Proton's Landscape at a Fermi Scale: Generalized Parton Distributions

Jianwei Qiu

Theory Center, Jefferson Lab

August 19, 2020

Jefferson Lab

TMD Collaboration

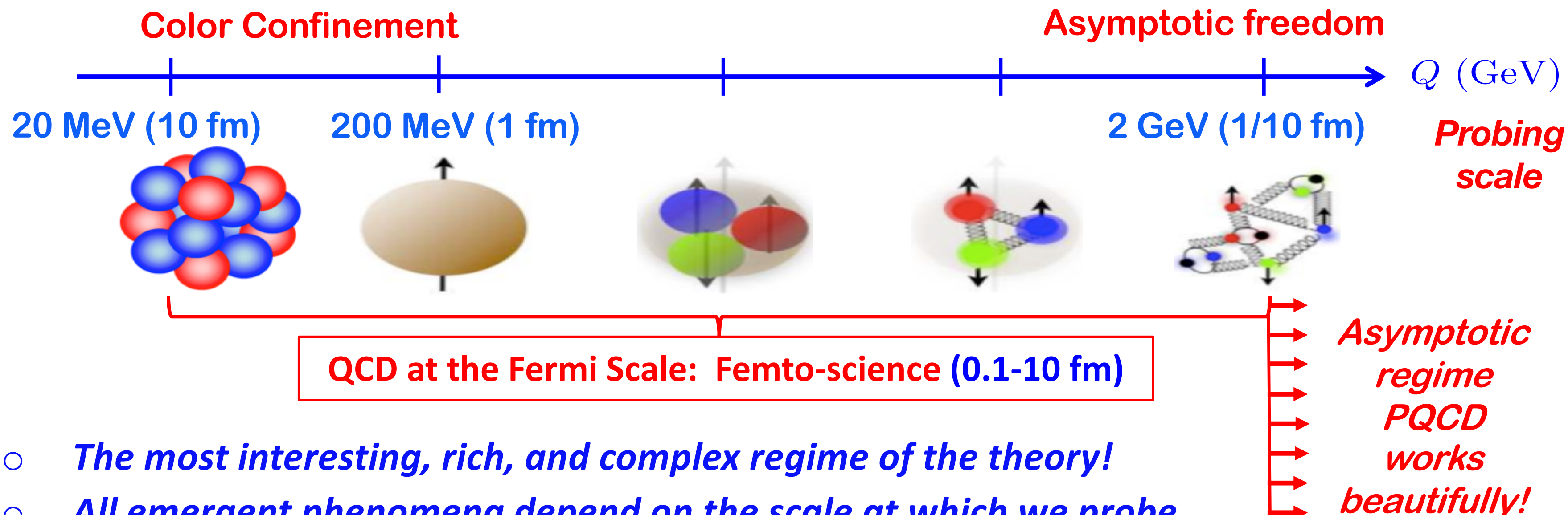
U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

QCD at a Fermi Scale

❑ QCD – Color Confinement:

- *Do not see any quarks and gluons in isolation*
- *The structure of nucleons and nuclei – emergent properties of QCD*



- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

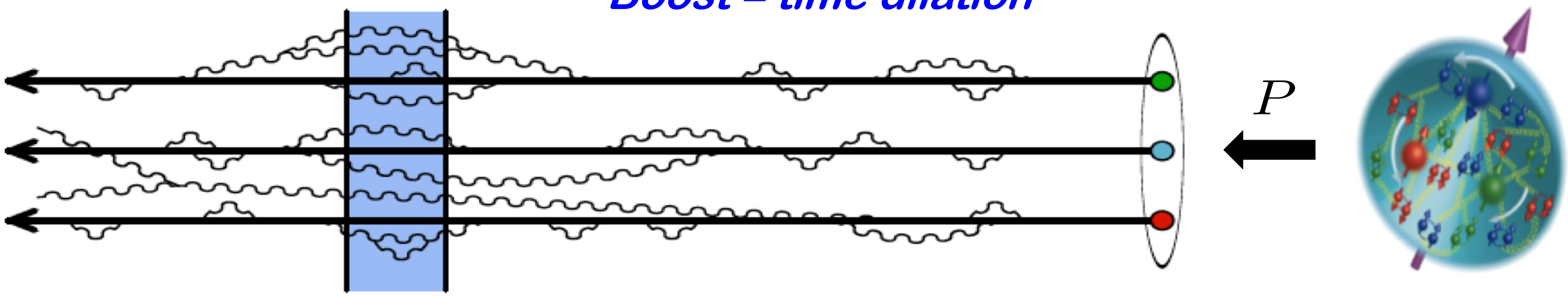
❑ QCD – Asymptotic Freedom:

- *Force becomes weaker at a shorter-distance – Controllable "Probes"*
- *Explore the structure of nucleons and nuclei indirectly by using "local", "sharp", and "controllable" probes, …*

How to "see" 3D partonic structure of hadrons?

- **Hard probes to "catch" the quantum fluctuation:**

Boost = time dilation

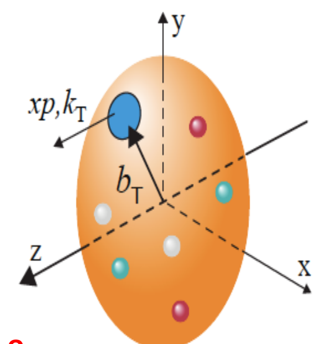


Hard probe ($t \sim 1/Q <$ fm) → Probability to "catch" the parton!

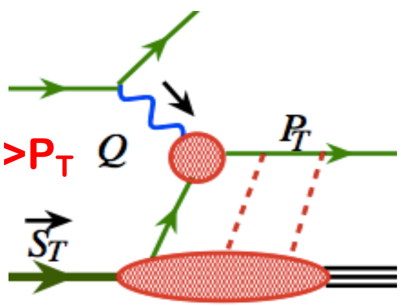
- **Observables with two momentum scales:** $Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$

  ✧ **Hard scale:** Q_1 "see" particle nature of "partons"

  ✧ **Soft scale:** Q_2 "sensitive" to the fermi-scale structure

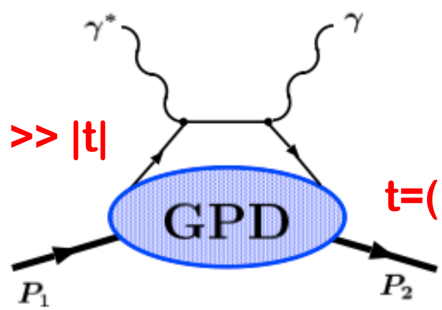


SIDIS: $Q \gg P_T$



TMDs – Confined motion

DVCS: $Q^2 \gg |t|$

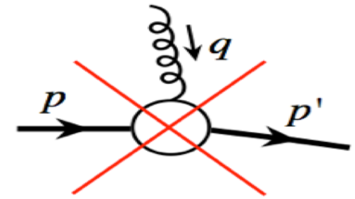


$t = (p_1 - p_2)^2$

GPDs – Spatial imaging

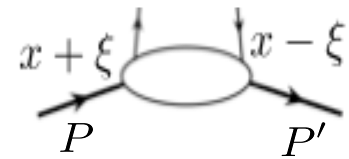
Definition of GPDs

- ❑ **No color nucleon elastic form factor:**



- ❑ **Quark "form factor":**

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+ q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$



