

Instability of N³LO at small x

Marco Bonvini

INFN, Rome 1 unit

EF05/EF06 meeting: NNLO and N3LO computations for PDF analyses

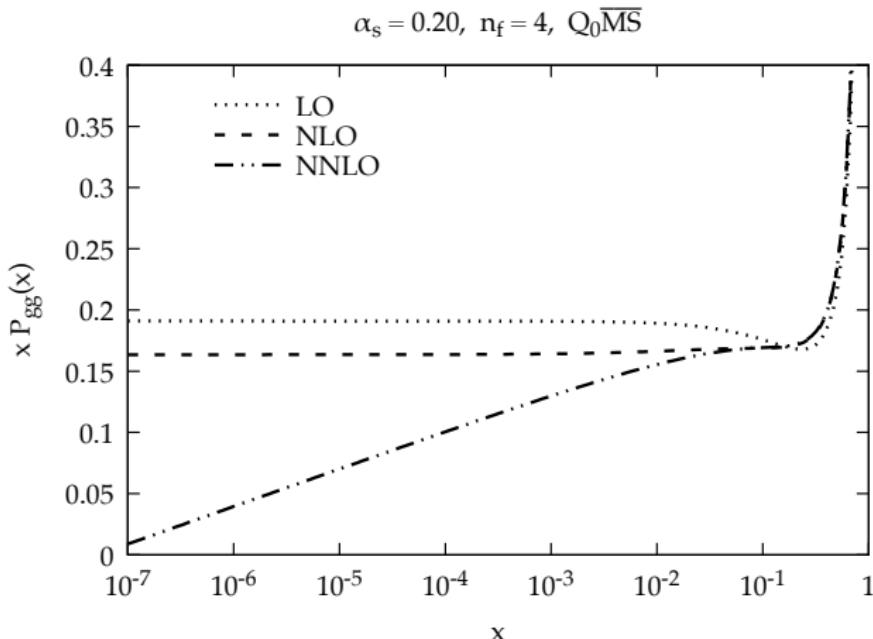
23 September 2020



Istituto Nazionale di Fisica Nucleare
Sezione di ROMA

Small- x logarithms in the splitting functions

Only **singlet sector** affected: P_{gg} , P_{gq} , P_{qg} , P_{qq}



Logarithms start to grow for $x \lesssim 10^{-2}$ (for $Q \sim 5\text{GeV}$)
→ very large resummation region!

fixed-order $xP_{gg}(x, \alpha_s)$
splitting function at
small x :

LO:

$$\alpha_s \times \text{const}$$

NLO:

$$\alpha_s^2 \left(\ln \frac{1}{x} + \text{const} \right)$$

NNLO:

$$\alpha_s^3 \left(\ln^2 \frac{1}{x} + \ln \frac{1}{x} + \text{const} \right)$$

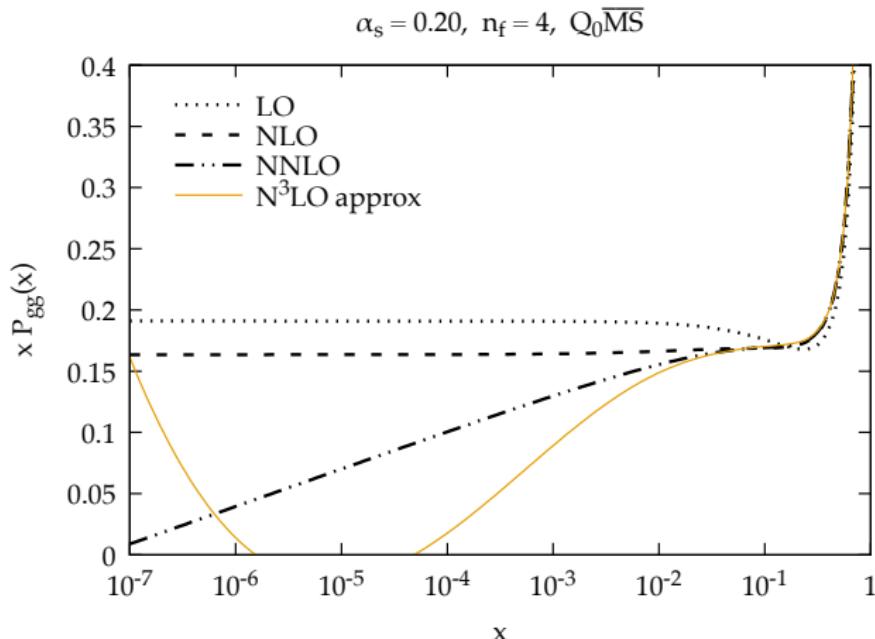
$N^3\text{LO}$:

$$\alpha_s^4 \left(\ln^3 \frac{1}{x} + \ln^2 \frac{1}{x} + \ln \frac{1}{x} + \dots \right)$$

accidental zeros!

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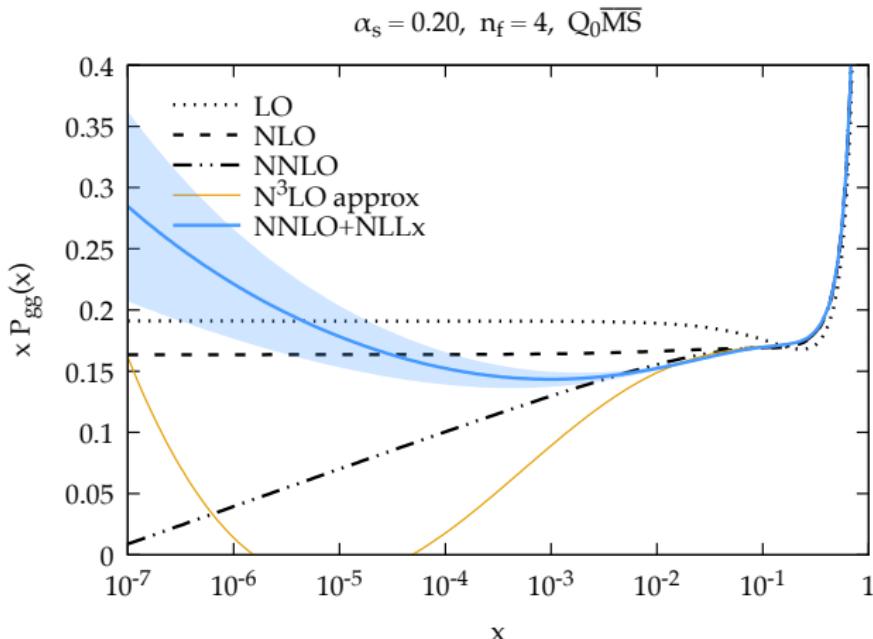
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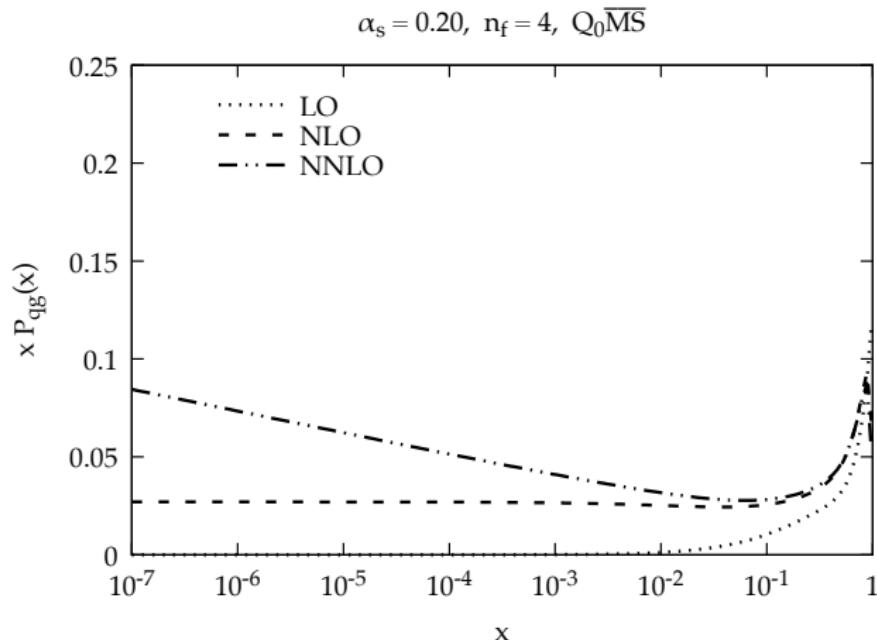
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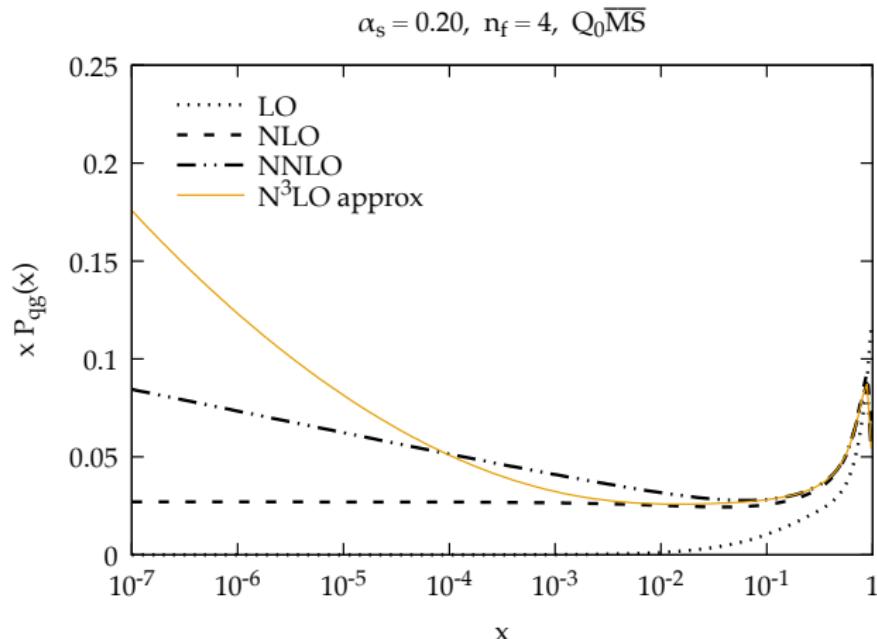
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P_{qg} and P_{qq} are NLL quantities, while P_{gg} and P_{gq} are LL
Small- x resummation is gluon-driven

