Some Recent Results on Models with $n - \bar{n}$ Oscillations

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Outline

- Motivations
- General formalism and current experimental limits
- A model in which $n - \bar{n}$ oscillations are dominant manifestation of baryon number violation (BNV): Standard-Model effective field theory analysis
- A corresponding model arising from a left-right symmetric theory
- Some results on $\Delta B = -2$ dinucleon decays
- Conclusions

References

This work has involved collaborations with S. Nussinov, S. Girmohanta; (earlier collab. with S. Rao (1982-84); recent Refs:

1. S. Girmohanta and RS, Phys. Lett. B 803, 135296 (2020) [arXiv:1910.08356].
2. S. Girmohanta and RS, Phys. Rev. D 101, 015017 (2020) [arXiv:1911.05102].
3. S. Girmohanta and RS, Phys. Rev. D 101, 095012 (2020) [arXiv:1911.05102].
4. S. Nussinov and RS, Phys. Rev. D, in press, arXiv:2005.12493.
5. S. Nussinov and RS, Phys. Rev. Lett. 88, 171601 (2002).

Almost 40 years since early discussion of $n - \bar{n}$ oscillations at the first Snowmass workshop, Snowmass-82; recent white papers and reviews:

Snowmass-2013: K. Babu et al., arXiv:1310.8593, 1311.5268,

D. Phillips et al. Phys. Rept. 612, 1 (2016) [arXiv:1410.1100].

A. Addazi et al., arXiv:2006.04907

Motivations

Producing the observed baryon asymmetry in the universe requires interactions that violate baryon number, B (as well as CP violation and deviation from thermal equilibrium) (Sakharov, 1967).

Suggestion of $n - \bar{n}$ transitions as a mechanism involved in generating baryon asymmetry in the universe (Kuzmin, 1970).

Standard Model (SM) conserves B perturbatively. SU(2) instantons produce nonperturbative violation of B and L, while conserving $B - L$ ('t Hooft, 1976), but this is negligible (exponentially small) at temperatures low compared with the electroweak scale (Kuzmin, Rubakov, Shaposhnikov, 1985).

Since (anti)quarks and (anti)leptons are placed in same representations in grand unified theories (GUT's), the violation of B and L is natural in these theories. Besides proton decay, $n - \bar{n}$ oscillations can occur and may be the dominant manifestation of baryon number violation (Glashow, 1980; Mohapatra and Marshak, 1980).

Some other early work: Chang+Chang, Kuo+Love, Cowsik+Nussinov, Rao+RS,...

A continuing question about B is whether it is just a global symmetry or whether it is gauged. In the SM and the SU(5) GUT, B is a global symmetry, while in the left-right symmetric (LRS) theory with gauge group (Mohapatra, Marshak, Senjanović, 1975...)

$$G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$$

$B - L$ is gauged. Electric charge in SM: $Q_{em} = T_{3L} + (Y/2)$; in LRS theory,

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

Further embedding of $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{B-L}$ in $\mathrm{SU}(4)$ (Pati-Salam): gauge group $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, and as $\mathrm{SO}(6) \otimes \mathrm{SO}(4)$ in SO(10) GUT.

Lepton number L is a global symmetry in the original SM. Neutrino masses and lepton mixing are confirmed physics beyond the SM; the most natural mechanism to explain light neutrino masses is the seesaw mechanism, which involves a combination of Dirac mass terms $\bar{\nu}_{iL} M^{(D)}_{ij} \nu_{j,R} + h.c.$ and Majorana mass terms $\nu^T_{i,R} C M^{(R)}_{ij} \nu_{j,R} + h.c.$; the Majorana terms break L, as $\Delta L = 2$ operators.

The occurrence of $\Delta L = 2$ operators, possibly at a low-scale, in neutrino mass models gives further motivation to explore the possibility that there might also be $\Delta B = 2$

operators at scales well below a GUT scale. This is particularly natural in models with a gauged $U(1)_{B-L}$, containing Higgs with $|B-L|=2$, whose vacuum expectation values (VEVs) thus lead to both $|\Delta L|=2$ and $|\Delta B|=2$ processes.

These are good motivations for new experimental searches for $n-\bar{n}$ oscillations and associated $\Delta B=-2$ dinucleon decays as well as proton and bound neutron decay, as manifestations of baryon number violation (BNV).

Plan for $n-\bar{n}$ search exp. at European Spallation Source, ESS, also including search for n conversion via mirror n': $n \to [n', \bar{n}'] \to \bar{n}$ (Adazzi et al., arXiv:2006.04907) and latter search also at High Flux Isotope Reactor, HIFR, at ORNL (e.g. Broussard et al., arXiv:1912.08264; talk by Yuri Kamyshkov); here we focus on $n-\bar{n}$ oscillations.

Continuing searches for $\Delta B=-2$ dinucleon decays at Super-K and in future at Hyper-K and DUNE.