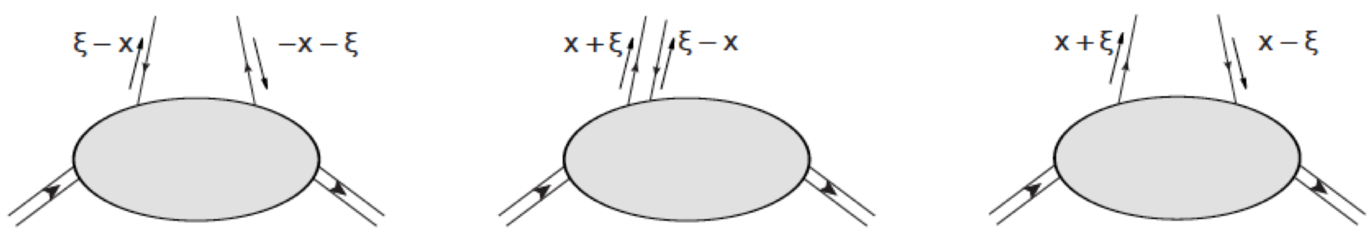
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

with $\xi = (P' - P)\cdot n/2$ and $t = (P' - P)^2 \Rightarrow -\Delta_\perp^2$ if $\xi \to 0$

Gauge link: $W[a,b] = P\exp\left(ig\int_b^a dx^- A^+(x^- n_-)\right)$

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

- ❑ **Kinematics:**



$-1 \le x \le -\xi$ $\quad$ $-\xi \le x \le \xi$ $\quad$ $\xi \le x \le 1$

Two more for quarks: $\tilde{H}_q(x,\xi,t,Q)$, $\tilde{E}_q(x,\xi,t,Q)$

with $\gamma\cdot n \longrightarrow \gamma\cdot n\,\gamma_5$

Definition of GPDs

- Gluon "form factor":

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

$$F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|G^{+\mu}(-\tfrac{1}{2}z)G_\mu{}^+(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Two more for gluons: $\tilde{H}^g(x,\xi,t)$ $\tilde{E}^g(x,\xi,t)$

with the two gluon field strength contracted anti-symmetrically

- Forward limit – connection to collinear PDFs:

$$H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x) \qquad \text{for } x > 0$$

$$H^q(x,0,0) = -\bar{q}(-x), \quad \tilde{H}^q(x,0,0) = \Delta\bar{q}(-x) \qquad \text{for } x < 0$$

$$H^g(x,0,0) = xg(x), \quad \tilde{H}^g(x,0,0) = x\Delta g(x) \qquad \text{for } x > 0$$

The factorization scale dependence is suppressed

Properties of GPDs

❑ Connection to Dirac and Pauli form factors:

$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \quad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t)$$

Where $$\langle p'|\bar{q}(0)\gamma^\mu q(0)|p\rangle = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{\mathrm{i}\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right]u(p)$$

And the axial and pseudoscalar version:

$$\int_{-1}^{1} \mathrm{d}x\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \quad \int_{-1}^{1} \mathrm{d}x\, \tilde{E}^q(x,\xi,t) = g_P^q(t)$$

Where $$\langle p'|\bar{q}(0)\gamma^\mu\gamma_5 q(0)|p\rangle = \bar{u}(p')\left[g_A^q(t)\gamma^\mu\gamma_5 + g_P^q(t)\,\frac{\gamma_5\Delta^\mu}{2m}\right]u(p)$$

❑ Some symmetry properties:

$$H^g(x,\xi,t) = H^g(-x,\xi,t) \qquad E^g(x,\xi,t) = E^g(-x,\xi,t)$$

$$\tilde{H}^g(x,\xi,t) = -\tilde{H}^g(-x,\xi,t) \qquad \tilde{E}^g(x,\xi,t) = -\tilde{E}^g(-x,\xi,t)$$

$$H^{q,g}(x,\xi,t) = H^{q,g}(x,-\xi,t), \ldots$$

$$H^{q,g}(x,\xi,t)^* = H^{q,g}(x,-\xi,t), \ldots$$

GPDs are real value functions

Properties of GPDs

- QCD energy-momentum tensor:

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \text{with} \quad \begin{aligned} T_q^{\mu\nu} &= \bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \\ T_g^{\mu\nu} &= G^{\mu\alpha} G_{\alpha}{}^{\nu} + \tfrac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta} \end{aligned}$$

- Form factors:

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = A_{q,g}(t) \bar{u} P^{(\mu}\gamma^{\nu)} u + B_{q,g}(t) \bar{u} \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2m} u + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{m} \bar{u}u + \bar{C}_{q,g}(t) m g^{\mu\nu} \bar{u}u$$

- Light-cone helicity operator:

$$J^3 = \int dx^- \, d^2\mathbf{x} \, M^{+12}(x) \quad \text{with} \quad M^{\alpha\mu\nu} = T^{\alpha\nu} x^\mu - T^{\alpha\mu} x^\nu$$

- Connection to the proton spin:

Ji, PRL78, 1997

$$\langle J_q^3 \rangle = \tfrac{1}{2}[A_q(0) + B_q(0)] \, , \quad \langle J_g^3 \rangle = \tfrac{1}{2}[A_g(0) + B_g(0)]$$

$$A_q(t) + B_q(t) = \int_{-1}^{1} dx \, x [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$A_g(t) + B_g(t) = \int_{0}^{1} dx [H_g(x,\xi,t) + E_g(x,\xi,t)]$$

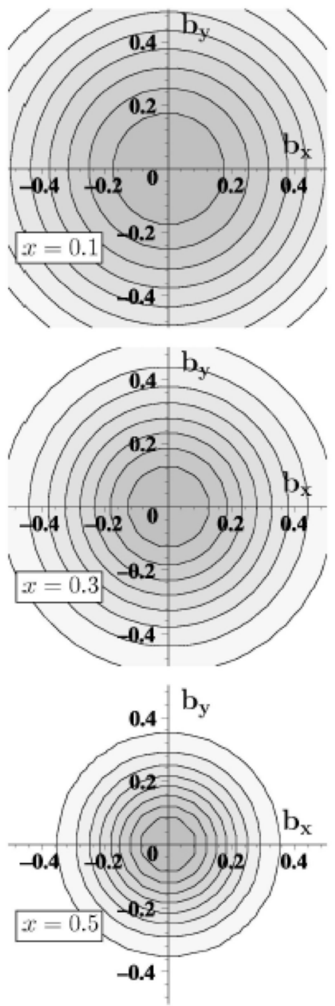
Spatial imaging from GPDs

M. Burkdart, PRD 2000

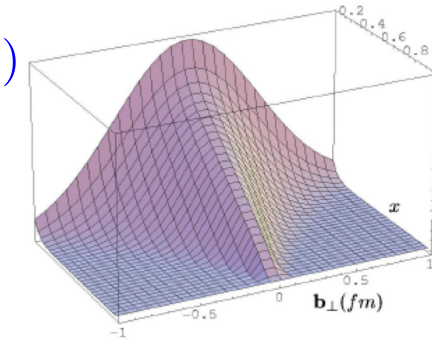
❑ Impact parameter dependent quark distribution:

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

$q(x, \mathbf{b}_\perp)$ for unpol. p



Unpolarized proton



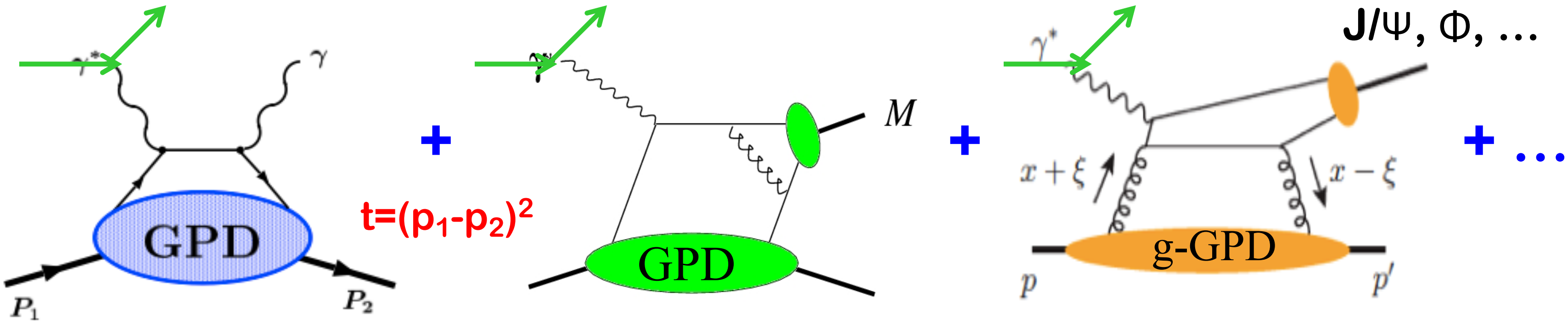
- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\to$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$

