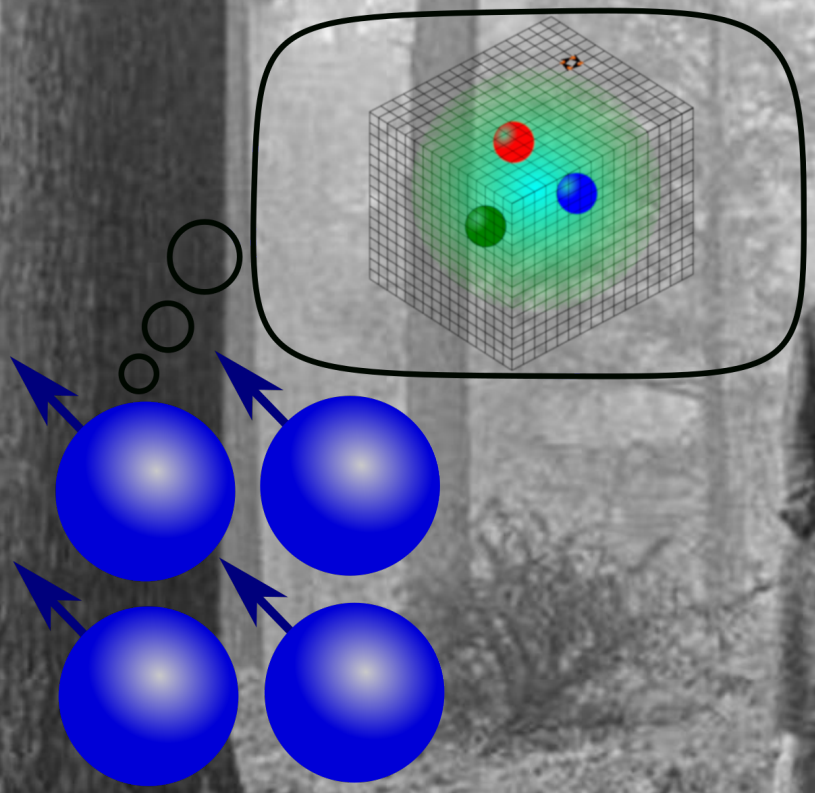
Gluon Field Digitization for Quantum Computers

Hank Lamm

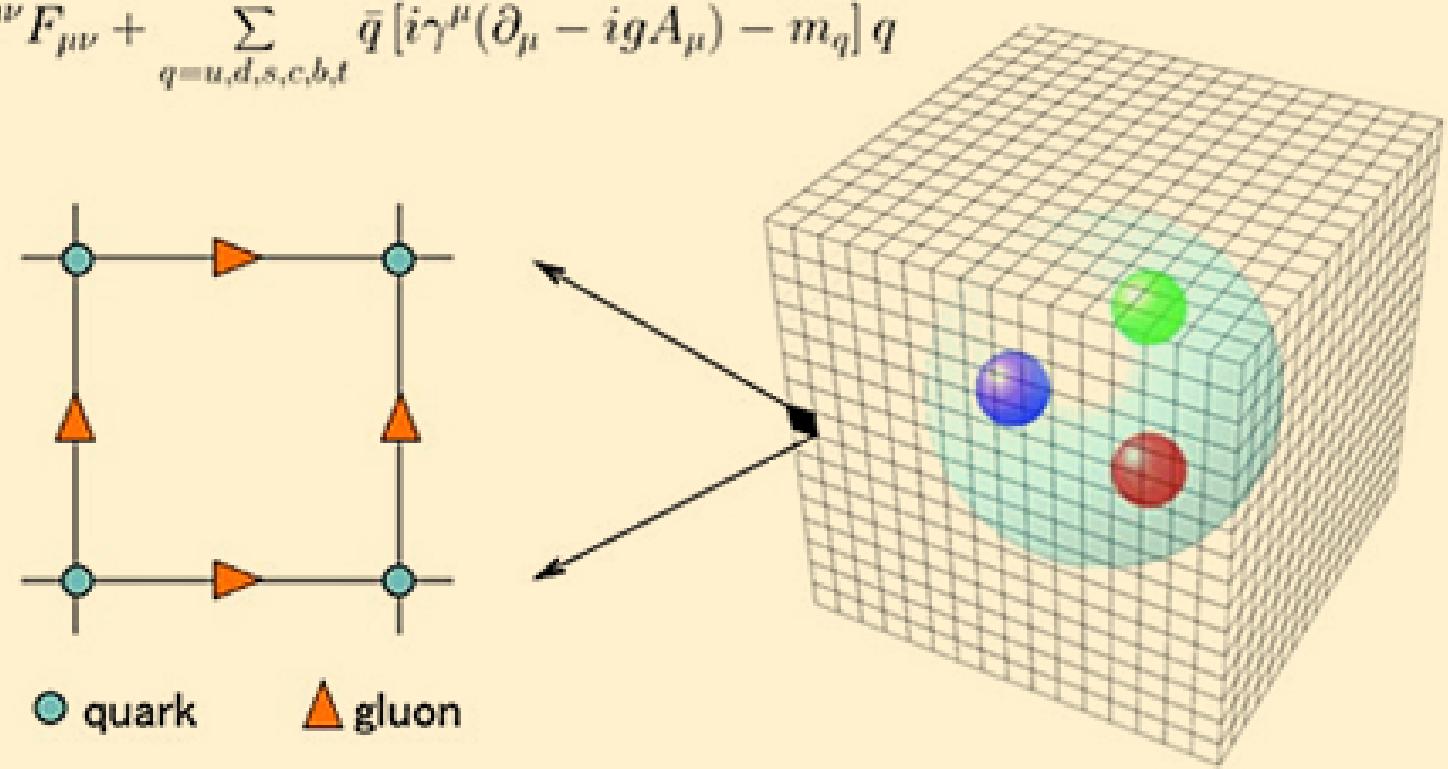
New Perspectives 2020 (2.0)