General Formalism

$n - \bar{n}$ Oscillations in Field-Free Vacuum:

CPT: $\langle n|H_{eff}|n\rangle = \langle\bar{n}|H_{eff}|\bar{n}\rangle = m_n - i\lambda_n/2$, where H_{eff} denotes relevant Hamiltonian and $\lambda_n^{-1} = \tau_n = 0.88 \times 10^3$ sec. H_{eff} may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle\bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the matrix in $(n, \bar{n})$ basis:

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates

$$|n_\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |\bar{n}\rangle)$$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$.

So if start with pure $|n\rangle$ state at $t = 0$, then there is a finite probability P for it to be an $|\bar{n}\rangle$ at $t \neq 0$:

$$P(n(t) = \bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t}$$

where $\tau_{n\bar{n}} = 1/|\delta m|$.

General Formalism for $n - \bar{n}$ Oscillations

In the $(n, \bar{n})$ basis, write

$$\mathcal{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

where

$$\tan(2\theta) = \frac{2\delta m}{\Delta M}$$

and $\Delta M = M_{11} - M_{22}$. The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2} \right]$$

Let $\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$; transition probability:

$$P(n(t) \to \bar{n}) = |\langle \bar{n}|n(t)\rangle|^2 = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda_n t}$$

$$= \left[\frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2}\right] \sin^2\left[\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t\right] e^{-\lambda_n t}$$

N.B.: if $\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t << 1$, then by expanding the sin, the quantity $(\Delta M/2)^2 + (\delta m)^2$ cancels, so

$$P(n(t) \to \bar{n}) \simeq [(\delta m)t]^2\, e^{-\lambda_n t} = (t/\tau_{n\bar{n}})^2\, e^{-\lambda_n t}$$

Although $\Delta M = 2\vec{\mu}_n \cdot \vec{B}$, where $\vec{B}$ is a small residual magnetic field in a reactor exp., this inequality enables exp. to be sensitive to δm.

Most sensitive reactor $n - \bar{n}$ exp. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994, obtaining limit $\tau_{n\bar{n}} \geq 0.86 \times 10^8$ sec (90 % CL).

$n - \bar{n}$ Oscillations in Matter:

For $n - \bar{n}$ oscillations involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

with

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with $V_{nR}, \ V_{\bar{n}R}, \ V_{\bar{n}I} \sim O(100)$ MeV (Dover, Gal, Richard; Friedman; recently work by Barrow, Golubeva, Ladd, Paryev, Richard for ^{12}C (ESS) and ^{40}Ar (DUNE)).

Mixing is thus strongly suppressed; $\tan(2\theta)$ is determined by

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} << 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated for by the large number of nucleons in a nucleon decay detector, $\sim 10^{33}$ n's in Super-K.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\left[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2}\;\right]$$

Expanding m_1 for the mostly n mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i\frac{(\delta m)^2\, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability, mainly via $\bar{n}n,\ \bar{n}p \to \pi$'s, with rate

$$\Gamma_{m.i.} = \frac{1}{\tau_{m.i.}} = \frac{2(\delta m)^2|V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_{m.i.} \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$.

Writing $\tau_{m.i.} = R\,\tau_{n\bar{n}}^2$, one has $R \sim O(100)$ MeV, dependent on nucleus.

With $\hbar = 6.6 \times 10^{-22}$ MeV-sec, equiv. $R \sim 10^{23}$ sec^{-1}.

Searches for matter instability due to $n - \bar{n}$ oscillations with large nucleon decay detectors are complementary to searches with free neutrons at reactors or spallation sources.

Current best published bound: $\tau_{m.i.} > 1.9 \times 10^{32}$ yrs, giving

$\tau_{n\bar{n}} > 2.7 \times 10^{8}$ sec from Super-K (2015)

preliminary new limit from Super-K: $\tau_{m.i.} > 3.6 \times 10^{32}$ yrs (talk by L. Wan for Super-K); with same R factor, yields $\tau_{n\bar{n}} \gtrsim 5 \times 10^{8}$ sec.

The future $n - \bar{n}$ search experiment at ESS should significantly improve this limit or observe a signal.

$n - \bar{n}$ Oscillations in an Extra-Dimensional Model

We discuss a model in which proton decay can easily be suppressed well below experimental limits while $n - \bar{n}$ oscillations can occur at level comparable to existing limits. (Nussinov and RS, PRL 88, 171601 (2002) and recent work with S. Girmohanta).

Extra spatial dimensions have been of interest since Kaluza and Klein and received renewed attention with the development of string theory.

Consider a model with a $d = 4 + n$ dimensional spacetime, with n extra spatial dimensions. Denote usual spacetime coords. as x_ν, $\nu = 0, 1, 2, 3$ and consider n extra compact coordinates, y_λ with $0 \le y_\lambda \le L$, i.e., size of extra dimension(s) is L.