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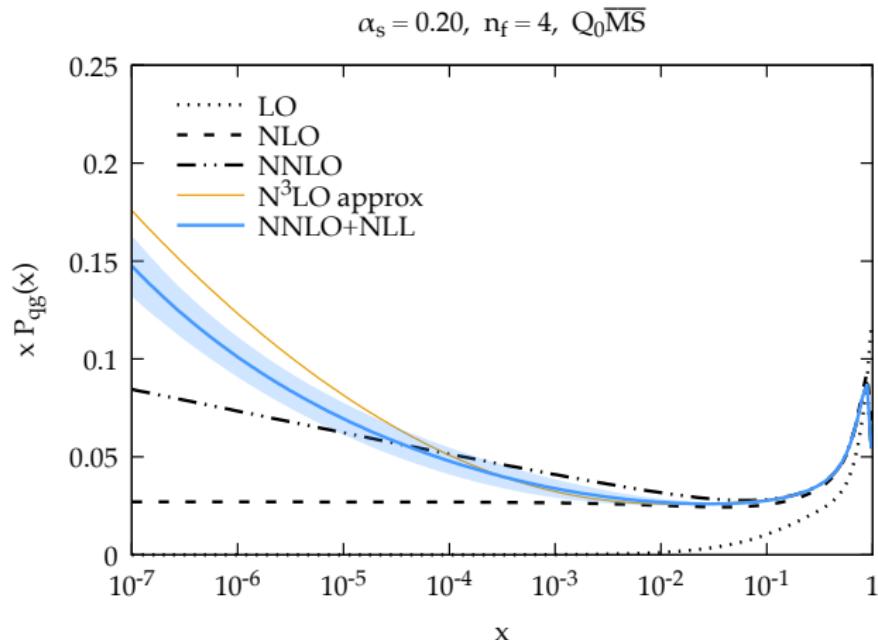
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Severe instability at small x in the singlet sector, the larger the higher the order

