# Measuring and Simulating $T_1$ and $T_2$ for Qubits

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Quantum computers are computational devices that employ quantum-mechanical phenomena in solving problems that are classically intractable. Future applications of quantum computers include prime factorization of large numbers, implementing more efficient search algorithms, among various applications. Most quantum algorithms utilize superposition of states in solving a given problem. Therefore, the decay of the superposition of states is a limitation to performing complex quantum algorithms. Quantum decoherence is usually characterized by measuring two constants: $T_1$ (thermal relaxation time) and $T_2$ (dephasing time). Here we present an overview of measuring and simulating $T_1$ and $T_2$ for qubits and qudits using QuTip and Qiskit.

## I. INTRODUCTION

Quantum computers perform computations exploiting quantum mechanics to a possible advantage, allowing us to prepare and manipulate states that do not have a classical equivalent. In particular, phenomena like superposition and entanglement may enable quantum computers to outperform their classical counterparts in certain applications. In fact, it has been shown that the number of steps required to find the prime factors of an integer increases exponentially as the integer increases [1]. Shor's factoring algorithm, however, can factor prime numbers in polynomial time. In fact, there has been promising results from the D-Wave 2000Q computer as it was able to factor the number 376289 using 94 logical qubit gates [2]. It is thus essential to develop new encryption protocols since the security of online transactions assumes the impossibility of factoring large numbers in a reasonable time using classical algorithms. Furthermore, quantum computers hold promise of efficiently simulating large atomic systems in order to understand their properties. The calculation time scales exponentially using classical computers as the number of atoms grows, while it grows polynomially on quantum computers [3].

Implementing these useful quantum algorithms is contingent upon building accurate quantum hardware that is not affected by noise. Environmental noise decreases coherence time of qubits, meaning that qubits do not stay in a desired state long enough to carry out a complex computation. Right now, the coherence time of qubits is on the order of 10's of microseconds, which is not long enough to solve interesting problems. Therefore, mitigating noise and designing noise-tolerant quantum computers is a necessity. To that end, harnessing the full power of quantum computers necessitate characterization and understanding of noise sources and how they impact a given quantum system.

Often times, $T_1$ and $T_2$ are used to quantify noise. In this report, we provide an approach as to how $T_1$ and $T_2$ values are calculated and simulated for quantum systems. In addition, we compare simulated values of $T_1$ and $T_2$ with those of a real quantum computer's measurements. IBMQ Experience is used to prepare, run and measure quantum states, while QuTip is used for simulation.

## II. BACKGROUND

The basic unit of information for quantum computers is a qubit. In classical computers, bits are manifested by transistors that are turned on or off through applying electrical pulses. The value of bit is deterministic and is measured to be either 0 or 1. For quantum computers, qubits are implemented as a two-state device exhibiting desired quantum-mechanical phenomena such as superposition or entanglement.

***Theoretically***, qubits are represented as vectors in Hilbert space, a complex vector space with an inner product. Qubits can exist in the state $|0\rangle$ or $|1\rangle$ or a superposition of both states such that $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha$ and $\beta$ are probability amplitudes, implying that $\alpha^2 + \beta^2 = 1$. Quantum gates are defined as operations on quantum states represented by unitary matrices, $U^\dagger$U=U$U^\dagger$=I, making quantum computations reversible. Gates may be visualized as rotations around the Bloch sphere in Figure 1. The Bloch sphere has basis states given by the x, y, and z axes. The direction of the state $\psi$, the red vector on the sphere, determines the state of the qubit. For example, if $\psi$ is pointing in the positive z-direction, the qubit is in the $|0\rangle$ state and if it is pointing in the negative z-direction, the qubit is in the $|1\rangle$ state. If $\psi$ is pointing anywhere between $|0\rangle$ or $|1\rangle$, the qubit is in a superposition of both states. It is also worth noting that the Bloch Sphere is a viable representation of a single qubit only, as states of higher dimensions cannot be visualized.

