Parameterization and applications of the low-Q^2 nucleon vector form factors

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Goal of the talk

- We present the proton and neutron vector form factors, uncertainties and correlations in a convenient parametric form that is model independent and optimized for $Q^2 \lesssim$ few GeV^2.
- The form factors are determined from a global fit to electron scattering data and precise charge radii measurements.
- Including high-precision data of A1@MAMI, charged current quasielastic cross sections change by 3-5 %
- Motivation for new measurement of the proton magnetic form factor

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Outline

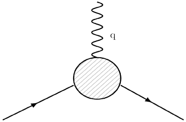
Theory
- Review of Form Factors
- The z Expansion Method

Application
- Neutrino-nucleon scattering
- Influence on Cross Sections
- Atomic Spectroscopy

Conclusion

Nucleon Form Factors in Scattering

- The nucleon electromagnetic current is expressed in terms of *Dirac* (F_1) and *Pauli* (F_2) form factors,



$$\Gamma^{\mu}(q^2) = F_1(q^2)\gamma^{\mu} + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2(q^2)$$

- F_1 and F_2 can be written in terms of *Sachs* electric and magnetic form factors G_E and G_M,

$$F_1 = \frac{G_E + \tau G_M}{1+\tau}, \quad F_2 = \frac{G_E - G_M}{1+\tau}, \quad \tau = -\frac{q^2}{4M^2}$$

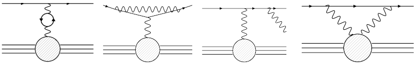
- The scattering cross-section of a relativistic electron off a recoiling point-like nucleus is given by the *Mott* formula

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{Z^2\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\frac{E'}{E}$$

- Structure-dependent part is expressed in terms of Sachs electric and magnetic form factors which is given by the *Rosenbluth* formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon}G_M^2\right\}, \quad \frac{1}{\epsilon} = 1 + 2(2+\tau)\tan^2\frac{\theta}{2}$$

- The form factors are defined from the matrix element of one-photon exchange.

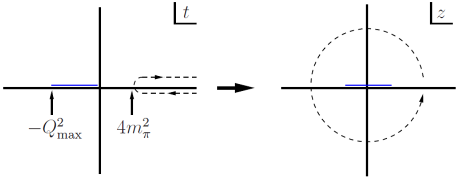


- To extract them with a percent precision or better, standard QED radiative corrections and modern calculations of structure-dependent two-photon exchange are included.

$$d\sigma_{\rm expt} = d\sigma_{\rm Born}(1+\delta_{\rm RC})$$

The Bounded z Expansion and Sum Rule

- According to QCD constraint, nucleon form factors must be analytic in $t \equiv q^2 \equiv -Q^2$ outside of a time-like cut starting at $t_{\text{cut}} = 4m_\pi^2$, the two-pion production threshold ($t_{\text{cut}} = 9m_\pi^2$ for isoscalar combinations). [Hill & Paz (2010)]



$$z(q^2) = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

- A conformal map gives a small expansion variable t_0 in kinematic region of scattering experiments that lies on the negative real axis. It is represented by the blue line for a set of data with maximum momentum transfer Q^2_{max}.

$$G_E = \sum_{k=0}^{k_{\text{max}}} a_k [z(q^2)]^k, \quad G_M = \sum_{k=0}^{k_{\text{max}}} b_k [z(q^2)]^k$$

- Perturbative QCD requires that the form factors fall off faster than $1/Q^3$ in the large Q^2 limit $\longrightarrow$ four sum rules [Lee, Arrington, Hill (2015)]

$$\sum_{k=n}^{k_{\text{max}}} k(k-1)\dots(k-n+1)a_k = 0, \quad n = 0, 1, 2, 3.$$

- We choose $k_{\text{max}} = 8$ and estimate fitting uncertainty as a difference to $k_{\text{max}} \to k_{\text{max}} + 1$. Four parameters (e.g., a_1, a_2, a_3, a_4) are determined by fitting to data.

νN CCQE Cross Section

- Neutrino-nucleon charged-current quasielastic cross section is expressed in terms of form factors as [Llewellyn-Smith (1972)]

$$\frac{d\sigma}{dQ^2}(Q^2, E_\nu) = \frac{G_F^2 |V_{ud}|^2}{8\pi} \frac{M^2}{E_\nu^2} \left[A(q^2) \frac{m_l^2 - q^2}{M^2} - B(q^2) \frac{s-u}{M^2} + C(q^2) \left(\frac{s-u}{M^2} \right)^2 \right]$$

- The parameters A, B and C depend on the nucleon isovector form factors $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$, axial form factor F_A and pseudoscalar form factor F_P

$$\begin{aligned} A = {} & 2\tau (F_1^V + F_2^V)^2 - (1+\tau) \left\{ (F_1^V)^2 + \tau (F_2^V)^2 - (F_A)^2 \right\} \\ & - r_l^2 \left\{ (F_1^V + F_2^V)^2 + (F_A + 2F_P)^2 - 4(1+\tau) F_P^2 \right\} \end{aligned}$$

$$B = 4\tau F_A (F_1^V + F_2^V) \qquad C = \frac{1}{4} \left\{ (F_1^V)^2 + \tau (F_2^V)^2 + (F_A)^2 \right\}$$

  - From PCAC, $F_P = \frac{2M^2}{m_\pi^2 + Q^2} F_A$
  - G_F is Fermi constant
  - V_{ud} is a CKM matrix element
  - $M = \frac{M_p + M_n}{2}$ is the average nucleon mass
  - m_l is the final-state lepton mass
  - E_ν is the incoming neutrino energy
  - $s - u = 4E_\nu M - Q^2 - m_l^2$, $r_l = \frac{m_l}{2M}$

