Exciting New Possibilities for Baryon Number Violation

Sudhakantha Girmohanta

C.N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, Stony Brook University

Theoretical Innovations for Future Experiments Regarding Baryon Number Violation by Two Units I, Aug 3-6, 2020

Overview

- Motivations for baryon-number-violation (BNV).

- BNV in a model with large extra dimensions: Standard model effective field theory analysis. **[S. Nussinov and R. Shrock, Phys. Rev. Lett. 88, 171601 (2002) ; S. Girmohanta and R. Shrock, Phys. Rev. D 101, 015017 (2020) (arXiv: 1911.05102)]**

- Nucleon decay and $n - \bar{n}$ oscillations in a left-right symmetric model with large extra dimensions. **[S. Girmohanta and R. Shrock, Phys. Rev. D 101, 095012 (2020) (arXiv: 2003.14185); S. Girmohanta (arXiv: 2005.12952)]**

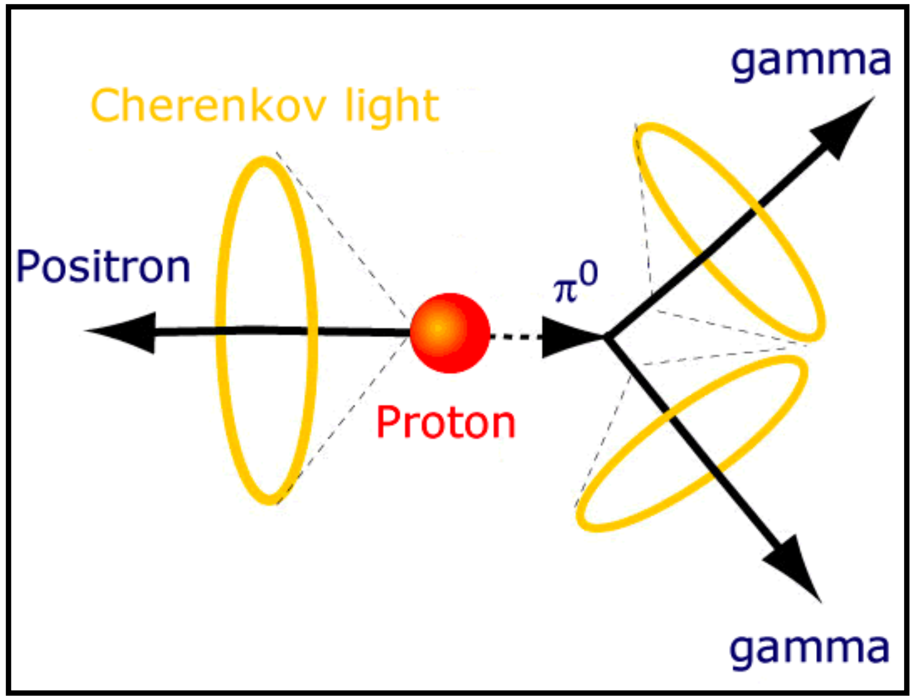
- Conclusion: $n - \bar{n}$ oscillations might be the main manifestation of BNV, which may occur at a rate comparable to current experimental limits.

Some recent reviews:

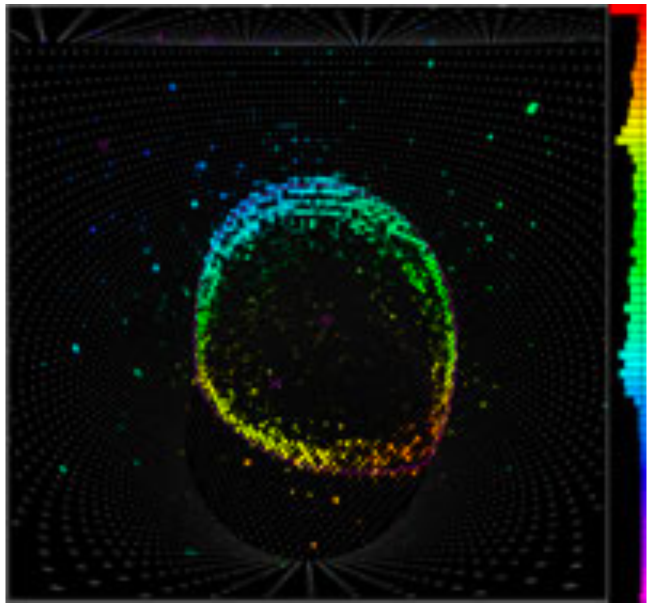
- **D. Phillips et al. Phys. Rept. 612, 1 (2016) [arXiv:1410.1100]**
- **A. Addazi et al., [arXiv:2006.04907]**

Why do we expect Baryon-Number-Violation (BNV) ?

- Baryon number (B) is an accidental global symmetry of the Standard model (SM)*, and is violated in many ultra-violet (UV) extensions of it (e.g. Grand unified theories (GUTs)).
- BNV is one of the necessary conditions for explaining the observed baryon asymmetry of the universe. [**Sakharov (1967)**]
- A number of dedicated experiments (e.g. SK, IMB, KamLAND, Fréjus and others) have searched for nucleon decay and obtained null results setting stringent upper limits for the rates of nucleon decays.



$p \to e^+\pi^0$ mode in a water Cherenkov detector like Super-Kamiokande (SK)



A μ-like event in SK detector (non-showering ring)

* violation of B by $SU(2)_L$ instantons in the SM is negligibly small at temperatures low compared with the electro-weak scale ['t Hooft ('76)]

$n - \bar{n}$ Oscillations

- $n - \bar{n}$ oscillations have $|\Delta B| = 2$ (and $|\Delta(B - L)| = 2$).

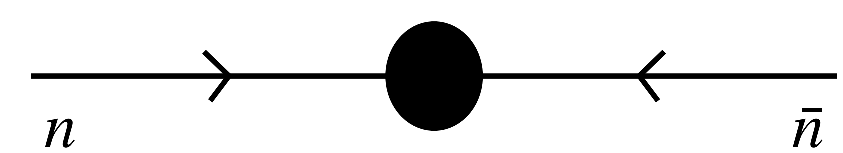
- This might provide the source of BNV for baryogenesis **[Kuzmin ('70)].**

- The 6-quark operators in the effective Hamiltonian $\mathscr{H}_{eff}^{(n\bar{n})}$ have coefficients with mass dimension -5, in contrast to the single-nucleon decay operators having coefficients with mass dimension -2.

- Naively one expects $n - \bar{n}$ oscillations to be highly suppressed when compared with single nucleon decay.

- Assumption of a single mass scale responsible for BNV (M_{BNV}) might be oversimplified and new scale(s) may suppress nucleon decay while mediating $n - \bar{n}$ at a rate comparable to current experimental limits. **[e.g. Mohapatra and Marshak PRL 44, 1316 (1980) ; Nussinov and Shrock PRL 88 171601 (2002) ; Rao and Shrock (1982, 1984)]**

$$\delta m = \langle \bar{n} | \mathscr{H}_{eff}^{(n\bar{n})} | n \rangle$$



Transition amplitude

↓ Matter-instability (m.i.)