Jefferson Lab

Hunting for GPDs – Exclusive DIS

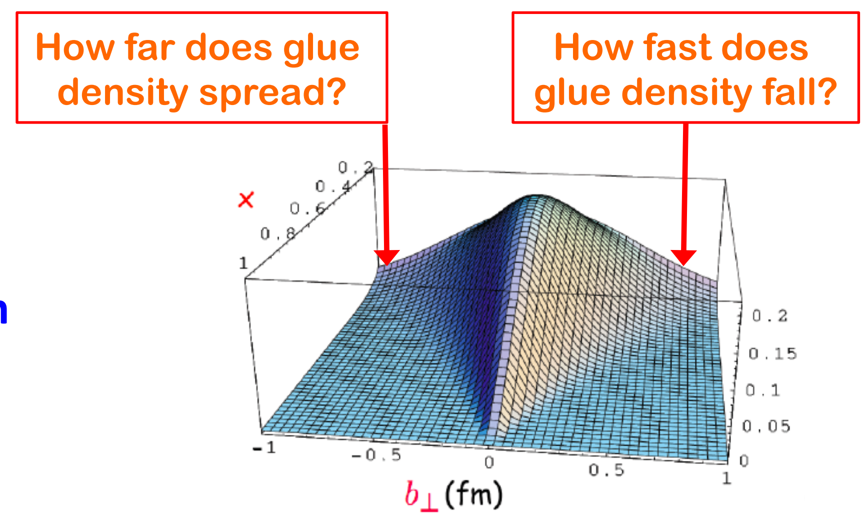
- Experimental access to GPDs:



DVCS: $Q^2 >> |t|$ DVEM: $Q^2 >> |t|$ EHMP: $Q^2 >> |t|$

- Much more complicated – (x, ξ, t) variables:
  - Challenge to derive GPDs from data

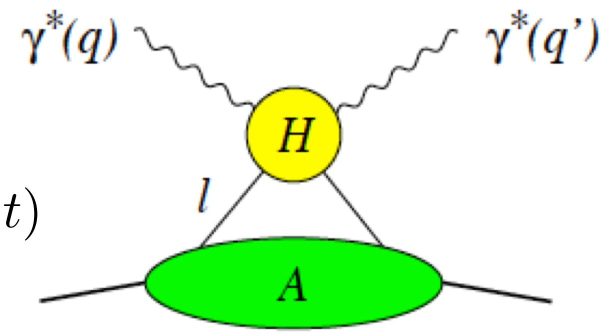
- GPDs could tell us:
  - Orbital contribution to proton’s spin
  - Proton radius of quark & gluon density
  - Hints for color confining radius/mechanism
  - Origin of nuclear force, …
  - …



QCD factorization

- Deep Virtual Compton Scattering (DVCS):

$$\gamma^*(q) + p(p) \rightarrow \gamma^*(q') + p(p')$$



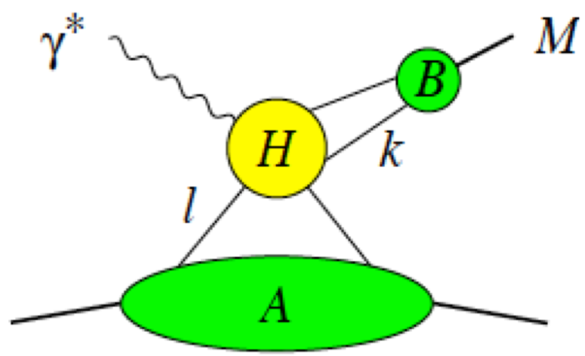
- Factorization:

$$\mathcal{A}(\gamma^* p \rightarrow \gamma p) = \sum_i \int_{-1}^{1} dx\, T^i(x, \xi, \rho, Q^2)\, F^i(x, \xi, t)$$

$$\rho = -(q+q')^2 / 2(p+p') \cdot (q+q')$$

- Deep Virtual Meson Production (DVMP):

$$\gamma^*(q) + p(p) \rightarrow M(q') + p(p')$$



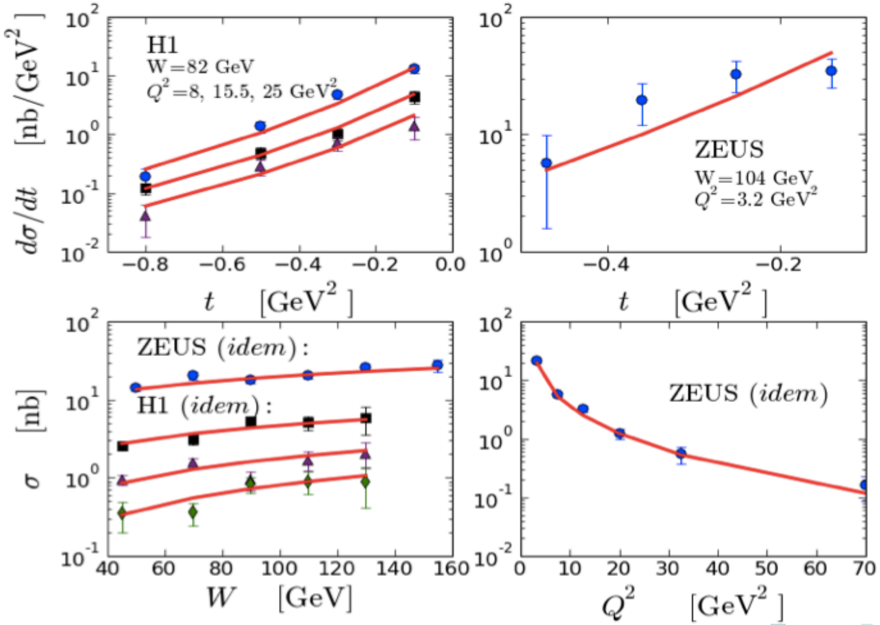
- Factorization:

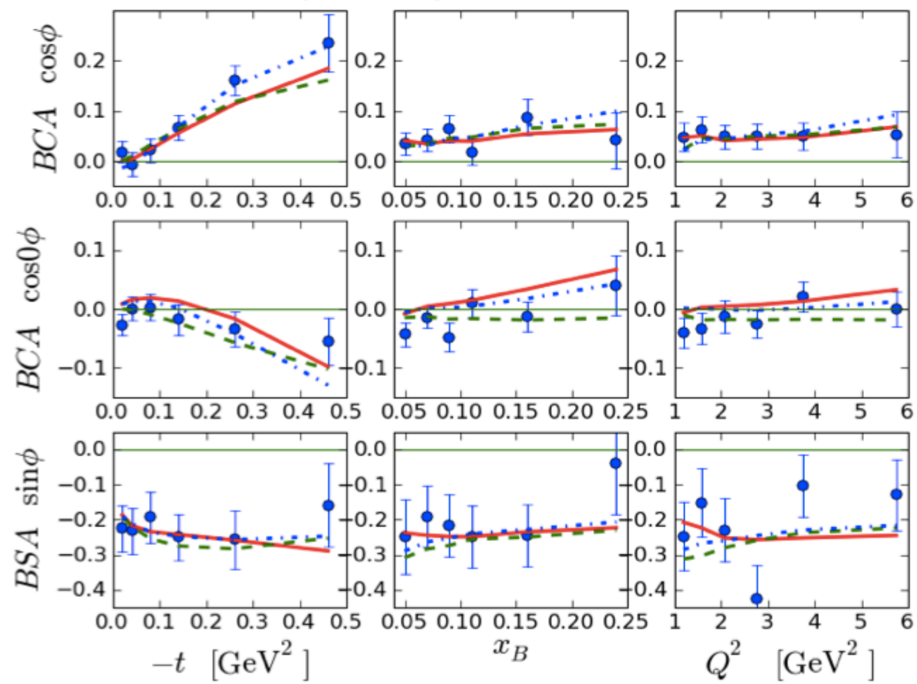
$$\mathscr{A}(\gamma_L^* p \rightarrow M_L p) = \frac{1}{Q} \sum_{ij} \int_{-1}^{1} \mathrm{d}x \times \int_0^1 \mathrm{d}z\, T^{ij}(x, \xi, z, Q^2) F^i(x, \xi, t) \Phi^j(z)$$

