Automatic Leptonic Tensor Calculation for Beyond the Standard Model (BSM) Models

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Introduction

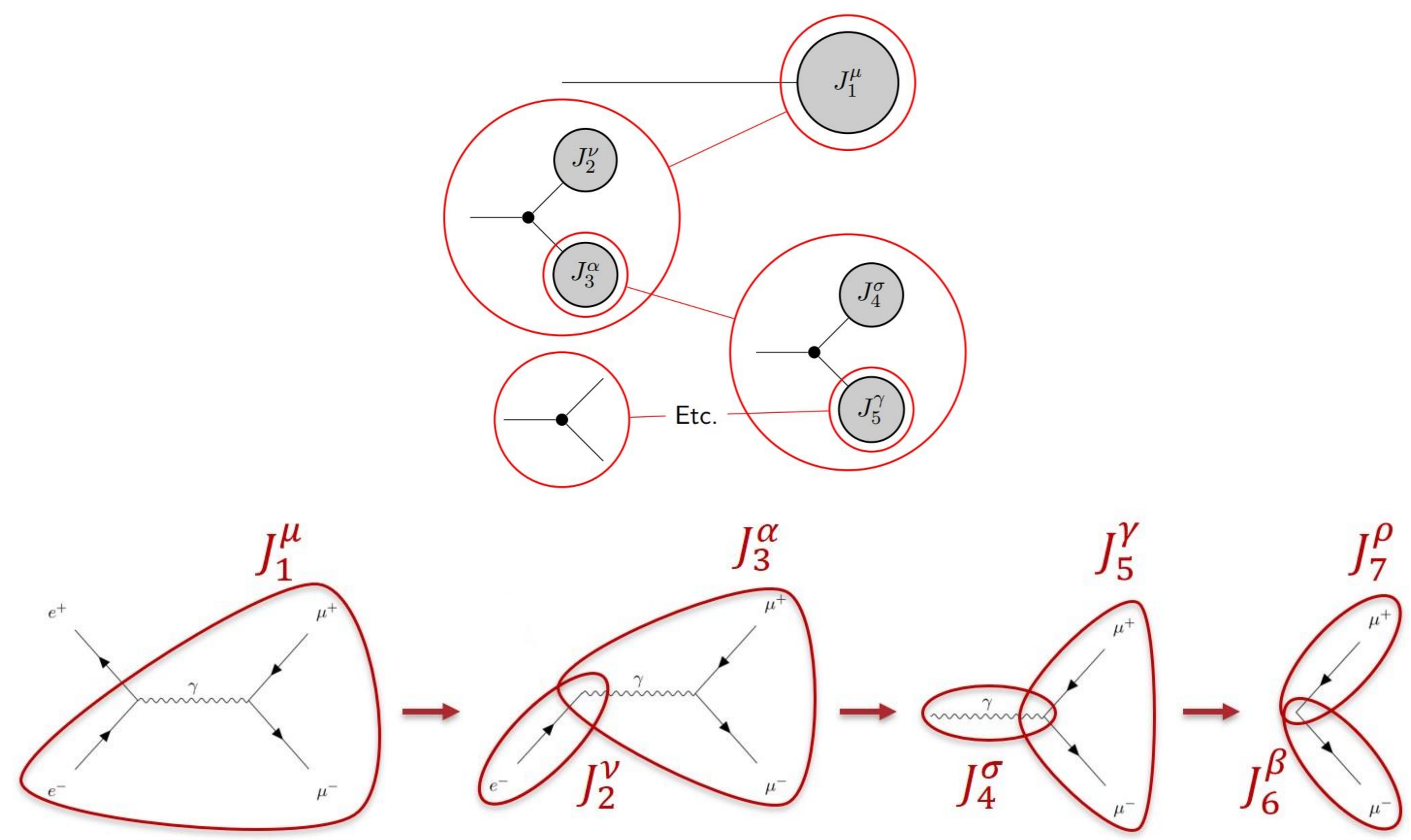
- Colossal data output from neutrino experiments (e.g. DUNE) will require testing of several BSM theories.
- Manual implementation of BSM theories in event generators is time-consuming and prone to errors.
- For neutrino events, we can always decompose the squared amplitude ($|M|^2$) into a hadronic ($H^{\mu\nu}$) and a leptonic ($L_{\mu\nu}$) tensor: $|M|^2 = H^{\mu\nu}L_{\mu\nu}$.
- $H^{\mu\nu}$ is complicated to calculate but event generators are good at doing it. Separation of amplitude into $H^{\mu\nu}$ and $L_{\mu\nu}$ allows easy calculation of effects of BSM theories on $L_{\mu\nu}$.
- Develop a program to automatically calculate leptonic tensors of BSM theories:
  - Requires only BSM Lagrangian.
  - Can be easily interfaced to several neutrino event generators.

