Calibration and Tsys Beam and directivity some comments

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Updated - May 2020 , July 2020

Calibration on Autocorrelation

- Assuming an unpolarized source with brightness in Jansky S^*, and an antenna with collecting area A and collection efficiency η, the total power collected per unit frequency (both polarisation) is

$$(P^*)_{2pol} = A\eta S^*$$

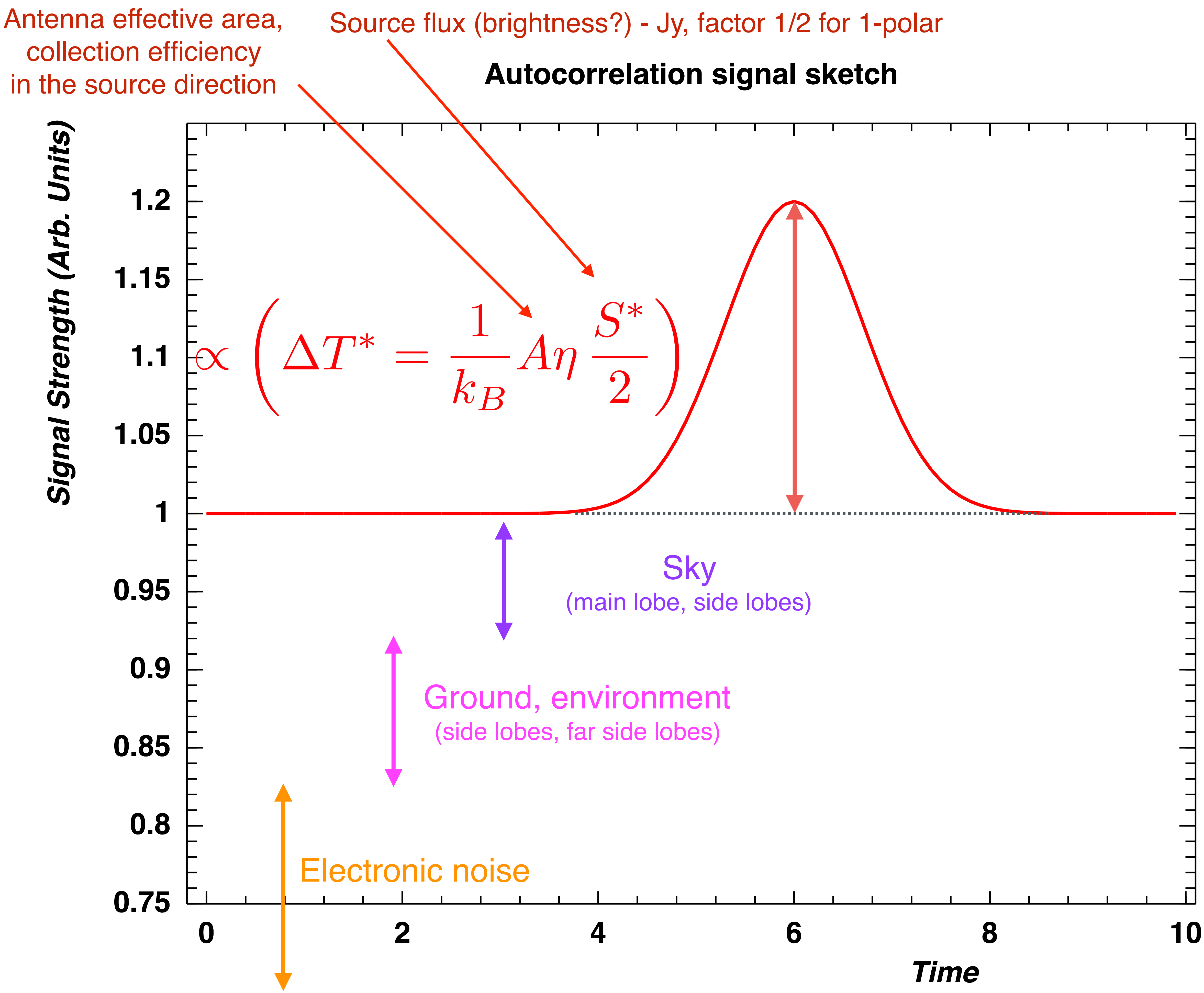
- for a single polarisation, the received power will be half, leading to increase in temperature ΔT^*. k_B is the Boltzmann constant ($1\text{Jy} = 10^{-26}\text{W/m}^2/\text{Hz} = 10^{-26}\text{J/m}^2$).

$$\begin{aligned}
(P^*)_{1pol} &= \frac{1}{2} A\eta\, S^* \\
(P^*)_{1pol} &= k_B \Delta T^* \\
k_B &= 1.38 \times 10^{-23}\,\text{J/K} = 1380 \times 10^{-26}\,\text{J/K}
\end{aligned}$$

