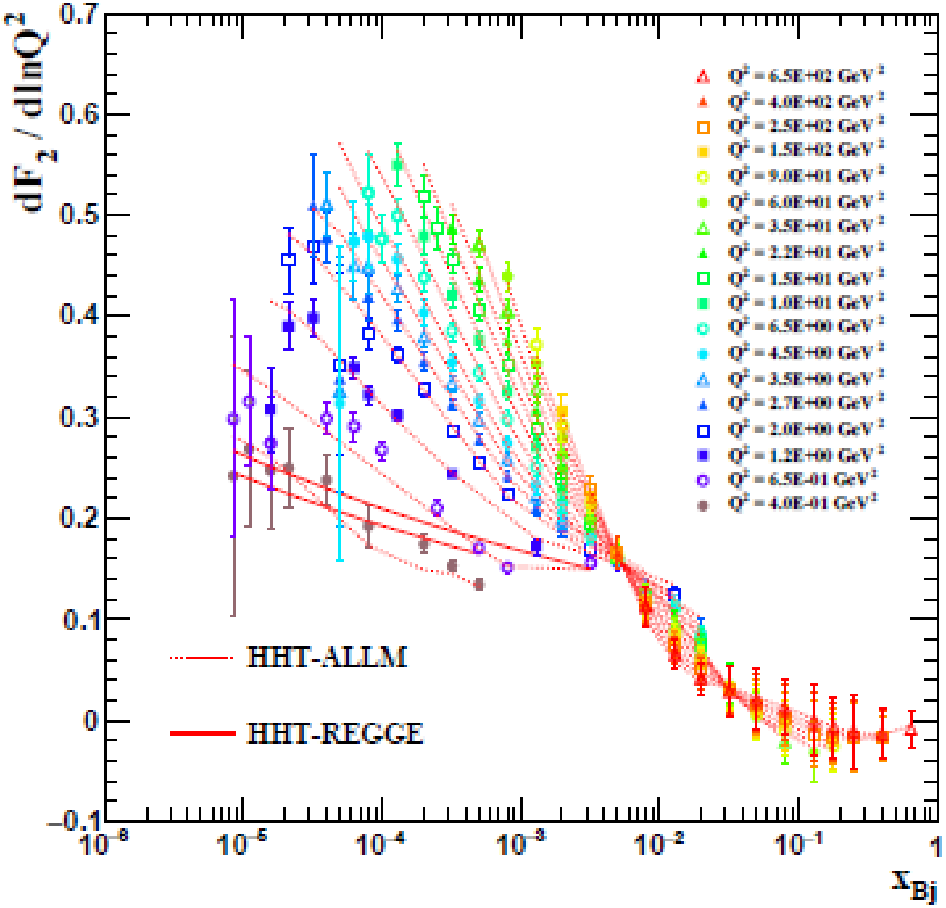
Very low Q2

Traditionally, HERA physics at low Q^2 and x_{Bj} is discussed in terms of F_2 and $\sigma^{\gamma^* p}$, defined as the cross section for virtual photon exchange. The values of F_2 have to be extracted from the reduced cross-section data. This cannot be done in an unbiased way and it cannot be done in the same way over the whole kinematic region. Indeed, on the contrary, very different models have to be used. However, in all cases, F_2 is extracted as

$$F_2^{\mathrm{extracted}} = F_2^{\mathrm{predicted}} \frac{\sigma_r^{\mathrm{measured}}}{\sigma_r^{\mathrm{predicted}}} . \quad (3)$$

Two different models were used to extract F_2 in two overlapping Q^2 ranges for this paper. The results of the HHT NNLO analysis [8] and Eq. 1 were used for $Q^2 \geq 1.2\,\mathrm{GeV}^2$. The contributions from γZ interference and Z exchange, which become important as Q^2 increases, were taken into account through Eq. 1. For $Q^2 \leq 2.7\,\mathrm{GeV}^2$, Eq. 2 was used to extract F_2 using estimates of $R = F_{\mathrm{L}}/(F_2 - F_{\mathrm{L}})$ from the Badelek–Kwiecinski–Stasto (BKS) model [19] for F_{L} at low x_{Bj} and low Q^2. This model is based on the kinematic constraint that $F_{\mathrm{L}} \propto Q^4$ as $Q^2 \rightarrow 0$ and on the photon–gluon fusion mechanism. The contribution of quarks having limited transverse momenta is treated phenomenologically, assuming the soft Pomeron exchange. The value of R was predicted by extrapolating F_2 to the region of low Q^2. In principle, R depends not only on Q^2, but also on x_{Bj}. However, the dependence on x_{Bj} is small and for the extraction of F_2, the average value of R over the x_{Bj} range relevant for each Q^2 value was used.

Parting remarks



arXiv:1704.03187

Here's a recently produced plot on $dF_2/d\ln Q^2$. At LO and x <~0.005 this quantity is directly related to the gluon PDF.

At very low x and Q^2 the turnovers could indicate saturation– a new state of high-density gluons- but one is also falling into the non-perturbative region. At HERA this is not definitive.

To really probe the high density region there are two ways:

- **A machine with lower x reach for higher Q^2 – the LHeC**
- **A machine with higher-density reach due to the use of nuclei -- the EIC**