$nn, np \rightarrow$ **multi-pions**

Experimental searches for $n - \bar{n}$ oscillations

Free neutron beam

Decay rate of free neutron

In $|n\rangle, |\bar{n}\rangle$ basis:

$$\mathscr{H}^{(n\bar{n})}_{eff} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Probability for $|n\rangle$ state to turn into $|\bar{n}\rangle$

$$P(n(t) = \bar{n}) = [\sin^2(\delta m\, t)]e^{-\lambda_n t}$$

$|\delta m| < 0.77 \times 10^{-29}$ MeV **(ILL, Grenoble '94)**

Bound neutron in nucleus

Imaginary part reflects the annihilation probability of $|\bar{n}\rangle$

$$\mathscr{H}^{(n\bar{n})}_{eff} = \begin{pmatrix} m_n + V_{nR} & \delta m \\ \delta m & m_n + V_{\bar{n}R} - iV_{\bar{n}I} \end{pmatrix}$$

The physical neutron state:

$$|n\rangle_{\text{phys.}} = \cos\theta_{n\bar{n}}|n\rangle + \sin\theta_{n\bar{n}}|\bar{n}\rangle$$

$$\tan(2\theta_{n\bar{n}}) = 2\delta m \Big/ |(V_{nR} - V_{\bar{n}R}) + iV_{\bar{n}I}|$$

From matter instability search inferred bound on δm:

$|\delta m| < 2.4 \times 10^{-30}$ MeV **(SK, 2015)**

$$\tau_{\text{m.i}} = \frac{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}{2(\delta m)^2|V_{\bar{n}I}|} > 3.6 \times 10^{32}\ \text{yr}\,.$$ **(talk by L. Wan: SK, preliminary)**

Nucl. effects: Gal, Richard, Barrow…

Baryon-Number-Violating Nucleon and Dinucleon Decays in a Model with Large Extra Dimensions

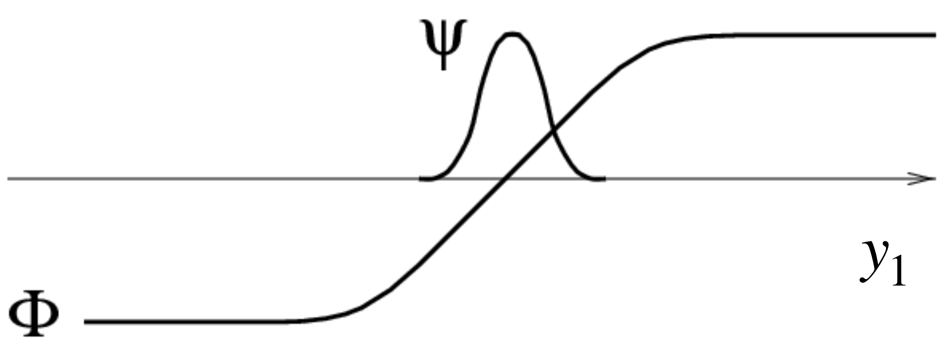
[Phys. Rev. D 101, 015017 (2020)] [arXiv: 1911.05102]

Large extra dimensional framework

[N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000)]

[Arkani-Hamed, Dimopoulos, Dvali PLB 429, 263 (1998) ; Antoniadis, Arkani-Hamed, Dimopoulos, Dvali PLB 436, 257 (1998) ; Dienes, Dudas, Gherghetta PLB 436, 55 (1998); Nussinov, Shrock PRD 59 105002 (1999)]

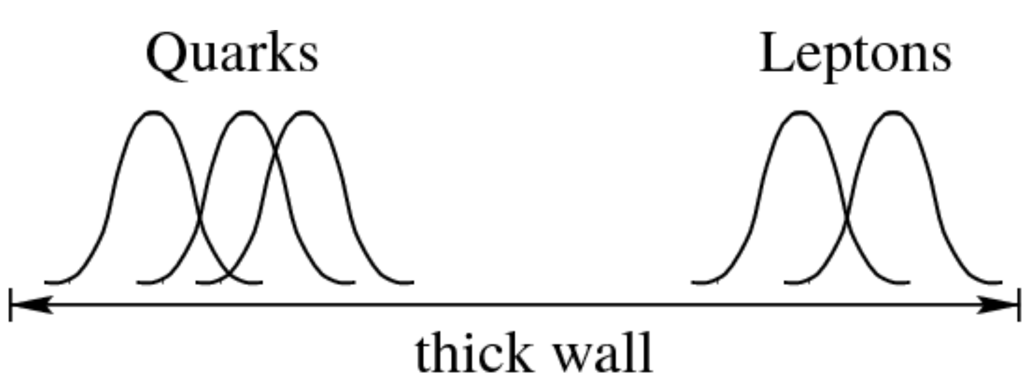
- SM fields can propagate in n extra dimensions (y_λ) and SM fermion wavefunctions have strong localization at various points in the extra-dimensional space (Gaussian profile).



Coupling fermions (ψ) to scalars (Φ) having kink solution (for $n = 1$) or vortex solution (for $n = 2$) localizes the zero-mode fermion solutions in the extra dimensions (y_λ)

- Can explain 4-dimensional Yukawa hierarchies in the long-distance effective theory from the “geography” of fermion wavefunction localization in the extra dimensions.

- Proton decay rate can be exponentially suppressed to safety by separating quark and lepton wavefunctions in the extra dimensions.



Theoretical Framework

Fermion wave function (Ψ) is taken to be in a factorized form :

$$\Psi_f(x,y) = \psi_f(x)\chi_f(y)$$

$\chi_f(y) \rightarrow A\, e^{-\mu^2 \|y-y_f\|^2}$

Gaussian profile

$$A = \left(\frac{2}{\pi}\right)^{n/4} \mu^{n/2}$$ Normalization

$$y_f = ((y_f)_1, \ldots, (y_f)_n)$$

$$\|y_f\| \equiv \left(\sum_{\lambda=1}^{n} y_{f,\lambda}^2\right)^{1/2} .$$

Fermions are restricted to the interval $0 \leq y_\lambda \leq L$ in the extra dimensions

$$L^{-1} \equiv \Lambda_L \gtrsim 100\,\mathrm{TeV}$$

- Adequate suppression of Flavor Changing Neutral Currents
- Consistent with precision electroweak constraints
- Consistent with collider searches

$$\xi \equiv \mu L \sim 30$$

- Choice to make sufficient separation of the fermion wave functions while still fitting within the wall of thickness L .
- $\mu \sim 3 \times 10^3\,\mathrm{TeV}$

- we use Wilsonian low-energy effective field theory, integrating over the short-distance physics associated with the extra dimensions to obtain the 4D effective Lagrangian.

$$f = f(x)\, A\, e^{-\|\eta-\eta_f\|^2};\ [\eta = \mu y]$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$: SM Higgs field

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$: Vacuum expectation value ($v = 246\,\mathrm{GeV}$)

Neglect off-diagonal Yukawa mixing

Yukawa interaction in higher dimension ($d = 4 + n$):

$$\mathscr{L}_{Yuk} = h^{(f)} \bar{f}_L \phi f_R$$

The coupling in the 4-dimensional long-distance theory:

$$= A^2 h^{(f)} \frac{v}{\sqrt{2}} \int d^n y\, e^{-\|\eta-\eta_{f_L}\|^2 - \|\eta-\eta_{f_R}\|^2}$$

Mass hierarchy from "geography"

$$m_f = h^{(f)} \frac{v}{\sqrt{2}} \exp\left[-\frac{1}{2} \|\eta_{f_L} - \eta_{f_R}\|^2 \right]$$

$h^{(f)} \sim O(1)$

In the higher-dimensional theory

Exponential suppression from the wavefunction separation

Assumption of a single M_{BNV} scale may be naive

BNV operators O having $k-$fold fermion fields in $d = 4 + n$ dimensions:

$$\mathscr{L}_{eff,4+n}(x, y) = \sum_r \kappa_{r,(k)} O_{r,(k)}(x, y) + h.c.$$

In 4-dimensions

$$\mathscr{L}_{eff}(x) = \sum_r c_{r,(k)} \mathscr{O}_{r,(k)}(x) + h.c.$$

From dimensional Analysis:

$$c_{r,(k)} = [\kappa_{r,(k)}]\,(I_{r,(k)}) = \left[\frac{\bar{\kappa}_{r,(k)}}{M_{BNV}^{(k(3+n)/2-4-n)}}\right](b_k\, e^{-S_{r,(k)}})$$

$$b_k = \left[2^{k/4}\pi^{-(k-2)/4}k^{-1/2}\mu^{(k-2)/2}\right]^n$$

Λ_{QCD} : QCD mass scale (~ 0.25 GeV)

