

# FORWARD JET PRODUCTION WITHIN SMALL-X ITMD FACTORIZATION

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BASED ON WORK IN COLLABORATION WITH:

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# PLAN

## 1. Introduction

A. Motivation

B. Dilute-dense collisions in Color Glass Condensate (CGC)

## 2. Framework

A. Limiting cases of CGC formulae for dilute-dense collisions

B. Small- $x$  Improved TMD factorization (ITMD) for pA and  $\gamma$ A

C. TMD gluon distributions at small  $x$

D. KaTie Monte Carlo generator

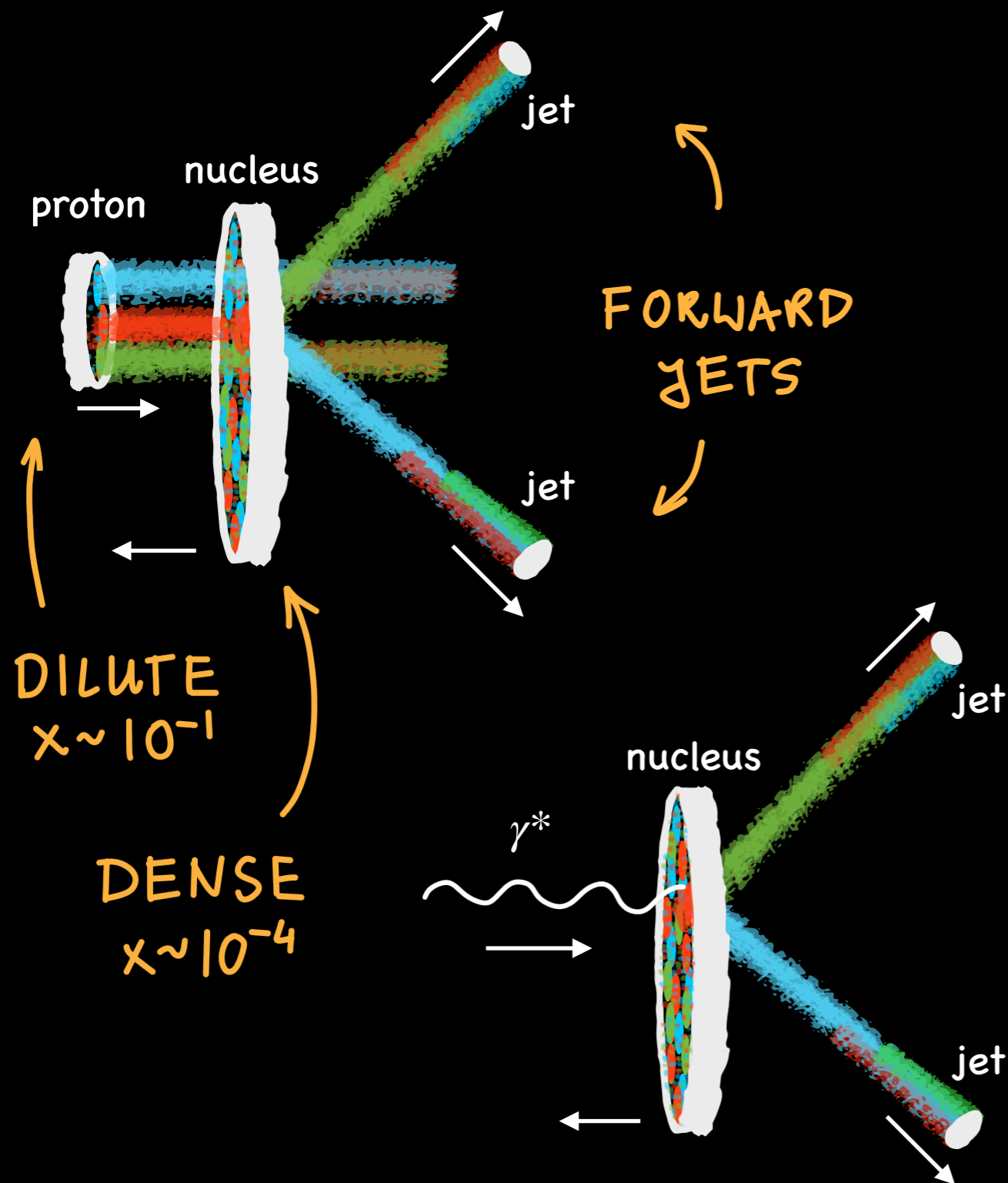
## 3. Phenomenology for LHC and EIC

A. Forward dijets and trijets in pA collisions

B. Forward dijets in  $\gamma$ A collisions

## 4. Summary & Outlook

# MOTIVATION



Study of high energy limit of QCD:

- saturation of gluon density

Nonlinear evolution of TMD PDFs.

Interplay of saturation and Sudakov resummation.

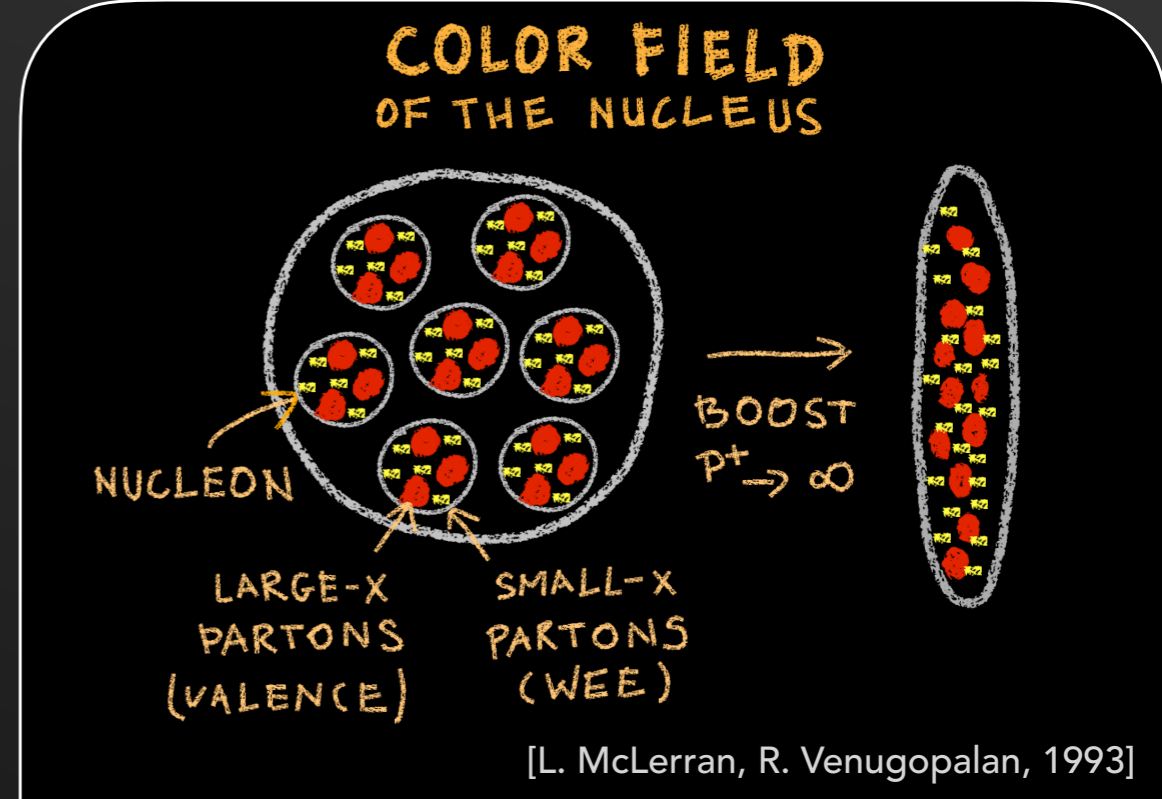
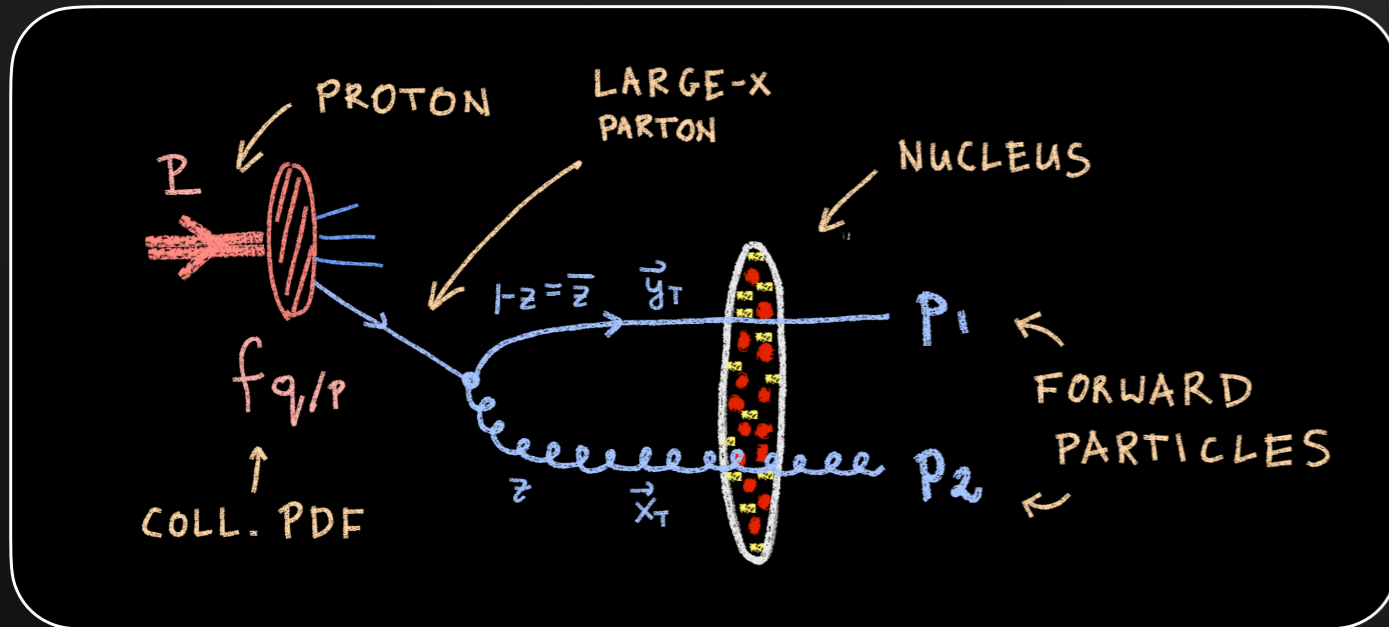
Nonuniversality of TMD gluon PDFs.

