Coulomb corrections for charged current scattering on nuclei

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Introduction & Background

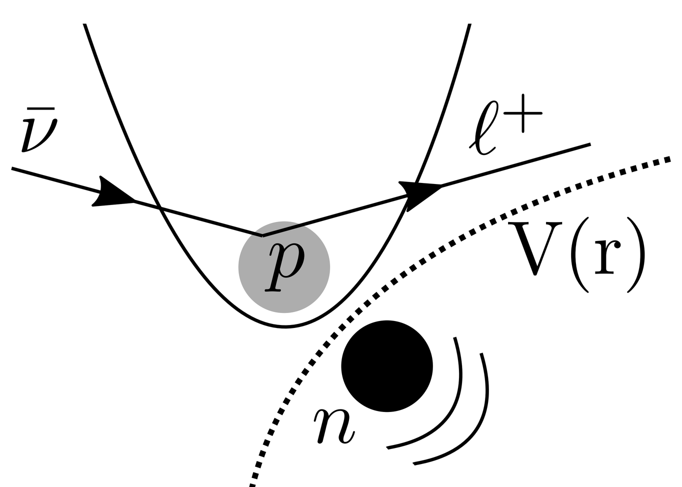
Neutrino nucleus scattering will soon require percent level input for neutrino nucleus cross sections.

QED effects can be enhanced by large nuclear charges. Rather than having α as a small parameter, corrections are controlled by $Z\alpha$.

These effects influence charged leptons that are produced in neutrino scattering on nuclei. Corrections can depend on the lepton mass and are therefore flavour dependent.

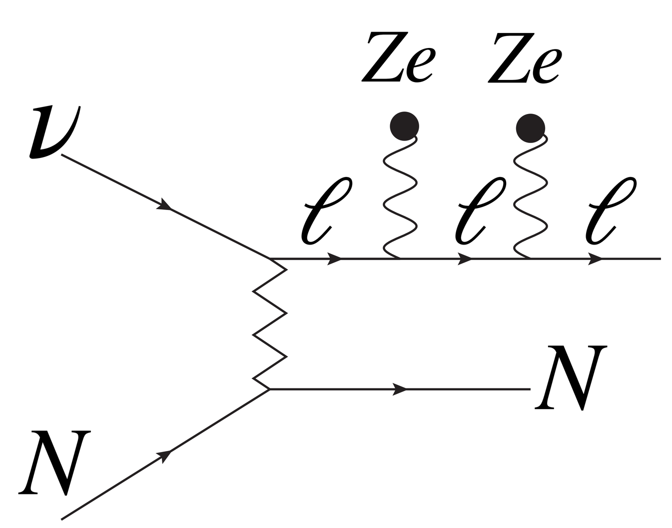
Our goal is to understand if/when Coulomb corrections can be factorized from nuclear physics effects.

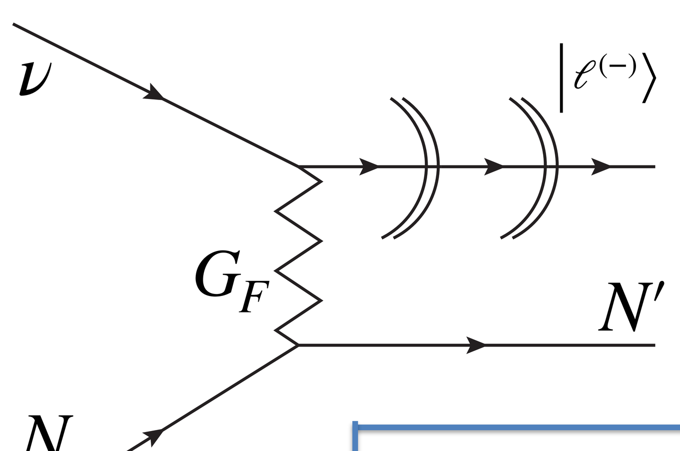
Distorted Wave Calculation



Consider toy model of a nucleus with a bound proton in the initial state.

Perturbative treatment expands in $Z\alpha$ by including background field diagramatically.





$$\mathscr{M} \sim \int \mathrm{d}^3x \; \bar{\mathscr{U}}_\ell(x)\Gamma^\mu u_\nu \mathscr{J}_\mu(x)$$

Incorporate bacgkround field effect to all orders in $Z\alpha$ by using ***distorted waves*** that solve the Dirac equation with a background field.