Euclidean Lattice Field Theory is wildy successful

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$



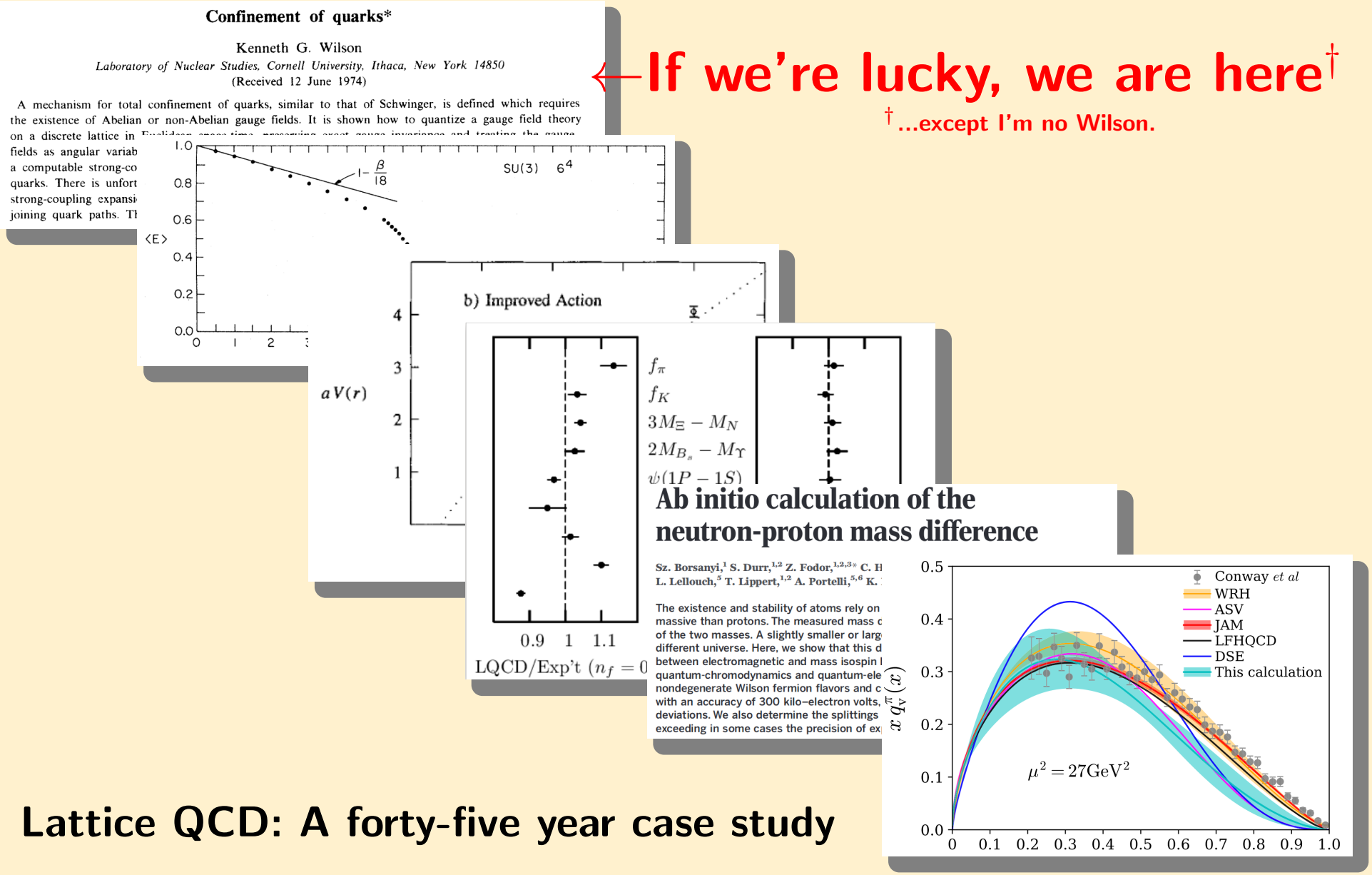
LFT can compute **most** $\langle\psi_i|\prod_n \mathcal{O}_n(\tau_n)|\psi_i\rangle$