$\langle S_{(k)} \rangle$: Typical size of wavefunction separation

The ratio of two BNV decay rates comprising of k_1 and k_2 fold fermion operators resp.

$$\frac{\Gamma_{(k_2)}}{\Gamma_{(k_1)}} \sim \left(\frac{\Lambda_{QCD}}{M_{BNV}}\right)^{3(k_2-k_1)} \left(\frac{\mu}{M_{BNV}}\right)^{(k_2-k_1)n} \left(\frac{2^{(k_2-k_1)/2}k_1}{\pi^{(k_2-k_1)/2}k_2}\right)^n e^{-2(\langle S_{(k_2)}\rangle - \langle S_{(k_1)}\rangle)}$$

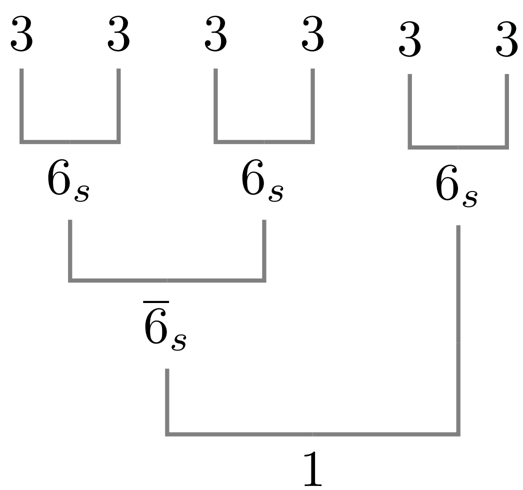
Interplay of different scales ($\mu, M_{BNV}, \Lambda_{QCD}, S_{(k)}$) determine the decay rate.

Can processes mediated by $\mathscr{O}_{r,(k_2)}$ be less suppressed than $\mathscr{O}_{r,(k_1)}$ even if $k_2 > k_1$?

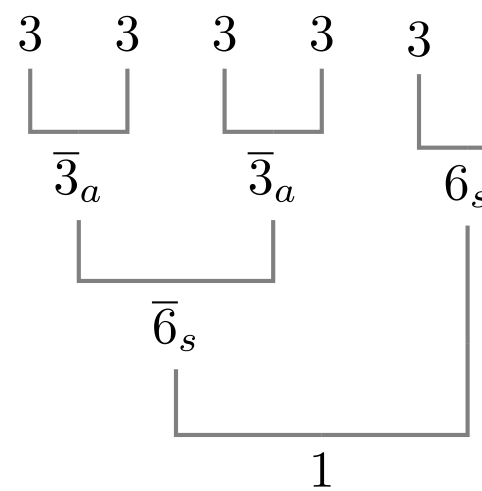
Some Formulas

BNV scale > $v(250\,\mathrm{GeV}) \Longrightarrow$ Effective operators are $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ invariant .

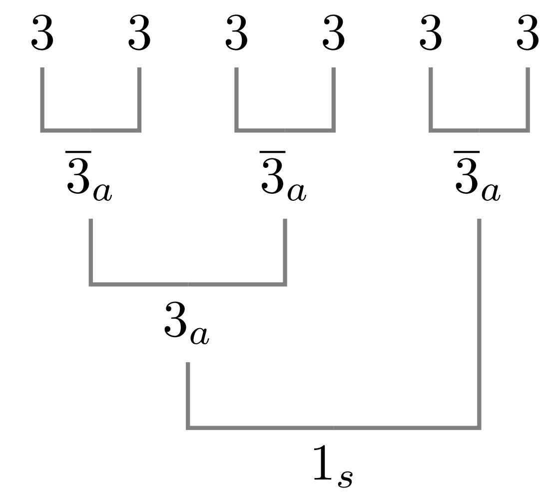
$SU(3)_c$ contraction tensors (for 6 fundamental reps.)



$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$



$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

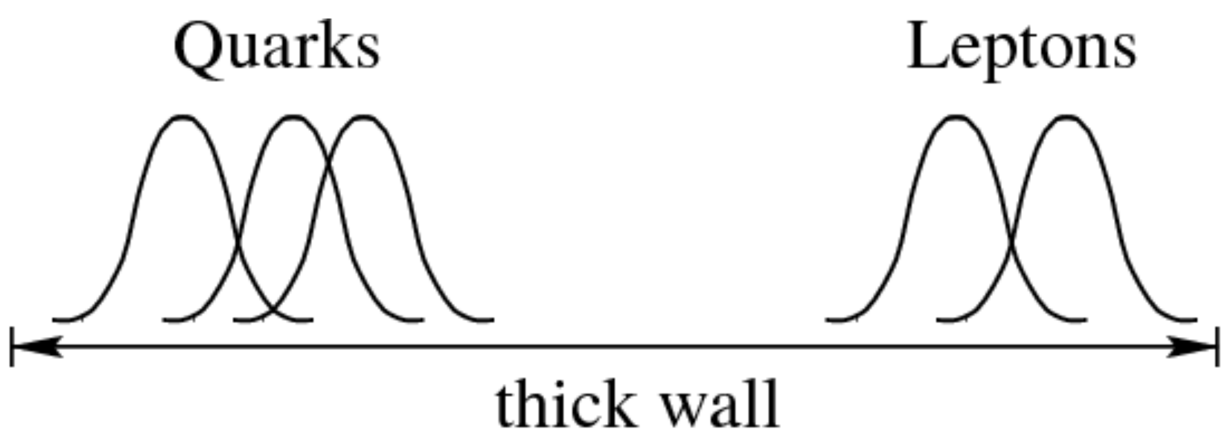


$$(T_{a3})_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} - \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$$\int d^n\eta \, \exp\left[-\sum_{i=1}^{m} a_i \|\eta - \eta_{f_i}\|^2\right] = \left[\frac{\pi}{\sum_{i=1}^{m} a_i}\right]^{n/2} \exp\left[\frac{-\sum_{j,k=1;\; j<k}^{m} a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^{m} a_s}\right] \qquad (A)$$

Given the field content in an operator, using this formula we can write down the exponential suppression. We call these operators giving rise to different exponential factors as included in different **classes**.

Proton Decay vs $n - \bar{n}$ Oscillations in Large Extra Dimensions



Proton decay can be easily suppressed by separating quarks and leptons in the extra dimensions

For example consider one operator mediating proton decay:

$$\mathcal{O}_4^{(Nd)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_{a_1,L}^{i\alpha\ T} C Q_{a_2,L}^{j\beta}][Q_{a_3,L}^{k\gamma\ T} C L_{a_4,L}^{m}]$$

$$\text{Suppression} \propto \exp\left[-\frac{3}{4}\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2\right]$$

From experimental bound $(\tau/B)_{p\to e^+\pi^0} > 1.6 \times 10^{34}$ yrs and $(\tau/B)_{p\to\mu^+\pi^0} > 0.77 \times 10^{34}$ yrs [**SK**]

$$\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 > 62 - \frac{8}{3}\ln\left(\frac{M_{BNV}}{100\ \text{TeV}}\right) - \frac{8}{3}\ln\left(\frac{M_{BNV}}{\mu}\right)$$

$(n = 2)$
Extra dimensions

For $M_{BNV} = 100\,\text{TeV}$ and $\mu = 3 \times 10^3\,\text{TeV}$, $\|\eta_{Q_L} - \eta_{L_{\ell,L}}\| > 8.4$

