Application of LCSR to $\tau^- \to \pi^- \nu_\tau \gamma$

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Abstract

The radiative tau decay ($\tau^- \to \pi^- \nu_\tau \gamma$) involves two contributions: the Internal Bremmstrahlung (IB) contribution and the Structure dependent (SD) contribution. The SD contribution encodes the non-perturbative QCD effects and depends on two form factors (F_A and F_V). Here, I will discuss the computation of these form factors in the framework of light cone sum rules. The momentum transferred square, t in this system is positive and can take values upto m_τ^2, which makes it phenomenologically rich. We have found the structure dependent parameter, γ, (i.e. $\frac{F_A}{F_V}$ at zero momentum transfer) in good agreement with the experimental determination and also present the decay width and invariant mass spectrum in the $\pi - \gamma$ system.

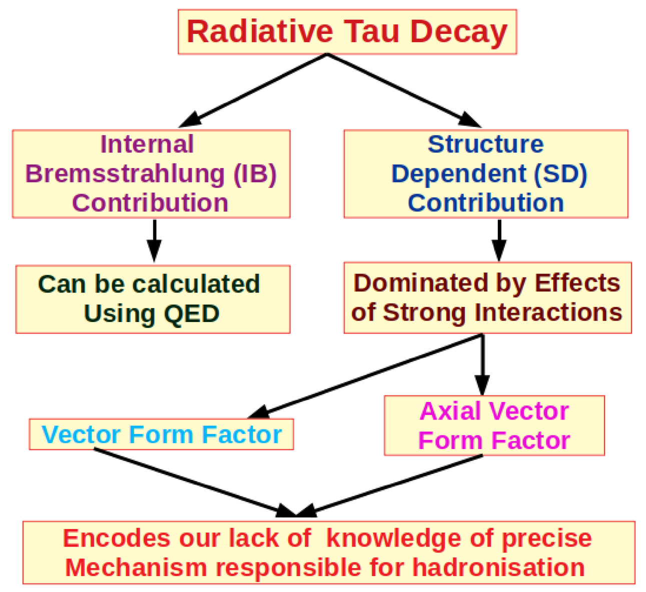
Introduction

τ being the heaviest lepton with $m_\tau = 1776.86 \pm 0.12 MeV$ has numerous decay channels. The study of hadronic τ decays helps in understanding the dynamics of strong interaction involved in the hadronization of QCD currents.

The branching ratio of $\tau^- \to \pi^- \nu_\tau$ is $(10.82 \pm 0.05)\%$. From a naive calculation, one expects the branching ration of $\tau^- \to \pi^- \nu_\tau \gamma$ to be $\mathcal{O}(10^{-3})$ i.e., roughly the product of the branching ratio of $\tau \to \rho \nu_\tau$ and $\rho \to \pi \gamma$. Even though this branching ratio is not very small, radiative decay is not observed experimentally yet which makes the study of this mode important.

The amplitude of radiative tau decay can be divided into two contributions:

1. ***Internal bremmtrahlung (IB):*** *Emission from either the incoming or the outgoing particles and can be calculated using QED.*
2. ***Structure Dependent (SD):*** *Captures the long disctance dynamics of strong interaction and can be parametrized by vector and axial-vector form factors (FFs) $F_V^{(\pi)}$ and $F_A^{(\pi)}$, respectively using Lorentz symmetry. It also includes a contact term (CT), which emerges as a consequence of gauge invariance. CT contribution turns out to be equal and opposite to the m_τ independent IB contribution and hence they cancel out each other in the total amplitude.*



