

Neutrino Mixing by Modifying the Yukawa Coupling structure of Constrained Sequential Dominance(arXiv:2005.04023)

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Abstract

- Tri-bimaximal mixing pattern in the neutrino sector has been explained by certain Yukawa coupling structure in the model of constrained sequential dominance (CSD). We propose a phenomenological model by modifying the CSD Yukawa coupling structure.
- Essentially we add small complex parameters to CSD Yukawa structure and demonstrate that neutrino mixing angles deviate from the TBM pattern.
- We compute numerical values of the small complex parameters in our analysis and we also construct a model based on flavor symmetry and flavon fields in order to justify our Yukawa coupling structure.

Sequential Dominance and CSD

Assuming the charged lepton mass matrix to be diagonal, we add three right handed neutrinos ν_R^{atm} , ν_R^{sol} and ν_R^{dec} to the Standard Model. Yukawa Lagrangian for neutrino mass is

$$\mathcal{L}^{Yuk} = \left(\frac{H_u}{v_u}\right)(d\bar{L}_e + e\bar{L}_\mu + f\bar{L}_\tau)\nu_R^{atm} + \left(\frac{H_u}{v_u}\right)(a\bar{L}_e + b\bar{L}_\mu + c\bar{L}_\tau)\nu_R^{sol} + \left(\frac{H_u}{v_u}\right)(a'\bar{L}_e + b'\bar{L}_\mu + c'\bar{L}_\tau)\nu_R^{dec} + H.c. \quad (1)$$

Majorana Lagrangian is given by

$$\mathcal{L}_\nu^M = M_{sol}\bar{\nu}_R^{sol}(\nu_R^{sol})^c + M_{atm}\bar{\nu}_R^{atm}(\nu_R^{atm})^c + M_{dec}\bar{\nu}_R^{dec}(\nu_R^{dec})^c. \quad (2)$$

So,

$$M_R = \begin{pmatrix} M_{atm} & 0 & 0 \\ 0 & M_{sol} & 0 \\ 0 & 0 & M_{dec} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}, \quad (3)$$

$$m_\nu = m_D M_R^{-1} m_D^T. \quad (3)$$

$$M_{atm} \ll M_{sol} \ll M_{dec}, \quad \frac{(e, f)^2}{M_{atm}} \gg \frac{(a, b, c)^2}{M_{sol}} \gg \frac{(a', b', c')^2}{M_{dec}}. \quad (4)$$

- Third column of m_D and m_R can be decoupled with the above conditions.
- Three right handed neutrino model \implies Two right handed neutrino model.
- m_D and M_R take the form

$$M_R = \begin{pmatrix} M_{atm} & 0 \\ 0 & M_{sol} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a \\ e & b \\ f & c \end{pmatrix} \quad (5)$$

$$d = 0, \quad e = f, \quad a = b = -c. \quad (5)$$

After performing above mentioned decoupling and above condition, Dirac and Majorana mass matrices take the form

$$m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{atm} & 0 \\ 0 & M_{sol} \end{pmatrix}. \quad (6)$$

Putting this m_D and M_R in m_ν of Eq.(4), we find

$$U_{TBM}^T m_\nu U_{TBM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{atm}} \end{pmatrix}, \quad U_{TBM} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{4}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (7)$$

From the unitary matrix U_{TBM} one can extract the three neutrino mixing angles and we can see that they have the TBM values.

Our Model and deviation from TBM pattern

We consider a phenomenological model where we modified the Dirac mass matrix as

$$m'_D = m_D + \Delta m_D, \quad m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad \Delta m_D = \begin{pmatrix} \epsilon_1 & a\epsilon_4 \\ \epsilon_2 & a\epsilon_5 \\ \epsilon_3 & a\epsilon_6 \end{pmatrix}, \quad (8)$$

Here ϵ_i , $i=1..6$ are complex parameters. Hence the seesaw formula for active neutrinos

$$m_\nu^s = m'_D M_R^{-1} (m'_D)^T \quad (9)$$

since we are in a basis where charged leptons are diagonalized, this form of mass matrix should be diagonalized by PMNS matrix which in terms of PDG convention is

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \quad (10)$$

Here $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. In order to simplify our calculations we parameterize s_{12} and s_{23} as

$$s_{12} = \frac{1}{\sqrt{3}}(1+r), \quad s_{23} = \frac{1}{\sqrt{2}}(1+s) \quad (11)$$

So, if we allow non-zero ϵ_i in our model, we can get non-zero r, s, s_{13} . The relation for diagonalization of the seesaw mass matrix is

$$m_\nu^d = U_{PMNS}^T m_\nu^s U_{PMNS} = \text{diag}(m_1, m_2, m_3) \quad (12)$$

In the limit where ϵ_i, r, s and s_{13} tends to zero, we get the leading order expressions for neutrino masses.