- The system temperature T_{sys} would determine the level of power (auto-correlation signal strength) when all external source contribution can be neglected. We denote V_{ii} the auto-correlation signal strength in arbitrary units, V_{ii}^K the calibrated signal in Kelvin, and $\mathbf{C}$ the calibration coefficient.

$$\begin{aligned}
V_{ii}^K &= \mathbf{C} \times V_{ii} \\
V_{ii}^K &= T_{sys} + \Delta T^* = \mathbf{C}\left(V_{ii}^{base} + \Delta V_{ii}\right)
\end{aligned}$$

For a dish $A = \frac{\pi}{4} D^2$, and $\eta \simeq 0.8 - 0.9$



Antenna effective area,
collection efficiency
in the source direction
Source flux (brightness?) - Jy, factor 1/2 for 1-polar
Autocorrelation signal sketch
$\propto \left(\Delta T^* = \frac{1}{k_B} A\eta \frac{S^*}{2} \right)$
Signal Strength (Arb. Units)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0
2
4
6
8
10
Time
Sky
(main lobe, side lobes)
Ground, environment
(side lobes, far side lobes)
Electronic noise

Beam and effective area - I

- The beam is defined by the antenna (reflector+feed) pattern or angular response to the electric field $R_E(\vec{\omega})$, and its square which is the intensity or power response:

$$
\begin{aligned}
s &= \int\int_{4\pi} R_E(\vec{\omega})\, E(\vec{\omega}) d\vec{\omega} \qquad \text{Voltage} \\
p = |s|^2 &= \int\int_{4\pi} R_I(\vec{\omega})\, P(\vec{\omega}) d\vec{\omega} \qquad \text{Power}
\end{aligned}
$$

- However, the EM radiation power is characterised by the Poynting vector, which give the power flow (per unit of area) and as function of the direction $\vec{\omega}$. One needs also the antenna collecting area A, which might be itself direction dependent $A(\vec{\omega})$.

- Antenna pattern in power $R_I(\vec{\omega})$ usually expressed as directivity $D(\vec{\omega})$ or (used as receptor) or gain $D(\vec{\omega})$ (used as transmitter)

$$
\begin{aligned}
D(\vec{\omega}) &= 4\pi \frac{p_r(\vec{\omega})}{P_r} \\
G(\vec{\omega}) &= 4\pi \frac{p_t(\vec{\omega})}{P_t}
\end{aligned}
$$

where P_r is the total radiation power incident on the antenna, $p_r(\vec{\omega})$ incident radiation from the direction $\vec{\omega}$, per unit of solid angle, and P_t is the total power fed to the antenna, and $p_t(\vec{\omega})$ the power radiated per unit solid angle, in the direction $\vec{\omega}$. 4π ensures that for a fully isotropic antenna, $D(\vec{\omega}) = G(\vec{\omega}) = 1$.

Beam and effective area - II

- The power received from a blank uniform sky $P^r_{1pol}(T_{sky})$, considered as a blackbody source with temperature T_{sky} - One has to consider the receiver effective area A, related to its resolution $\delta\Omega$ by the diffraction pattern, to obtain that the received power, in the Rayleigh Jeans regime is: (the factor $\frac{1}{2}$ is to account for a single polarisation)

