# N3LO computations for Drell-Yan processes 

Claude Duhr in collaboration with Falko Dulat and Bernhard Mistlberger

EF05/EF06 meeting:
NNLO and N3LO computations for PDF analyses

23 September 2020

## Towards N3LO accuracy

- Standard approach to LHC computations: QCD factorisation + perturbation theory.

$$
\begin{array}{rlrl}
\sigma_{p p \rightarrow X}(S) & =\sum_{i, j} \int_{0}^{1} d x_{1} d x_{2} f_{i}\left(x_{1}, \mu_{F}^{2}\right) f_{j}\left(x_{2}, \mu_{F}^{2}\right) \hat{\sigma}_{i j \rightarrow X}\left(\hat{s}, \mu_{F}^{2}\right) & S & =E_{C o M}^{2} \\
\hat{s} & =x_{1} x_{2} S \\
\hat{\sigma}_{i j \rightarrow X} & =\hat{\sigma}_{0}+\alpha_{s}\left(\mu_{R}\right) \hat{\sigma}_{1}+\alpha_{s}\left(\mu_{R}\right)^{2} \hat{\sigma}_{2}+\ldots & \alpha_{s}\left(m_{Z}^{2}\right) & =0.118
\end{array}
$$

- Naive counting: NLO $\longrightarrow 10 \%$ NNLO $\longrightarrow 1 \%$
$\Rightarrow$ We know several examples where this naive counting fails (e.g. Higgs production).
- Goal: Compute N3LO corrections (at least for a selected class of processes.


## The cross section

- The NLO cross section:


Virtual corrections ('loops')


Real emission

- The NNLO cross section:


Double virtual


Real-virtual


Double real

## The cross section

- The N3LO cross section:


Triple virtual


Real-virtual
squared


Double virtual real


Double real virtual


Triple real

## State-of-the-art

- Available (inclusive) results at N3LO at hadron colliders
[Anastasiou, CD, Dulat, Herzog, Mistlberger;
$\Rightarrow$ Higgs production in gluon-fusion. Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger]
$\Rightarrow$ Higgs production in bottom-fusion in 5 flavour scheme.
(+ matching to 4FS)
[CD, Dulat, Mistlberger; CD, Dulat, Hirschi, Mistlberger]
$\Rightarrow$ Drell-production (photon and W). [CD, Dulat, Mistlberger]
$\Rightarrow$ di-Higgs in gluon fusion.
$\Rightarrow$ VBF (in DIS approach).


## Energy variation

Higgs production:
Nice convergence of perturbative expansion.



Choice of central scales:
ggH: $\mu_{F}=\mu_{R}=m_{H} / 2$
$\mathrm{bbH}: \mu_{F}=m_{H} / 4, \quad \mu_{R}=m_{H}$

## Q-variation (photon)


$\begin{array}{lllllllllllllll}10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 & 110 & 120 & 130 & 140 & 150\end{array}$ Q [GeV]

Q-variation (W)



## Scale dependence (photon)



## 左 <br> Scale dependence (W)




Scale dependence

- For Higgs (ggH \& bbH): Scale bands overlap very well (for smallish $\mu_{F}$ ).
- For DY (photon $\&$ W): Scale bands do not overlap over a large range of virtualities.
$\Rightarrow$ Difference in central values: few \%.
$\Rightarrow$ For both $\mu_{F}$ and $\mu_{R}$.
- All results obtained with pdf4lhc_nnlo_mc.
- Observation: Large cancellation between channels for DY at NNLO and N3LO (both photon and W).
$\Rightarrow$ No cancellation for Higgs.


## Cross section ratios

$$
R_{X Y}(Q)=\frac{\sigma_{X}(Q)}{\sigma_{Y}(Q)}, \quad X, Y \in\left\{W^{ \pm}, \gamma^{*}\right\}
$$

- Prescriptions for ratios:
$\Rightarrow$ A: Ratio of expansions, correlated scales.
$\Rightarrow$ B: Ratio of expansions, uncorrelated scales.
$\Rightarrow A^{\prime}$ : Expansion of ratio, correlated scales.
$\Rightarrow$ B': Expansion of ratio, uncorrelated scales.
$\Rightarrow$ C: Progression of series, correlated scales:

$$
\delta(\text { pert. })= \pm\left|1-\frac{R_{X Y}^{(n)}(Q)}{R_{X Y}^{(n-1)}(Q)}\right| \times 100 \%
$$