- $k_T$ -factorization

TMD factorization beyond leading power.

# INTRODUCTION

## pA (dilute-dense) collisions within CGC



$$\frac{d\sigma_{qA \rightarrow 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2y}{(2\pi)^2} \frac{d^2y'}{(2\pi)^2} e^{-i\vec{p}_{T1} \cdot (\vec{x}_T - \vec{x}'_T)} e^{-i\vec{p}_{T2} \cdot (\vec{y}_T - \vec{y}'_T)}$$

← QUARK WAVE FUNCTION

$$\times \psi_z^* (\vec{x}'_T - \vec{y}'_T) \psi_z (\vec{x}_T - \vec{y}_T)$$

$$\times \left\{ S_x^{(6)} (\vec{y}_T, \vec{x}_T, \vec{y}'_T, \vec{x}'_T) - S_x^{(4)} (\vec{y}_T, \vec{x}_T, \vec{z}\vec{y}'_T + z\vec{x}'_T) \right.$$

← CORRELATORS OF WILSON LINES

$$\left. - S_x^{(4)} (\vec{z}\vec{y}_T + z\vec{x}_T, \vec{y}'_T, \vec{x}'_T) - S_x^{(2)} (\vec{z}\vec{y}_T + z\vec{x}_T, \vec{z}\vec{y}'_T + z\vec{x}'_T) \right\}$$

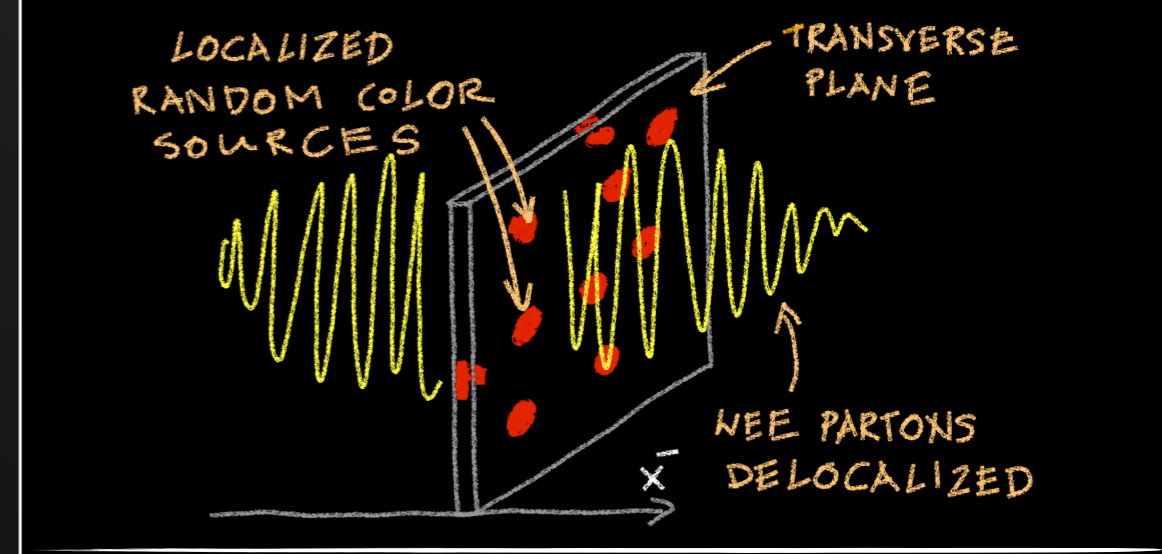
$$S_x^{(2)} (\vec{y}_T, \vec{x}_T) = \frac{1}{N_c} \langle \text{Tr} U(\vec{y}_T) U^\dagger(\vec{x}_T) \rangle_x$$

$$S_x^{(4)} (\vec{z}_T, \vec{y}_T, \vec{x}_T) = \frac{1}{2C_F N_c} \langle \text{Tr} [U(\vec{z}_T) U^\dagger(\vec{y}_T)] \text{Tr} [U(\vec{y}_T) U^\dagger(\vec{x}_T)] \rangle_x$$

etc...

$$U(\vec{x}_T) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, \vec{x}_T) t^a \right\}$$

[C. Marquet, 2007]



Large-x partons — the color source for wee partons:  
 $(D_\mu F^{\mu\nu})_a(x^-, \vec{x}_T) = \delta^{\nu+} \rho_a(\vec{x}_T) \delta(x^-)$   
 RANDOM DISTRIBUTION OF COLOR SOURCES

AVERAGE OVER COLOR SOURCES  
 GAUSSIAN FUNCTIONAL →  $\mathcal{W}_x[\rho]$   
 B-JIMWLK EVOLUTION IN X  
 [Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner, 1996-2002]

only full CGC  
calculations included

### Single inclusive hadron production in pA (NLO)

[G.A. Chirilli, B.-W. Xiao, F. Yuan, 2012]

[A. Stasto, B.-W. Xiao, D. Zaslavsky, 2014]

[T. Altinoluk, N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, 2015]

[E. Iancu, A.H. Mueller, D.N. Triantafyllopoulos, 2016]

[B. Dudoe, T. Lappi, Y. Zhu, 2017]

PHENO

### Dijet+photon production in $\gamma^*A$ (NLO)

[K. Roy, R. Venugopalan, 2018, 2019]

### Dijet production in diffractive $\gamma^*A$ (NLO)

[R. Boussarie, A. Grabovsky, L. Szymanowski, S. Wallon, 2019]

PHENO

### Dijet/di-hadron production in $\gamma^*A$

[F. Salazar, B. Schenke, 2020]

[H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019]

PHENO

### Dijet/di-hadron production in pA

[C. Marquet, 2007] [H. Fujii, F. Gelis, R. Venugopalan, 2005]

[E. Iancu, J. Leidet, 2013] [H. Fujii, C. Marquet, K. Watanabe, 2020]

PHENO

### Heavy quark pair production in pA

[C. Marquet, C. Roiesnel, P. Taels, 2018]

### Trijet production in $\gamma^*A$

[A. Ayala, M. Hentchinski, J. Jalilian-Marian, M.E. Tejeda-Yeomans, 2016]

[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2020]

### Dijet+photon production in pA

[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2019]

[T. Altinoluk, N. Armesto, A. Kovner, M. Lublinsky, E. Petreska, 2018]

### Trijet production in pA

[E. Iancu, Y. Mulian, 2018]

# INTRODUCTION

## Limiting cases of CGC in dilute-dense collisions

**CGC**  
*dilute-dense*

three scales:

$Q_s \gg \Lambda_{\text{QCD}}$  — saturation scale

$k_T$  — jet transverse momentum imbalance

$P_T$  — jet average transverse momentum

$$P_T \gg k_T \sim Q_s$$



**TMD**  
**GENERALIZED**  
**FACTORIZATION**

leading twist

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]  
[C. Marquet, E. Petreska, C. Roiesnel, 2016]  
[C. Marquet, C. Roiesnel, P. Taels, 2018]  
[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2019]  
[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2020]

$$P_T \sim k_T \gg Q_s$$



**DILUTE**

**$k_T$ -FACTORIZATION**  
**BFKL dynamics**

[S. Catani, M. Ciafaloni, F. Hautmann, 1991]  
[M. Deak, F. Hautmann, H. Jung, K. Kutak, 2009]  
[E. Iancu, J. Leidet, 2013]

$$P_T \gg Q_s$$



**ITMD**  
**"IMPROVED"**  
**TMD factorization**

all kinematic twists

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]  
[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]  
[T. Altinoluk, R. Boussarie, PK, 2019]

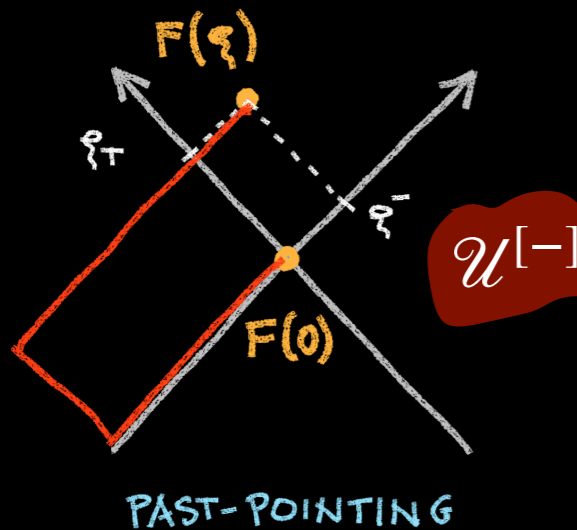
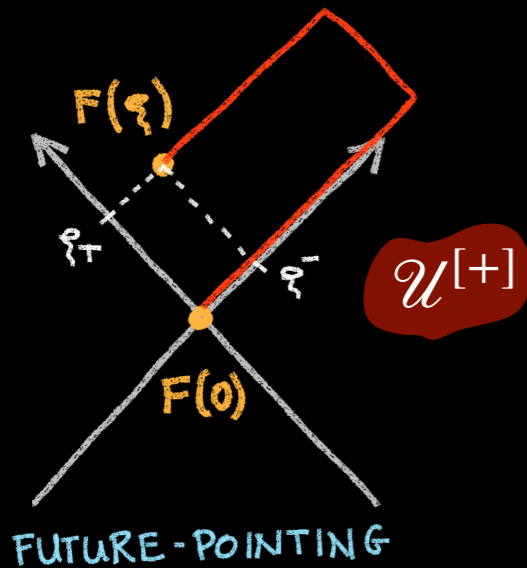