$n - \bar{n}$ Oscillations in Large Extra Dimensions

S. Nussinov and R. Shrock, Phys. Rev. Lett. 88, 171601 (2002)

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

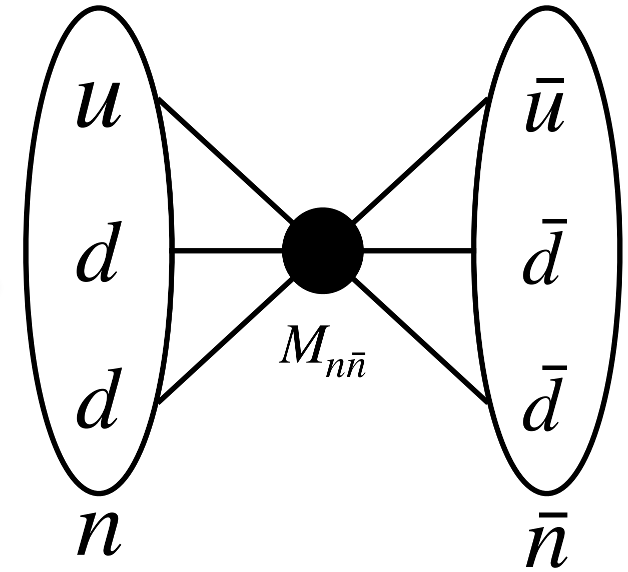
$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_3^{(n\bar{n})} = \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_4^{(n\bar{n})} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathscr{L}_{eff}^{(n\bar{n})}(x) = \sum_{r=1}^{4} c_r^{(n\bar{n})} \mathcal{O}_r^{(n\bar{n})}(x)$$

Operators mediating $n - \bar{n}$ oscillations



Suppression from Extra Dimensional wavefunction separation

$$I_{\mathcal{O}_1}^{(n\bar{n})} = I_{\mathcal{O}_2}^{(n\bar{n})} = I_{C_1}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{u_R} - \eta_{d_R}\|^2\right]$$

$$I_{\mathcal{O}_3}^{(n\bar{n})} = I_{C_2}^{(n\bar{n})} = b_6 \exp\left[-\frac{1}{6}\left\{2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2\right\}\right]$$

$$I_{\mathcal{O}_4}^{(n\bar{n})} = I_{C_3}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2\right]$$ (Dominates)

(Known from the down quark mass)

From quark mass matrix

$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75$

$\|\eta_{Q_L} - \eta_{d_R}\| = 4.60$

$\|\eta_{u_R} - \eta_{d_R}\| = 7$

Mirabelli, Schmaltz

$$|\delta m| = (\tau_{n\bar{n}})^{-1} = |\langle \bar{n} | \mathcal{L}_{eff}^{(n\bar{n})} | n \rangle| < 2.4 \times 10^{-30} \, \mathrm{MeV}$$

$$M_{n\bar{n}} > (44 \text{ TeV}) \left(\frac{\tau_{n\bar{n}}}{2.7 \times 10^8 \text{ sec}} \right)^{1/9} \left(\frac{\mu}{3 \times 10^3 \text{ TeV}} \right)^{4/9} \left(\frac{|\langle \bar{n} | \mathcal{O}_4^{(n\bar{n})} | n \rangle|}{\Lambda_{QCD}^6} \right)^{1/9}$$

$n - \bar{n}$ oscillations are the main manifestation of BNV in this case

- Λ_{QCD} is taken to be 0.25 GeV. The bound is not very sensitive to the precise value of the matrix element due to $1/9$ power dependence.

- $\mathcal{O}_4^{(n\bar{n})} = -Q_3$ in notation of recent lattice QCD calculation **(Rinaldi, Syritsyn et al. 2019).** With corresponding matrix element $\langle n | Q_3 | \bar{n} \rangle \simeq 5 \times 10^{-4} \text{ GeV}^6 = 2\Lambda_{QCD}^6$; substituting this yields a factor of $2^{1/9}$ and the lower bound changes to $\simeq 48$ TeV.

- In the extra-dimensional model, while nucleon decay can easily be suppressed well below experimental bounds, this does not suppress $n - \bar{n}$ oscillations, which can occur near experimental limits. This provided motivation for further experimental searches for $n - \bar{n}$ oscillations and the associated dinucleon decays.

Constraints on the distances of the wavefunction centers in the extra dimensions from nucleon decay ($\Delta L = -1$)

From the experimental bounds on the decay rate ($\Gamma_{N\to f.s.}$) of single nucleon (N) mediated by BSM Physics at a scale M_{BNV} by operators $\mathcal{O}_r^{(Nd)}$ in the context of extra dimensional model, we get the constraints on fermion wavefunction separations :

$$\Gamma_{N\to f.s.} = \frac{1}{2m_N}\frac{1}{(M_{BNV})^4}\left(\frac{\mu}{\pi^{1/2}M_{BNV}}\right)^{2n}\left|\sum_r \bar{\kappa}_r^{(Nd)}\, e^{-S_r^{(Nd)}}\langle f.s.\,|\,\mathcal{O}_r^{(Nd)}\,|N\rangle\right|^2 R_2^{(f.s.)}$$

Using formula (A) and considering no destructive interference we get the conservative bound ($\ell = e, \mu$):

$$\Big\{ \|\eta_{u_R}-\eta_{\ell_R}\|,\ \|\eta_{d_R}-\eta_{\ell_R}\|,\ \|\eta_{Q_L}-\eta_{\ell_R}\|,\ \|\eta_{Q_L}-\eta_{L_{\ell,L}}\|,\ \|\eta_{L_{\ell,L}}-\eta_{u_R}\|,\ \|\eta_{L_{\ell,L}}-\eta_{d_R}\|,\ \|\eta_{u_R}-\eta_{\nu_{s,R}}\|,\ \|\eta_{d_R}-\eta_{\nu_{s,R}}\|, \|\eta_{Q_L}-\eta_{\nu_{s,R}}\| \Big\} > [(S_r^{(Nd)})_{\min}]^{1/2}$$

where: $$(S_r^{(Nd)})_{\min} = 48 - \frac{n}{2}\ln\pi - 2\ln\Big(\frac{M_{BNV}}{100\ \text{TeV}}\Big) - n\ \ln\Big(\frac{M_{BNV}}{\mu}\Big)$$

From bounds on modes like $p \to \bar{\nu}\pi^+$, where τ can appear as an internal line we get:

$$\Big\{ \|\eta_{q_R}-\eta_{\tau_R}\|, \|\eta_{Q_L}-\eta_{\tau_R}\|, \|\eta_{q_R}-\eta_{L_{\tau,L}}\|, \|\eta_{Q_L}-\eta_{L_{\tau,L}}\| \Big\} > [(S_r^{(Nd)})_{min} - 2]^{1/2} \quad \text{for } q = u,\, d\ .$$

Are other $\Delta L = -3\,, 1\,, -2\,, 2$ nucleon and dinucleon decays sufficiently suppressed in this model ?