## Cross section ratios

$$
R_{W^{+} W^{-}}\left(Q=m_{W}\right)
$$

|  | NLO |  | NNLO |  | $\mathrm{N}^{3} \mathrm{LO}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mu^{\text {cent }}$ | $m_{W}$ | $m_{W} / 2$ | $m_{W}$ | $m_{W} / 2$ | $m_{W}$ | $m_{W} / 2$ |
| A | $1.342_{-0.00 \%}^{+0.10 \%}$ | $1.342_{-0.05 \%}^{+0.07 \%}$ | $1.348_{-010 \%}^{+0.12 \%}$ | $1.349_{-0.11 \%}^{+0.15 \%}$ | $1.350_{-0.06 \%}^{+0.05 \%}$ | $1.350_{-005 \%}^{+0.04 \%}$ |
| $\mathrm{A}^{\prime}$ | $1.343^{-0.13 \%}$ | $1.344_{-0.10 \%}^{+0.010 \%}$ | $1.349^{-0.13 \%}$ | $1.351_{-013 \%}^{+0.33 \%}$ | $1.350^{+0.022 \%}$ | $1.350^{+0.00 \%}$ |
| B | $1.342_{-8.08}^{+8.82}$ | $1.342_{-11.4 \%}^{+12.9 \%}$ | $1.348_{-2.31 \%}^{-+.26 \%}$ | $1.349_{-2.27 \%}^{++.24 \%}$ | $1.350_{-114 \%}^{+2.21 \%}$ | $1.350_{-1.14 \%}^{+2.21 \%}$ |
| $\mathrm{B}^{\prime}$ | $1.343_{-7.40 \%}^{+5.28 \%}$ | $1.344_{-9.97 \%}^{+18.09 \%}$ | $1.349_{-2.63 \%}^{+1.85 \%}$ | $1.351_{-2.24 \%}^{+2.22 \%}$ | $1.350_{-2.25 \%}^{+2.26 \% \%}$ | $1.350_{-2.70 \%}^{+4.45 \%}$ |
| C | $1.342_{-0.99 \%}^{+0.99 \%}$ | $1.342_{-0.58 \%}^{+0.58 \%}$ | $1.349_{-0.52 \%}^{+0.52 \%}$ | $1.349_{-0.53 \%}^{+0.53 \%}$ | $1.350_{-0.15 \%}^{+0.15 \%}$ | $1.350_{-0.11 \%}^{+0.112 \%}$ |

$\Rightarrow$ Almost no difference between "expansion of ratio" or "ratio of expansions" already at lower orders.
$\Rightarrow$ Large difference for scale variation between correlated and uncorrelated.
$\Rightarrow$ Ratio is extremely stable in perturbation theory.

## Cross section ratios




$$
\begin{gathered}
R_{W^{+} W^{-}}(Q) \\
\mu^{\mathrm{cent} .}=Q
\end{gathered}
$$

pdf4lhc_nnlo_mc


## $\mathrm{PDF}+\alpha_{s}$-uncertainty


$\Rightarrow$ Central set: pdf4lhc_nnlo_mc.
$\Rightarrow$ Uncertainty band obtained following PDF4LHC recommendation.

## $\mathrm{PDF}+\alpha_{s}$-uncertainty


$\Rightarrow$ Red band obtained scale variation (pdf4lhc_nnlo_mc).
$\Rightarrow$ Strong coupling taken from PDF set.

## Missing N3LO PDFs

- We do not have N3LO PDFs
- This introduces a mismatch in our calculation.
- Estimate of the uncertainty:

$$
\delta_{\mathrm{PDF}}^{\mathrm{N}^{3} \mathrm{LO}}=\frac{1}{2}\left|\frac{\sigma_{\mathrm{NNLO}-\mathrm{PDFs}}^{\mathrm{NNLLO}}-\sigma_{\mathrm{NLO}}^{\mathrm{NNLO}}}{\sigma_{\mathrm{NNLO}}^{\mathrm{NNLO}} \mathrm{PDFs}}\right|
$$

- The factor $1 / 2$ takes into account that this estimate is most likely overly conservative.
$\Rightarrow c f$. convergence pattern of DIS.

[Moch, Vermaseren, Vogt]


## 园 <br> Missing N3LO PDFs



## Summary

$$
Q[\mathrm{GeV}] \mathrm{K}_{\mathrm{QCD}}^{\mathrm{N}^{3} \mathrm{LO}} \quad \delta_{\text {scale }} \quad \delta_{\mathrm{PDF}} \quad \delta_{\mathrm{PDF}}^{\mathrm{N}^{3} \mathrm{LO}}
$$


VBF (DIS, 14 TeV ) $\quad 0.999{ }_{-0.05 \%}^{+0.05 \%}$
[Dreyer, Karlberg]