Early on, there were competiting methods

Real-t QFT have sign problems[1]

[1] Feynman, R. P. In: *Int.J.Theor.Phys.* 21 (1982).

So ahead of the curve, the curve becomes a sphere



If we're lucky, we are here†

†...except I'm no Wilson.

Lattice QCD: A forty-five year case study

How do I digitize gluons, all things considered?

Lots of choices for bosons:

- Loop-String-Hadrons[2] – Tensor Networks[3] —Quantum Link Models[4] – **Discrete Subgroups**[5]→Matrices with floats[6]

What qualities make a GOOD scheme?

- What is the qubit cost per degree of freedom?
- What is the circuit depth per trotterization step?
- What is the rate of approach to the physical point?
- How easily can they be analyzed?
- **Can the scheme be simulated classically?**

[2] Raychowdhury, I. and J. R. Stryker. In: *Phys. Rev. D* 101 (2020). arXiv: 1912.06133 [hep-lat].

[3] Butt, N., S. Catterall, Y. Meurice, R. Sakai, and J. Unmuth-Yockey. In: *Phys. Rev. D* 101 (2020).

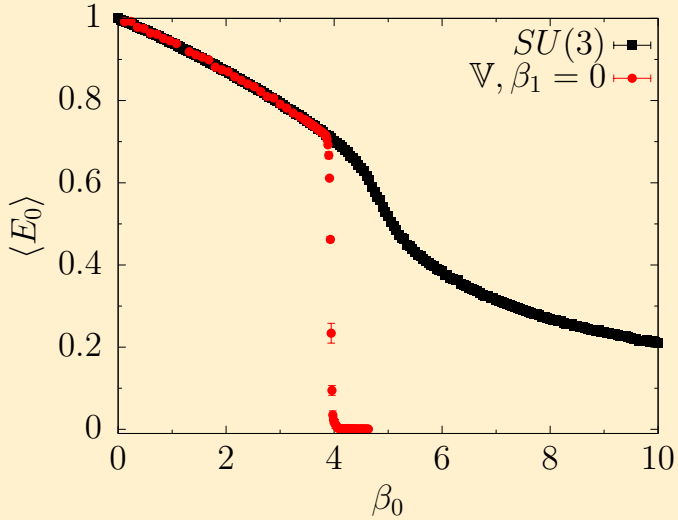
[4] Luo, D. et al. In: (Dec. 2019). arXiv: 1912.11488 [quant-ph].