$\Delta L = -2$ Dinucleon Decays to Dileptons $[pp \to \ell^+\ell'^+, np \to \ell^+\bar{\nu}, nn \to \bar{\nu}\bar{\nu}](k=8)$

class $C_k^{(NN')}$	N_d	structure
$C_1^{(pp)}$	0	$u^4d^2\ell^2$
$C_2^{(np)}$	0	$u^3d^3\ell\nu$
$C_3^{(nn)}$	0	$u^2d^4\nu^2$
$C_4^{(pp)}$	2	$Q^2u^3d\ell^2$
$C_5^{(np)}$	2	$Q^2u^2d^2\ell\nu$
$C_6^{(nn)}$	2	$Q^2ud^3\nu^2$
$C_7^{(pp,np)}$	2	$QLu^3d^2\ell$
$C_8^{(np,nn)}$	2	$QLu^2d^3\nu$
$C_9^{(np)}$	2	$L^2u^3d^3$
$C_{10}^{(pp)}$	4	$Q^4u^2\ell^2$
$C_{11}^{(np)}$	4	$Q^4ud\ell\nu$
$C_{12}^{(nn)}$	4	$Q^4d^2\nu^2$
$C_{13}^{(pp,np)}$	4	$Q^3Lu^2d\ell$
$C_{14}^{(np,nn)}$	4	$Q^3Lud^2\nu$
$C_{15}^{(pp,np,nn)}$	4	$Q^2L^2u^2d^2$
$C_{16}^{(pp,np,nn)}$	6	Q^4L^2ud
$C_{17}^{(pp,np)}$	6	$Q^5Lu\ell$
$C_{18}^{(np,nn)}$	6	$Q^5Ld\nu$
$C_{19}^{(pp,np,nn)}$	8	Q^6L^2

How to find different classes?

$$\mathcal{O}^{(NN')} = Q_L^{n_Q}\, L_L^{n_L}\, u_R^{n_u}\, d_R^{n_d}\, \ell_R^{n_\ell}\, \nu_{s,R}^{n_{\nu_s}}$$

Solve:

$$n_Q + n_L + n_u + n_d + n_\ell + n_{\nu_s} = 8 \qquad (8-\text{fermion operator})$$

$$n_Q + n_u + n_d = 2N_c = 6 \qquad \text{(dinucleon initial state)}$$

$$n_Q\left(\frac{1}{3}\right) + n_L(-1) + n_u\left(\frac{4}{3}\right) + n_d\left(-\frac{2}{3}\right) + n_\ell(-2) = 0 \qquad Y\text{-cancellation}$$

$$N_d = n_Q + n_L = \text{even} \qquad SU(2)_L \text{ invariant}$$

$$I_{C_k}^{(NN')} = b_8 \exp\left[-\frac{1}{8}\sum_{f,f';\ f\neq f',ord.} n_f n_{f'} \|\eta_f - \eta_{f'}\|^2\right] \quad \text{Highly suppressed}$$

Example of an operator $\in C_{19}$

$$\mathcal{O}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}\epsilon_{np}\epsilon_{st}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[\underbrace{Q_L^{i\alpha\,T} C Q_L^{j\beta}}_{u_L\ d_L}][\underbrace{Q_L^{k\gamma\,T} C Q_L^{m\delta}}_{u_L\ d_L}][\underbrace{Q_L^{n\rho\,T} C L_{\ell,L}^{p}}_{u_L\ \ell_L \text{ or } d_L\ \nu_\ell}][\underbrace{Q_L^{s\sigma\,T} C L_{\ell',L}^{t}}_{u_L\ \ell_L \text{ or } d_L\ \nu_\ell}]$$

$\Delta L = -3$ Nucleon Decays to Trileptons [$p \to \ell^+ \bar{\nu}\bar{\nu}'$ and $n \to \bar{\nu}\bar{\nu}'\bar{\nu}''$] ($k = 6$)

Each class consists of several operators

class $C_k^{(Nm3)}$	N_d	structure
$C_1^{(pm3)}$	0	$u^2 d\ell\nu^2$
$C_2^{(nm3)}$	0	$ud^2\nu^3$
$C_3^{(pm3)}$	2	$Q^2 u\ell\nu^2$
$C_4^{(nm3)}$	2	$Q^2 d\nu^3$
$C_5^{(pm3,nm3)}$	2	$QLud\nu^2$
$C_6^{(pm3,nm3)}$	2	$QLu^2\ell\nu$
$C_7^{(pm3,nm3)}$	4	$Q^3 L\nu^2$
$C_8^{(pm3,nm3)}$	4	$Q^2 L^2 u\nu$

$$\mathcal{O}_1^{(pm3)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\,T} C d_R^\beta][u_R^{\gamma\,T} C \ell_R][\nu_{s,R}^T C \nu_{s',R}] \in C_1^{(pm3)}$$

$$\mathcal{O}_3^{(nm3)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\,T} C d_R^\beta][d_R^{\gamma\,T} C \nu_{s,R}][\nu_{s',R}^T C \nu_{s'',R}] \in C_2^{(nm3)}$$

$$\mathcal{O}_4^{(pm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C Q_L^{j\beta}][u_R^{\gamma\,T} C \ell_R][\nu_{s,R}^T C \nu_{s',R}] \in C_3^{(pm3)}$$

$$\mathcal{O}_6^{(nm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C Q_L^{j\beta}][d_R^{\gamma\,T} C \nu_{s,R}][\nu_{s',R}^T C \nu_{s'',R}] \in C_4^{(nm3)}$$

$$\mathcal{O}_7^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C L_{\ell,L}^j][u_R^{\beta\,T} C d_R^\gamma][\nu_{s,R}^T C \nu_{s',R}] \in C_5^{(pm3,nm3)}$$

$$\mathcal{O}_9^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C L_{\ell',L}^j][u_R^{\beta\,T} C \ell_R][u_R^{\gamma\,T} C \nu_{s,R}] \in C_6^{(pm3,nm3)}$$

$$\mathcal{O}_{10}^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C Q_L^{j\beta}][Q_L^{k\gamma\,T} C L_{\ell,L}^m][\nu_{s,R}^T C \nu_{s',R}] \in C_7^{(pm3,nm3)}$$

$$\mathcal{O}_{11}^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha\,T} C L_{\ell,L}^j][Q_L^{k\beta\,T} C L_{\ell',L}^m][u_R^{\gamma\,T} C \nu_{s,R}] \in C_8^{(pm3,nm3)}$$

Using formula (A) we can easily write down the exponential suppression. For example:

$$I_{C_4}^{(nm3)} = b_6 \exp\Big[-\frac{1}{6}\Big\{ 2\|\eta_{Q_L} - \eta_{d_R}\|^2 + 2\|\eta_{Q_L} - \eta_{\nu_{s,R}}\|^2 + 2\|\eta_{Q_L} - \eta_{\nu_{s',R}}\|^2 + 2\|\eta_{Q_L} - \eta_{\nu_{s'',R}}\|^2 + \|\eta_{d_R} - \eta_{\nu_{s,R}}\|^2$$
$$+ \|\eta_{d_R} - \eta_{\nu_{s',R}}\|^2 + \|\eta_{d_R} - \eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s,R}} - \eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}} - \eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s',R}} - \eta_{\nu_{s'',R}}\|^2 \Big\}\Big]$$

An experiment won't distinguish outgoing ν or $\bar{\nu}$. We similarly analyzed $\Delta L = 1$ nucleon decay by replacing $\nu_{s,R}$ with $\nu_{\mathrm{s,R}}^{\mathrm{c}} \equiv (\nu^{\mathrm{c}})_{\mathrm{s,L}}$. We also analyzed $nn \to \nu\nu'$ similarly.

These decays are heavily suppressed, hence in accord with experimental limits

Nucleon decay and $n - \bar{n}$ oscillations in a left-right symmetric model with large extra dimensions

[Phys. Rev. D 101, 095012 (2020)] [arXiv: 2003.14185]

Brief Summary

- It is of interest to investigate this physics in a extra-dimension model with an enlarged gauge group, the left-right-symmetric (LRS) gauge group:

$$G_{LRS} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

(Mohapatra, Marshak, Senjanović)

- We show that, similar to the earlier study by Nussinov and Shrock, while one can easily suppress baryon-number-violating nucleon decays far below existing experimental bounds by separating quark and lepton wavefunction centers in the extra dimensions, this does not suppress $n - \bar{n}$ oscillations, which can occur at levels comparable to present limits.

- An interesting new feature that we find in the LRS extra-dim. model is that for some $n - \bar{n}$ transition operators, the integration over the extra dimensions **does not yield any exponential suppression factors**, in contrast to the model with a SM effective field theory (SMEFT), where all such integrations led to exponential suppression factors.