### Generic operator definition (unpolarized)

$$\mathcal{F}_g(x, k_T) = 2 \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi^+, \vec{\xi}_T, \xi^- = 0) \mathcal{U}_{C_1} \hat{F}^{i-}(0) \mathcal{U}_{C_2} \right] | P \rangle$$

GLUON FIELD  
 $\hat{F} = F_a t^a$

WILSON LINES  
 IN FUNDAMENTAL  
 REPRESENTATION



Gauge links  $\mathcal{U}_{C_1}, \mathcal{U}_{C_2}$  depend on the color structure of the hard process. They are built from two basic Wilson lines:

[C. Bomhof, P. Mulders, F. Pijlman, 2004]

$$\begin{aligned} \mathcal{U}^{[\pm]} = & [0, (\pm\infty, \vec{0}_T, 0)] \\ & \times [(\pm\infty, \vec{0}_T, 0), (\pm\infty, \vec{\xi}_T, 0)] \\ & \times [(\pm\infty, \vec{\xi}_T, 0), (\xi^+, \vec{\xi}_T, 0)] \end{aligned}$$

$$[x, y] = \mathcal{P} \exp \left\{ ig \int_{\overline{xy}} dz_\mu A_a^\mu(z) t^a \right\}$$

STRAIGHT LINE  
 SEGMENT

Light-cone basis:

$$v^\pm = v^\mu n_\mu^\pm, \quad n^\pm = (1, 0, 0, \mp 1)$$

$$v^\mu = \frac{1}{2} v^+ n^- + \frac{1}{2} v^- n^+ + v_T^\mu$$

### All possible operators

$$\mathcal{F}_{qg}^{(1)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{qg}^{(2)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[ \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(4)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[-]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle P | \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

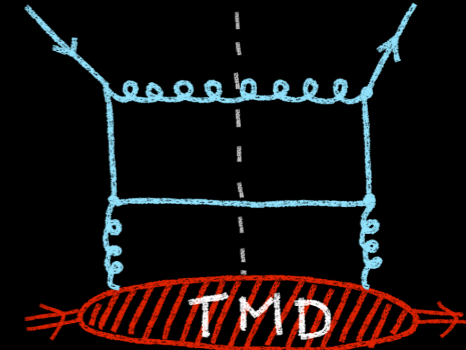
$$\mathcal{F}_{gg}^{(7)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[ \hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

WILSON LOOP  $\rightarrow \mathcal{U}^{[\square]} = \mathcal{U}^{[+]} \mathcal{U}^{[-]\dagger}$

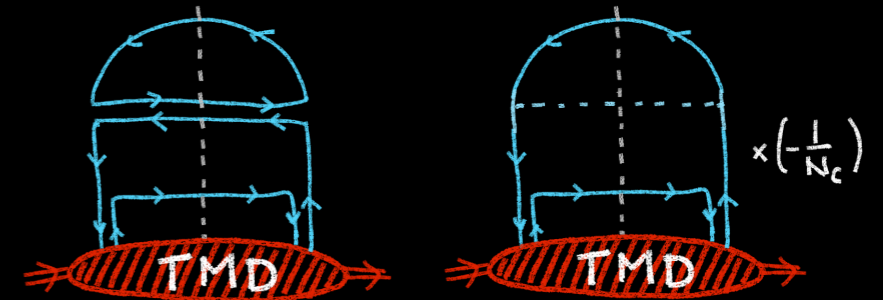
[M. Bury, PK, K. Kutak, 2018]

### Example

What is the TMD gluon distribution for the following process:



Two independent color flows:



$$\rightsquigarrow \frac{N_c}{2C_F} \mathcal{F}_{qg}^{(2)} - \frac{1}{2N_c C_F} \mathcal{F}_{qg}^{(1)}$$

Gluon TMD for any multiparticle process is given by a linear combination of these "basis" TMDs.



### Small-x limit of TMD gluon distributions

$$\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[ \hat{F}^{i-} \left( \xi^+, \vec{\xi}_T, \xi^- = 0 \right) \mathcal{U}_{C_1} \hat{F}^{i-} (0) \mathcal{U}_{C_2} \right] | P \rangle$$

LIMIT  
 $x \rightarrow 0.$

Dependence on  $x$  is only via the small- $x$  evolution equations:

- BFKL (Balitsky-Fadin-Kuraev-Lipatov).
- BK (Balitsky-Kovchegov) and modifications
- JIMWLK (Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner)

### Correspondence to CGC

Example:

$$\mathcal{F}_{qg}^{(1)} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4} k_T^2 e^{-i\vec{k}_T \cdot (\vec{x}_T - \vec{y}_T)} \langle \text{Tr} [U(\vec{x}_T) U^\dagger(\vec{y}_T)] \rangle_x$$

DIPOLE  
GLUON DISTRIBUTION

WILSON LINES

AVERAGE OVER  
CGC COLOR SOURCES

$$U(\vec{x}_T) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, \vec{x}_T) t^a \right\}$$

$$\langle \dots \rangle_x \rightarrow \frac{\langle \text{Tr} \dots \text{Tr} \rangle}{\langle \text{Tr} \mathbb{1} \rangle}$$

Intensively studied:

- [D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
- [B. Xiao, F. Yuan, 2010]
- [F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
- [A. Metz, J. Zhou, 2011]
- [E. Akcakaya, A. Schafer, J. Zhou, 2012]
- [C. Marquet, E. Petreska, C. Roiesnel, 2016]
- [I. Balitsky, A. Tarasov, 2015, 2016]
- [D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
- [C. Marquet, C. Roiesnel, P. Taels, 2018]
- [Y. Kovchegov, D. Pitonyak, M. Sievert, 2017, 2018]
- [T. Altinoluk, R. Boussarie, 2019]
- [R. Boussarie, Y. Mehtar-Tani, 2020]

# FRAMEWORK Small-x Improved TMD Factorization (ITMD)

Factorization formula for forward dijets in p-p and p-A collisions

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]

$$\frac{d\sigma_{pA \rightarrow 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1, \mu) \sum_{i=1,2} K_{ag \rightarrow cd}^{(i)}(k_T) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_T)$$

RAPIDITY  $x_2 \ll x_1$       TRANSVERSE MOMENTA  $|\vec{p}_{T1} + \vec{p}_{T2}| = k_T$   
 COLLINEAR PROTON PDF  
 GAUGE INVARIANT OFF-SHELL HARD FACTORS  
 TMD GLUON DISTRIBUTIONS AT SMALL-X

TWO PER CHANNEL  
 $(g^*q \rightarrow qq, qg^* \rightarrow gg, g^*g \rightarrow q\bar{q})$

ITMD factorization formula has been proven from the Color Glass Condensate (CGC) theory.

⇒ RESUMMATION OF KINEMATIC TWISTS AND NEGLECTING GENUINE TWISTS.

$$\Lambda_{\text{QCD}} \ll Q_s \ll P_T$$

SATURATION SCALE

[T. Altinoluk, R. Boussarie, PK, 2019]

# PHENOMENOLOGY Obtaining small-x TMD gluon distributions

Using CGC theory one can derive a relation between the small-x TMDs using:

- (i) large  $N_c$  limit
- (ii) mean field (Gaussian) approximation.

All TMDs needed for dijet production can be calculated from

the dipole gluon distribution  $\mathcal{F}_{qg}^{(1)}$ .

We used  $\mathcal{F}_{qg}^{(1)}$  obeying the BK equation (with subleading corrections based on the Kwiecinski-Martin-Stasto equation) and fitted to HERA data.

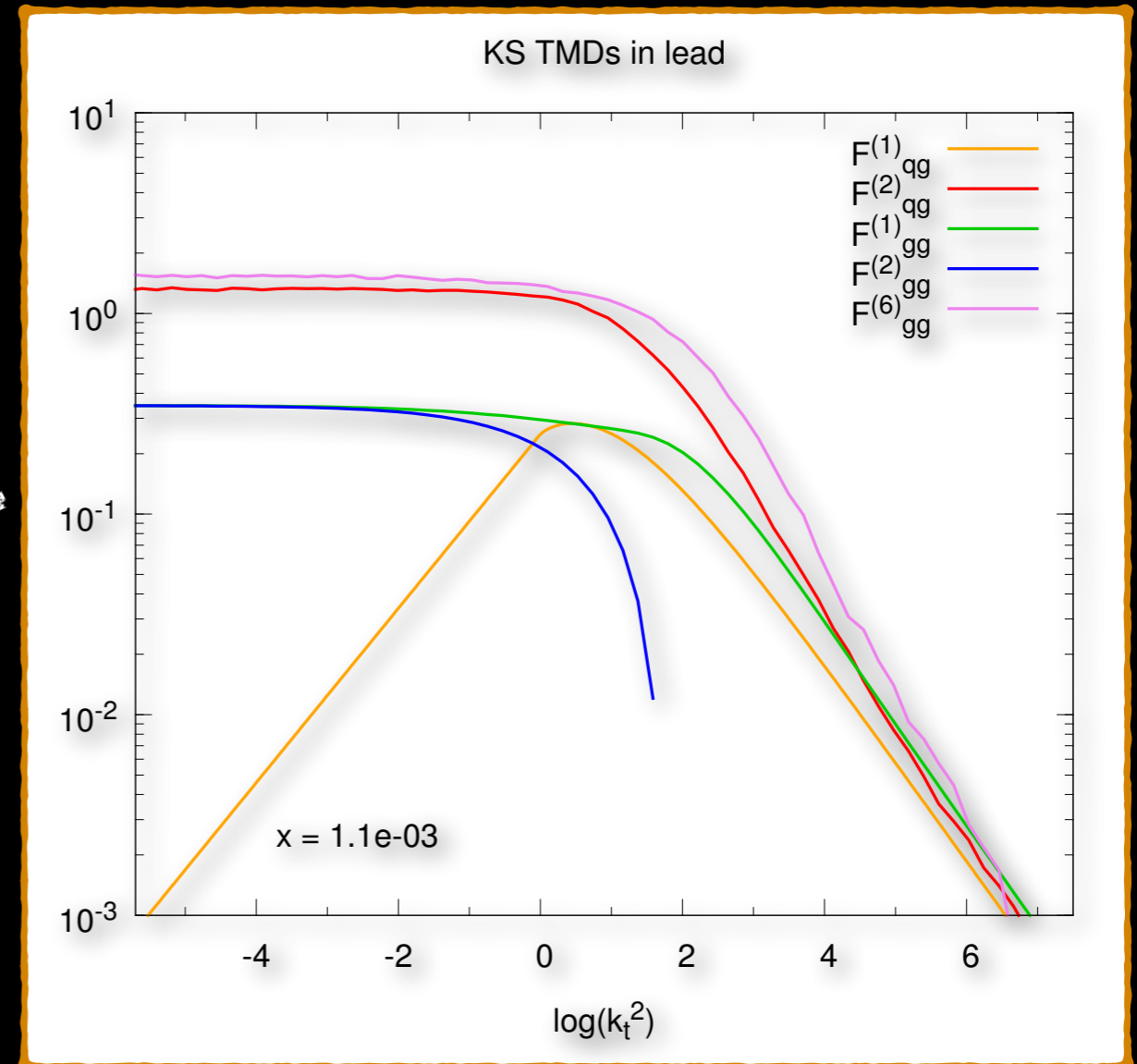
[K. Kutak, J. Kwieciński, 2003]