Each SM fermion f has the form

$$\Psi_f(x, y) = \psi_f(x)\chi_f(y)$$

with strong localization at a point y_f in the extra dimensions, with a Gaussian profile of half-width $L_\mu \equiv \mu^{-1} << L$:

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2} = Ae^{-\|\eta-\eta_f\|^2}$$

where $\|y_f\| = (\sum_{\lambda=1}^{n} y_{f,\lambda}^2)^{1/2}$, A is a normalization constant, and we define a convenient dimensionless variable $\eta_f = \mu y_f$.

Such models are of interest partly because they can provide a mechanism for obtaining a generational hierarchy in fermion masses and quark mixing.

We use a low-energy effective field theory (EFT) approach with an ultraviolet cutoff M_*, where $M_* > \mu$ for self-consistency. Consider only lowest relevant mode in the Kaluza-Klein (KK) mode decompositions of each Ψ field.

Starting from the Lagrangian in the d-dimensional spacetime, one obtains the resultant low-energy EFT in 4D by integrating over the extra n dimension(s). For canonical normalization of thd 4D fermion kinetic term,

$$A = \left(\frac{2}{\pi}\right)^{n/4} \mu^{n/2}$$

The localization is achieved by coupling to auxiliary "localizer" scalar fields with kink form for $n = 1$, and similarly for higher n (Arkani-Hamed + Schmaltz; Mirabelli+Schmaltz, 2000). Higgs fields are taken flat in extra dims.

Define $\Lambda_L = 1/L$; take $\Lambda \sim 10^2$ TeV, $L_\mu \sim L/30$; this gives adequate separation of fermions while fitting in interval $[0, L]$, consistent with precision electroweak data, collider bounds, flavor-changing neutral current constraints.

With $\Lambda_L = 10^2$ TeV, this yields $\mu \sim 3 \times 10^3$ TeV.

Given the localization of fermion wavefunctions on scale $L_\mu << L$, in the integration over the extra dimensions, can extend $\int_0^L \to \int_{-\infty}^{\infty}$ to good approximation.

Integrals over extra dimensions have the general form (with $\int d^n\eta = \int_{-\infty}^{\infty} d^n\eta$)

$$\int d^n\eta \, \exp\Big[-\sum_{i=1}^{m} a_i\|\eta - \eta_{f_i}\|^2\Big] = \left[\frac{\pi}{\sum_{i=1}^{m} a_i}\right]^{n/2} \exp\left[\frac{-\sum_{j,k=1;\ j<k}^{m} a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^{m} a_s}\right].$$

For example, for $m = 3$,

$$\int d^n\eta \, \exp\Big[-\Big(a_1\|\eta - \eta_{f_1}\|^2 + a_2\|\eta - \eta_{f_2}\|^2 + a_3\|\eta - \eta_{f_3}\|^2\Big)\Big] =$$

$$= \left[\frac{\pi}{a_1 + a_2 + a_3}\right]^{n/2} \exp\left[\frac{-\Big(a_1a_2\|\eta_{f_1} - \eta_{f_2}\|^2 + a_2a_3\|\eta_{f_2} - \eta_{f_3}\|^2 + a_3a_1\|\eta_{f_3} - \eta_{f_1}\|^2\Big)}{a_1 + a_2 + a_3}\right].$$

If only one fermion involved in integrand, then no exponential suppression:

$$\int d^n\eta \, \exp\Big[-a_1\|\eta - \eta_{f_1}\|^2\Big] = \left[\frac{\pi}{a_1}\right]^{n/2}$$

A Yukawa interaction in the d-dimensional space with coefficients of order unity and moderate separation of localized fermion wavefunction centers yields a strong hierarchy in the low-energy 4D Yukawa interaction,

$$\int d^n y\, \bar{\chi}(y_{f_L})\chi(y_{f_R}) \sim \int d^n \eta\, e^{-\|\eta-\eta_{f_L}\|^2} e^{-\|\eta-\eta_{f_R}\|^2} \sim e^{-(1/2)\|\eta_{f_L}-\eta_{f_R}\|^2}$$

Resultant fermion masses m_f:

$$m_f \simeq h^{(f)} \frac{v}{\sqrt{2}} \exp\Big[-\frac{1}{2}\|\eta_{f_L} - \eta_{f_R}\|^2 \Big] ,$$

where $v/\sqrt{2}$ is SM Higgs VEV. With $h^{(f)} \simeq 1$, produce fermion generational hierarchy via different separation distances $\|\eta_{f_L} - \eta_{f_R}\|$ for different generations.

Leading nucleon decay operators are of the form $qqq\ell$. Hence, one can suppress nucleon decay well below experimental limits by arranging that the wavefunction centers of the u and d quarks are separated far from those of the leptons.