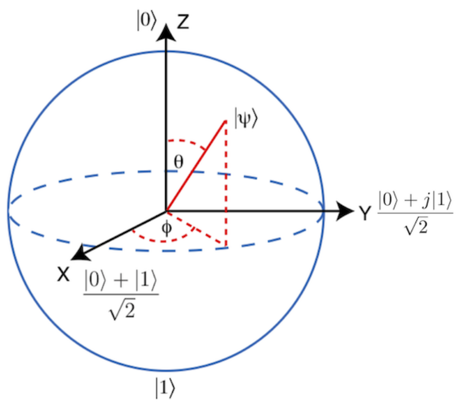


Figure 1: **The Bloch Sphere**

***Experimentally***, there are numerous techniques to fabricate qubits. Generally, in order for a qubit to perform well, it must satisfy some requirements such as being loosely coupled to their environment and strongly coupled to a classical control system. It is also imperative for the qubit to be an anharmonic oscillator. A harmonic oscillator would not work because all levels are uniformly spaced, so a pulse that excites the first transition would also excite the second (and third and any others)[4]. Hence, the goal is to design an anharmonic oscillator where the two lowest energy levels are used as qubit.For the purpose of this report, only the realization of transmission line shunted plasma oscillation qubits (transmon qubits) will be discussed. The transmon circuit, as shown in Figure 2, consists of a Josephson Junction shunted by a relatively large capacitor so that noise charge is reduced [5]. The Josephson Junction is responsible for anharmonicity, which allows for selective qubit control. The circuit then could be conceived as an artificial atom. This design minimizes energy dissipation, resulting in longer coherence time ≈100$\mu$s and allowing for longer computations to take place [6].

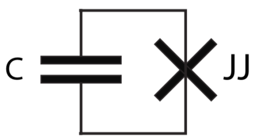


Figure 2: **Transmon Qubit**:the transmon qubit circuit contains a Josephson Junction shunted by a relatively large capacitor so that $E_J \gg E_C$, where $E_c$ is the charging energy (kinetic energy in capacitor) and $E_J$ is the potential energy stored in the Josephson Junction

***Measurement*** After applying gates to qubits, a measurement should be done in order to identify the state of the system. Measurement is performed in a given computational basis i.e. the $|+\rangle$,$|-\rangle$ basis or the $|0\rangle$,$|1\rangle$ basis. Measuring the state of the system causes the wave function to collapse into one state, destroying the quantum state. However, creative techniques, such as quantum non-demolishing measurement(QND), have been developed to observe the state of the system without causing its state decay. That way, it is possible to continuously monitor the state of the qubit for some time before it eventually decays due to energy dissipation.

## III. METHODS

In order to simulate quantum systems, the following tools were used

- ***Qiskit***: IBMQ Experience is an online platform that enables the general public to run quantum algorithms on real quantum computers. Qiskit, an open-source software development kit, provides users with pulse-level control. The used model of computation is quantum circuits, where quantum algorithms consist of consecutive gates. IBM's quantum computers consist of superconducting transmon qubits that are based in their research centers [7].
- ***QuTip***: an open-source software for simulating the dynamics of quantum systems. It contains built-in functions that represent different noise models, offering an easy-to-use simulation of open quantum systems. Finally, QuTip is Python-based and can be run on Jupyter notebooks [8].

***Measuring*** $T_1$: thermal relaxation time is defined as the time needed for a qubit to move from the excited state $|1\rangle$ to the ground state $|0\rangle$. The process could be formulated by the density matrix $\rho = \alpha|\psi\rangle\langle\psi| + \beta|\text{g}\rangle\langle\text{g}|$ where $\alpha^2$ is the probability that the qubit is in state $|\psi\rangle$ whereas $\beta^2$ is the probability that a qubit is in the ground state $|\text{g}\rangle$. As time goes by, the value of $\beta^2$ approaches 1. Experimentally, $T_1$ is the time by which the population of excited state decays to 1/e of its initial value, $P_e(\text{t}) = P_e(0)\text{e}^{-t/T_1}$[5].