Amplitude Calculation

The amplitude of $\tau^-(p_1) \to \pi^-(p_2)\nu_\tau(p_3)\gamma(k)$ is,

$$\mathcal{A}(\tau^- \to \pi^- \nu_\tau \gamma) = \frac{G_F}{\sqrt{2}} V_{ud} \left\langle \pi^- \nu_\tau \gamma | (\bar{\nu}_\tau \Gamma^\mu \tau)(\bar{d}\Gamma_\mu u) | \tau^- \right\rangle \tag{1}$$

where, $\Gamma^\mu = \gamma^\mu(1-\gamma_5)$. Emission from pion is dictated by the hadronic matrix element

$$T^{\alpha\mu}(p_2, k) = i \int d^4x e^{ikx} \langle \pi^- | T\{j^\alpha_{em}(x) \bar{d}\Gamma^\mu u(0)\} | 0 \rangle \tag{2}$$

where, $j^\alpha_{em}(x) = Q_\psi \bar{\psi}(x)\gamma^\alpha \psi(x) = -\bar{\tau}\gamma^\alpha \tau + Q_u \bar{u}\gamma^\alpha u + Q_d \bar{d}\gamma^\alpha$. It can be parametrized in terms on two gauge invariant scalar functions of $P^2 = (p_2 + k)^2$ as,

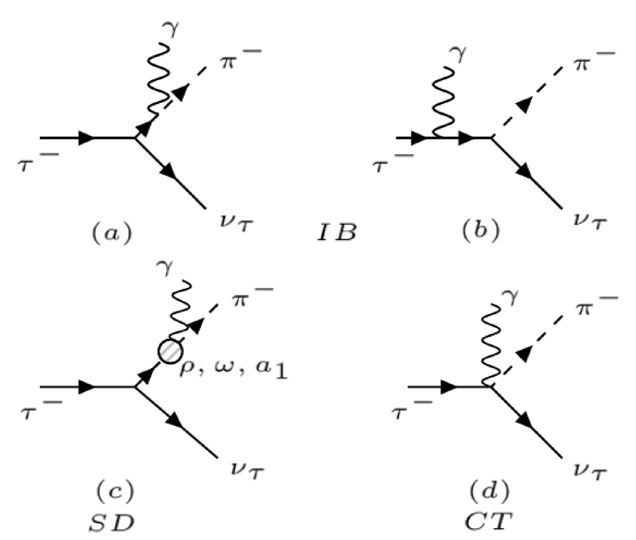


Figure 1: Feynman diagrams showing different possibilities of photon emission.

$$T^{\alpha\mu}(p_2, k) = F_A^{(\pi)} [g^{\alpha\mu}(P.k) - P^\alpha k^\mu] + i F_V^{(\pi)} \epsilon^{\alpha\mu\beta\nu} P_\beta k_\nu - i f_\pi g^{\alpha\mu} + i f_\pi \frac{P^\alpha P^\mu}{P.k}. \tag{3}$$

The final form of amplitude turns out to be $\mathcal{A}(\tau^- \to \pi \nu_\tau \gamma) = \mathcal{A}_{IB} + \mathcal{A}_{SD}$ with,

$$\mathcal{A}_{IB} = \frac{G_F}{\sqrt{2}} V_{ud} \left[i e f_\pi m_\tau \bar{u}_\nu \left\{ \frac{\epsilon^*.p_1}{p_1.k} - \frac{\not{k}\not{\epsilon}^*}{2p_1.k} - \frac{\epsilon^*.p_2}{p_2.k} \right\} (1+\gamma_5) u_\tau \right]$$