Key point: this does not suppress $n - \bar{n}$ oscillations because the $n - \bar{n}$ transition operators do not involve leptons.

For example, one nucleon decay operator is (with $\ell = e,\ \mu$)

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\ T} C d_R^{\beta}][u_R^{\gamma\ T} C \ell_R]$$

where $\alpha,\ \beta,\ \gamma$ are SU(3)$_c$ color indices.

The product of y-dependent fermion wavefunctions in this operator is

$$A^4 \exp\left[-\left\{2\|\eta-\eta_{u_R}\|^2 + \|\eta-\eta_{d_R}\|^2 + \|\eta-\eta_{\ell_R}\|^2\right\}\right]$$

The integral over y yields

$$I_1^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\left\{2\|\eta_{u_R}-\eta_{d_R}\|^2 + 2\|\eta_{u_R}-\eta_{\ell_R}\|^2 + \|\eta_{d_R}-\eta_{\ell_R}\|^2\right\}\right]$$

where $b_4 = (\mu/\sqrt{\pi})^n$.

One can guarantee that this is sufficiently small by taking the distances between wavefunction centers $\|\eta_{u_R}-\eta_{\ell_R}\|$ and/or $\|\eta_{d_R}-\eta_{\ell_R}\|^2$ sufficiently large.

Similarly for other nucleon decay operators.

Analyze $n-\bar{n}$ oscillations: with $H_{eff}^{(n\bar{n})} = \int d^3x \mathcal{H}^{(n\bar{n})}$, $\delta m = \langle \bar{n}|H_{eff}^{(n\bar{n})}|n\rangle$.

In $d=4$ dims., effective Lagrangian

$$\mathcal{L}_{eff}^{(n\bar{n})}(x) = \sum_r c_r^{(n\bar{n})}\,\mathcal{O}_r^{(n\bar{n})}(x) + h.c.\ .$$

Correspondingly, in $d=4+n$ dimensions,

$$\mathcal{L}_{eff,4+n}^{(n\bar{n})}(x,y) = \sum_r \kappa_r^{(n\bar{n})} O_r^{(n\bar{n})}(x,y) + h.c.\ .$$

where the $\mathcal{O}_r^{(n\bar{n})}(x)$ and $O_r^{(n\bar{n})}(x,y)$ are 6-quark operators in $d=4$ and $d=4+n$ dims. Coeffs. $\kappa_r^{(n\bar{n})} = \bar{\kappa}_r^{(n\bar{n})}/M_{n\bar{n}}^{5+2n}$, where $M_{n\bar{n}}$ is an effective mass characterizing the physics responsible for the $n-\bar{n}$ oscillation. Can set $\bar{\kappa}_r^{(n\bar{n})}=1$ for the dominant $O_r^{(n\bar{n})}$ in defining $M_{n\bar{n}}$.

Integration of fermion wavefunctions in the $O_r^{(n\bar{n})}(x,y)$ over y yield the coeffs. $c_r^{(n\bar{n})}$ in terms of $\kappa_r^{(n\bar{n})}$

Operators $\mathcal{O}_r^{(n\bar{n})}$ must be color singlets and, for $M_{n\bar{n}}$ larger than the electroweak symmetry breaking scale, also $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-singlets. Relevant operators in SM EFT:

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

where $Q_L = \binom{u}{d}_L$, $i, j..$ are SU(2)$_L$ indices, and color tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ is symmetric in the indices $(\alpha\beta)$, $(\gamma\delta)$, $(\rho\sigma)$.

$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ is antisymmetric in $(\alpha\beta)$ and $(\gamma\delta)$ and symmetric in $(\rho\sigma)$.

The integrals of these operators over y comprise three classes: operators $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ yield the integral

$$I_{r12}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3} \|\eta_{u_R} - \eta_{d_R}\|^2 \right] ,$$

$O_3^{(n\bar{n})}$ yields the integral

$$I_{r3}^{(n\bar{n})} = b_6 \exp\left[-\frac{1}{6} \Big\{ 2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2 \Big\} \right] .$$

$O_4^{(n\bar{n})}$ yields the integral

$$I_{r4}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3} \|\eta_{Q_L} - \eta_{d_R}\|^2 \right] .$$

where $b_6 = (2 \cdot 3^{-1/2} \pi^{-1} \mu^2)^n$.

The coeffs. $c_r^{(n\bar{n})} = \bar{\kappa}_r^{(n\bar{n})}/(M_{n\bar{n}})^5$ times these $I_r^{(n\bar{n})}$ integrals.