Experimentally, the value of $T_1$ is measured through the following sequence of operations [5]:

1. Prepare the qubit in the excited state by sending a $\pi$-pulse to the qubit.

2. Wait some time t.

3. Measure the state of the qubit.

***Measuring*** $T_2$: dephasing time is defined as the elapsed time before a qubit's resonance frequency becomes unidentified. $T_2$ could be thought of as the loss of quantum coherence over time. The measurement of $T_2$ is achieved through the following sequence of operations [5]:

1. Prepare the qubit in superposition state $\frac{1}{\sqrt{2}}(|\text{g}\rangle + \text{i}|\text{e}\rangle)$ by applying a $\frac{\pi}{2}$-pulse to the qubit.

2. Wait some time t.

3. Apply another $\frac{\pi}{2}$-pulse to bring back the qubit to ground state.

4. measure the state of the qubit.

## IV. RESULTS

***Using Qiskit***: for the experiments presented in Figures 1 and 2, the ibmqx2 device was used. Ibmqx2 is a five-qubit computer that is based in Yorktown, New York. Both experiments, measuring $T_1$ and $T_2$, were prepared, run and measured on the first qubit (qubit 0) with 1024 shots (number of times the system was prepared, run, and measured). According to IBM's specifications of the device, on average, the qubit's $T_1 \approx 52$ $\mu$s, while the value of $T_2 \approx 77$ $\mu$s. Though our experiments did not yield these exact numbers, it still provided insights into the order of magnitude for both values, which is $\mu$s. The values provided by IBM were obtained by averaging numerous measurements, so it is highly likely that the exact numbers will not obtained from one experiment.

In Figure 3, the probability of the qubit being in the $|0\rangle$ state (or graound state) increases as time progresses. After about 70 $\mu$s, the probably of measuring the system in the ground state is very close to 1.

In Figures 4 and 5, the envelope of the curve exponentially decreases as time progresses. It is usually simulated as an exponential decay. The probability of measuring the qubit in the $|+\rangle$ state increases as time goes. After some time, probability of finding the qubit in the $|+\rangle$ state is 0.5, meaning that there we lost track of the qubit's resonance frequency.

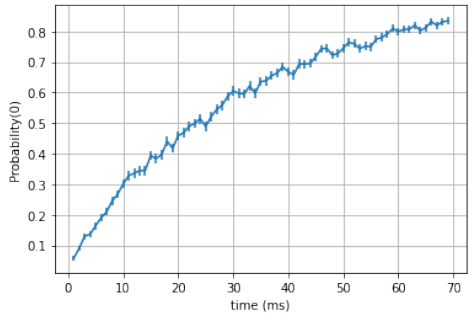


Figure 3: **A measurement of** $T_1$ for a qubit in $\mu$s.

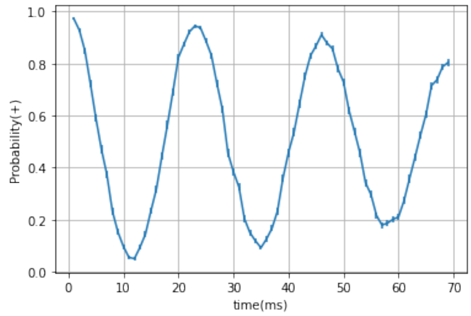


Figure 4: **A measurement of** $T_2$ for a qubit in $\mu$s.

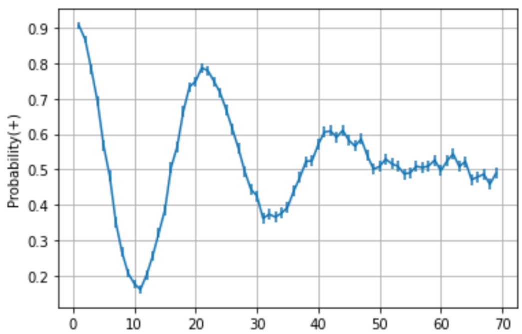


Figure 5: **Simulation of** $T_2$ **of the first excited state** for a qubit.