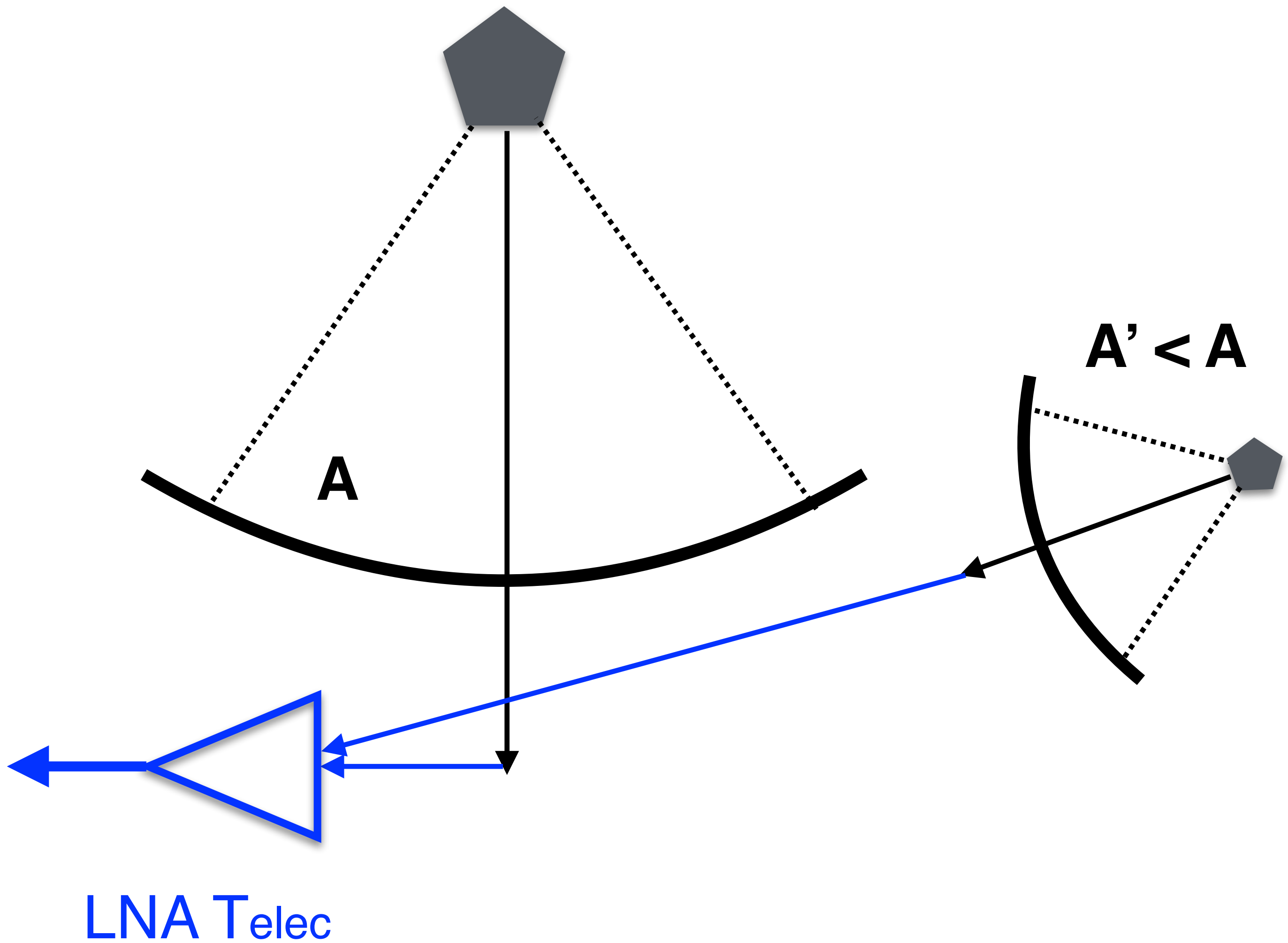
$$\begin{aligned} \delta\Omega &= \frac{\lambda^2}{A} \\ P^r_{1pol}(T_{sky}) \times \Delta\nu &= \frac{1}{2} B(T,\nu)\, A\, \delta\Omega\Delta\nu = k_B T_{sky} \times \Delta\nu \end{aligned}$$

- For a point source with brightness in Jansky S^* , the received power would be

$$P^*_{1pol} \times \Delta\nu = \frac{1}{2} S^*\, A^* \times \Delta\nu$$

- The effective area to be used for source is A^* , in the direction of the source , if the effective collecting area is direction dependent. Maybe we should use the main beam only to get the effective area to be used for point source (for calibration).

What will happen in such a configuration ?



The power from a blank, uniform sky with temperature T_{sky} will be **doubled** : $P^r_{1pol}(T_{sky}) = P + P' = 2k_B T_{sky}$