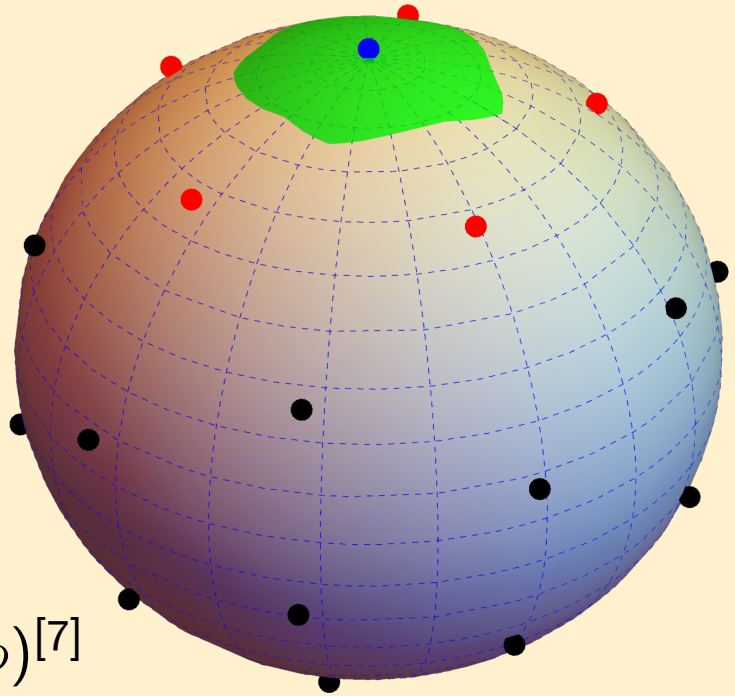
[5] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

[6] Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019). arXiv: 1811.03629 [quant-ph].

Discrete subgroups allow plug-and-play framework[8]

- Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$
- H has $\Delta S > 0$ so 1st PT at $\beta_f < \infty$...





- $\mathcal{L} \approx -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$[7]
- UV difference scales $\propto \left(\frac{\phi}{\Lambda}\right)^n$
- If $\beta_f > \beta_s \implies$ rough approx. in EFT sense

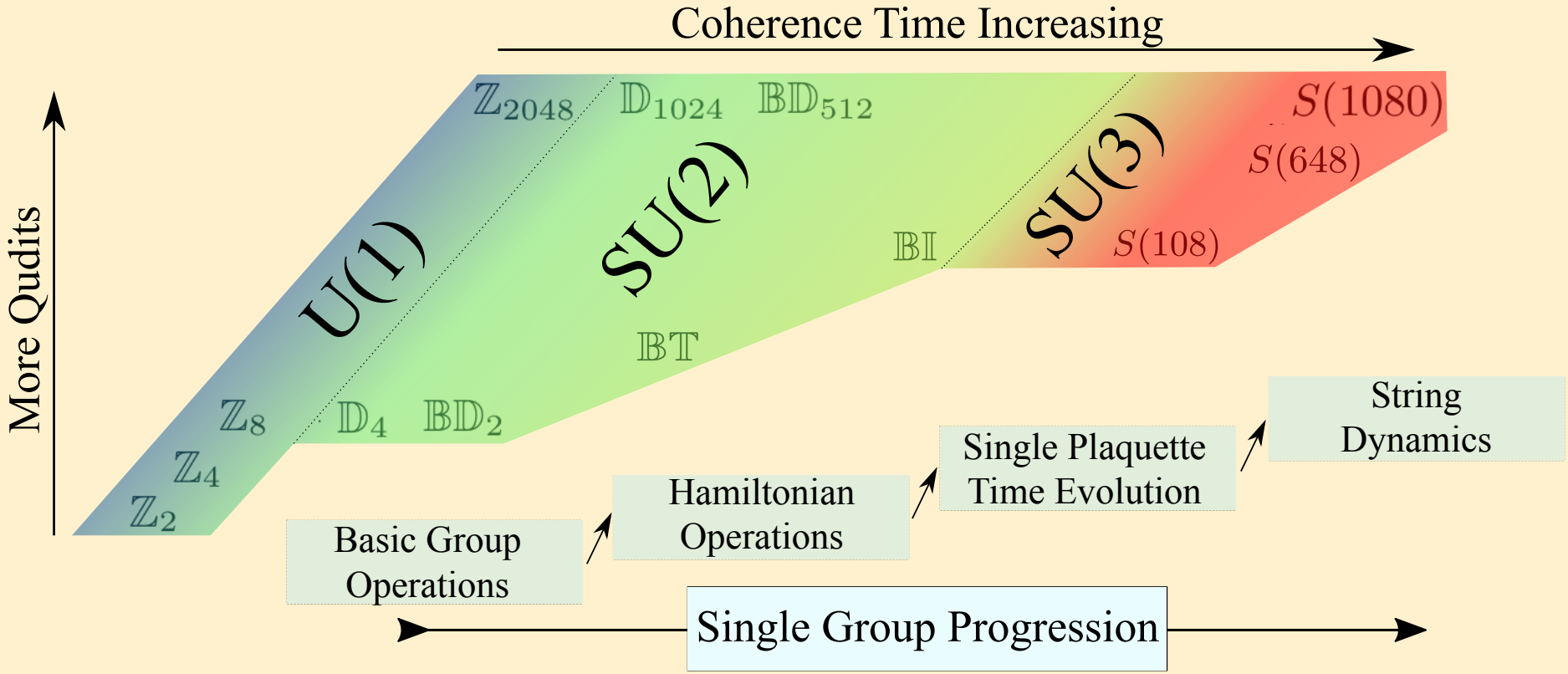
[7] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).