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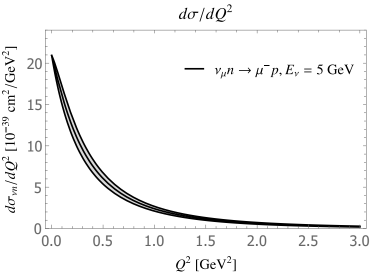
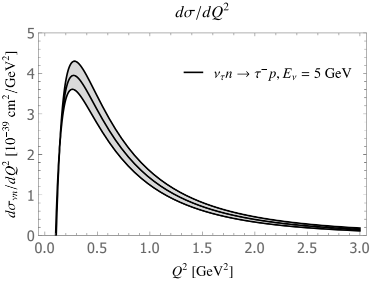
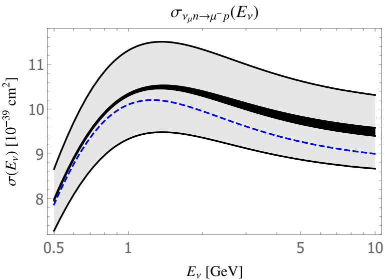
Relevant kinematics for DUNE and HYPER-K

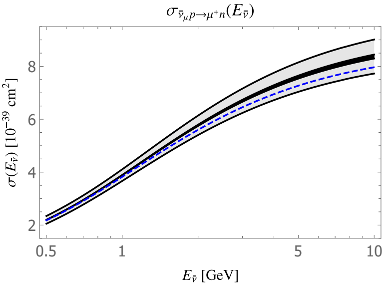
- Electron and muon neutrino cross sections are sensitive to $Q^2 \lesssim 1\ \mathrm{GeV}^2$ while tau neutrino requires larger Q^2.
- Two isospin-decomposed fits with data below $Q^2 < 1\ \mathrm{GeV}^2$ and $Q^2 < 3\ \mathrm{GeV}^2$ are performed.





νN CCQE Cross Section Results



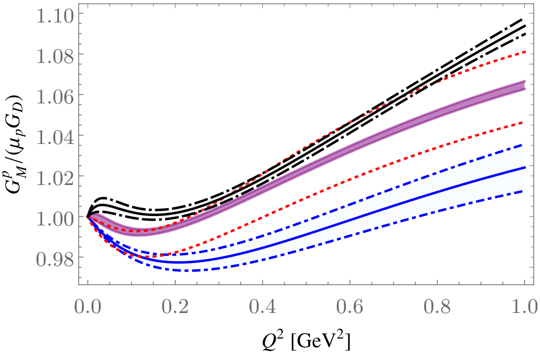


- iso 1: A1@MAMI and neutron data, $Q^2 < 1$ GeV^2; iso 3: all data, $Q^2 < 3$ GeV^2

$E_\nu = 1$ GeV	$\sigma_{\nu_\mu n \to \mu^- p}$ [10^{-39} cm^2]	$\sigma_{\bar{\nu}_\mu p \to \mu^+ n}$ [10^{-39} cm^2]
p, n	$10.312(987)_A(11)_p(22)_n(5)_t$	$3.886(220)_A(5)_p(8)_n(3)_t$
iso 1	$10.319(988)_A(24)_V(6)_t$	$3.887(220)_A(9)_V(3)_t$
iso 3	$10.200(981)_A(20)_V(3)_t$	$3.851(225)_A(7)_V(1)_t$
BBBA	$10.10(98)_A(18)_V$	$3.82(23)_A(8)_V$
Meyer et al. (2016)	10.1(9)	3.83(23)

- CCQE cross section differs by 3–5% compared to commonly-used form factor models (dashed line) when the vector form factors are constrained by recent high-statistics electron–proton scattering data from A1@MAMI.

Proton Magnetic Form Factors including A1@MAMI data



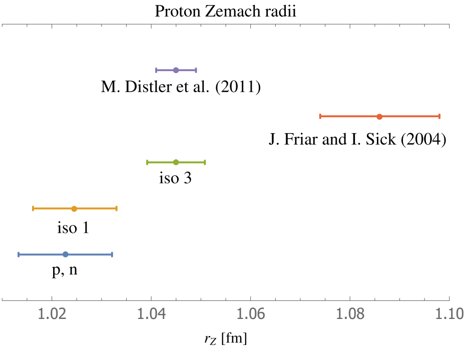
- 1σ bands of G_M^p normalized to dipole form.
  - The black long dash-dotted curves: A1@MAMI data with electric charge radius constraint
  - the purple bands: world data including A1@MAMI
  - the red dotted curves: Ye et al.
  - the blue dash-dotted curve: BBBA2005.
- G_M^p from A1@MAMI is significantly different to previous results.

Hyperfine Splitting and Zemach Radius

- Two-photon exchange provides the leading finite-size correction to hyperfine splitting.
- The dominant piece of two-photon exchange is given by, $\Delta E_Z = -2\alpha m_r E_F r_Z$
- The Zemach radius r_Z is calculated as,

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\frac{G_M(Q^2)G_E(Q^2) - G_M(0)G_E(0)}{G_M(0)}\right]$$

- Proton Zemach radii are compared below,



- The Zemach radius is sensitive to charge and magnetic radii, and both electric and magnetic form factor.

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Conclusion

- Nucleon electromagnetic form factors are a crucial input for precise probes with nucleons and nuclei.
- Form factor fit from relevant kinematical region is presented in a convenient form for applications, i.e., neutrino event generators like GENIE.
- Including data of A1@MAMI Collaboration, CCQE cross sections shift by 3-5 % triggered by proton magnetic form factor.