Below $x \sim 10^{-3}$ fixed-order result from NNLO onwards unreliable

Need to supplement NNLO and N³LO results with small- x resummation

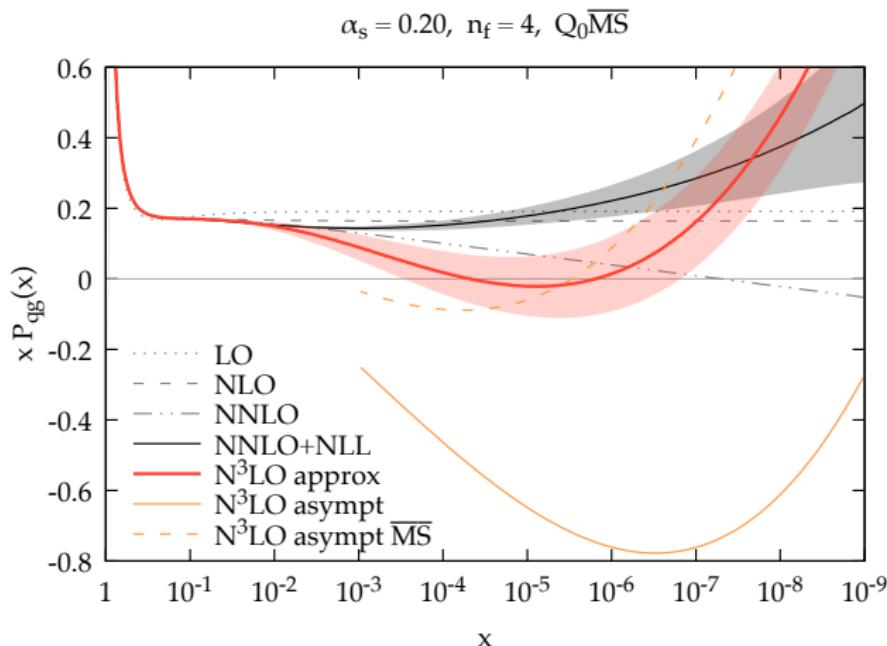
Towards N³LO evolution

Recent impressive progress towards N³LO splitting functions

[Davies,Vogt,Ruijl,Ueda,Vermaseren 1610.07477] [Moch,Ruijl,Ueda,Vermaseren,Vogt 1707.08315]

At small x , approximate predictions from NLL resummation

[MB,Marzani 1805.06460]



Large uncertainties from subleading logs

Logarithmic accuracy

	LL	NLL	NNLL	N^3LL
LO:	const			
NLO:	$\ln \frac{1}{x}$	const		
NNLO:	$\ln^2 \frac{1}{x}$	$\ln \frac{1}{x}$	const	
N^3LO :	$\ln^3 \frac{1}{x}$	$\ln^2 \frac{1}{x}$	$\ln \frac{1}{x}$	const

For N^3LO , NNLL accuracy would be needed $\rightarrow N^3LO + NNLL$

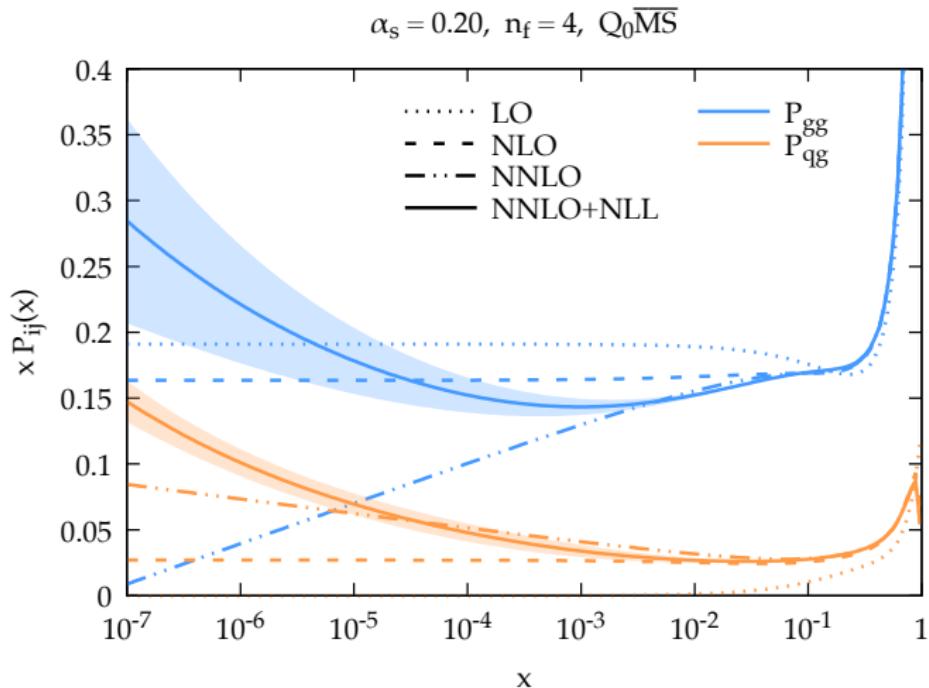
Work in (slow) progress, any help welcome!