[8] Bhanot, G. and C. Rebbi. In: *Phys. Rev.* D24 (1981), Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019). arXiv: 1811.03629 [quant-ph].

A Ladder from Discrete Subgroups

What do we know from Wilson Action?

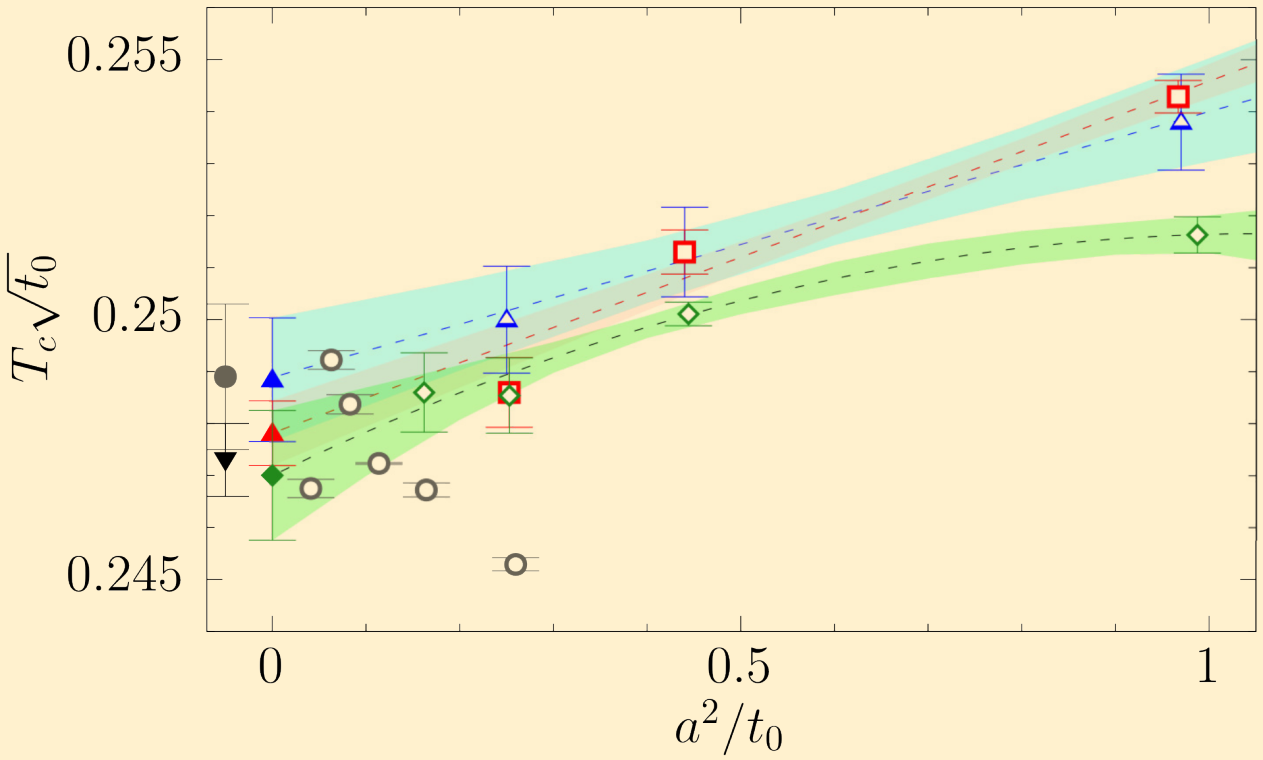
- $U(1) \to \mathbb{Z}_N$, $N > 4$
- $SU(2) \to \mathbb{BO}, \mathbb{BI}$
- $SU(3) \to \mathbb{V}$ has $\beta_f = 3.935(5) < \beta_s \approx 6$
- One **1152** qubit $SU(3)$ link vs $\sim 4^3$ lattice of **11** qubits for $\mathbb{V}$ link



$T_c\sqrt{t_0}$ suggests $a \approx 0.07$ fm ≈ 2 GeV^{-1} possible[10]

$$S = \sum \frac{\beta_0}{3} \mathrm{Re}\,\mathrm{Tr}\, U + \beta_1 f(U) \text{ with } f(U) = \{\mathrm{Tr}^2\, U + \mathrm{Tr}\, U^2, |TrU|^2\}$$

Compare to SU(3)[9]



On-going work to extract quenched spectroscopy

[9] Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev. D* 91 (2015). arXiv: 1503.05652 [hep-lat].