- Our results in PRD 101, 095012 (2020) [arXiv:2003.14185] thus provides further motivation for further experimental searches for $n - \bar{n}$ oscillations (e.g., at the European Spallation Source, ESS) and the associated $\Delta B = -2$ dinucleon decays (at SK, and future Hyper-K and DUNE)

Left-right symmetric model

The left-right symmetric gauge group: $G_{LRS} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Gauge coupling: g_L g_R

Matter content

Fermions :

$$Q_L = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L : (3,2,1)_{1/3} \qquad Q_R = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_R : (3,1,2)_{1/3}$$

$$L_{\ell,L} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L : (1,2,1)_{-1} \qquad L_{\ell,R} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_R : (1,1,2)_{-1}$$

Higgs :

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} : (1,2,2)_0$$

$$\Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} : (1,3,1)_2 \qquad \Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} : (1,1,3)_2$$

The minimization of the Higgs potential yields the following VEVs:

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}\kappa_1 & 0\\ 0 & \kappa_2 e^{i\theta_\Phi}\end{pmatrix} \quad ; \quad \langle\Delta_L\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ v_L e^{i\theta_\Delta} & 0\end{pmatrix} \quad ; \quad \langle\Delta_R\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix}$$

Spontaneous symmetry breaking pattern:

$$SU(2)_R \otimes U(1)_{B-L} \xrightarrow{\langle\Delta_R\rangle_0} U(1)_Y$$

; W_R gets a large mass : $m_{W_R} = g_R v_R/\sqrt{2}$ (leading order)

$$SU(2)_L \otimes U(1)_Y \xrightarrow{\langle\Phi\rangle_0} U(1)_{em}$$

; $m_{W_L} = g_L\sqrt{\kappa_1^2+\kappa_2^2}/2 = g_L v_{EW}/2$

- $v_L \ll \kappa_{1,2}$ to maintain $\rho = m_{W_L}^2/(m_Z^2 \cos^2\theta_W) = 1$.
- The electric charge $Q_{em} = T_{3L} + T_{3R} + (B-L)/2$; ($\vec{T}_{L,R}$ are $SU(2)_{L,R}$ generators).
- The non-observation of $W_R^\pm, Z'$ can be accommodated by making v_R sufficiently large.
- G_{LRS} can be naturally embedded into Pati-Salam group $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$, which is maximal subgroup of $SO(10)$.

$$SO(10) \supseteq SO(6) \otimes SO(4) \approx SU(4) \otimes SU(2) \otimes SU(2)$$

- SM fields can propagate in n extra dimensions ($y_\lambda \in [0,L]$) and SM fermion wavefunctions have strong localization at various points in the extra-dimensional space (Gaussian profile). Δ_R is taken to have flat VEV in the extra dimensions.

Fermions: $$\Psi_f(x,y) = \psi_f(x)\chi_f(y)$$ $\chi_f(y) \to A\, e^{-\mu^2 \|y-y_f\|^2}$

Gaussian profile

- The VEV of Δ_R breaks $B-L$ by two units. For a process having $\Delta L = 0$ this implies : $|\Delta B| = 2$, such as $n-\bar{n}$ oscillations, which has a characteristic mass scale $M_{n\bar{n}} = v_R$.

- We shall write down G_{LRS} invariant six-quark operators multiplied by $\Delta_R^\dagger$. The resultant operators after the Δ_R field gets a VEV mediate $B-L$ violating $n-\bar{n}$ oscillations.

Constraints from nucleon decay searches

We assume nucleon decay mass scale M_{Nd} is greater than v_R

$$\mathcal{O}^{(Nd)}_{LL} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha\,T}CQ_L^{j\beta}][Q_L^{k\gamma\,T}CL^m_{\ell,L}]$$

$$\mathcal{O}^{(Nd)}_{RR} = \epsilon_{\alpha\beta\gamma}\epsilon_{i'j'}\epsilon_{k'm'}[Q_R^{i'\alpha\,T}CQ_R^{j'\beta}][Q_R^{k'\gamma\,T}CL^{m'}_{\ell,R}]$$

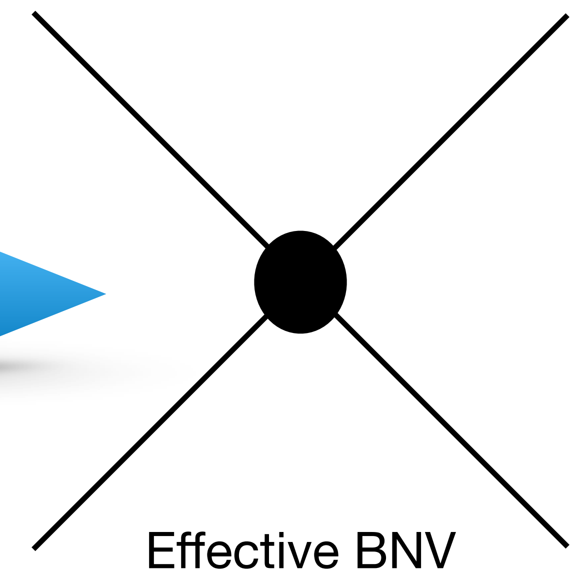
$$\mathcal{O}^{(Nd)}_{LR} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{i'j'}[Q_L^{i\alpha\,T}CQ_L^{j\beta}][Q_R^{i'\gamma\,T}CL^{j'}_{\ell,R}]$$

$$\mathcal{O}^{(Nd)}_{RL} = \epsilon_{\alpha\beta\gamma}\epsilon_{i'j'}\epsilon_{ij}[Q_R^{i'\alpha\,T}CQ_R^{j'\beta}][Q_L^{i\gamma\,T}CL^{j}_{\ell,L}]$$

α, β, γ : Color indices ; i, j : $SU(2)_L$ indices ; i', j' : $SU(2)_R$ indices

$$\mathcal{L}^{(Nd)}_{eff} = \sum_r c_r \mathcal{O}_r$$

G_{LRS} invariant
4-fermion vertex



Effective BNV vertex

$$\Gamma_{N\to \mathrm{f.s.}} = \frac{1}{2m_N}\frac{1}{(M_{Nd})^4}\left(\frac{\mu}{\pi^{1/2}M_{Nd}}\right)^{2n}\left|\sum_r \bar{\kappa}_r^{(Nd)}\, e^{-S_r^{(Nd)}}\langle \mathrm{f.s.}\,|\,\mathcal{O}_\mathrm{r}^{(\mathrm{Nd})}\,|\,\mathrm{N}\rangle\right|^2 \mathrm{R}_2^{(\mathrm{f.s.})}$$

Using the experimental bound on $(\tau/B)_{N\to\mathrm{f.s.}} = (\Gamma_{N\to\mathrm{f.s.}})^{-1}$ we have these conservative bounds:

$$\left\{\|\eta_{Q_L}-\eta_{L_{\ell_L}}\|,\ \|\eta_{Q_R}-\eta_{L_{\ell_R}}\|,\ \|\eta_{Q_L}-\eta_{L_{\ell_R}}\|, \|\eta_{Q_R}-\eta_{L_{\ell_L}}\|\right\} > [(S_r^{(Nd)})_{\min}]^{1/2}$$

Where: $$(S_r^{(Nd)})_{\min} = 39 - \frac{n}{2}\ln\pi - 2\ln\left(\frac{M_{Nd}}{10^4\ \mathrm{TeV}}\right) - n\ \ln\left(\frac{M_{Nd}}{\mu}\right)$$

$n-\bar{n}$ Oscillations in LRS with large extra dimensions

$$\mathcal{O}_1^{(n\bar{n})}=(T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{i'k'}\epsilon_{j'm'}+\epsilon_{j'k'}\epsilon_{i'm'})(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_R^{i'\alpha T}CQ_R^{j'\beta}][Q_R^{k'\gamma T}CQ_R^{m'\delta}][Q_R^{p'\rho T}CQ_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$\mathcal{O}_2^{(n\bar{n})}=(T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{i'j'}\epsilon_{k'm'}(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_R^{i'\alpha T}CQ_R^{j'\beta}][Q_R^{k'\gamma T}CQ_R^{m'\delta}][Q_R^{p'\rho T}CQ_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