Consider, e.g., case $n = 2$: one can fit data on quark masses, mixing with

$$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75, \quad \|\eta_{Q_L} - \eta_{d_R}\| \simeq 4.60$$

$$\|\eta_{u_R} - \eta_{d_R}\| \simeq 7$$

We find that the $|c_r^{(n\bar{n})}|$ for $r = 1, 2, 3$ are $<<$ $|c_4^{(n\bar{n})}|$, and hence we focus on $c_4^{(n\bar{n})}$:

To leading order (neglecting small CKM mixings), $\|\eta_{Q_L} - \eta_{d_R}\|$ is determined by m_d via relation (with Higgs vev $v = 246$ GeV)

$$m_d = h_d \frac{v}{\sqrt{2}}$$

with

$$h_d = h_{d,0} \exp[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2]$$

where $h_{d,0}$ is the Yukawa coupling in $(4 + n)$-dims. so that

$$\exp\left[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2\right] = \frac{2^{1/2} m_d}{h_{d,0} v}$$

With $h_{d,0} \sim 1$

$$\delta m \simeq c_4^{(n\bar{n})} \langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_{n\bar{n}}^9} \right) \left(\frac{2^{1/2} m_d}{v} \right)^{8/3} \langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle$$

Requiring that $\tau_{n\bar{n}} = 1/|\delta m|$ agree with the published lower limit from Super-K, $\tau_{n\bar{n}} > 2.7 \times 10^8$ sec. yields the lower bound on the mass scale of $n - \bar{n}$ oscillations:

$$M_{n\bar{n}} > (44\,\mathrm{TeV}) \left(\frac{\tau_{n\bar{n}}}{2.7 \times 10^8\ \mathrm{sec}} \right)^{1/9} \left(\frac{\mu}{3 \times 10^3\ \mathrm{TeV}} \right)^{4/9} \left(\frac{|\langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle|}{\Lambda_{QCD}^6} \right)^{1/9} .$$

where $\Lambda_{QCD} = 0.25$ GeV. This bound is not very sensitive to the precise size of $\langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle$ because of the 1/9 power in the exponent.

$\mathcal{O}_4^{(n\bar{n})} = -Q_3$ in notation of lattice QCD calculation (Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, Wasem, 2019), with LQCD matrix element $|\langle \bar{n} | Q_3 | n \rangle| \simeq 5 \times 10^{-4}$ GeV$^6 = 2\Lambda_{QCD}^6$; substituting this yields factor $2^{1/9} = 1.08$ so lower bound is (1.08)44 TeV = 48 TeV.

Previous Super-K lower limit $\tau_{m.i.} > 1.9 \times 10^{32}$ yrs, yielding $\tau_{n\bar{n}} > 2.7 \times 10^8$ sec. New preliminary limit $\tau_{m.i.} > 3.6 \times 10^{32}$ yrs gives ~ 2 increase in lower limit on $\tau_{n\bar{n}}$, yielding a factor of $2^{1/9}$ increase, so LHS becomes 51 TeV.

Hence, for relevant values of $M_{n\bar{n}}$ in this model, $n - \bar{n}$ oscillations could occur at levels that are close to the current limit.

This model also illustrates how baryon number violation can occur via $n - \bar{n}$ oscillations with strongly suppressed proton decay.

Other models can predict $n - \bar{n}$ oscillations near to current limits (Mohapatra+Marshak, 1980; Rao-RS, 1984; Babu+Mohapatra, 2001; Babu, Mohapatra, Nasri, Bhupal Dev 2006-present (talks by Mohapatra, Babu); Arnold, Fornal, Wise (2013); Fileviez Perez... In several of these models, nucleon decay is absent or suppressed so that $n - \bar{n}$ oscillations are main manifestation of BNV.

An interesting question is what are the implications in this model for nucleon and dinucleon decays to various dilepton and trilepton final states with $\Delta L = -3, -2, 1, 2$. See talk by S. Girmohanta.

$n - \bar{n}$ Oscillations in an Extra-Dimensional Model with G_{LRS} Gauge Group

We have also studied $n - \bar{n}$ oscillations in an extra-dimensional model with the gauge group $G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$

in Girmohanta + RS, PRD 101, 095012 (2020) [arXiv:2003.14185].

This model provides a useful contrast to the previous study because in the SM the $n - \bar{n}$ oscillations do not break the SM gauge symmetry, while in the LRS model, they occur via the breaking of the U$(1)_{B-L}$ gauge symmetry.