[10] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

Systematics from Group Space Decimation[11]

Decimate via $U = u \cdot \epsilon$ in analogy to Wilsonian renormalization:

$$Z = \int_G DU\, e^{-S[U]} = \sum_{u \in H} \int_\Omega D\epsilon\, e^{-S[u,\epsilon]} = \sum_{u \in H} e^{-S[u]}\,, \tag{1}$$

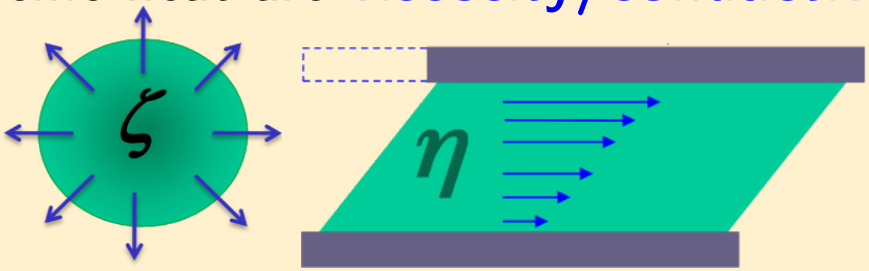
Match moments with cumulants in terms of χ_r with $V_r \propto \langle \chi_r(\epsilon) \rangle$

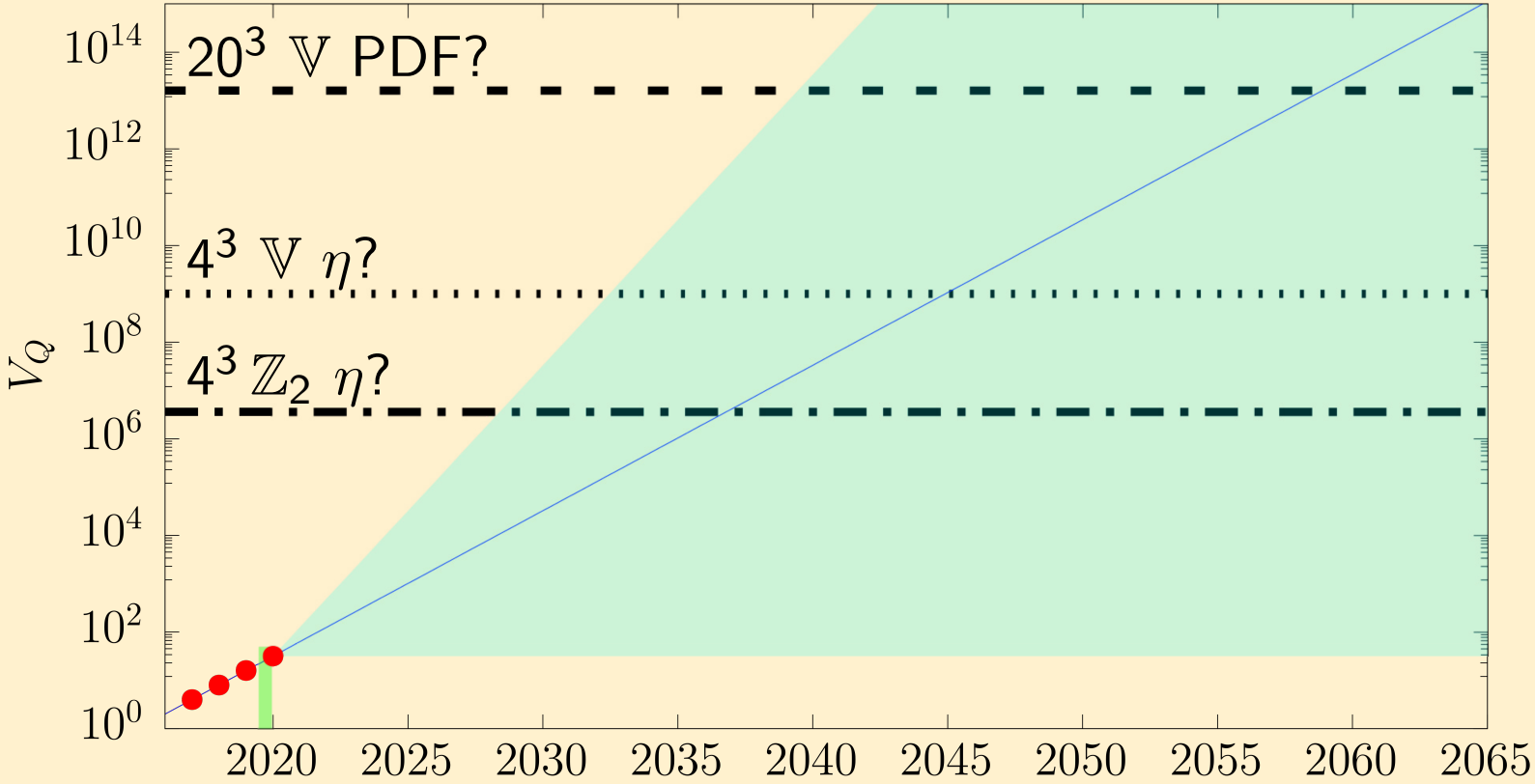
$$\begin{aligned} S[u] = \sum_p & \left(\beta_{\{1\}} + \beta_{\{1,1\}}\right) \frac{1}{3} \operatorname{Re} \chi_{\{1\}} + \left(\beta_{\{0\}} + \beta_{\{1,1,1\}}\right) \\ & + \left(\beta_{\{2\}} + \beta_{\{1,1,-1\}}\right) \frac{1}{6} \operatorname{Re} \chi_{\{2\}} + \left(\beta_{\{1,-1\}} + \beta_{\{2,1\}}\right) \frac{1}{8} \chi_{\{1,-1\}} \\ & + \frac{\beta_{\{3\}}}{10} \operatorname{Re} \chi_{\{3\}} + \frac{\beta_{\{2,-1\}}}{15} \operatorname{Re} \chi_{\{2,-1\}}\,. \end{aligned} \tag{2}$$