[MB,Marzani,Ridolfi,Rinaudo,Silvetti]

Anyway, in the meantime, $N^3LO + NLL$ is certainly better than N^3LO alone!

Backup slides

Some representative HELL results: splitting functions

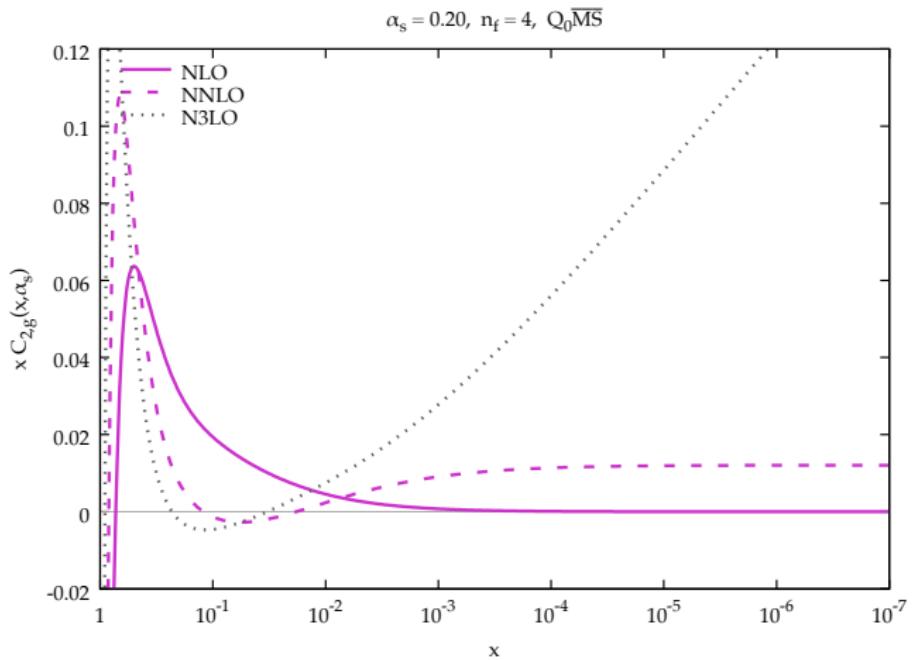


All-order behaviour rather different from fixed order (especially for P_{gg})

$P_{gg} > P_{qg}$ at resummed level (at NNLO they swap at some x)

Small- x logarithms in DIS coefficient functions

Only **singlet sector** affected: $C_{a,g}$, $C_{a,q}^S$, $a = \mathbf{2}, L, 3$



fixed-order
 $x C_{2,g}(x, \alpha_s)$ splitting
function at small x :