Recall field content of LRS model (Mohapatra, Pati, Senjanović, 1975...) for fermions (first gen.):

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : \ (3,2,1)_{1/3,L} \ , \quad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R : \ (3,1,2)_{1/3,R}$$

$$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : \ (1,2,1)_{-1,L} \ , \quad L_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R : \ (1,1,2)_{-1,R} \ ,$$

Higgs sector:

$$\Phi : (1,2,2)_0 : \quad \Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \ .$$

$$\Delta_L : (1,3,1)_2, \quad \Delta_R : (1,1,3)_2$$

$$\Delta_{L,R} = \begin{pmatrix} \Delta_{L,R}^+/\sqrt{2} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\Delta_{L,R}^+/\sqrt{2} \end{pmatrix} \ ,$$

Minimization of Higgs potential yields VEVs

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\theta_\Phi} \end{pmatrix} \ ,$$

$$\langle\Delta_L\rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_\Delta} & 0 \end{pmatrix}$$

$$\langle\Delta_R\rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \ .$$

At highest scale, v_R breaks $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \to \mathrm{U}(1)_Y$ with $|\Delta(B-L)| = 2$. This naturally yields $n - \bar{n}$ oscillations and connects them with the Majorana neutrino mass generation. So in this model,

$$M_{n\bar{n}} = v_R$$

At electroweak level, $\kappa,\ \kappa'$ break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$. Take $v_L << \kappa,\ \kappa'$ to preserve $\rho = 1$ where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$.

Fermion fields are localized, while Higgs fields are taken flat in extra dims., as in SM EFT (Arkani-Hamed + Schmaltz; Mirabelli+Schmaltz).

As in the SM EFT, nucleon decay can be suppressed well below experimental limits by separating the wavefunction centers of the quarks from those of the leptons.

Since the adjoint rep. of SU(2) is the rank-2 symmetric tensor, can write Δ_L as $(\Delta_L)^{ij}$ and Δ_R as $(\Delta_R)^{i'j'}$, where i, j are SU(2)$_L$ indices and i', j' are SU(2)$_R$ indices.

$O_r^{(n\bar{n})}$ operators:

$$O_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}\,(\epsilon_{i'k'}\epsilon_{j'm'} + \epsilon_{j'k'}\epsilon_{i'm'})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_2^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{i'j'}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{km}\,(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_5^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{ik}\epsilon_{jm} + \epsilon_{jk}\epsilon_{im})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

After sym. bk. of U(1)$_{B-L}$, replace Δ_R by VEV, v_R.

In the same way as before, we obtain the low-energy 4D EFT by integrating the operator products over the n extra dimensions.

Because $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ involve only one kind of fermion field (namely, Q_R), we find that for these two operators the integral over y does not yield any exponential (Gaussian) suppression factor. Coeffs. $\bar{\kappa}_r^{(n\bar{n})}$ can naturally be $\sim$ O(1) in the model for these operators.

This is in contrast to the SM EFT, where the integrals of all $n-\bar{n}$ operators involved exponential suppression factors.

Because of this, the constraint that this model should agree with the experimental lower limit on $\tau_{n\bar{n}}$ imposes a more stringent lower bound on the scale $M_{n\bar{n}}$ in this model than in the SM EFT analysis:

$$M_{n\bar{n}} > \max\Bigg[(1\times 10^3\ \mathrm{TeV})\Big(\frac{\tau_{n\bar{n}}}{2.7\times 10^8\ \mathrm{sec}}\Big)^{1/9} \times \Big(\frac{\mu}{3\times 10^3\ \mathrm{TeV}}\Big)^{4/9}\Bigg(\frac{|\bar{\kappa}_r^{(n\bar{n})}\ \langle \bar{n}|\mathcal{O}_r^{(n\bar{n})}|n\rangle|}{\Lambda_{QCD}^6}\Bigg)^{1/9}\Bigg], \quad r=1,2$$

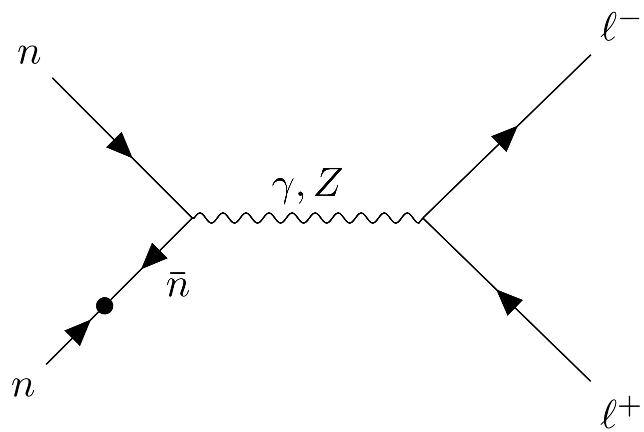
Some Results on Dinucleon Decays

As before, let the physics beyond the SM responsible for $n - \bar{n}$ oscillations be encoded in $H_{eff}^{(n\bar{n})}$.

The presence of a nonzero $\delta m = \langle \bar{n} | H_{eff}^{(n\bar{n})} | n \rangle$ transition matrix element gives rise to some $\bar{n}$ in a nucleus. The $\bar{n}$ annihilates with a neighboring nucleon. The dominant final states are hadronic, consisting of multiple pions, which then propagate out of the nucleus.

The annihilation can also lead to leptonic final states. One can write down Feynman diagrams for these by adding appropriate SM vertices and propagators to the basic $\bar{n}$ annihilation process.