[11] Flyvbjerg, H. In: *Nucl. Phys.* B243 (1984), Ji, Y., **H. Lamm**, and S. Zhu. In: (May 2020). arXiv: 2005.14221 [hep-lat].

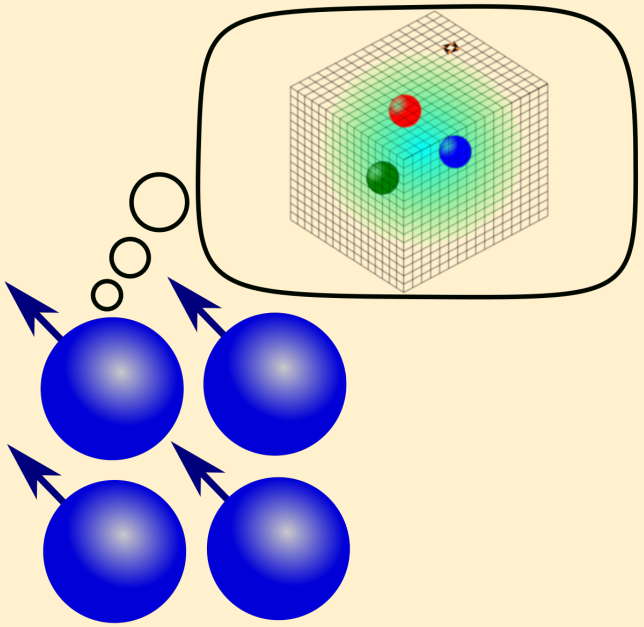
What will it take for practical quantum advantage?

- Find **Bulk properties** with **fast continuum limit** $\Longrightarrow$ smaller V
- Analogs to specific heat are **viscosity/conductivity**





We are at the start of an exciting era!

Many things to do!

- Digitizing SU(3)
  - **Spectroscopy** for $\mathbb{V}$
  - $\mathbb{V}$ **circuits**
- Error Analysis
  - e.g. Finite volume, decimation errors, fidelity to obtain **realisitic** resource estimates
- Investigate desirable properties
  - **Viscosity?**