[K. Kutak, S. Sapeta, 2012]

It is possible to relax the assumptions (i) and (ii) using the JIMWLK equation.

Prove of concept:

[C. Marquet, E. Petreska, C. Roiesnel, 2016]



[A. Van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]

# PHENOMENOLOGY

## ITMD vs ATLAS data

Measurement of dijet azimuthal correlations in p+p and p+Pb. [ATLAS, Phys. Rev. C100 (2019)]

$\sqrt{s} = 5.02 \text{ TeV}$  rapidity:  $2.7 < y_1, y_2 < 4.5$

$$C_{12} = \frac{1}{N_1} \frac{dN_{12}}{d\Delta\phi}$$

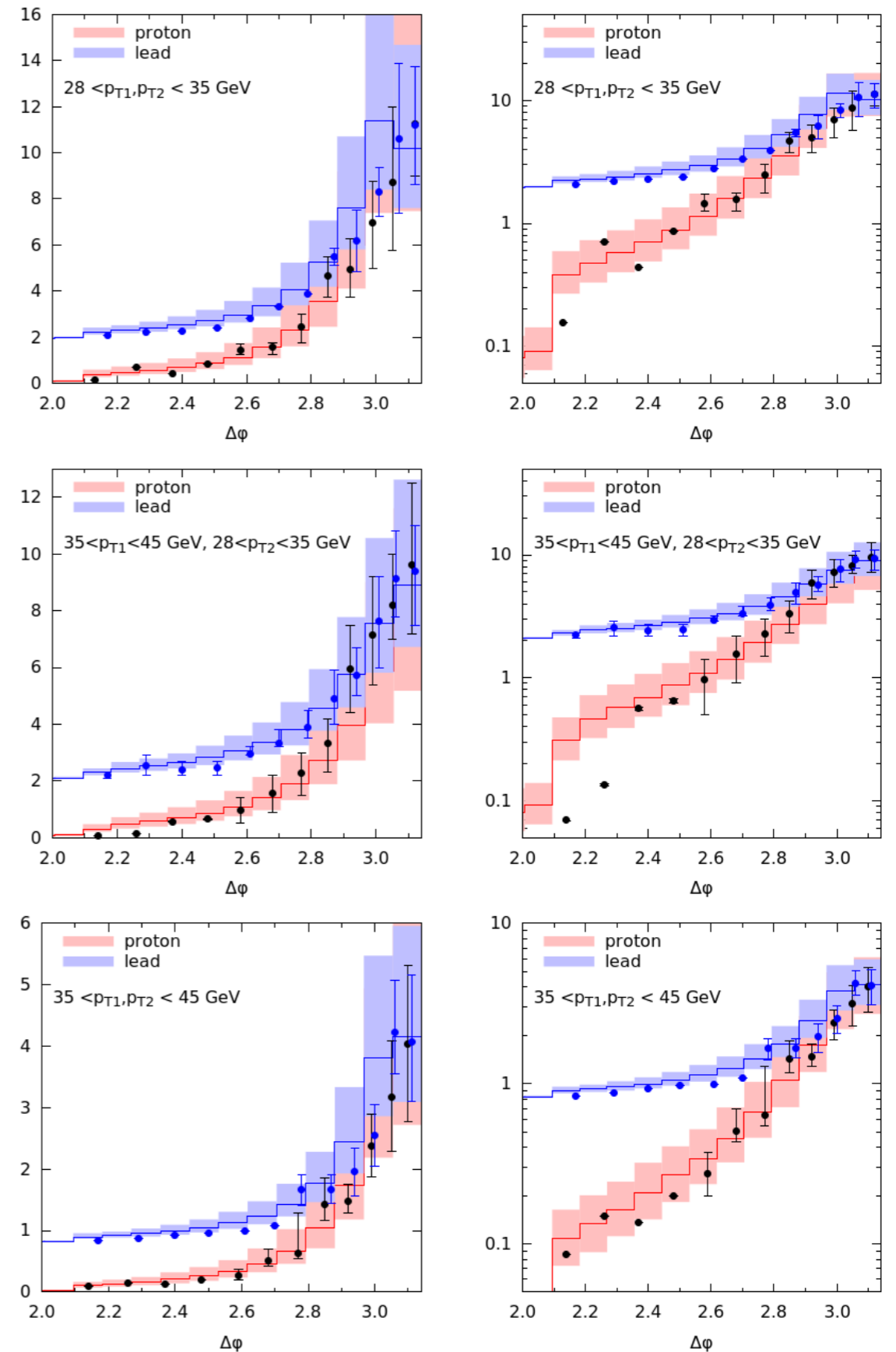
NUMBER OF DIJETS  
AZIMUTHAL ANGLE BETWEEN JETS  
NUMBER OF LEADING JETS

We study an interplay of saturation and Sudakov resummation vs the **shape** of  $C_{12}$ .

Good description of the broadening effects

Similar study for RHIC in the back-to-back limit:

[A. Stasto, S-Y. Wei, B-W. Xiao, F. Yuan, 2018]



A. Van Hameren, P. Kotko, K. Kutak, S. Sapeta, Phys. Lett. B795 (2019) 511

# FRAMEWORK ITMD for jets in DIS

ITMD factorization formula for DIS is almost the same as the  $k_T$ -factorization formula in inclusive DIS, but probes different TMD gluon distribution

[PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, 2017]

$$d\sigma_{\gamma^*A \rightarrow 2j+X} \sim \int \frac{dx}{x} \int d^2k_T \mathcal{F}_{gg}^{(3)}(x, k_T, \mu) d\sigma_{\gamma^*g^* \rightarrow j_1j_2}(x, k_T, \mu)$$

WEIZSÄCKER-WILLIAMS  
TMD GLUON DISTRIBUTION

OFF-SHELL  
PHOTON-GLUON FUSION

Weizsacker-Williams TMD gluon distribution is the true gluon number distribution.

It is not probed in inclusive processes nor in jet production in pA (at large  $N_c$ ).

Multi jet/hadron production at EIC will be a unique probe.