For example, for the $\Delta B = -2$ dinucleon decays $nn \to \ell^+ \ell^-$, where $\ell = e, \ \mu$, one has the Feynman diagrams:



Here the $n - \bar{n}$ transition is indicated, followed by the annihilation to a virtual photon or Z in the s channel. The diagram with the Z is $\sim G_F m_N^2$, and hence the photon contribution is expected to be larger. So one has the estimate

$$A_{nn\to\ell^+\ell^-} \simeq (\delta m)\, e^2 \, \langle 0|J^{\lambda}_{em}|n\bar{n}\rangle \, \frac{1}{q^2} \, [\bar{u}(p_2)\gamma_\lambda v(p_1)] \ ,$$

The momenta q, p_1, and p_2 are of hadronic scales. The small $n - \bar{n}$ factor that produces the $\bar{n}n$ state and is common to hadronic and leptonic decays.

no phase space suppression for $nn \to 2\pi^0$ or $nn \to \ell^+\ell^-$, $\ell = e,\ \mu$.

Hence, roughly, $\Gamma_{nn \to \ell^+\ell^-} \sim e^4\,\Gamma_{nn \to 2\pi^0}$.

Using lower bounds on partial lifetimes $\tau/B = \Gamma^{-1}$ for this and other hadronic dinucleon decay modes, one can then derive rough lower bounds on partial lifetimes for these dinucleon-to-dilepton decays, and similarly for other leptonic final states.

We present estimates for these lower bounds in Girmohanta + RS, PLB 803, 135296 (2020) [arXiv:1910.08356] (see talk by Girmohanta); e.g., using Super-K limit $\Gamma^{-1}_{nn \to 2\pi^0} > 4.04 \times 10^{32}$ yr,

$$\Gamma^{-1}_{nn \to \ell^+\ell^-} \gtrsim e^{-4}\Gamma^{-1}_{nn \to 2\pi^0} \gtrsim 5 \times 10^{34} \text{ yr for } \ell = e,\ \mu$$

Strengthened bounds on $nn \to e^+e^-$ and $nn \to \mu^+\mu^-$ in in Nussinov + RS, PRD, in press [arXiv:2005.12493].

Direct experimental limits are not this strong, e.g., $\Gamma^{-1}_{nn \to e^+e^-} > 4.2 \times 10^{33}$ yrs, $\Gamma^{-1}_{nn \to \mu^+\mu^-} > 4.4 \times 10^{33}$ yrs (Super-K, arXiv:1811.12430).

Conclusions

- General theoretical expectation that baryon number is violated, and this is borne out in many BSM scenarios.

- $n - \bar{n}$ oscillations are an interesting possible manifestation of baryon number violation, of $|\Delta B| = 2$ type, complementary to proton decay. A discovery of $n - \bar{n}$ oscillations would be of profound significance.

- We have discussed two models that show how new physics beyond the SM can produce $n - \bar{n}$ oscillations at rates comparable with current limits. These models also show that $n - \bar{n}$ oscillations can be the main manifestation of baryon number violation, since proton decay is strongly suppressed.

- These results provide motivation for new experiments to search for $n - \bar{n}$ oscillations, including exp. at ESS - input to Snowmass 2021

- Further results on dinucleon decays

further questions/comments: can email to robert.shrock@stonybrook.edu as well as Slack.

Extra Slides

$n-\bar{n}$ Oscillations in a Magnetic Field $\vec{B}$:

Relevant to Institut Laue-Langevin (ILL) and planned ESS experiments searching for $n-\bar{n}$ oscillations

n and $\bar{n}$ interact with $\vec{B}$ via magnetic moment $\vec{\mu}=\mu\,\vec{\sigma}$, $\mu_n=-\mu_{\bar{n}}=\kappa\mu_N$, where $\kappa=-1.91$, $\mu_N=e/(2m_N)=3.15\times10^{-14}$ MeV/Tesla, so

$$\mathcal{M}=\begin{pmatrix} m_n-\vec{\mu}_n\cdot\vec{B}-i\lambda_n/2 & \delta m \\ \delta m & m_n+\vec{\mu}_n\cdot\vec{B}-i\lambda_n/2 \end{pmatrix}$$

So $\Delta M = M_{11}-M_{22}=-2\vec{\mu}_n\cdot\vec{B}$ and diagonalization yields mass eigenstates $|n_1\rangle$, $|n_2\rangle$, with mixing

$$\tan(2\theta)=-\frac{\delta m}{\vec{\mu}_n\cdot\vec{B}}$$

and energy eigenvalues

$$E_{1,2}=m_n\pm\sqrt{(\vec{\mu}_n\cdot\vec{B})^2+(\delta m)^2}\;-i\lambda_n/2$$

ILL experiment reduced $|\vec{B}| = B$ to $B \sim 10^{-4}$ G $= 10^{-8}$ T, so

$$|\mu_n|B = (6.03 \times 10^{-22}\ \text{MeV}) \left(\frac{B}{10^{-8}\ \text{T}}\right)$$