Eikonal Expansion in a Lightcone Basis

Suppose massless fermion with $E \gg V$ and introduce lightlike vectors

$$n^2 = 0 \quad , \quad \bar{n}^2 = 0 \quad , \quad \bar{n}\cdot n = 2$$

Dirac equation in lightcone coordinates (see SCET literature)

$$\left[\mathrm{i}(n\cdot\partial) - \mathrm{i}\not{\partial}_\perp \frac{1}{\mathrm{i}\bar{n}\cdot\partial - V}\mathrm{i}\not{\partial}_\perp\right]\mathscr{U}_{\mathbf{p}}(x) = 0$$

Solve wth Eikonal ansatz order by order in V/E

$$\mathscr{U}_{\mathbf{p}}(x) = \mathrm{e}^{\mathrm{i}En\cdot x}\mathrm{e}^{\mathrm{i}\chi(x)} \times u_{\mathbf{p}}$$

$$\chi(x) = \chi_0(x) \; + \; \frac{1}{E}\chi_1(x) \; + \; \ldots$$

Solve wth Eikonal ansatz order by order in V/E

See also

1. Yennie, Boos, Ravenhall, Physical Review 137 (1965)
2. Tjon & Wallace, Physical Review C 74 (2006)
3. J. Engel, Physical Review C 54 (1998)

Power Counting for Matrix Elements

We would like a reliable method to predict how different terms in the eikonal expansion contribute to matrix elements order by order in $1/E$.

Different orders in the eikonal expansion "mix" when computing matrix distorted wave matrix elements. due to ***rapidly oscillating phase.***

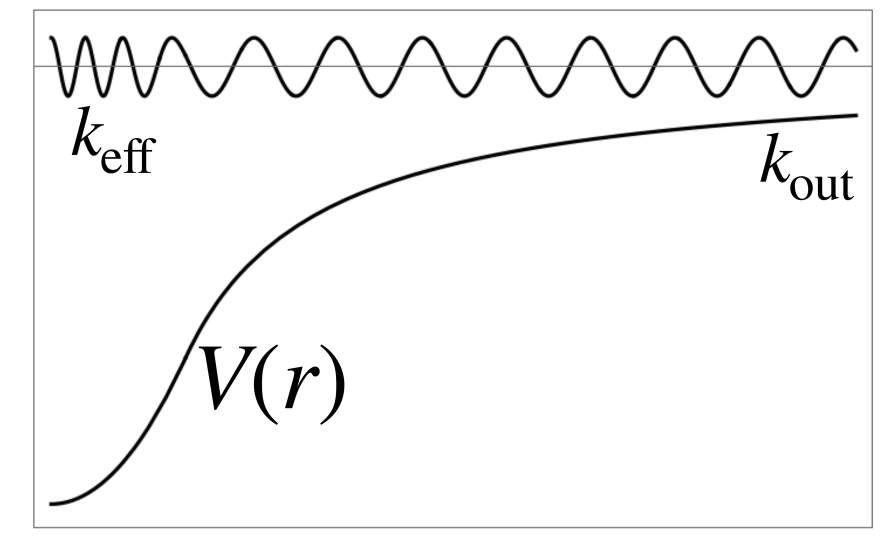
$$\int \mathrm{d}^3x \;\; \exp\left[\mathrm{i}\chi_0(x) + \frac{1}{E}\chi_1(x) + \ldots\right] \quad \mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot\mathbf{x}}$$

Solution: Taylor expand eikonal phases and count $x^n \sim O(1/E^n)$ by a generalization of the Reimann-Lebesque lemma

$$\int \mathrm{d}z \; z^n \;\; \mathrm{e}^{\mathrm{i}\xi z} \sim O(1/E^{n+1}) \qquad \text{as} \qquad \xi \to \infty$$

Modified Effective Momentum Approximation

Phenomenological ansatz introduced by Engel can derived ***systematically*** justified via matrix element **power counting**.



Retain the following terms

1. First deriivative of $\chi_0(x)$. evaluated at the origin.
2. Imaginary part of $\chi_1(x)$ evaluated at the origin.

1. Effective momentum $\quad k \to k_{\text{eff}} = k + [\partial_z \chi_0](0)$
2. Focusing factor $\quad u_p \to \mathrm{e}^{-\mathrm{Im}\chi_1(0)/E} \times u_p$

Transverse Momentum Fluctuations

Working to higher order in 1/E we find Gaussian integrals that lead to fluctuations in the lepton's transverse momentum.

$$\langle k_\perp^2 \rangle \sim Z\alpha \left\langle r^{-2} \right\rangle$$

Controlled by ``inverse moments'' of charge distribution

$$\left\langle r^{-2}\right\rangle = \int \mathrm{d}^3x \; \rho_Z(x) \; r^{-2}$$

Conclusions

Controllable high-energy expansion valid for large $Z\alpha$.

Developed within a systematic and improvable framework.

Phenomenological applications ongoing. Papers coming soon.

This work was supported by the DOE, under Award Number DE-SC0019095. R.P. was supported by funds from the Intensity Frontier Fellowship at Fermilab. O.T. is supported by the Visiting Scholars Award Program of the URA.

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