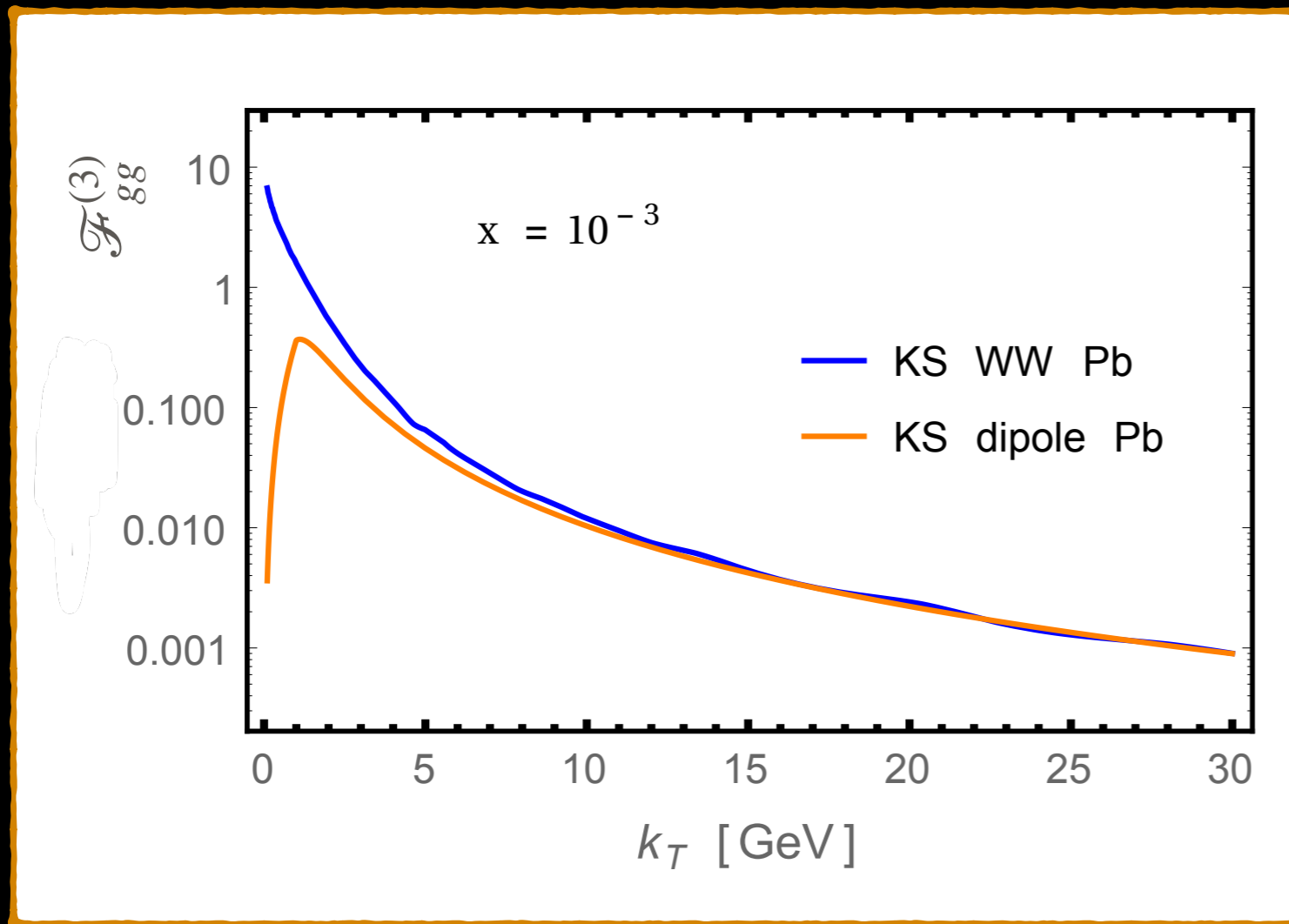
Also studied in the back-to-back regime

[L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014]

[L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, Z-B. Yin, 2018]

# PHENOMENOLOGY Weizsacker-Williams TMD gluon distribution

Using the same approximations as for other gluon TMDs we can calculate the Weizsacker-Williams TMD from the dipole distribution.

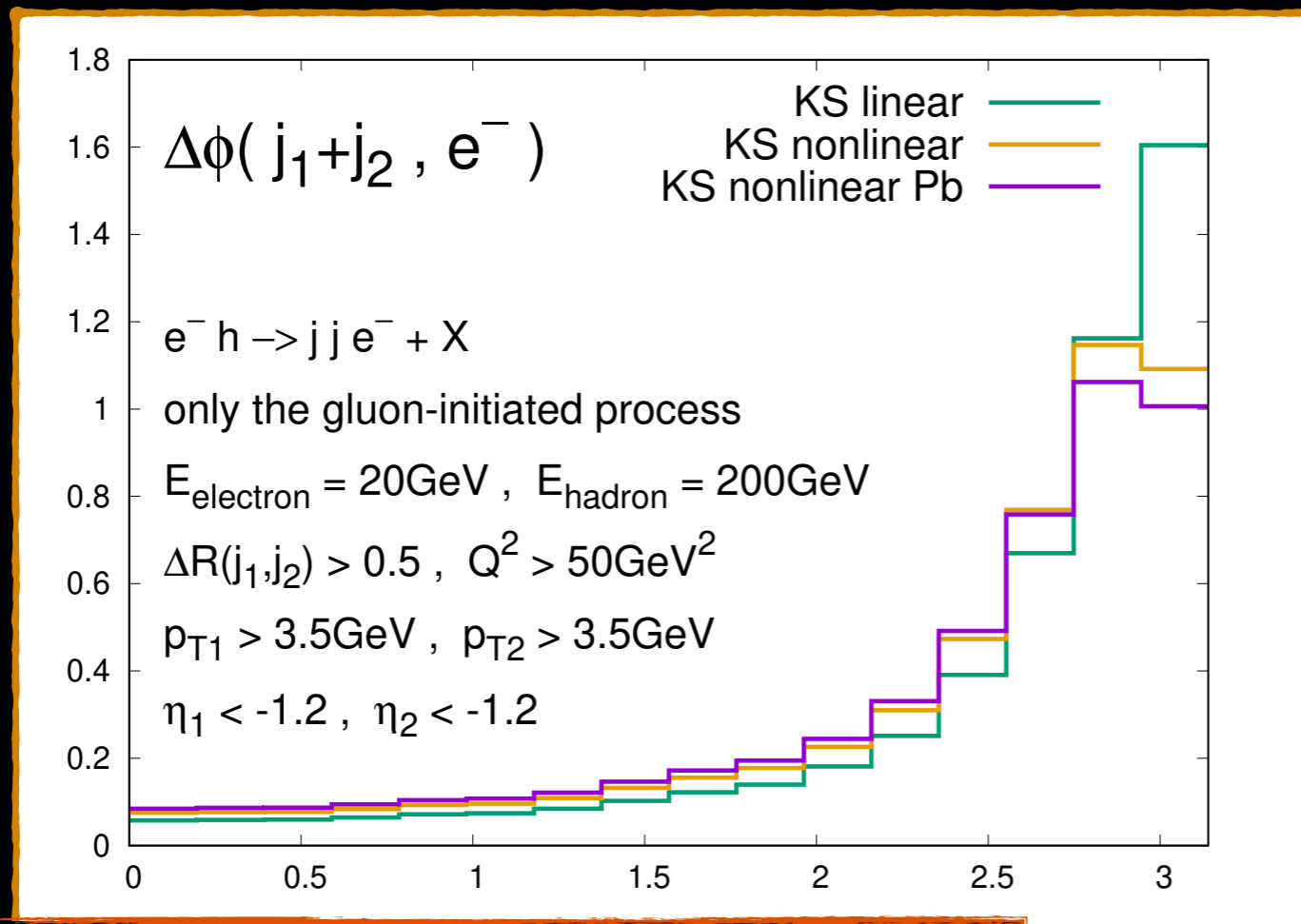


CALCULATED  
FROM THE  
KUTAK-SAPETA (KS)  
DIPOLE TMD

- Very different behavior at small  $k_T$
- Convergence at large  $k_T$

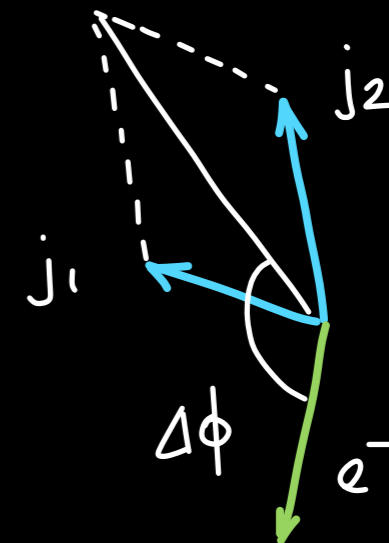
[PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, 2017]

ITMD for dijets in  $\gamma A$  collisions has been so far studied in ultraperipheral heavy ion collisions at LHC for dijet azimuthal imbalance. [PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, 2017]



A. van Hameren, EIC yellow report seminar

At EIC one can study also angle between final states and the final state lepton.



Detailed study in progress

# FRAMEWORK ITMD\* for three and more jets in pA collisions

If we ignore linearly polarized gluon contribution for multi-jet processes, the ITMD framework (called ITMD\*) can be formulated (and automatized).

[M. Bury, PK, K. Kutak, 2018]

[M. Bury, A. van Hameren, PK, K. Kutak, 2020]

$$\frac{d\sigma_{pA \rightarrow 3j+X}}{dP \cdot S} \sim \int \frac{dx_1 dx_2}{x_1 x_2} \int d^2 k_T \sum_{\text{partons}} f_{a/p}(x_1, \mu) \vec{\mathcal{A}}^\dagger \Phi(x_2, k_T, \mu) \vec{\mathcal{A}}$$