Since $|\delta m| \lesssim 10^{-29}$ MeV $<< |\mu_n|B$ from exp., it follows that $|\theta| \lesssim 10^{-8} << 1$ and

$$\Delta E = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$$

In a reactor $n - \bar{n}$ experiment, arrange that n's propagate a time t such that

$$|\vec{\mu}_n \cdot \vec{B}|t = 0.92 \left(\frac{B}{10^{-8}\ \text{T}}\right)\left(\frac{t}{1\ \text{sec}}\right) << 1 \quad \text{and } t << \tau_n$$

Then

$$P(n(t) \to \bar{n}) \simeq (2\theta)^2 \left(\frac{\Delta E t}{2}\right)^2 \simeq \left(\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}}\right)^2 \left(\vec{\mu}_n \cdot \vec{B}\, t\right)^2 = [(\delta m)\, t]^2 = (t/\tau_{n\bar{n}})^2$$

and the experiment is sensitive to δm.

Strengthened Lower Bounds on partial lifetimes $\tau/B = \Gamma^{-1}$ for Dinucleon Decays to Dileptons $nn \to e^+e^-$ and $nn \to \mu^+\mu^-$ (Nussinov and RS, Phys. Ref. D, in press (arXiv:2005.12493])

We use e^+e^- and $\bar{p}p$ annihilation data to improve the lower bounds on these partial lifetimes for the dinucleon decays to dileptons

$$\frac{\Gamma_{\bar{n}n\to\ell^+\ell^-}}{\Gamma_{\bar{n}n\to 2\pi^0}} = \left[\frac{\frac{\Gamma_{\bar{n}n\to\ell^+\ell^-}}{\Gamma_{\bar{p}p\to\ell^+\ell^-}}}{\frac{\Gamma_{\bar{n}n\to 2\pi^0}}{\Gamma_{\bar{p}p\to\ 2\pi^0}}}\right] \frac{\Gamma_{\bar{p}p\to\ell^+\ell^-}}{\Gamma_{\bar{p}p\to 2\pi^0}} = \left[\frac{\frac{\Gamma_{\bar{n}n\to\ell^+\ell^-}}{\Gamma_{\bar{p}p\to\ell^+\ell^-}}}{\frac{\Gamma_{\bar{n}n\to 2\pi^0}}{\Gamma_{\bar{p}p\to\ 2\pi^0}}}\right] \frac{BR(\bar{p}p \to \ell^+\ell^-)}{BR(\bar{p}p \to 2\pi^0)} \ .$$

From isospin invariance of strong ints.

$$\frac{\Gamma_{\bar{n}n\to 2\pi^0}}{\Gamma_{\bar{p}p\to 2\pi^0}} = 1 \ ,$$

Exp. measurements (most recently, from BABAR and VEPP/Novosibirsk) in the region of $\sqrt{s}$ slightly above threshold give

$$\sigma(e^+e^- \to \bar{p}p) \simeq 0.9 \pm 0.1 \ \text{nb.}$$

$$\sigma(e^+e^- \to \bar{n}n) \simeq 0.85 \pm 0.20 \ \text{nb}$$

for this interval in $\sqrt{s}$, using time reversal invariance of EM interactions,

$$\frac{\Gamma_{\bar{n}n\to e^+e^-}}{\Gamma_{\bar{p}p\to e^+e^-}} \simeq \frac{\sigma_{e^+e^-\to\bar{n}n}}{\sigma_{e^+e^-\to\bar{p}p}} \simeq \ 0.9$$

From exps. at LEAR (CERN), $BR(\bar{p}p \to e^+e^-) = (3.58 \pm 0.10) \times 10^{-7}$.
Super-K event simulation:

$$BR(\bar{p}p \to 2\pi^0)_{^{16}\mathrm{O}} = BR(\bar{n}n \to 2\pi^0)_{^{16}\mathrm{O}} = 1.5 \times 10^{-2}$$

Combining these inputs,

$$\frac{\Gamma_{\bar{n}n\to\ell^+\ell^-}}{\Gamma_{\bar{n}n\to 2\pi^0}} \simeq \frac{(0.9)(3.58 \times 10^{-7}\ \mathrm{yrs})}{1.5 \times 10^{-2}} = 2 \times 10^{-5}$$

Super-K lower bound:

$$\Gamma^{-1}_{nn\to 2\pi^0} > 4.04 \times 10^{32}\ \mathrm{yrs}$$

Combining these,

$$\Gamma^{-1}_{nn\to\ell^+\ell^-} \gtrsim 2 \times 10^{37}\ \mathrm{yrs} \quad \mathrm{for}\ \ell = e,\ \mu$$