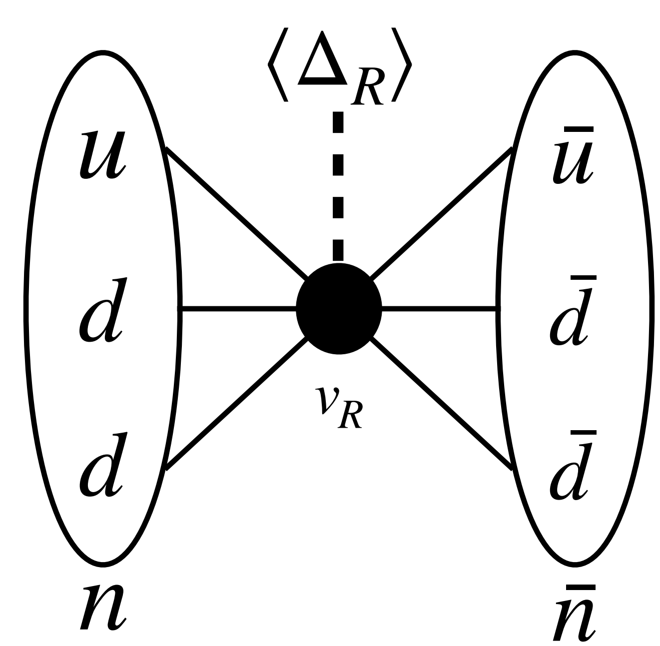
$$\mathcal{O}_3^{(n\bar{n})}=(T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{k'm'}(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_R^{k'\gamma T}CQ_R^{m'\delta}][Q_R^{p'\rho T}CQ_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$\mathcal{O}_4^{(n\bar{n})}=(T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{km}(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CQ_L^{m\delta}][Q_R^{p'\rho T}CQ_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$\mathcal{O}_5^{(n\bar{n})}=(T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{ik}\epsilon_{jm}+\epsilon_{jk}\epsilon_{im})(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CQ_L^{m\delta}][Q_R^{p'\rho T}CQ_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

Interestingly the suppression factor $S_r^{(n\bar{n})}$ associated with operator $\mathcal{O}_r^{(n\bar{n})}$ in the low energy effective Lagrangian:

$$S_r^{(n\bar{n})}=\begin{cases}0 & \text{, for } r=1,\,2\\ -\frac{4}{3}\|\eta_{Q_L}-\eta_{Q_R}\|^2 & \text{, for } r=3,4,5\end{cases}$$



No suppression for $\mathcal{O}_{1,2}^{(n\bar{n})}$ from extra dimensional integrals $\Longrightarrow$ will dominate !

In this model, the transition amplitude:

$$|\delta m| = |\langle \bar{n} | \mathcal{L}_{eff}^{(n\bar{n})} | n \rangle| = \frac{1}{v_R^5} \left(\frac{\mu}{v_R}\right)^{2n} \left(\frac{2}{3^{1/2}\pi}\right)^{n} \times \left| \sum_r \bar{\kappa}_r^{(n\bar{n})} e^{-S_r^{(n\bar{n})}} \langle \bar{n} | \mathcal{O}_r^{(n\bar{n})} | n \rangle \right|$$

For dominant operators $\mathcal{O}_{1,2}^{(n\bar{n})}$ the suppression factor $S_r^{(n\bar{n})} = 0$. From experimental bound:

$$|\delta m| = (\tau_{n\bar{n}})^{-1} < 2.4 \times 10^{-30}\,\mathrm{MeV}$$ **(SK, 2015)**

$$v_R > (1 \times 10^3\ \mathrm{TeV}) \left(\frac{\tau_{n\bar{n}}}{2.7 \times 10^8\ \mathrm{sec}}\right)^{1/9} \times \left(\frac{\mu}{3 \times 10^3\ \mathrm{TeV}}\right)^{4/9} \left(\frac{|\langle \bar{n} | \mathcal{O}_{1,2}^{(n\bar{n})} | n \rangle|}{\Lambda_{QCD}^6}\right)^{1/9}$$

$\Delta B = -1$ nucleon decay can be easily suppressed while $|\Delta B| = 2$ $n - \bar{n}$ oscillations may occur at a level comparable to current experimental limits.

- Other models can also predict $n - \bar{n}$ oscillations occurring at a level close to observable limits. This list includes:

  Mohapatra, Marshak (1980) ; Rao, Shrock (1982, 1984) ; Babu, Mohapatra (2001) ; Babu, Mohapatra, Nasri (2006) (talks by Mohapatra, Babu) ; Arnold, Fornal, Wise (2013) ; Babu, Dev, Fortes, Mohapatra (2013)…

- Another interesting possibility for BNV is n to mirror n' oscillations **(talk by Kamyshkov, Berezhiani, Young)**. Plan for $n - \bar{n}$ searches include n converting into $\bar{n}$ via $[n', \bar{n}']$ **(Addazi et al. arXiv: 2006.04907)**, and search also at the High Flux Isotope Reactor, HFIR at ORNL **(e.g. Broussard et al. , arXiv: 1912.08264).**

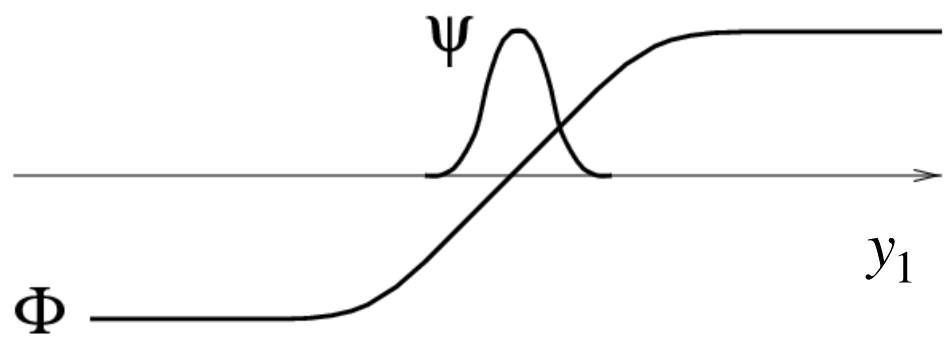
Conclusions

- Several nucleon and dinucleon decay modes have been considered in a large extra dimensional model and we find that they are in accord with experimental constraints.

- $n - \bar{n}$ oscillations are special in the context of large extra dimensions as it only involves quark fields. Separating out quark and lepton wavefunctions in extra dimension(s) to suppress proton decay does not suppress $n - \bar{n}$ oscillations, which may occur at a rate comparable to current experimental limits.

- We have shown that dominating operators mediating $n - \bar{n}$ oscillations in LRS models in the extra dimensions can be even less suppressed than their SMEFT counterpart.

Thank you for your time!
Questions?