Methods

- Universal FeynRules Output (UFO) files:
  - Use BSM Lagrangian to calculate Feynman vertices. Output in Python.
- Lark package:
  - Parser for string outputs of UFO files.
- Berends-Giele algorithm:
  - Recursive break down of Feynman diagrams.
  - Allows recycling of diagrams' components. Highly-efficient.
  - BG Equations: Current $J_i(\pi)$ and amplitude $M(\pi)$ for set of particles π. Base case for $J_i(\pi)$ is the particle's wavefunction.

$$J_i(\pi) = \underbrace{P_i(\pi)}_{\text{Propagator term}} \underbrace{\sum_{V_i^{j,k}} \sum_{P_2(\pi)}}_{\text{Sum over all possible vertices and permutations}} \underbrace{S(\pi_1,\pi_2)}_{\text{Symmetry factor}} \underbrace{V_i^{j,k}(\pi_1,\pi_2)}_{\text{Interaction vertex}} \underbrace{J_j(\pi_1) J_k(\pi_2)}_{\text{Adjacent currents}}$$

$$M(\pi) = \underbrace{J_n(n)}_{\text{Current for } n} \cdot \underbrace{\frac{1}{P_{\bar{n}}(\pi\backslash n)}}_{\text{Reversed particle properties; Propagator term}} \cdot \underbrace{J_{\bar{n}}(\pi\backslash n)}_{\text{Current for } \bar{n}}$$



Schematic of Berends-Giele algorithm. Breakdown of a Feynman diagram into its constituent currents J_i^μ. Example for $e^+e^- \to \mu^+\mu^-$.

Results and Discussion

- Validation results of squared amplitude of three SM processes ($e^+e^- \to \mu^+\mu^-, e^-\mu^- \to e^-\mu^-, e^+e^- \to e^+e^-$) plotted versus $\cos(\theta)$ and for randomly generated azimuthal angles ϕ.
- Our results show percentage deviations of order 10^{-14} with respect to analytic calculations of our SM processes.
- Work can be extended to more complex processes and to BSM theories.
- To illustrate how $|M|^2$ can be split into $H^{\mu\nu}$ and $L_{\mu\nu}$, we perform the calculation for $e^-N \to e^-N$, with N being an atomic nucleus:

1. Consider simpler case $e^-\mu^- \to e^-\mu^-$ with Feynman diagram shown in 2). Diagram is composed by upper e^- part and lower μ^- part.
2. Label $e^-_{in}: 1$, $\mu^-_{in}: 2$, $e^-_{out}: 3$, $\mu^-_{out}: 4$. Matrix element $|M|^2$ of $e^-\mu^-$ scattering given by:

$$= \underbrace{\frac{2e^2}{(p_1-p_3)^2} \cdot \left[p_{3\mu}p_{1\nu} + p_{3\nu}p_{1\mu} + (m_e^2 - p_1 \cdot p_3)g_{\mu\nu} - i\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_3^\beta\right]}_{L_{\mu\nu,e^-}}$$