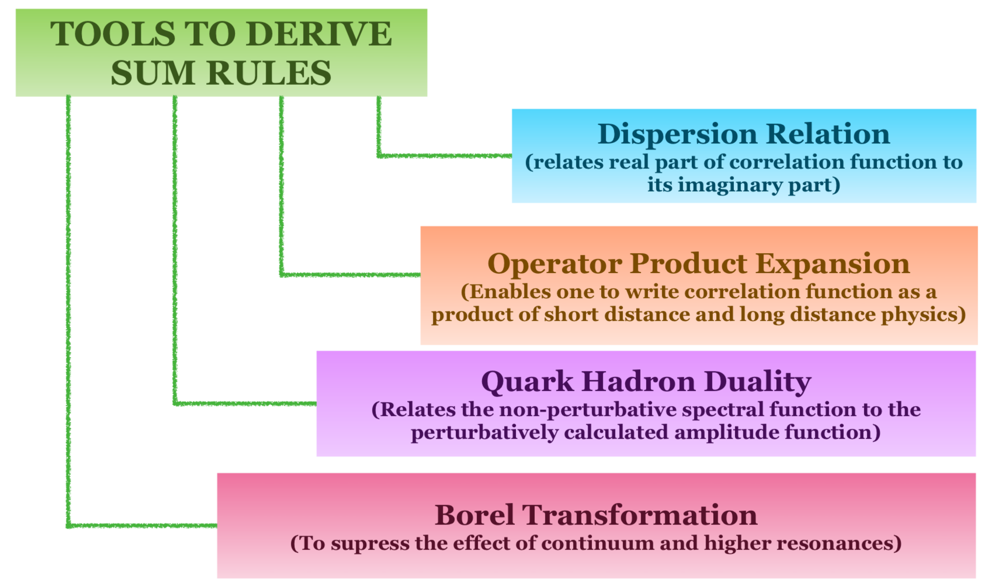
$$\mathcal{A}_{SD} = \frac{G_F}{\sqrt{2}} V_{ud} \left[i e \epsilon^{*\alpha} (\bar{u}_\nu \Gamma^\mu u_\tau) \left(i F_A^{(\pi)} [g_{\alpha\mu}(P.k) - P_\mu k_\alpha] - F_V^{(\pi)} \epsilon_{\alpha\mu\beta\nu} P^\beta k^\nu \right) \right]$$

Main Objectives

1. To calculate $F_V^{(\pi)}(P^2)$ and $F_A^{(\pi)}(P^2)$ using light cone sum rules.
2. To calculate the structure dependent parameter, $\gamma = \frac{F_A^{(\pi)}(0)}{F_V^{(\pi)}(0)}$.
3. To compute the radiative tau decay width and the invariant mass spectrum of the $\pi - \gamma$ system.

Light cone sum rules (LCSR)

- The method of QCD sum rules was developed by Shifman, Vainstein and Zakharov (SVZ) in 1979.
- **Basic idea:** To derive the hadronic parameters using the analytical properties of the correlation function.
- In the lightcone limit, the bilocal operator sandwiched between the pion state and vacuum is expressed as,



$$\left\langle \pi^0(p) | \bar{u}(y)\gamma_\mu\gamma_5 u(x) | 0 \right\rangle_{x^2=0} = -i f_\pi p_\mu \int_0^1 du\, e^{i(up_2.y + \bar{u}p.x)} \phi(u,\mu) \tag{4}$$

where, $\bar{u} = 1 - u$ and $\phi(u, \mu)$ is leading twist-2 distribution amplitude.

Form factor calculation using LCSR

We first compute $T^{\alpha\mu}(P, k)$ in perturbative QCD (pQCD):

$$F_V^{QCD}(P^2) = \frac{i f_\pi}{3} \int_0^1 du \frac{\phi(u,\mu)}{P^2\bar{u} + k^2 u} \quad \text{and} \quad F_A^{QCD}(t) = -i f_\pi \int_0^1 du \phi(u,\mu) \left(\frac{1 - 2\bar{u}}{P^2 \bar{u} + k^2 u} \right) \tag{5}$$

Then, using the analytic properties of $T^{\alpha\mu}(P, k)$ and considering the contribution from the lowest resonance states i.e. ρ, ω and a_1-mesons, we get the dispersion relation,

$$T^{\alpha\mu}(p_2, k) = \frac{2 i m_\rho f_\rho \epsilon^{\alpha\lambda\beta\nu} g^\mu_\lambda p_{2\beta} k_\nu F_{\rho\pi}(k^2)}{m_\rho^2 - (p_2+k)^2 - i m_\rho \Gamma_\rho} + \frac{i m_{a_1} f_{a_1} \left[2p_2.k g^{\alpha\mu} - 2p_2^\alpha k^\mu \right] G_{a_1\pi}(k^2)}{m_{a_1}^2 - (p_2+k)^2 - i m_{a_1}\Gamma_{a_1}} + \frac{1}{\pi} \int_{s_0^h}^\infty ds \frac{Im\{T^{\alpha\mu}(s,k)\}}{s - k^2 - i\epsilon}. \tag{6}$$

Here, s_0^h is the threshold of the lowest continuum state and Γ_ρ and Γ_{a_1} are the decay widths of ρ and a_1 mesons, respectively. $F_{\rho\pi}$ ($G_{a_1\pi}$) captures the physics of transition of ρ (a_1)-meson to the π-meson. The factor of 2 in the first term corresponds to $\rho + \omega$ contribution.