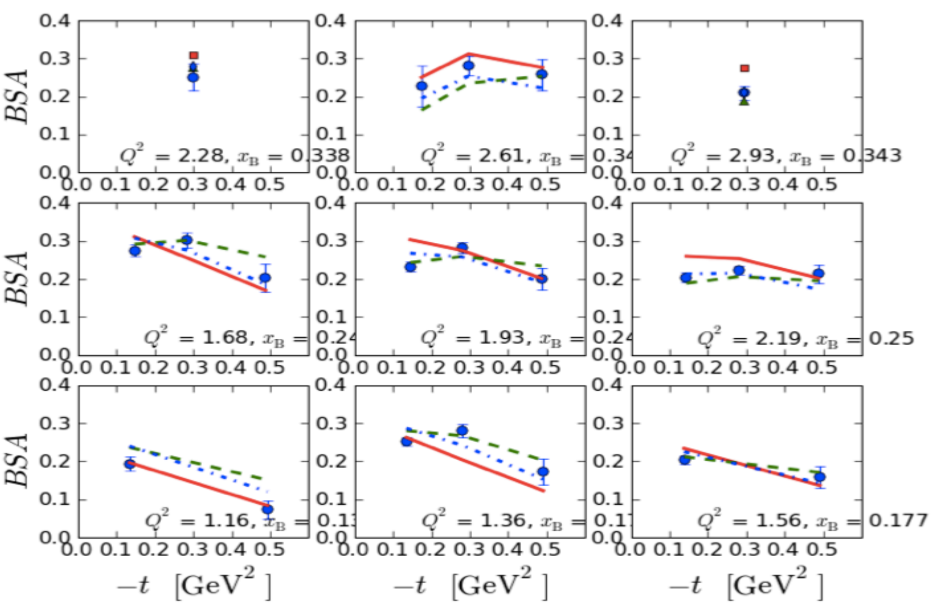
- Evolution:

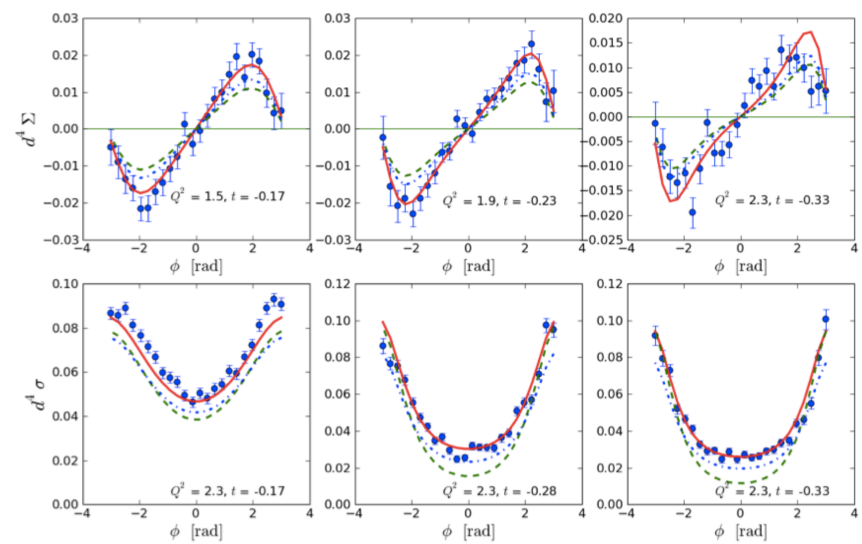
Factorization naturally leads to evolution equations for GPDs