VECTOR OF  
COLOR ORDERED  
OFF-SHELL AMPLITUDES

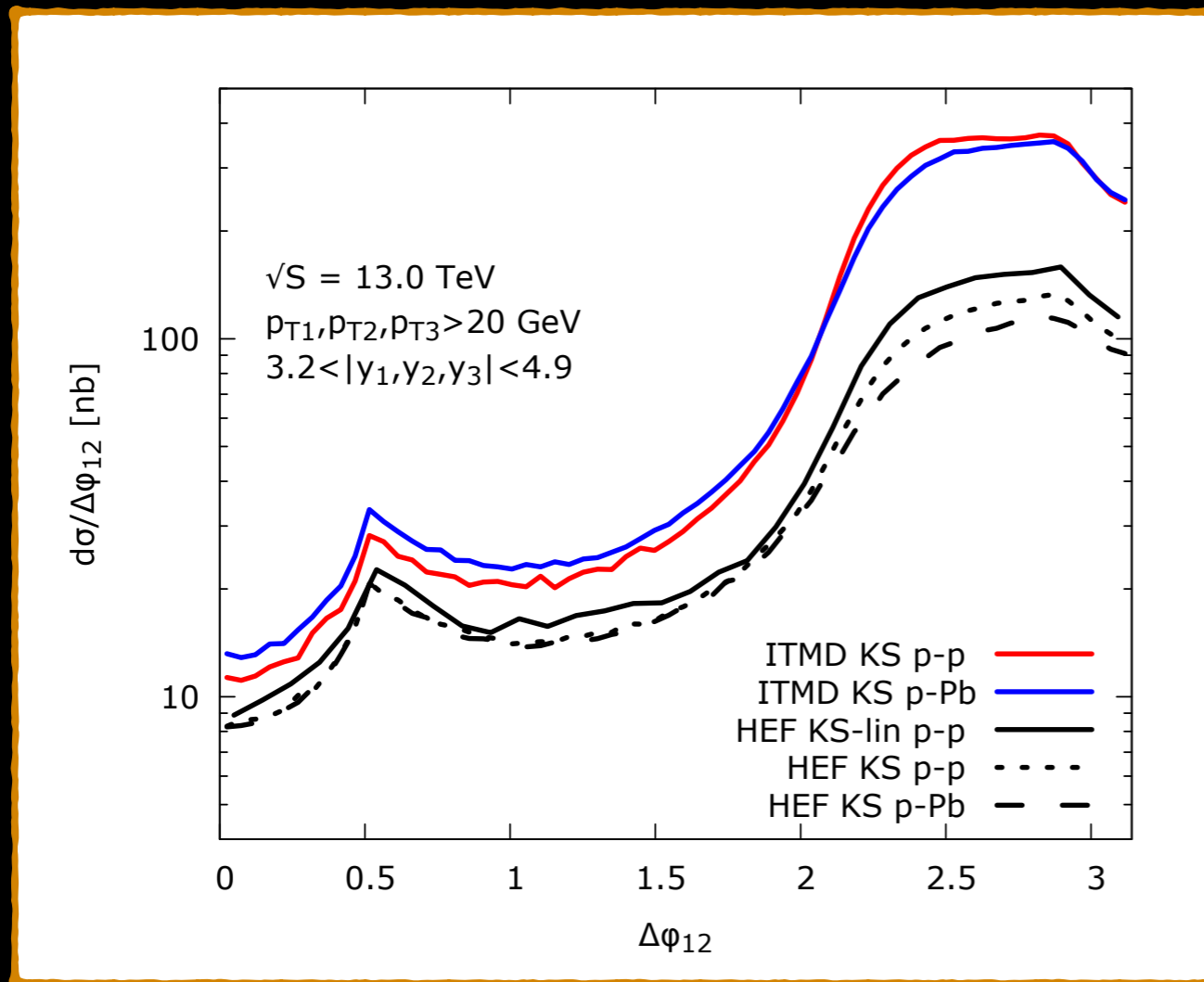
MATRIX  
OF TMD GLUON  
DISTRIBUTIONS



| CHANNEL                         | DIM          |
|---------------------------------|--------------|
| $g^* q \rightarrow q q q$       | $6 \times 6$ |
| $g^* q \rightarrow q \bar{q} q$ | $6 \times 6$ |
| $g^* q \rightarrow q q \bar{q}$ | $6 \times 6$ |
| $g^* q \rightarrow q q \bar{q}$ | $4 \times 4$ |

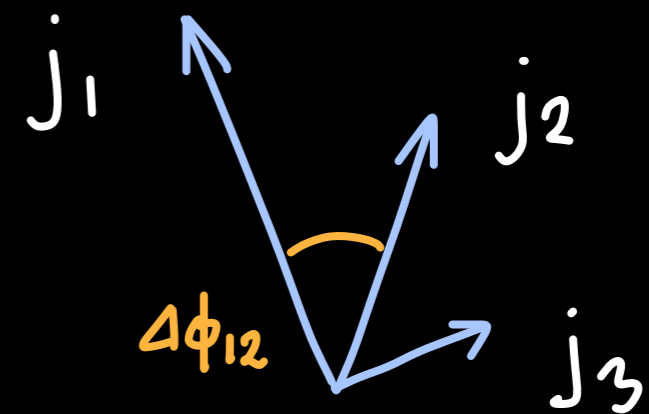


Using the large  $N_c$  limit to the TMD matrix, and in the mean field approximation, the same TMD operators contribute as for dijets.

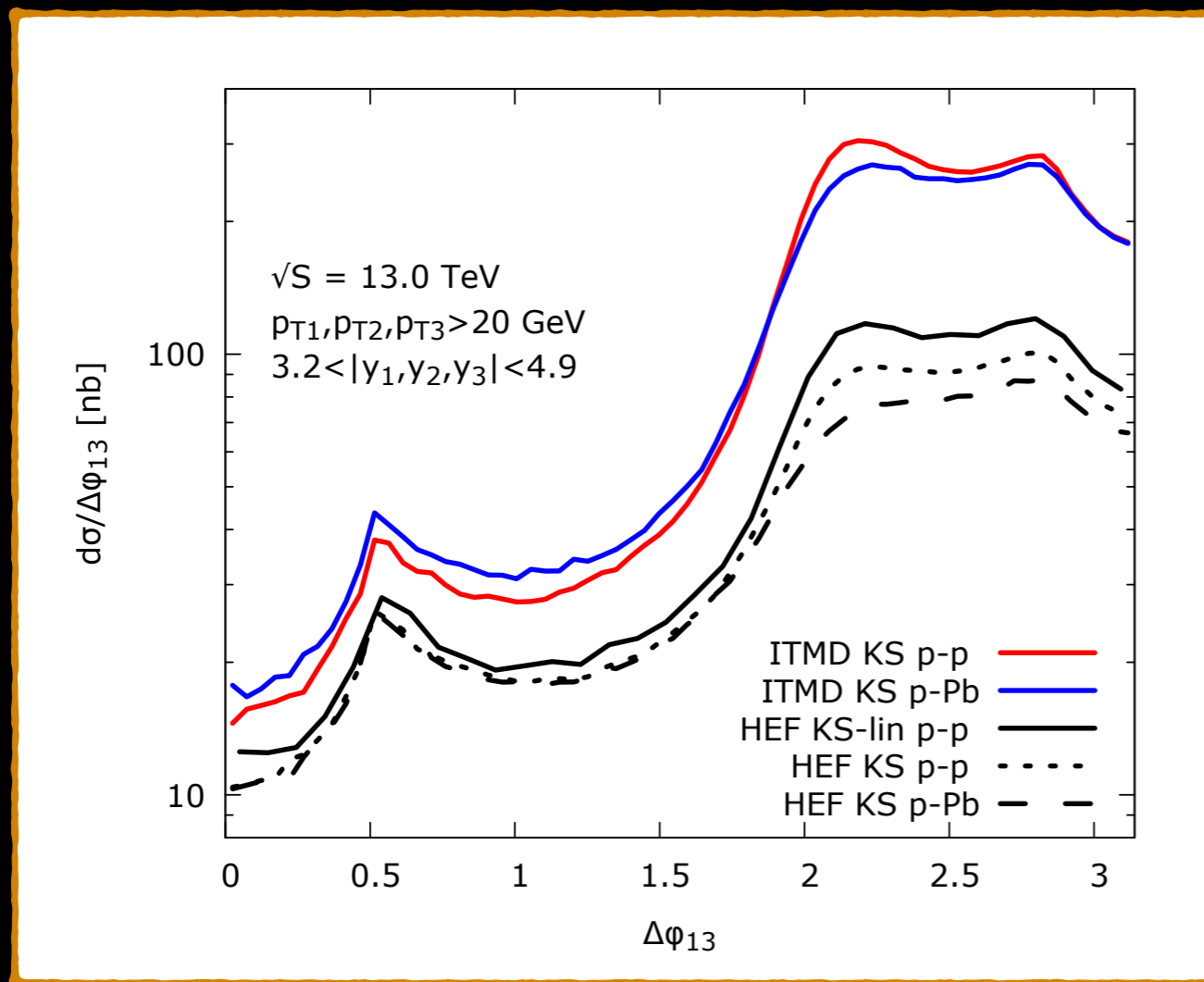


[M. Bury, A. van Hameren, PK, K. Kutak, 2020]

Azimuthal angle between the leading jets

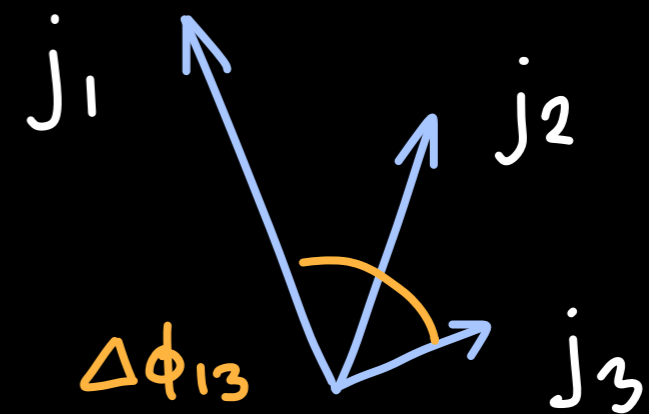


Using the large  $N_c$  limit to the TMD matrix, and in the mean field approximation, the same TMD operators contribute as for dijets.