Coupling fermions (Ψ) to scalars (Φ) having kink solution (for $n = 1$) or vortex solution (for $n = 2$) localizes the zero-mode fermion solutions in the extra dimensions (y_λ)

[Rubakov, Shaposhnikov Phys. Lett. 125B 136 (1983) ; Kaplan, Schmaltz Phys. Lett. B368 44 (1996); Dvali, Shifman Phys. Lett. B 475 295 (2000)]

5-dimensional action S : $S = \int d^4x \int dy_1 \bar{\Psi} \left[i\gamma_\mu \partial^\mu + i\gamma_5 \partial^5 + \Phi(y_1) \right] \Psi$

$$S = \int d^4x \times \int dy_1 \begin{bmatrix} \Psi_L^\dagger & \Psi_R^\dagger \end{bmatrix} \cdot \begin{bmatrix} i\partial_\mu \bar{\sigma}^\mu & -\partial_5 + \Phi(y_1) \\ \partial_5 + \Phi(y_1) & i\partial_\mu \sigma^\mu \end{bmatrix} \cdot \begin{bmatrix} \Psi_L & \Psi_R \end{bmatrix}$$

Decomposing into KK modes $\Psi(x_\mu, y_1) = \sum_i \begin{bmatrix} \Psi_{L(i)}(x_\mu)\chi_i^L(y_1) \\ \Psi_{R(i)}(x_\mu)\chi_i^R(y_1) \end{bmatrix}$

$$S_{eff} = \int d^4x \; \psi_{L(0)}^\dagger [i\bar{\sigma}_\mu \partial^\mu] \psi_{L(0)} + \psi_{R(0)}^\dagger [i\sigma_\mu \partial^\mu] \psi_{R(0)} + \sum_i \bar{\psi}_{(i)} [i\gamma_\mu \partial^\mu - m_i] \psi_{(i)}$$

$$(-\partial_5 + \Phi(y_1))\, \chi_i^R(y_1) = m_i\, \chi_i^L(y_1)$$

$$(+\partial_5 + \Phi(y_1))\, \chi_i^L(y_1) = m_i\, \chi_i^R(y_1)$$

$$\chi_0^R(y_1) = A_0^R\, e^{+\int \Phi(y_1) dy_1}$$

$$\chi_0^L(y_1) = A_0^L\, e^{-\int \Phi(y_1) dy_1}$$

→ Zero-mode physical solution

Large extra dimensions

[Arkani-Hamed, Dimopoulos, Dvali PLB 429, 263 (1998) ; Antoniadis, Arkani-Hamed, Dimopoulos, Dvali PLB 436, 257 (1998) ; Dienes, Dudas, Gherghetta PLB 436, 55 (1998); Nussinov, Shrock PRD 59 105002 (1999)]

- The idea of possible existence of extra spatial dimensions dates back to the work of Kaluza and Klein (1920s).

- An intriguing possibility is that the size of the extra dimensions could be large enough to have observable consequences.

- M_{Pl} becomes a derived constant which depends on the scale of compactification r_c , number of extra dimensions n, and the fundamental Planck scale M_*: which can be much lower than 10^{19} GeV.

Matching condition for 4D-effective theory: $\boxed{M_{Pl}^2 \sim r_c^n \, M_*^{2+n}}$

$$r_c = M_*^{-1}\left(\frac{M_{Pl}}{M_*}\right)^{2/n} = (2\times 10^{-17}\ \mathrm{cm})\left(\frac{1\ \mathrm{TeV}}{M_*}\right)\left(\frac{M_{Pl}}{M_*}\right)^{2/n}.$$

$n - \bar{n}$ Oscillations

In the $|n\rangle$, $|\bar{n}\rangle$ basis, the mass matrix:

$$\mathscr{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates:

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

The mixing angle: $\tan 2\theta = \dfrac{2\delta m}{\Delta M}$; $\Delta M = M_{11} - M_{22}$

The eigenvalues : $E_{1,2} = \dfrac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2}\right]$

Let $\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$

The probability for an initial $|n\rangle$ state to turn into $|\bar{n}\rangle$ after time t :

$$P(n(t) \rightarrow \bar{n}) = |\langle \bar{n} | n(t) \rangle|^2$$

$$= \sin^2(2\theta)\sin^2\left[\frac{\Delta E t}{2}\right] e^{-\lambda_n t}$$

$$= \left[\frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2}\right] \sin^2\left[\sqrt{(\Delta M/2)^2 + (\delta m)^2}\; t\right] e^{-\lambda_n t}$$

If we arrange t such that: $\sqrt{(\Delta M/2)^2 + (\delta m)^2}\; t \ll 1$

$$P(n(t) \rightarrow \bar{n}) \simeq [(\delta m\, t)^2] e^{-\lambda_n t}$$

$n - \bar{n}$ Oscillations in a magnetic field $\overrightarrow{B}$

$n, \bar{n}$ can interact with a magnetic field via magnetic moment $\overrightarrow{\mu}$, the mass matrix becomes:

$$\mathcal{M} = \begin{pmatrix} m_n - \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 & \delta m \\ \delta m & m_n + \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 \end{pmatrix}$$

The mixing: $$\tan(2\theta) = -\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}}$$

Eigenvalues: $$E_{1,2} = m_n \pm \sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} - i\lambda_n/2$$

ILL experiment reduced $|\overrightarrow{B}| = B \sim 10^{-8}\,\mathrm{T}$, so :

$$|\mu_n| B = (6.03 \times 10^{-22}\ \mathrm{MeV}) \left(\frac{B}{10^{-8}\ \mathrm{T}}\right) \gg |\delta m|$$

$$\Delta E = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$$

In a reactor $n - \bar{n}$ experiment, arrange that n's propagate a time t such that:

$$|\vec{\mu}_n \cdot \vec{B}|t = 0.92 \left(\frac{B}{10^{-8}\ \mathrm{T}}\right)\left(\frac{t}{1\ \mathrm{sec}}\right) << 1 \quad \text{and } t << \tau_n$$

Then:

$$P(n(t) \to \bar{n}) \simeq (2\theta)^2 \left(\frac{\Delta E t}{2}\right)^2 \simeq \left(\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}}\right)^2 \left(\vec{\mu}_n \cdot \vec{B}\, t\right)^2$$

$$P(n(t) \to \bar{n}) \simeq [(\delta m\, t)^2]$$

Nucleon decay operators: SMEFT

Established experimental bounds $\Longrightarrow$BNV scale > Electroweak symmetry breaking scale ($\simeq 250$ GeV) $\Longrightarrow$ The effective Lagrangian $\mathscr{L}_{eff}$ is invariant under the **full SM group** $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$.

Weinberg ('79) ; Wilczek & Zee ('79)

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_{a_1,R}^{\alpha\,T} C d_{a_2,R}^{\beta}][u_{a_3,R}^{\gamma\,T} C \ell_{a_4,R}]$$

$$\mathcal{O}_2^{(Nd)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_{a_1,L}^{i\alpha\,T} C Q_{a_2,L}^{j\beta}][u_{a_3,R}^{\gamma\,T} C \ell_{a_4,R}]$$

$$\mathcal{O}_3^{(Nd)} = \epsilon_{km}\epsilon_{\alpha\beta\gamma}[u_{a_1,R}^{\alpha\,T} C d_{a_2,R}^{\beta}][Q_{a_3,L}^{k\gamma\,T} C L_{a_4,L}^{m}]$$

$$\mathcal{O}_4^{(Nd)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_{a_1,L}^{i\alpha\,T} C Q_{a_2,L}^{j\beta}][Q_{a_3,L}^{k\gamma\,T} C L_{a_4,L}^{m}]$$

$$\mathcal{O}_5^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_{a_1,R}^{\alpha\,T} C d_{a_2,R}^{\beta}][d_{a_3,R}^{\gamma\,T} C \nu_{s,R}]$$

$$\mathcal{O}_6^{(Nd)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_{a_1,L}^{i\alpha\,T} C Q_{a_2,L}^{j\beta}][d_{a_3,R}^{\gamma\,T} C \nu_{s,R}]$$

$$Q_{a,L}^{\alpha} = \begin{pmatrix} u_a^\alpha \\ d_a^\alpha \end{pmatrix}_L \quad ; \quad L_{a,L} = \begin{pmatrix} \nu_{\ell_a} \\ \ell_a \end{pmatrix}_L$$

α, β, γ : Color indices ; i, j : Weak indices

a : Generational indices

C : Dirac Charge conjugation matrix

$$\mathscr{L}_{eff} = \sum_r c_r \mathcal{O}_r$$