Data: just the beginning



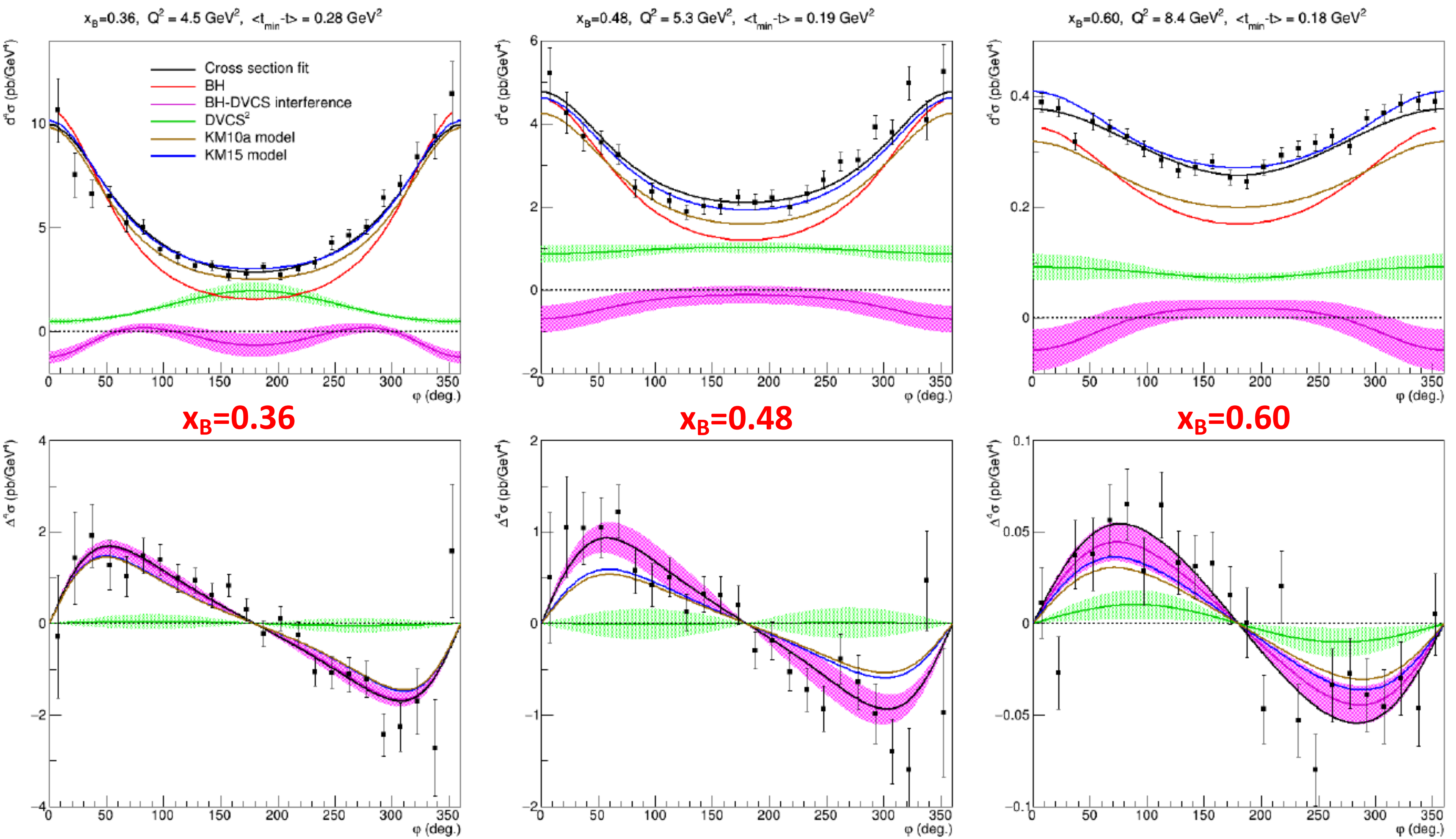






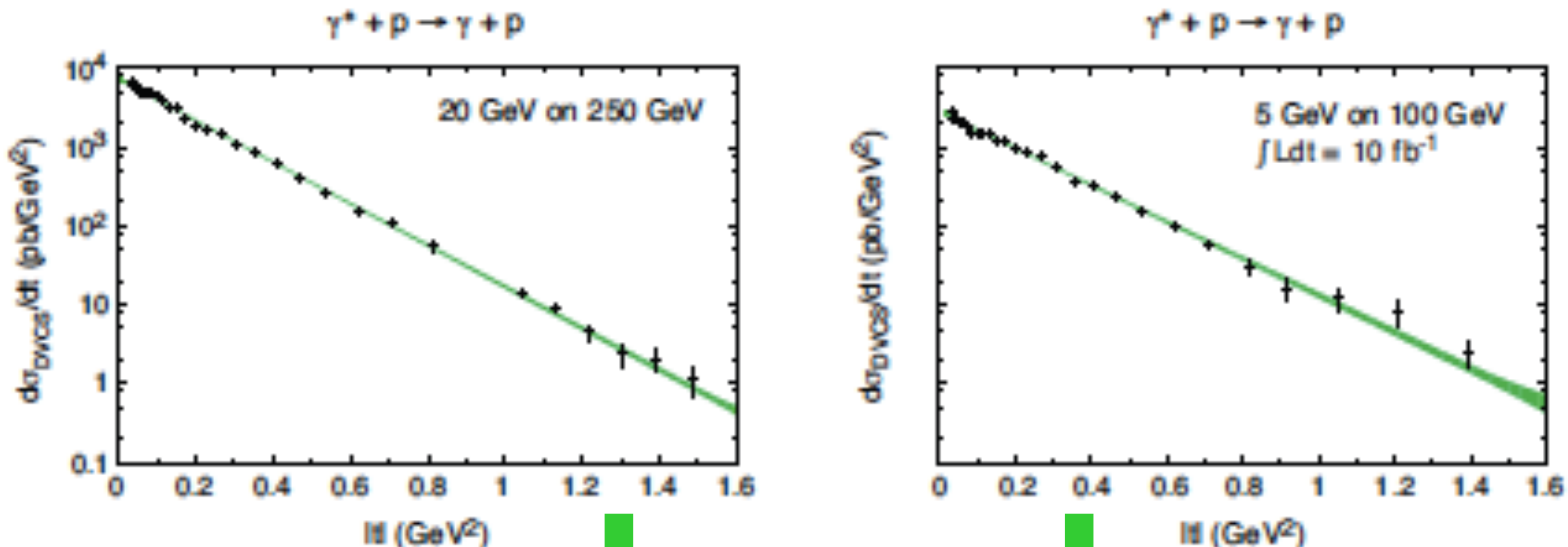
JLab E12-06-114 DVCS/Hall A experiment at 11 GeV

Sample of cross-section results:

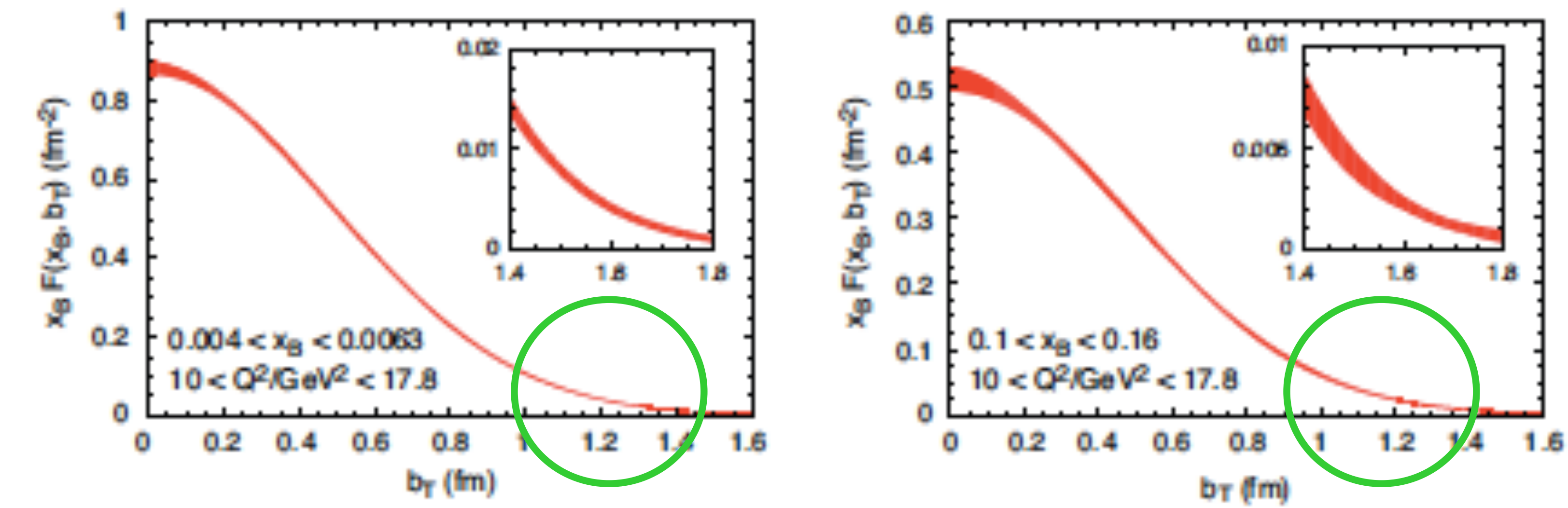


DVCS at future EIC (White Paper)

- Cross Sections:

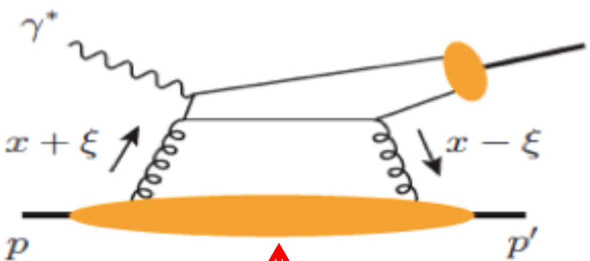


- Spatial distributions:



Imaging the gluon (White Paper)