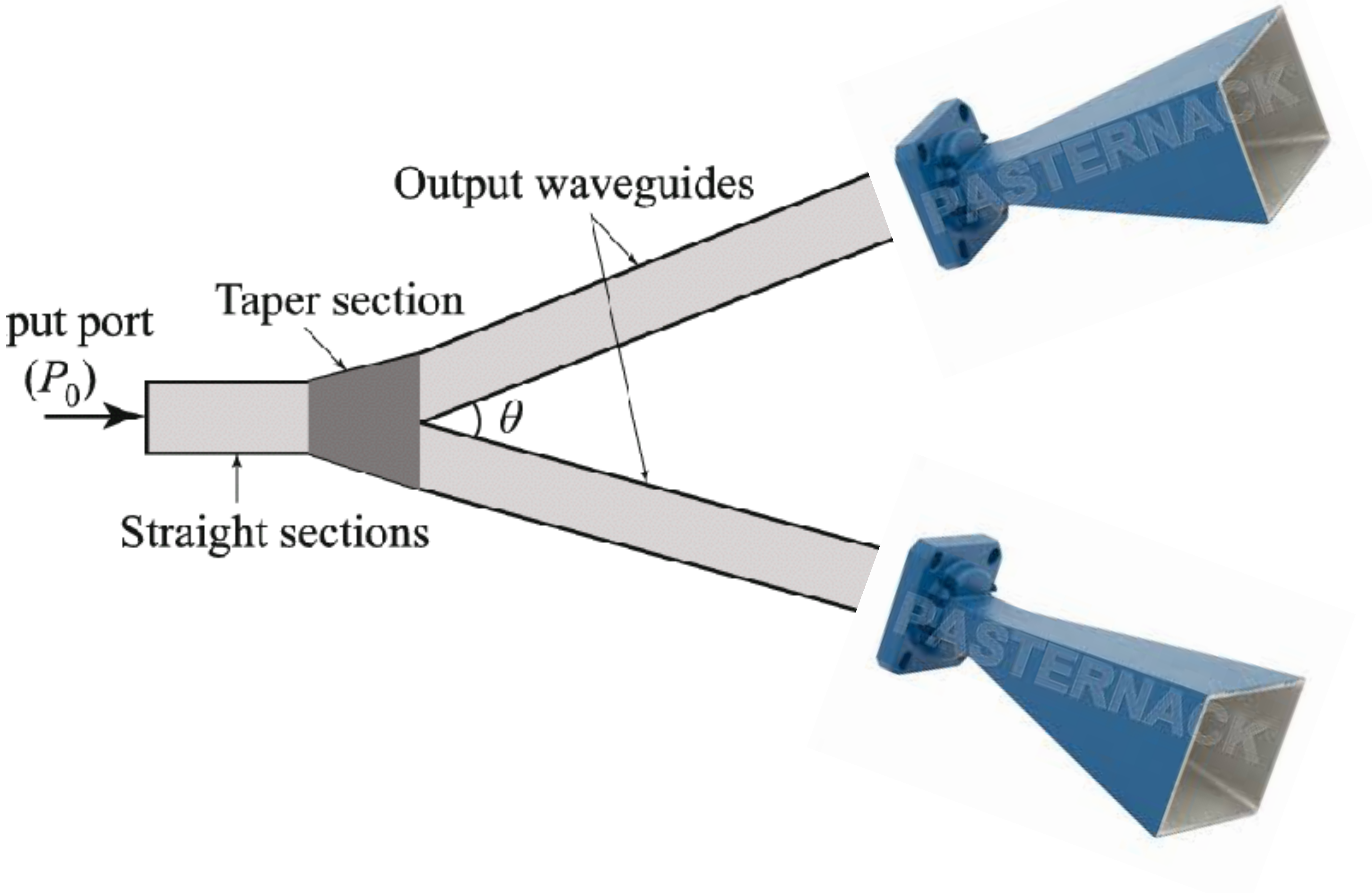
The contribution from the electronic noise, as expressed as a blank sky Temp **divided by 2** $\longrightarrow Telec \times \frac{1}{2}$

The signal received from the point source, when in the direction of the primary beam will be unchanged:

$$P^*_{1pol} = \frac{1}{2}\, S^*\, A$$

while the directivity, hence the collecting area will be divided by two

Does this make sense ?



We can make it using horns and waveguides, so we could also make it using mirrors …

- In a configuration where the global beam response can not be explained by a single collecting area, the power received from a blank sky at temperature T would be larger than $k_B\ T_{sky}$ by a factor ξ , $\xi>1$

- The beam directivity is defined as the maximum response, when the integral response is normalised to $4\ \pi$, adding side lobes decreases the directivity (proportional to effective area A)

- We can thus have a beam response such that the effective area associated with the main lobe would be different from the one derived from the beam directivity

- However, to convert the signal increase from a point source to the one corresponding to the increase of a blank sky temperature, one should use the area derived from the directivity (the source signal is a smaller fraction of the power from a blank sky)

- But, the contribution from the LNA noise to Tsys would also be smaller by a factor ξ, compared to have the same electronics put in the focus of an antenna with the main beam, without the additional side lobes

Backup

T_{sys} from cross-correlations

- To get T_{sys} from cross correlations, one can use the RMS flucutations on the baseline to determine it. In the following $\langle\rangle$ denotes average, while σ denotes RMS (Standard deviation) of fluctuations. N is the number of samples averaged , which should be:

$$N \sim \Delta\nu * t_{int}$$

  where $\Delta\nu$ is the frequency band, and t_{int} is the integration time or binning time for computing the average of visibilities.

- Outside bright sources, neglecting diffuse sky contribution , Autocorrelation signal V_{ii}:

$$\begin{aligned} \langle V_{ii} \rangle &\propto T_{sys} \\ \sigma_{ii} &= \frac{\langle Vii \rangle}{\sqrt{N}} \end{aligned}$$

- For the cross-correlations signals V_{ij}, outside point like sources and neglecting correlated noise :

$$\begin{aligned} \langle V_{ij}.real \rangle &= \langle V_{ij}.imag = 0 \\ \sigma_{ij.real} &= \sigma_{ij.imag} \simeq \frac{\sigma_{ii}}{\sqrt{2}} \end{aligned}$$

  Note that the factor $\sqrt{2}$ in the last equation has been determined numerically (montecarlo simulation)