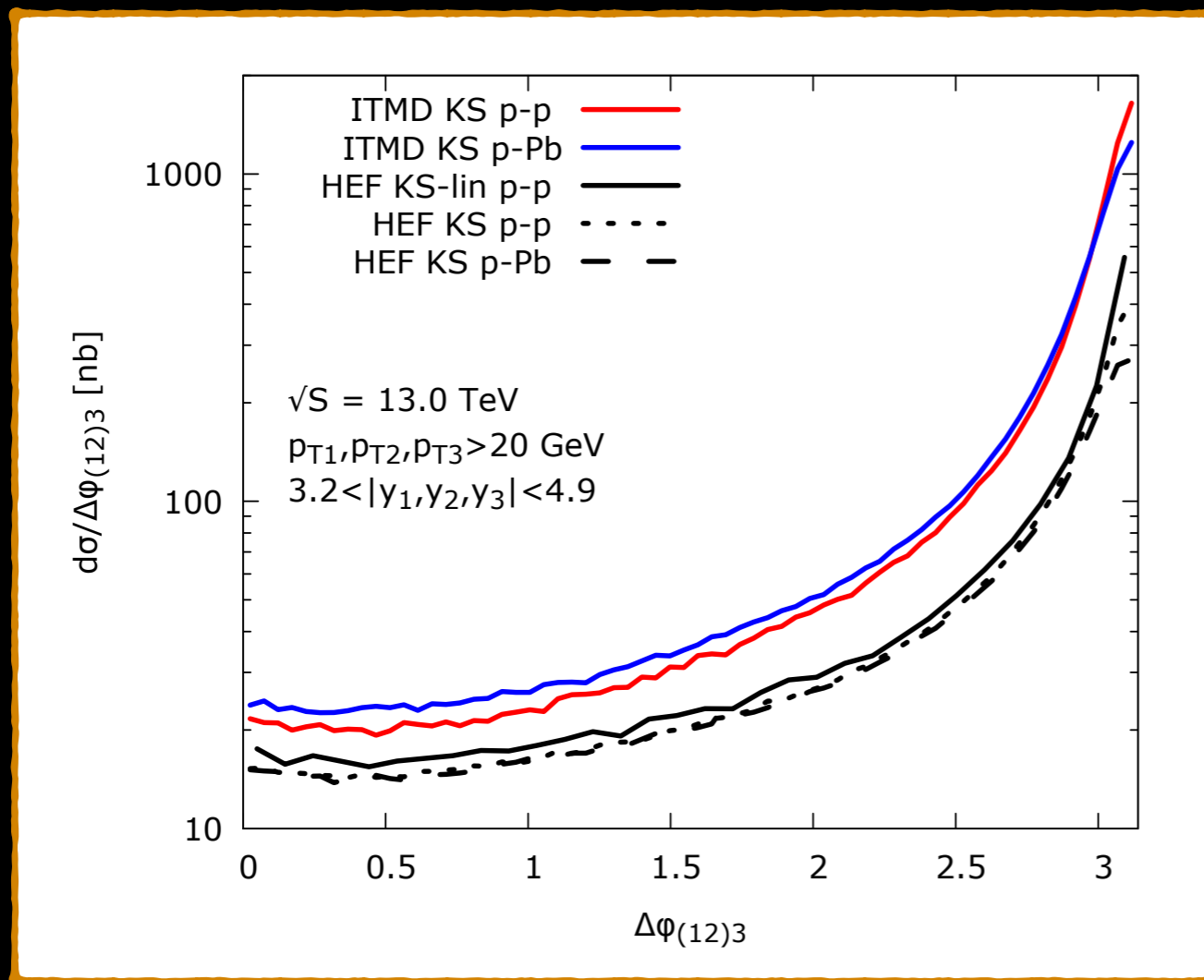


[M. Bury, A. van Hameren, PK, K. Kutak, 2020]

Azimuthal angle between the leading and the soft jet

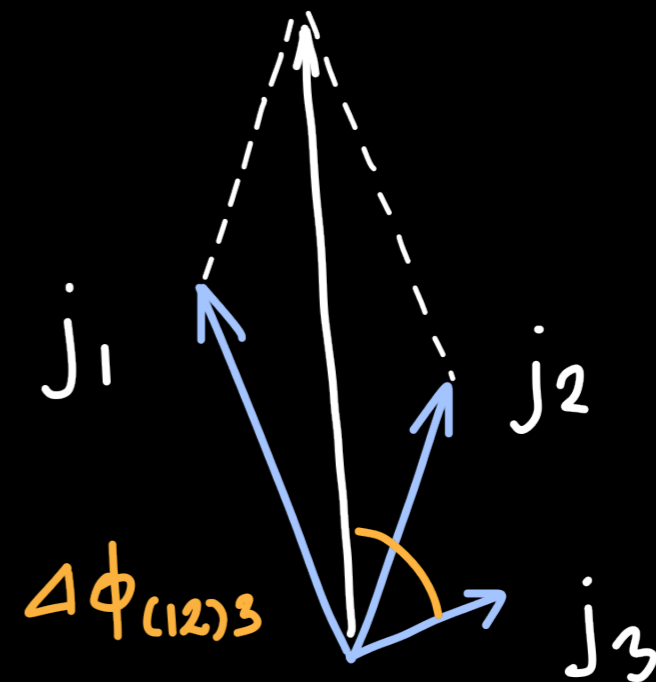


Using the large  $N_c$  limit to the TMD matrix, and in the mean field approximation, the same TMD operators contribute as for dijets.



[M. Bury, A. van Hameren, PK, K. Kutak, 2020]

Azimuthal angle between the plane spanned by the leading jets and the soft jet



KATIE

<https://bitbucket.org/hameren/katie>

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Higgs-gluon coupling) with several final-state particles.
- 0, 1, or 2 off-shell initial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- event files can be processed further by parton-shower program like CASCADE.
- (evaluation of) matrix elements now separately available, including C++ interface.

# SUMMARY

## PARTICLE PRODUCTION in dilute-dense collisions

### CGC

- cross section structure:
  - projectile wave function
  - color averages of straight infinite Wilson lines
- all kinematic twists
- multiple interactions (genuine twist)
- hard to compute and automatize
- domain: jets of any hardness

### ITMD

- cross section structure:
  - off-shell gauge invariant amplitudes
  - many TMD gluon distributions
- all kinematic twists
- no genuine twists
- has been automatized and implemented into MC codes
- domain: quite hard jets, but not neglecting saturation scale

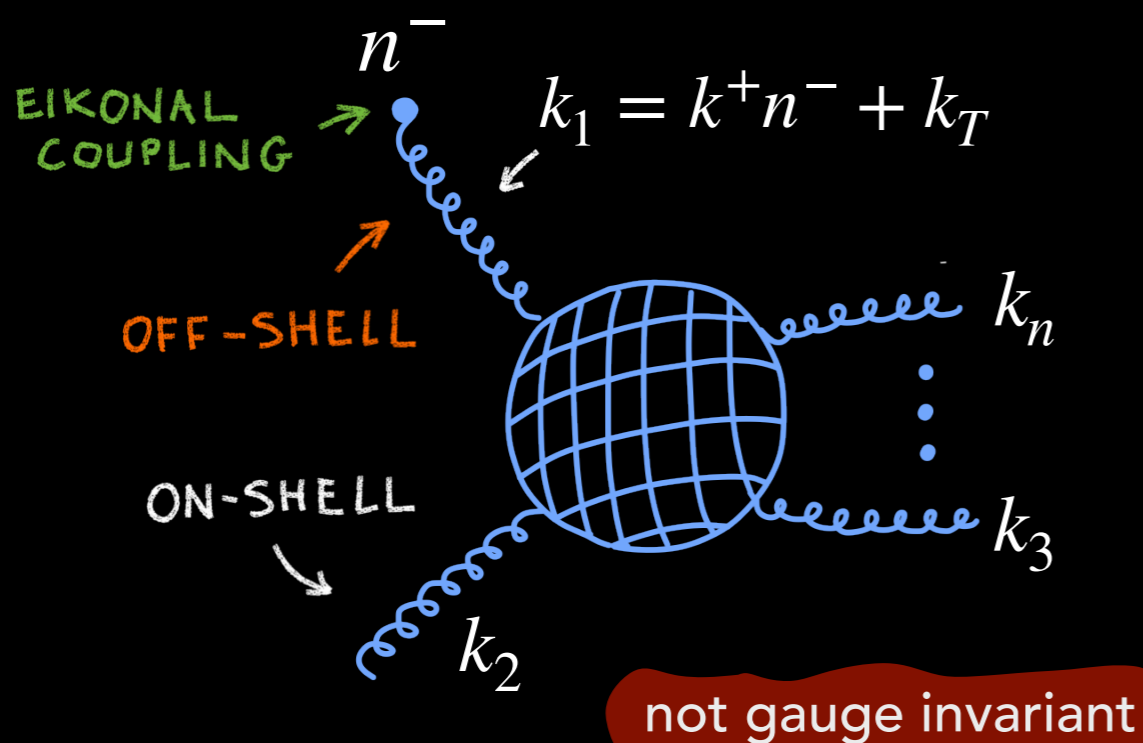
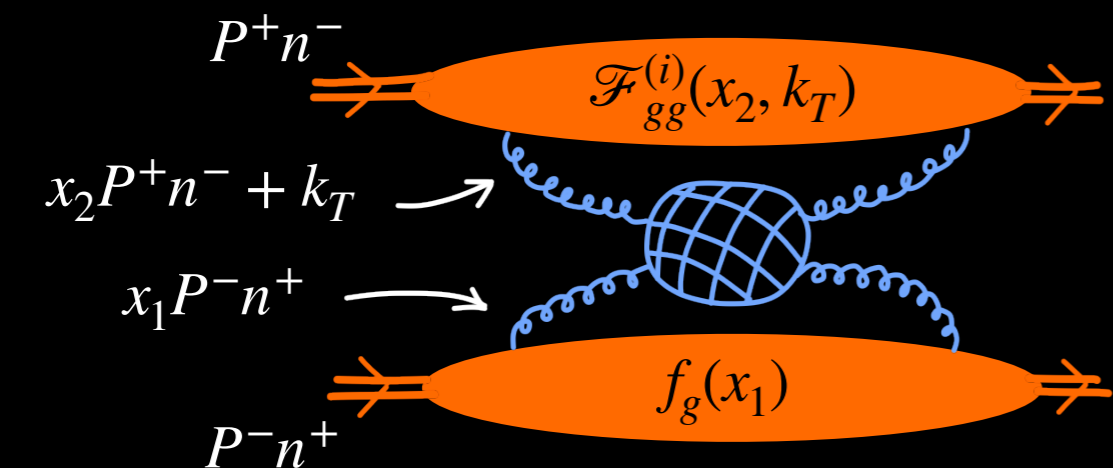
# FUTURE PLANS

- Data-driven small- $x$  TMD gluon distributions from JIMWLK equation (collaboration with lattice QCD experts K. Cichy and P. Korcyl)
- Improvement of the Sudakov resummation (basing on calculations by A. Mueller, B. Xiao, F. Yuan)
- Inclusion of linearly polarized gluons in higher multiplicity jet calculations (extending ITMD\* to full ITMD)
- Automated NLO calculations for off-shell gauge invariant amplitudes (result for any number of gluons with same helicity at NLO is ready)