❑ **Exclusive vector meson production:**



J/Ψ, Φ, …

↑ t-dep

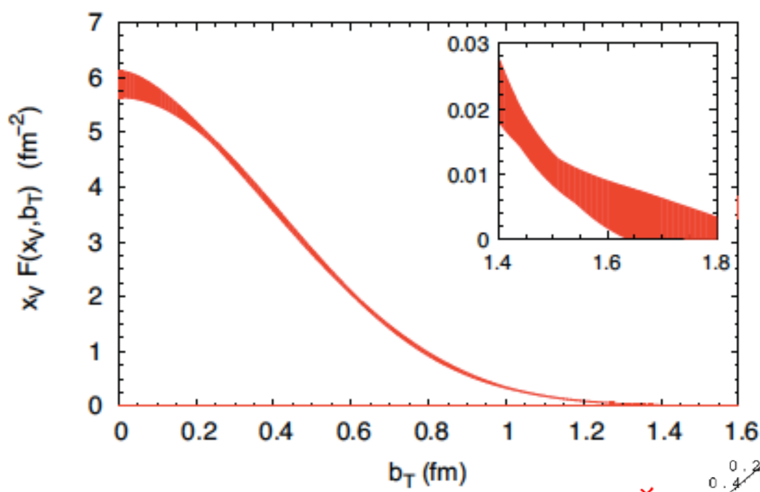
$$\frac{d\sigma}{dx_B dQ^2 dt}$$

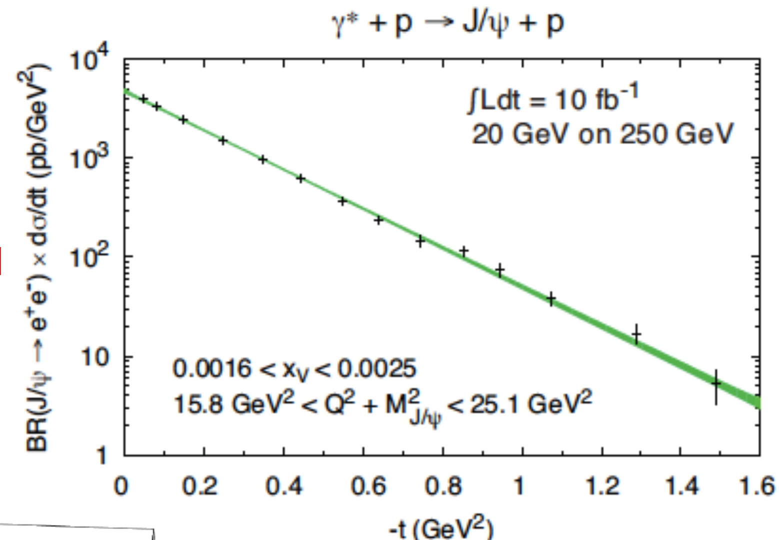
✧ **Fourier transform of the t-dep**

➡ **Spatial imaging of glue density**

✧ **Resolution ~ 1/Q or 1/M_Q**

❑ **Gluon imaging from simulation:**

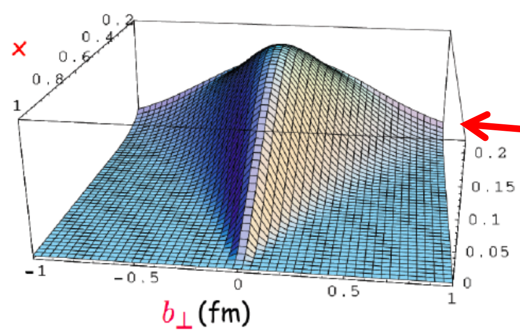




Only possible at the EIC

Gluon radius?

Gluon radius (x)!

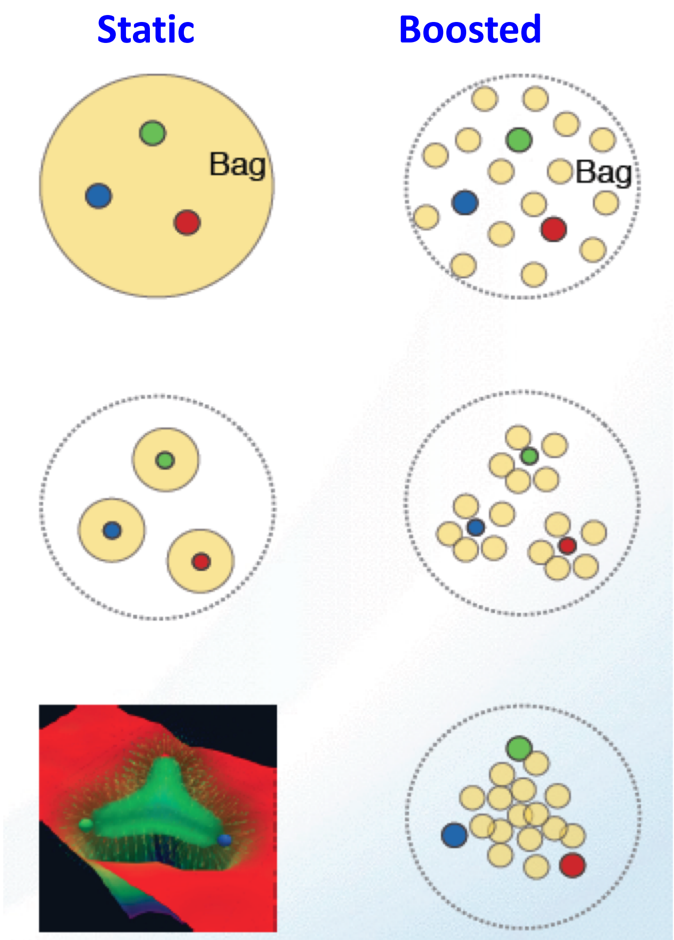


How spread at small-x?
Color confinement

Jefferson Lab

Beyond the 3D picture – confining radius of color?

❑ Spatial distributions of quarks and gluons:



Bag Model:

Gluon field distribution is wider than the fast moving quarks.

Gluon radius > Charge Radius

Constituent Quark Model:

Gluons and sea quarks hide inside massive quarks.

Gluon radius ~ Charge Radius

Lattice Gauge theory (with slow moving quarks):

Gluons more concentrated inside the quarks

Gluon radius < Charge Radius

3D confined motion (TMDs) + spatial distribution (GPDs)

Hints on the color confining mechanism

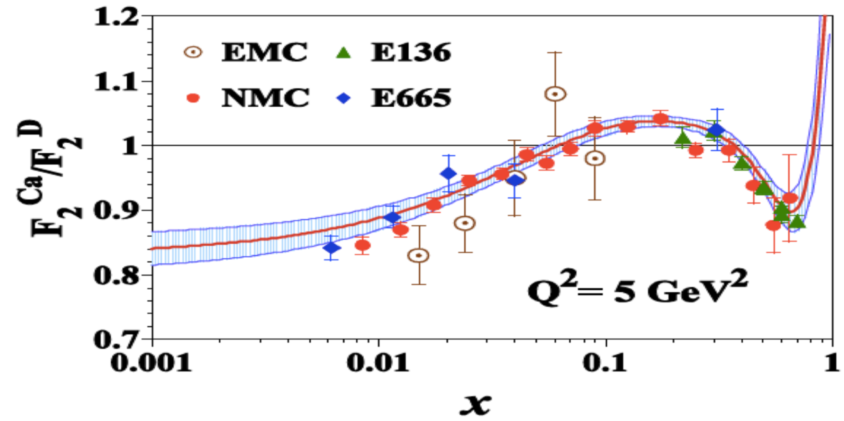
Relation between charge radius, quark radius (x), and gluon radius (x)?

Beyond the 3D picture – Nuclear Landscape?

- ❑ **EMC discovery:**

  Nuclear landscape

  ≠ Superposition of nucleon landscape



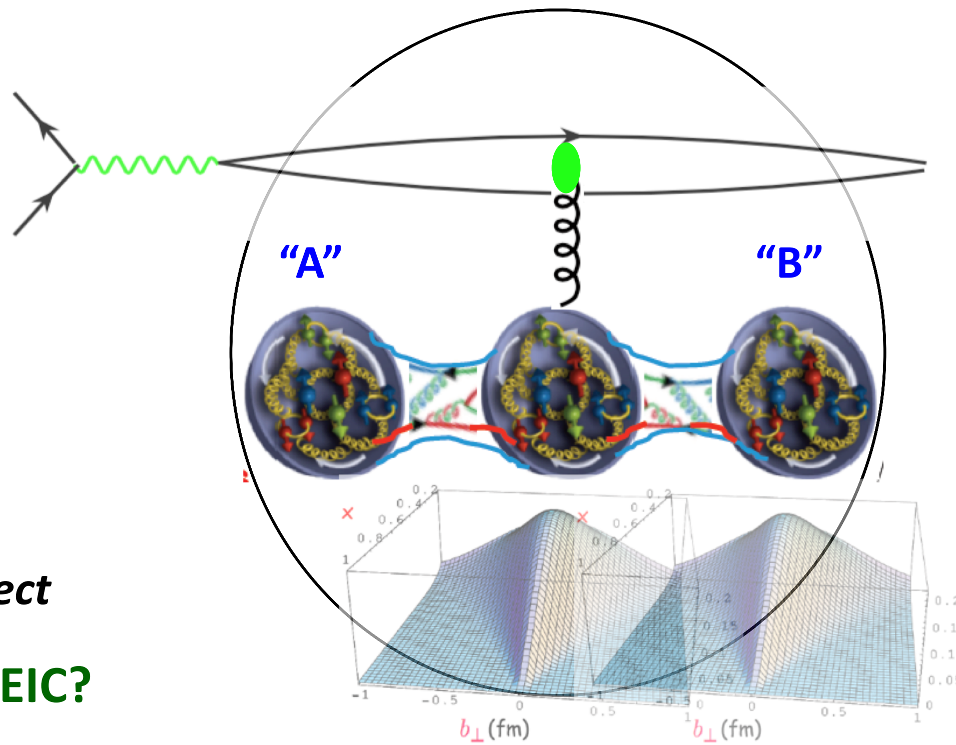
- ❑ **Simple, but fundamental, questions:**