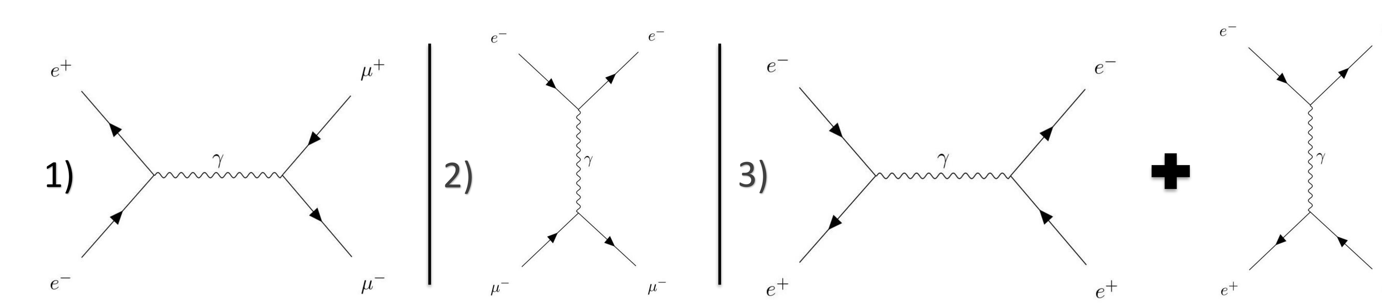
$$\underbrace{\frac{2e^2}{(p_2-p_4)^2} \cdot \left[p_4^\mu p_2^\nu + p_4^\nu p_2^\mu + (m_\mu^2 - p_2 \cdot p_4)g^{\mu\nu} + i\epsilon^{\mu\nu\alpha\beta}p_{2\alpha}p_{4\beta}\right]}_{L^{\mu\nu}_{\mu^-}}$$

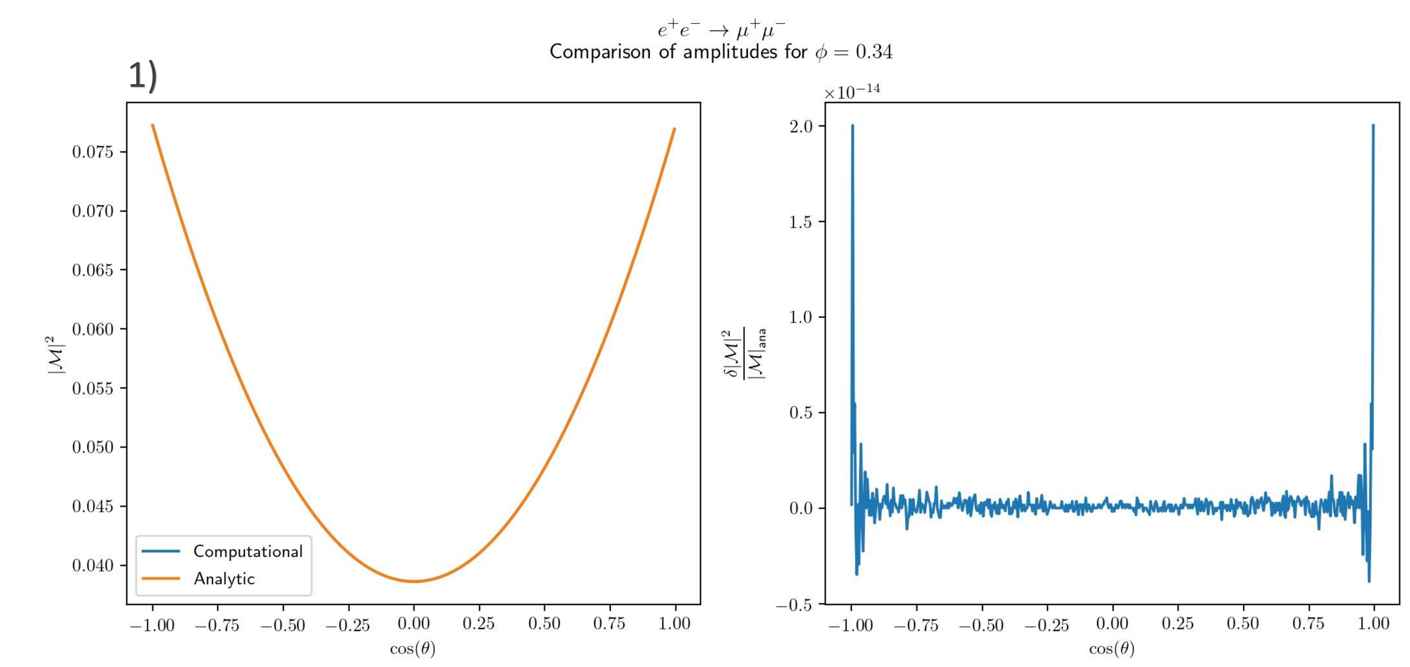
3. Similarly, matrix element $|M|^2$ of e^-N scattering contains:

$$\underbrace{\frac{2e^2}{(p_1-p_3)^2} \cdot \left[p_{3\mu}p_{1\nu} + p_{3\nu}p_{1\mu} + (m_e^2 - p_1 \cdot p_3)g_{\mu\nu} - i\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_3^\beta\right]}_{L_{\mu\nu,e^-}}$$

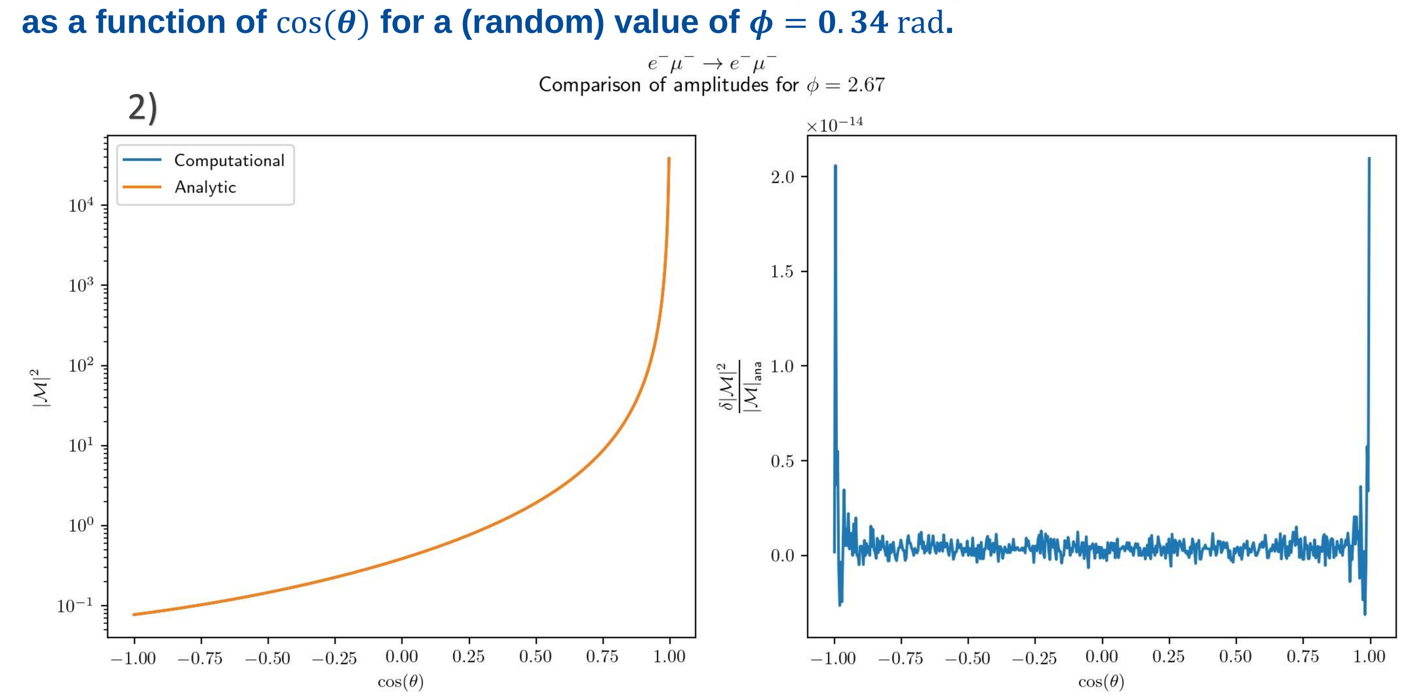
Indeed:

$$|M|^2 = L_{\mu\nu,e^-}H_N^{\mu\nu}$$

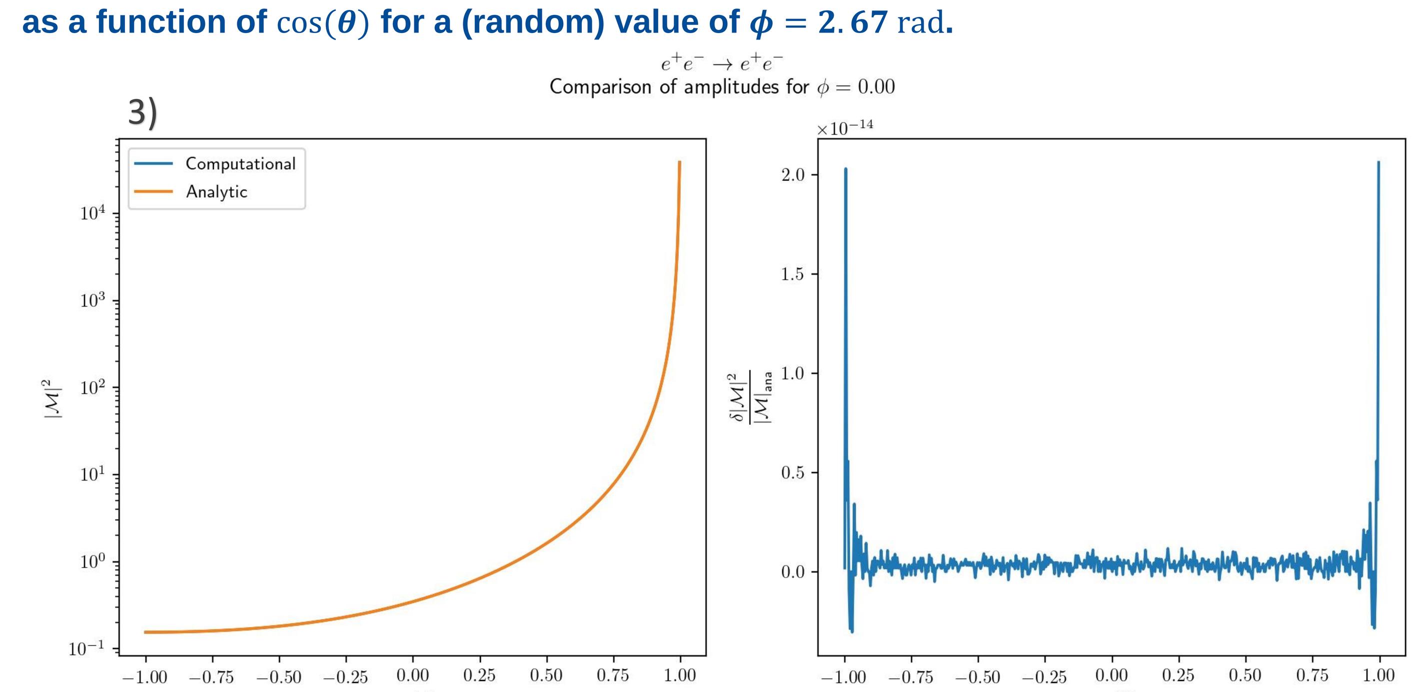




Comparison of our computational and analytic squared amplitudes for $e^+e^- \to \mu^+\mu^-$ as a function of $\cos(\theta)$ for a (random) value of $\phi = 0.34$ rad.



Comparison of our computational and analytic squared amplitudes for $e^-\mu^- \to e^-\mu^-$ as a function of $\cos(\theta)$ for a (random) value of $\phi = 2.67$ rad.



Comparison of our computational and analytic squared amplitudes for $e^+e^- \to e^+e^-$ as a function of $\cos(\theta)$ for a (random) value of $\phi = 0.00$ rad.

Conclusion and Future Steps

- Validation results are promising, proving our method works for SM processes.
- Convert amplitude into leptonic tensor to be interfaced with event generators.
- Perform tests in DIS events as well as with some BSM theories using leptonic tensor.

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