On equating the form of $F_V^{(\pi)}(t)$ and $F_A^{(\pi)}(t)$ from this dispersion relation with the pQCD result and applying the quark hadron duality and Borel transformation (M being the Borel parameter), we get,

$$F_V^{(\pi)}(P^2) = -i \frac{f_\pi}{3(m_\rho^2 - P^2 - i m_\rho \Gamma_\rho)} \int_0^1 du \frac{\phi(u)}{\bar{u}} e^{\frac{m_\rho^2}{M^2}}, \tag{7}$$

$$F_A^{(\pi)}(P^2) = -i \frac{f_\pi}{m_{a_1}^2 - P^2 - i m_{a_1}\Gamma_{a_1}} \int_0^1 \frac{\phi(u)}{\bar{u}} (1 - 2\bar{u}) e^{\frac{m_{a_1}^2}{M^2}}. \tag{8}$$

For the following computation, we have used the asymptotic and Chernyak-Zhitnisky form of the pion distribution amplitude,

$$\phi_\pi^{asym}(u,\mu) = 6u\bar{u} \quad \text{and} \quad \phi_\pi^{CZ}(u,\mu) = 6u\bar{u}\left[1 + \frac{3a_2(\mu)}{2}\{5(u-\bar{u})^2 - 1\}\right], \tag{9}$$

where, $a_2(\mu) = 0.12 \left(\frac{\alpha_s(\mu)}{\alpha_s(1GeV)} \right)^{\frac{50}{9\beta_0}}$ with α_s and β_0 being the QCD coupling strength and leading order beta function, respectively.

Results

- The IB contribution suffers from infrared(IR) divergences which can be taken care by putting a threshold on the photon energy (50 MeV here).
- Structure dependent parameter, $\gamma|_{CZ}(M = 3.35 GeV)$ is 0.469.
- Radiative tau decay width is found to decrease with an increase in Γ_{a_1}. The effect of P^2 dependence of decay widths is also studied.
- The value of normalised decay width i.e. $\bar{\Gamma} = \Gamma(\tau \to \pi\nu\gamma)/\Gamma(\tau \to \pi\nu)$) is found to be,

	Using ϕ^{asym}	Using ϕ^{CZ}
$\bar{\Gamma}_{IB}$	1.36×10^{-2}	1.36×10^{-2}
$\bar{\Gamma}_{SD}$	$(1.87 \pm 0.30) \times 10^{-3}$	$(2.29 \pm 0.43) \times 10^{-3}$
$\bar{\Gamma}_{int}$	$(3.82 \pm 2.14) \times 10^{-4}$	$(4.90 \pm 2.60) \times 10^{-4}$
$\bar{\Gamma}_{all}$	$(1.56 \pm 0.04) \times 10^{-2}$	$(1.61 \pm 0.06) \times 10^{-2}$

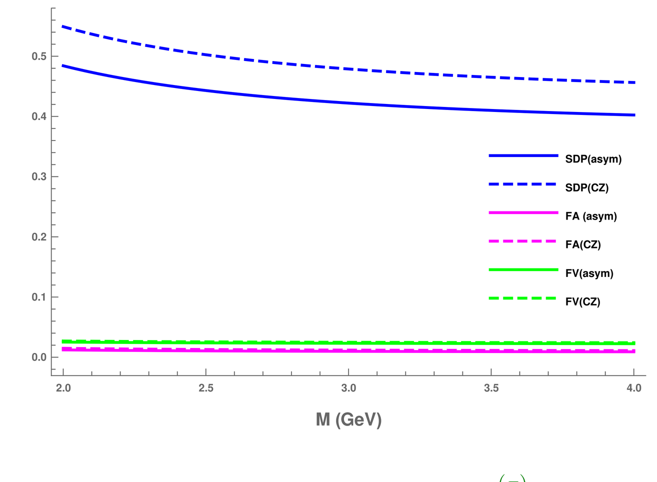


Figure 2: The variation of $F_V^{(\pi)}(0)$(Green), $F_A^{(\pi)}(0)$(Magenta) and structure dependent parameter (Blue) vs the Borel parameter M (in GeV).