***Using QuTip***: Owing to the fact that our system is an open quantum system, it is described by the Linblad Master Equation. Environmental noise, which is not a unitary operation, is represented as coupling terms between the qubit and its environment. In QuTiP, the function Mesolve is used to evolve a given system in time according to the Schrödinger equation and to the Master Equation [10]. The Mesolve function automatically determines if it is sufficient to use the Schrödinger equation (if no collapse operators were given) or if it has to use the aster equation (if collapse operators were given). Evolving the system according to the Schrödinger equation is less computationally intensive than using the Master Equation, so when possible, the solver will use the Schrödinger equation. The arguments of the Mesolve function are the system Hamiltonian, initial state, collapse operators, and expectation operators. Hamiltonians need to be input as a matrix, the initial state as a vector or matrix, and operators as lists.

Figure 6 shows the probability of finding the qubit in the first excited state $|1\rangle$. The probability of finding the qubit in the $|1\rangle$ state decreases effectively to zero as time goes by.

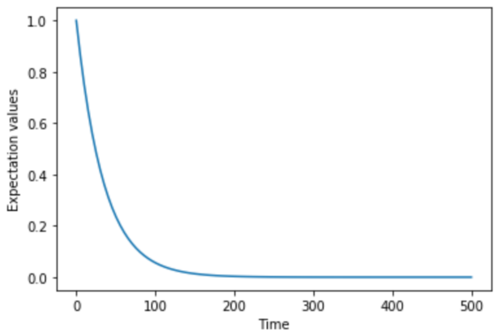


Figure 6: **Simulation of $T_1$** for a qubit.

In Figure 7, as the number of energy levels increases, the stability of the qudit decreases, resulting in a shorter $T_1$. A qu-d-it is a generalization of a qubit to a d-level or d-dimension system [9]. As the dimensions of qudits increase, more information will be held and processed. However, due to the instability of higher energy systems (qudits), they decay much quicker than two-level systems (qubits).

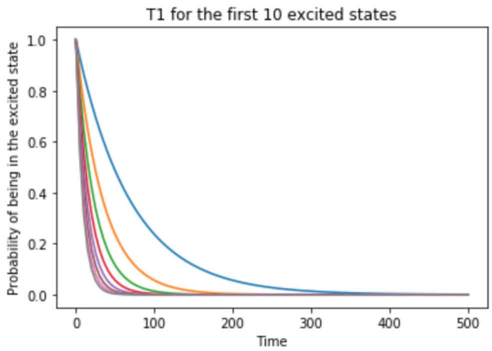


Figure 7: **Simulation of $T_1$ of 10 excited states** for a qudit. The blue curve represents $T_1$ for the first excited state. The curves to the left represent higher energy levels.

In Figure 8, the blue curve represents $T_2$ (the dephasing) of a qubit. The envelope of the curve could be efficiently simulated as a decaying exponential.

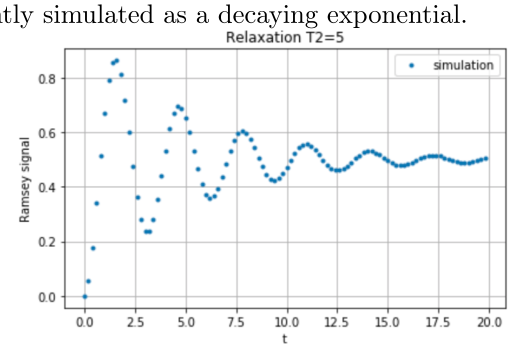


Figure 8: **Simulation of $T_2$ of the first excited state** for a qubit.


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