LO: 0

NLO: $\alpha_s \times 0$

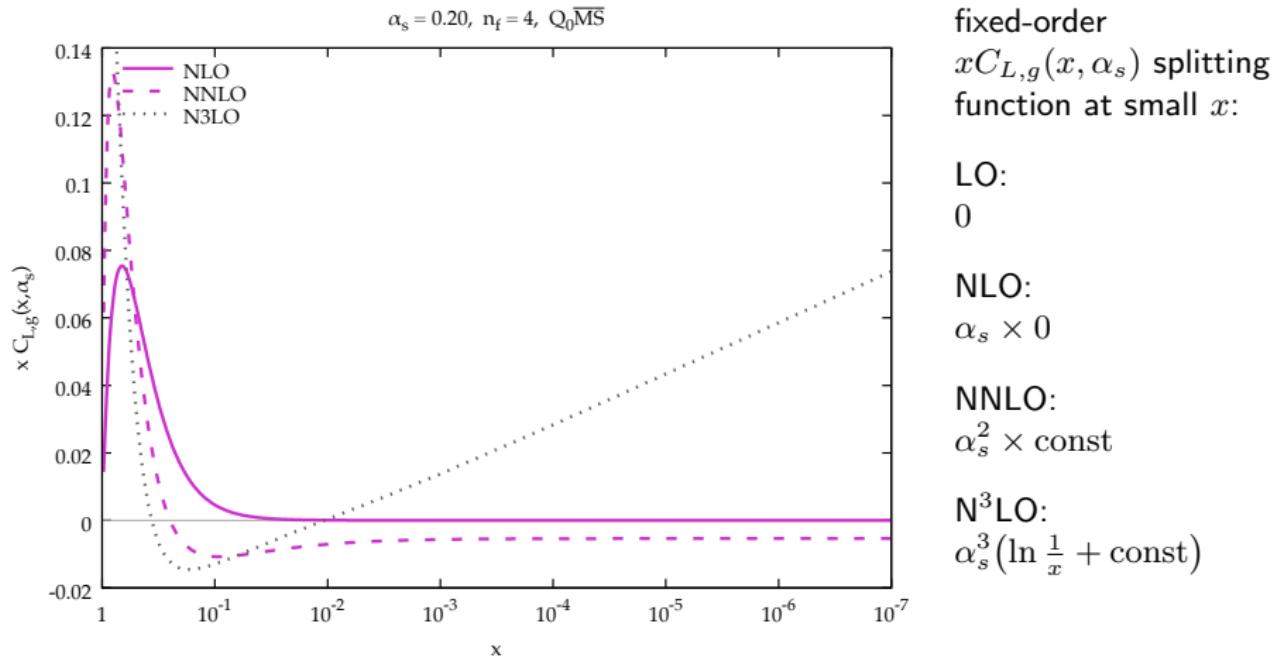
NNLO: $\alpha_s^2 \times \text{const}$

$N^3\text{LO}$: $\alpha_s^3 \left(\ln \frac{1}{x} + \text{const} \right)$

DIS coefficient functions are NLL quantities

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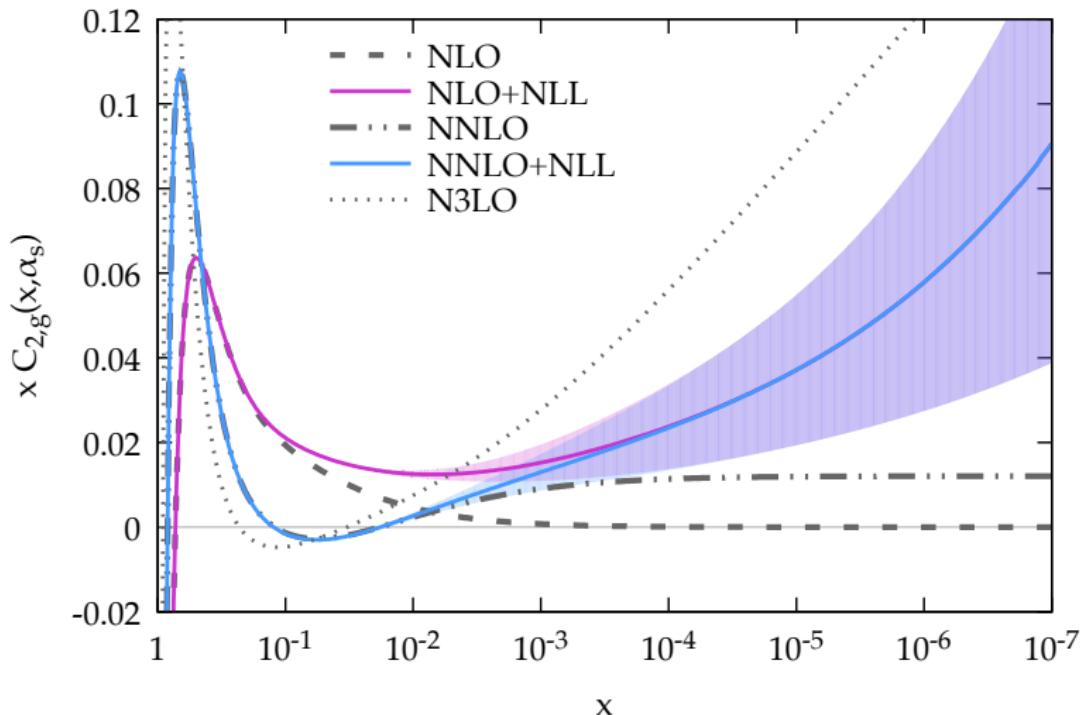