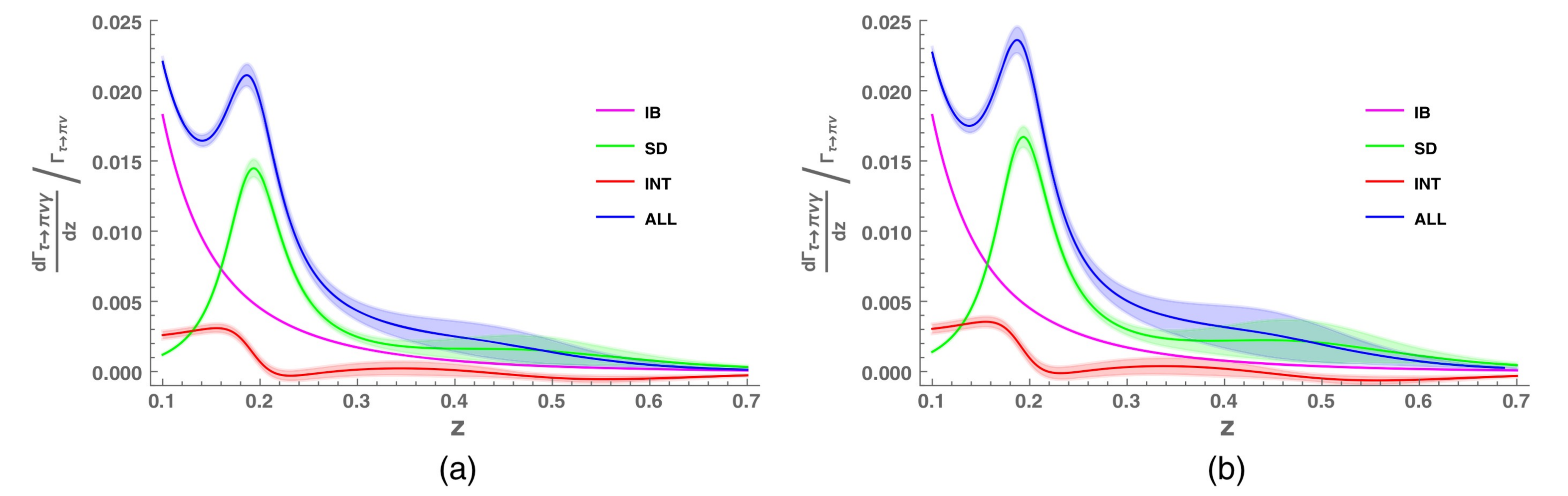


Figure 3: The invariant mass spectrum of the $\pi - \gamma$ system considering (a)asymptotic and (b)CZ distribution amplitude.z is $\frac{P^2}{m_\tau^2}$. Shaded region shows the uncertainities.

- The uncertainities are found to be about 10% and are dominated by SD contribution.
- A rough estimate for kaon in the final state gives an approximate normalised decay width to be roughly half of that for the pion.

Conclusions and Discussions

- We present a detailed prediction for the rate and photon spectrum for radiative tau decay.
- A contact term appears as a consequence of gauge invariance.
- The decay includes two timelike FFs which are calculated in the framework of LCSR upto twist-2 accuracy.
- The vector (axial-vector) FF gets contribution from ρ and $\omega(a_1)$-mesons.
- The structure dependent parameter is been calculated and is found to be in good agreement with experimental value (including sign) obtained from radiative pion decay.
- The normalised rate values are consistent with the study done using resonance χPT by Roig et. al (Phys. Rev. D 82 (2010) 113016).
- This is the first application of LCSR to such a mode and a study of higher twist contribution and radiative corrections will be interesting.