  ✧ **What does a nucleus look like *if we only see quarks and gluons ?***

  ✧ **Does the color of nucleon "A" know the color of nucleon "B"?**

  IF YES, Nucleus could act like a bigger proton at small-x, and could reaching the saturation much sooner!

  IF NOT, Observed nuclear effect in cross-section is a coherent collision effect



- ❑ **GPDs of nuclei – diffractive events at EIC?**

EIC can tell !

What lattice QCD can do?

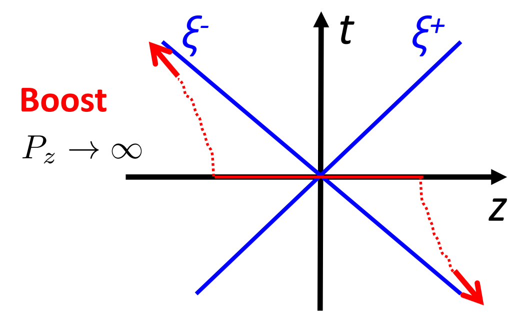
- **LQCD is formulated in Euclidean space:**

  LQCD *cannot* calculate the *leading power* x-dependent PDFs, TMDs, GPDs, …, *directly*!

- **New idea – quasi-PDFs (equal-time correlator):**

  Ji, arXiv:1305.1539

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \equiv \int \frac{d\xi_z}{4\pi}\, e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$



Conjecture:

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \longrightarrow q(x,\mu^2) \quad \textbf{when} \quad P_z \to \infty$$

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Based on the Large Momentum Effective Theory (LaMET) – Taylor expansion in $1/P_z$

Cautions:

- **P_z is limited by 1/a – lattice spacing – P_z < 2-3 GeV**
- **Power corrections could be of the form:** $\dfrac{\Lambda_{QCD}^2 R}{x^2(1-x)P_z^2}$ Braun et al. arXiv: 1810.00048
- **Power UV divergence: Non-perturbative renormalization impacts large z behavior**
- **Mixing with gluon and other flavor contribution beyond LO**

- **Pseudo-PDFs, Lattice x-section, …** **New ideas, new calculations, …**

Jefferson Lab

What lattice QCD can do?

Ma, Qiu, 1404.6860, 1709.03018

❑ Factorization:

$$\langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(-\frac{\xi_z}{2})|P\rangle = \sum_a \int_{-1}^{1}\frac{dx}{x} f_a(x,\mu^2) C_a(x\omega,\xi^2,\mu^2) + \mathcal{O}(\xi^2\Lambda_{\rm QCD}^2)$$

$$\omega = P\cdot\xi$$

Necessary condition: Need a hard scale to "see" particle nature of the parton field

$$\xi^2 \ll 1/\Lambda_{\rm QCD}^2 \sim R_N^2$$

❑ Fourier transform:

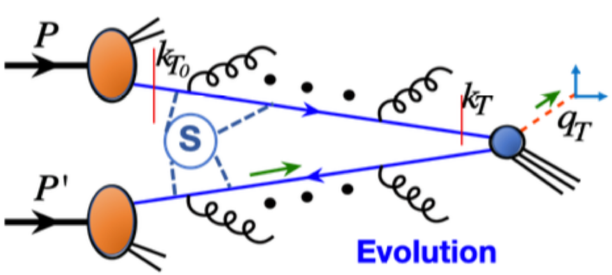
$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \propto \int_{1/(xP_z)<|\xi_z|<\infty}\frac{d\xi}{2\pi}\, e^{-i\tilde{x}P_z\xi_z}\, Z(\xi_z,\mu_R^2)\langle P|\mathcal{O}(\xi_z)|P\rangle$$

Not factorized,
Hard to control the size,
Sensitive to the ξ_z tail of
NP UV renormalization of the power divergence

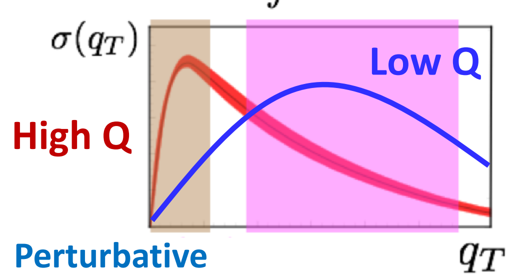
$$+\sum_a\int_{-1}^{1}\frac{dx}{x}f_a(x,\mu^2)\int_{|\xi_z|<1/(xP_z)}\frac{d\xi}{2\pi}\,e^{-i\tilde{x}P_z\xi_z}\,C_a(x\omega,\xi^2,\mu^2) + \int_{|\xi_z|<1/(xP_z)}\frac{d\xi}{2\pi}\,e^{-i\tilde{x}P_z\xi_z}\,\mathcal{O}(\xi^2\Lambda_{\rm QCD}^2)$$

Factorized contribution — **A small correction**

Similar example: ***Transverse momentum part of the TMD factorization***



$$\sigma(q_T) = H(Q,\mu)\int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_a,\vec{b}_T,\mu,\zeta_a)\, f_q(x_b,\vec{b}_T,\mu,\zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$



$$Q \sim xP_z \sim 0.2 - 2\ \text{GeV}$$
$$\text{for}\quad x\sim 0.1-1$$
$$\text{and}\quad Q\sim 2\ \text{GeV}$$

Jefferson Lab

GPDs from Lattice QCD

Y.-S. Liu et al.
arXiv:1902.00307

❑ Definition:

$$\widetilde{F}(\Gamma, x, \widetilde{\xi}, t, P^z, \widetilde{\mu}) = \frac{1}{N}\int \frac{dz}{4\pi} e^{ixzP^z} \langle P'', S''|\bar{\psi}\left(\frac{z}{2}\right)\Gamma\lambda^a W_z\left(\frac{z}{2}, -\frac{z}{2}\right)\psi\left(-\frac{z}{2}\right)|P', S'\rangle$$

GPDs could be extracted from LQCD calculation of j-j correlation

❑ Decomposition:

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \widetilde{\mu}) = \frac{1}{2P^t}\bar{u}(P'', S'')\Big\{\widetilde{H}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\Gamma + \widetilde{E}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\frac{[\not{\Delta}, \Gamma]}{4M}$$

$$+ \widetilde{H}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{P^{[z}\Delta^{\perp]}}{M^2} + \widetilde{E}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{\gamma^{[z}P^{\perp]}}{M}\Big\}u(P', S')$$

❑ Factorization - conjecture:

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \mu_R, p_R^z) = \int_{-1}^{1}\frac{dy}{|y|}C_\Gamma\left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z}\right)F(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}\right)$$