DIS coefficient functions are NLL quantities

Some representative HELL results: DIS coefficient functions

$F_{2,g}$ and $F_{L,g}$ massless DIS coefficient functions

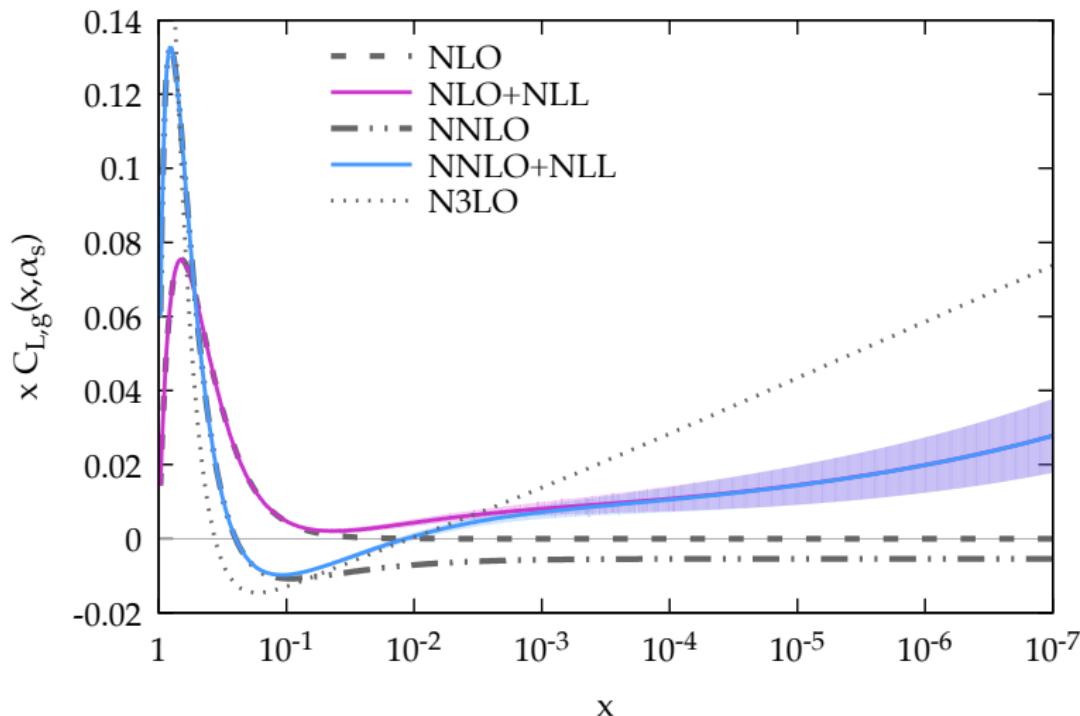
$$\alpha_s = 0.20, n_f = 4, Q_0 \overline{MS}$$