**BACKUP**

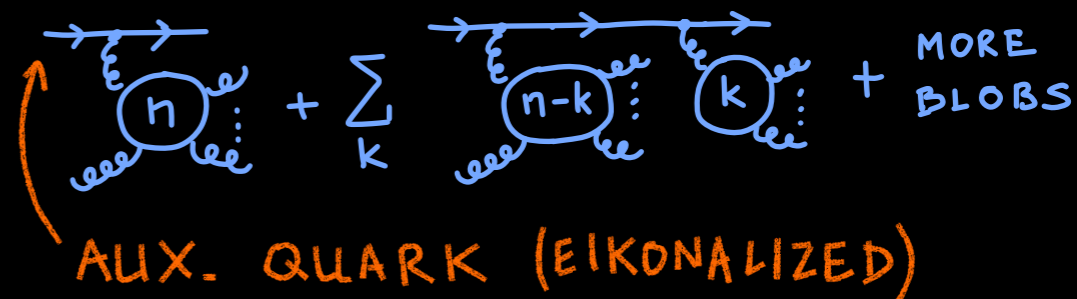
# FRAMEWORK Off-shell hard factors

## Partonic amplitudes at high energy



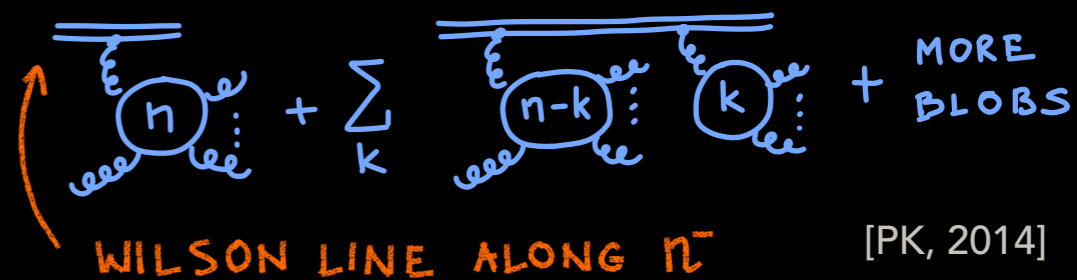
## Tree-level (automatic) techniques

- "embedding"



[A. van Hameren, PK, K. Kutak, 2013]

- ME of straight infinite Wilson line



[PK, 2014]

- Berends-Giele + gauge inv. restoration

[A. Van Hameren, PK, K. Kutak, 2012]

- Off-shell BCFW

[A. Van Hameren, 2014]

[A. Van Hameren, M. Serino, 2014]

Consistent with the Lipatov's high energy effective action.

[L. Lipatov, 1995]



# FRAMEWORK Off-shell hard factors

## Off-shell MHV tree amplitudes

$$\mathcal{M}(1^*, 2^-, 3^+, \dots, n^+) \sim g^{n-2} \frac{\langle 1^* 2 \rangle^4}{\langle 1^* 2 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n 1^* \rangle}$$

## SPINOR PRODUCTS

$$\langle ij \rangle = \langle k_i^- | k_j^+ \rangle$$

$$|k_j^\pm\rangle = \frac{1}{2}(1 \pm \gamma_5)u(k_j)$$

FOR OFF-SHELL LEG:

$$\langle 1^* j \rangle = \langle k^+ n^- - | k_j^+ \rangle$$

gauge invariance is essential

[A. Van Hameren, PK, K. Kutak, 2012]

[A. Van Hameren, 2014]

## Beyond tree level

There exist low multiplicity analytic results within the Lipatov's effective action approach.

[M. Nefedov, V. Saleev, 2017]

[M. Nefedov, 2019]

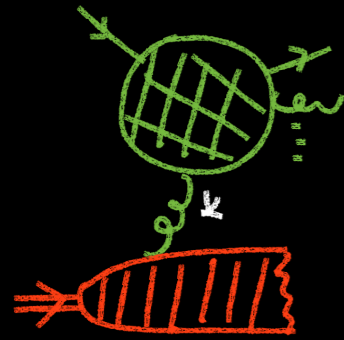
On going project towards automated one-loop corrections in "embedding" approach.

[A. Van Hameren, 2017]

[E. Blanco, A. Van Hameren, PK, K. Kutak, in preparation... ]

### TERMINOLOGY

#### ONE-BODY



$$\mathcal{O}_1 \sim \langle P | F_a^{-i}(x) | X \rangle$$

$$\mathcal{H}_1(k_T) \otimes \tilde{\mathcal{O}}_1(k_T)$$

#### TWO-BODY, etc.



$$\mathcal{O}_2 \sim \langle P | F_a^{-i}(x) F_b^{-j}(y) | X \rangle$$

$$\mathcal{H}_2(k_{T1}, k_{T2}) \otimes \tilde{\mathcal{O}}_2(k_{T1}, k_{T2})$$

#### LEADING TWIST

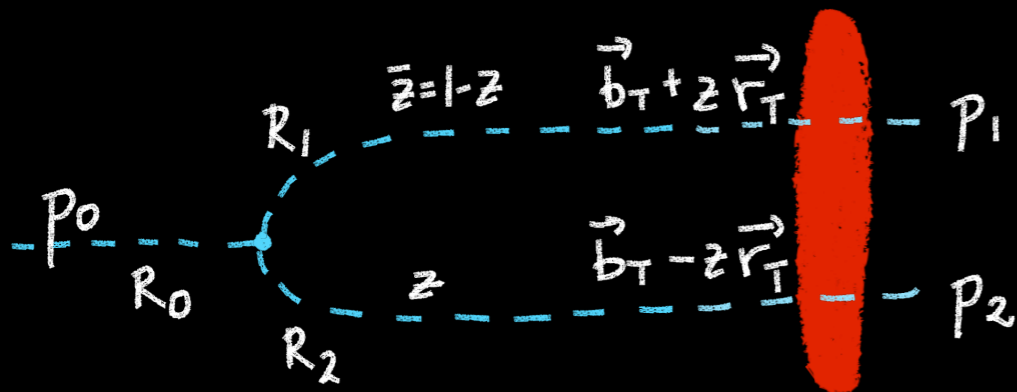
$$\mathcal{A}_{LT} = \mathcal{H}_1(0) \otimes \tilde{\mathcal{O}}_1(k_T)$$

#### NEXT-TO LEADING TWIST

$$\mathcal{A}_{NLT} = \vec{k}_T \cdot (\vec{\partial}_T \mathcal{H}_1)(0) \otimes \tilde{\mathcal{O}}_1(k_T) + \mathcal{H}_2(0,0) \otimes \tilde{\mathcal{O}}_2(k_{T1}, k_{T2})$$

KINEMATIC TWIST  
GENUINE TWIST

### GENERIC 1 → 2 CGC AMPLITUDE



$$\vec{k}_T = \vec{p}_{T1} + \vec{p}_{T2}$$

$$\vec{P}_T = \bar{z} \vec{p}_{T1} - z \vec{p}_{T2}$$

$R_i$  - COLOR REPRESENTATION

$$\mathcal{A} = \delta(p_1^+ + p_2^+ - p_0^+) \int d^2 b_T d^2 r_T e^{-i(\vec{P}_T \cdot \vec{r}_T + \vec{k}_T \cdot \vec{b}_T)} \times \frac{r_T^\mu}{r_T^2} \left\{ U^{R_1}(\vec{b} + \bar{z}\vec{r}) T^{R_0} U^{R_2}(\vec{b} - z\vec{r}) - U^{R_1}(\vec{b}) T^{R_0} U^{R_2}(\vec{b}) \right\} \Gamma_\mu$$

WILSON LINE IN REPR.  $R_i$       COLOR GENERATORS      DIRAC STRUCTURE

STEP #1 TAYLOR EXPANSION IN  $\vec{r}_T$ 

$$\mathcal{A}^{(n)} = \delta(p_1^+ + p_2^+ - p_0^+) \int d^2b_T d^2r_T e^{-i(\vec{P}_T \cdot \vec{r}_T + \vec{k}_T \cdot \vec{b}_T)} \frac{r_T^\mu \Gamma_\mu}{r_T^2} \\ \frac{1}{n!} r_T^{\alpha_1} \dots r_T^{\alpha_n} \sum_{i=0}^n \binom{n}{i} \bar{z}^i (-z)^{n-i} \left( \partial_{\alpha_1} \dots \partial_{\alpha_i} U^{R_1}(\vec{b}) \right) T^{R_0} \left( \partial_{\alpha_{i+1}} \dots \partial_{\alpha_n} U^{R_2}(\vec{b}) \right)$$

## STEP #2 ISOLATION OF 1-BODY CONTRIBUTIONS

$$\mathcal{A}_{1\text{-body}}^{(n)} = \delta(p_1^+ + p_2^+ - p_0^+) \int d^2b_T d^2r_T e^{-i(\vec{P}_T \cdot \vec{r}_T + \vec{k}_T \cdot \vec{b}_T)} \frac{r_T^\mu \Gamma_\mu}{r_T^2} \\ \vec{r}_T^\alpha \sum_{j=0}^n \frac{(i \vec{k}_T \cdot \vec{r}_T)^j}{(j+1)!} \left\{ \partial_\alpha U^{R_1}(\vec{b}) T^{R_0} U^{R_2}(\vec{b}) \bar{z}^{(j+1)} + U^{R_1}(\vec{b}) T^{R_0} \partial_\alpha U^{R_2}(\vec{b}) (-z)^{(j+1)} \right\}$$

## STEP #3 RESUMMATION &amp; INTEGRATION

$$\mathcal{A}_{1\text{-body}} = \delta(p_1^+ + p_2^+ - p_0^+) \int d^2b_T e^{-i \vec{k}_T \cdot \vec{b}_T} \frac{\Gamma_i}{k_T^2} (k_T^i \delta^{jl} + k_T^j \delta^{il} - k_T^l \delta^{ij}) \\ \left\{ \left( \frac{P_T^l}{P_T^2} + \frac{p_{2T}^l}{p_{2T}^2} \right) \partial_j U^{R_1}(\vec{b}) T^{R_0} U^{R_2}(\vec{b}) + \left( \frac{P_T^l}{P_T^2} - \frac{p_{1T}^l}{p_{1T}^2} \right) U^{R_1}(\vec{b}) T^{R_0} \partial_j U^{R_2}(\vec{b}) \right\}$$

STEP #4 SQUARE THE AMPLITUDE  
COLOR ALGEBRA
 $\Rightarrow$  ITMD