❑ Matching coefficient – NLO – RI/MOM renormalization:

$$C_\Gamma\left(x, \xi, r, \frac{p^z}{\mu}, \frac{p^z}{p_R^z}\right) = \delta(1-x) + \left[f_1\left(\Gamma, x, \xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_R^z}\right| f_2\left(\Gamma, \frac{p^z}{p_R^z}(x-1)+1, r\right)\right]_+$$

$$+ \delta_{\Gamma, i\sigma^{z\perp}}\delta(1-x)\frac{\alpha_s C_F}{4\pi}\ln\left(\frac{\mu^2}{\mu_R^2}\right) + \mathcal{O}(\alpha_s^2)$$

where the functions f1, f2 and δ_Γ are given in the paper (1902.00307)

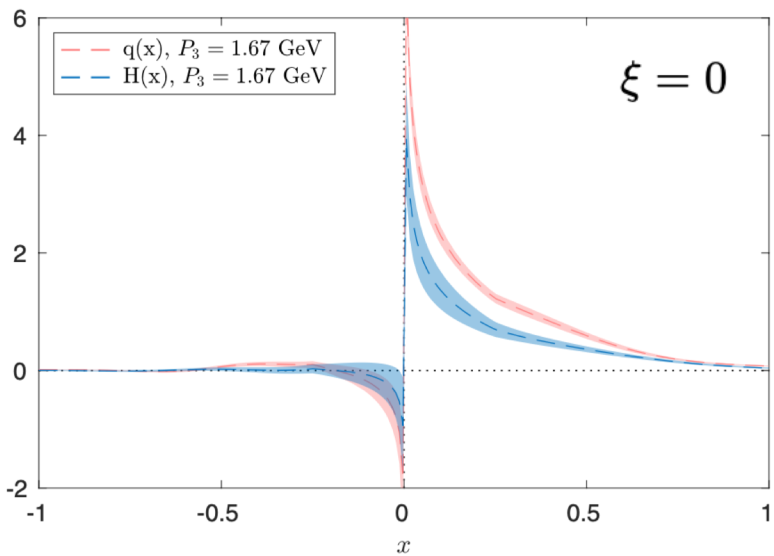
GPDs from Lattice QCD

- **Matrix element calculated on the lattice:**

$$\langle N(P3+Q/2)|\bar{\psi}(z)\,\Gamma W(0,z)\psi(0)\,|N(P_3-Q/2)\rangle$$

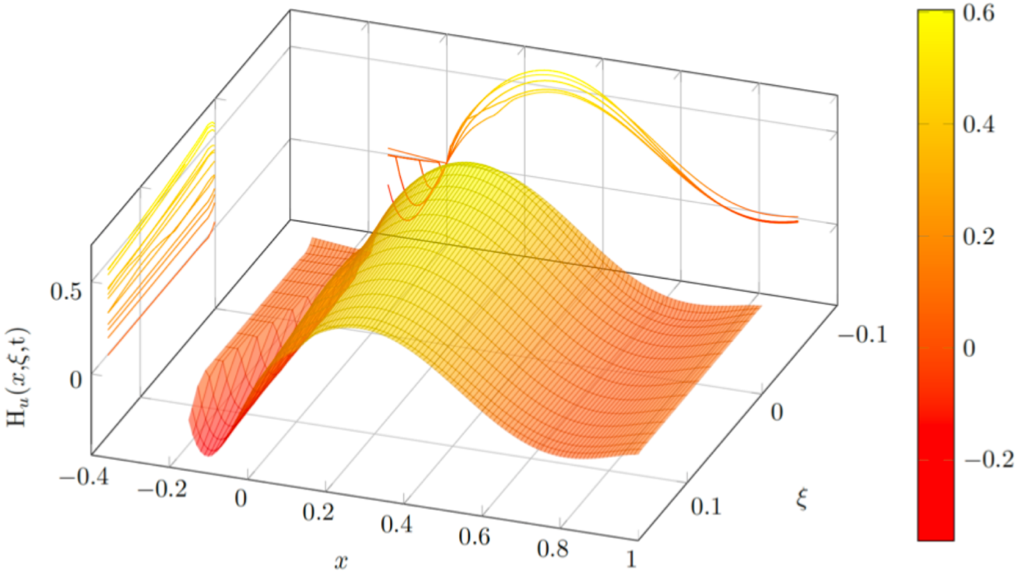
$$W[a,b] = P\exp\left(ig\int_b^a dx^-\,A^+(x^-n_-)\right)$$

- **GPDs from the lattice calculation:**



H-GPD for $P_3 = 1.67$ GeV, $Q^2 = 0.69$ GeV^2

C. Alexandrou, et al. arXiv:1910.13229



GPD H for the u quark at $t = -0.1GeV^2$

M. Constantinou, et al. arXiv: 2006.08636

Jefferson Lab

Summary

- ❑ **QCD at the Fermi scale:**

  The most interesting, rich, and complex regime of the theory

- ❑ **GPDs are fundamental quantum probability distributions**

  Carry important information on spatial imaging of hadron's partonic structure

- ❑ **Need exclusive processes with a unbroken hadron under hard collisions**

  - ✧ **Need lepton-hadron facilities**
  - ✧ **Need well-controlled exclusive processes in lepton-hadron collisions**
  - ✧ **DVCS, DDVCS, DVMP, Diffractive heavy vector boson production, …**
  - ✧ **JLab12, COMPASS, and future EIC will produce a lot of data on GPDs**

- ❑ **GPDs could be extracted from LQCD calculations**

  Work just got started – more efforts are needed!

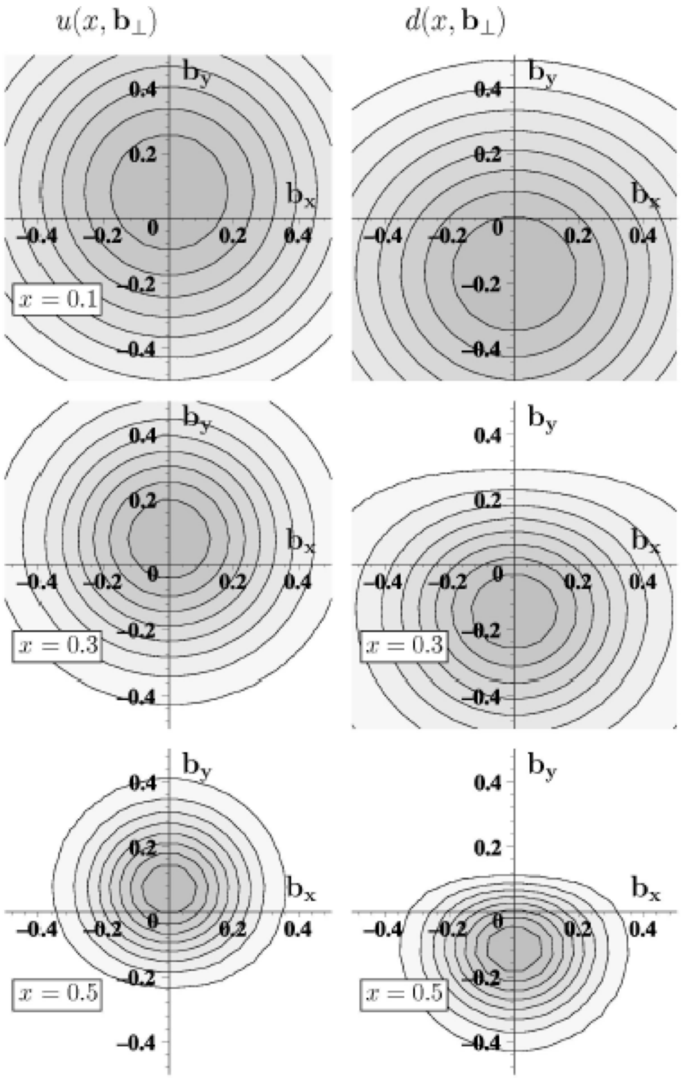
Thank you!

Spatial imaging from GPDs

M. Burkdart, PRD 2000

❑ Impact parameter dependent quark distribution:

Proton polarized in +x direction



$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

Sign and magnitude of the averaged shift related to the hadron's magnetic moment:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$

Jefferson Lab