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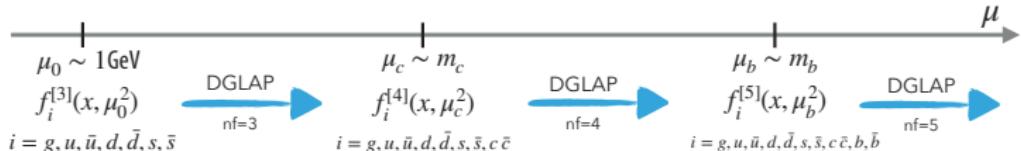
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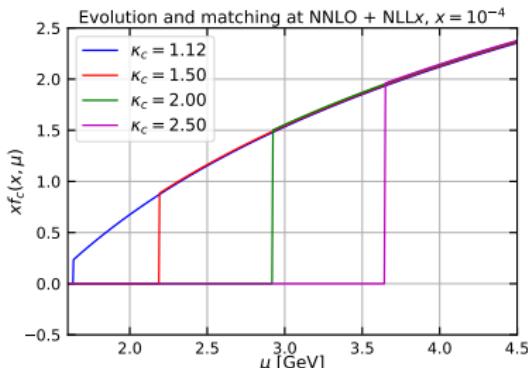
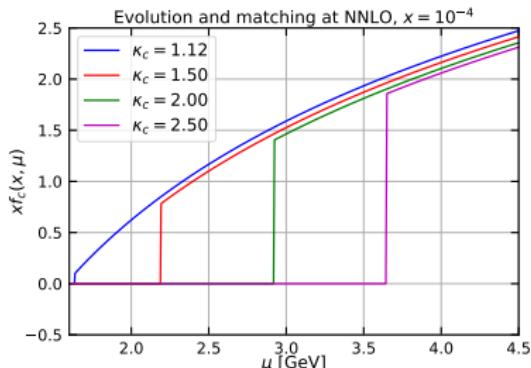
Matching conditions at the charm threshold

The number n_f of “active” flavours changes during the evolution (factorization scheme choice to resum large collinear mass logarithms from heavy quark pair production)



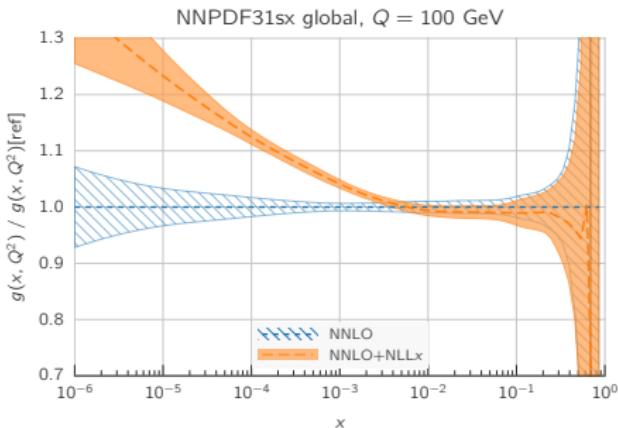
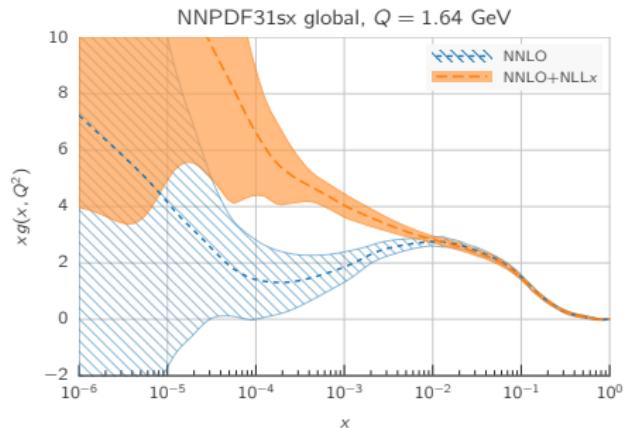
Matching relation between PDFs in schemes with different n_f

$$f_i^{[n_f+1]}(\mu^2) = \sum_{j=\text{light}} A_{ij}(m^2/\mu^2) \otimes f_j^{[n_f]}(\mu^2) \quad A_{ij} = \text{perturbative matching coefficients}$$



The perturbatively generated charm PDF is much less dependent on the (unphysical) matching scale when small- x resummation is included!

Fit results: impact on gluon PDF



Note: future higher energy colliders will probe smaller values of x ($x_{\min} \sim Q^2/s$)
→ small- x resummation will be even more important in future!

Impact of subleading logs (with xFitter)

First fit with HELL 3.0

[MB, Giuli 1902.11125]

Red and yellow curves differ by subleading logs