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}}, \quad m_3 = \frac{2e^2}{M_{atm}}. \quad (13)$$

Here up to leading order m_1 is zero. As a result of this we can have only normal hierarchy of neutrino masses. So, we can fit m_2 and m_3 to be square root of solar ($\sqrt{\Delta m_{sol}^2}$) and atmospheric ($\sqrt{\Delta m_{atm}^2}$) mass squared differences. We expect the following order of estimation

$$\frac{a^2}{M_{sol}} \sim \sqrt{\Delta m_{sol}^2}, \quad \frac{e^2}{M_{atm}} \sim \sqrt{\Delta m_{atm}^2} \quad (14)$$

First Order Correction

From the neutrino oscillation data one can notice that $\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} = s_{13}$. This would imply $m_2/m_3 = s_{13}$ in our model. To incorporate this order of estimation we reexpress our diagonalization formula as

$$\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d \equiv \frac{1}{\sqrt{\Delta m_{atm}^2}} U_{PMNS}^T m_\nu^s U_{PMNS} = \text{diag}\left(\frac{m_1}{\sqrt{\Delta m_{atm}^2}}, \frac{m_2}{\sqrt{\Delta m_{atm}^2}}, \frac{m_3}{\sqrt{\Delta m_{atm}^2}}\right) \quad (15)$$

Up to first order in ϵ_i , m_ν^s can be expanded as

$$m_\nu^s = m_{\nu(0)}^s + m_{\nu(1)}^s, \quad m_{\nu(0)}^s = m_D M_R^{-1} m_D^T, \quad m_{\nu(1)}^s = m_D M_R^{-1} (\Delta m_D)^T + \Delta m_D M_R^{-1} m_D^T \quad (16)$$

Similarly, up to first order in r, s and s_{13} , the expansion for U_{PMNS} is

$$U_{PMNS} = U_{TBM} + \Delta U, \quad \Delta U = \begin{pmatrix} -\frac{r}{\sqrt{6}} & \frac{r}{\sqrt{3}} & e^{-i\delta_{CP}} s_{13} \\ -\frac{r+s}{\sqrt{6}} - \frac{e^{i\delta_{CP}} s_{13}}{\sqrt{3}} & -\frac{r+2s+\sqrt{2}e^{i\delta_{CP}} s_{13}}{2\sqrt{3}} & \frac{s}{\sqrt{2}} \\ \frac{r+s}{\sqrt{6}} - \frac{e^{i\delta_{CP}} s_{13}}{\sqrt{3}} & \frac{r-2s-\sqrt{2}e^{i\delta_{CP}} s_{13}}{2\sqrt{3}} & -\frac{s}{\sqrt{2}} \end{pmatrix} \quad (17)$$

Terms up to first order in $\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d$ are given below.

$$\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d = \frac{1}{\sqrt{\Delta m_{atm}^2}} (m_{\nu(0)}^d + m_{\nu(1)}^d), \quad m_{\nu(0)}^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{atm}} \end{pmatrix}, \quad m_{\nu(1)}^d = \begin{pmatrix} x'_{11} & x'_{12} & x'_{13} \\ x'_{12} & x'_{22} & x'_{23} \\ x'_{13} & x'_{23} & x'_{33} \end{pmatrix}, \quad (18)$$

$$x'_{11} = 0, \quad x'_{12} = 0, \quad x'_{13} = \frac{e^2}{\sqrt{6}M_{atm}} [\sqrt{2}(2\epsilon_1 - \epsilon_2 + \epsilon_3 + 2s) - 4e^{i\delta_{CP}} s_{13}], \quad x'_{22} = 0,$$

$$x'_{23} = \frac{e^2}{\sqrt{3}M_{atm}} [\sqrt{2}(\epsilon_1 + \epsilon_2 - \epsilon_3 - 2s) - 2e^{i\delta_{CP}} s_{13}], \quad x'_{33} = \frac{2e^2}{M_{atm}} (\epsilon_2 + \epsilon_3)$$

Now, equating the diagonal elements on both sides of Eq. (15), we get the expressions for the three neutrino masses, which are given below

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}}, \quad m_3 = \frac{2e^2}{M_{atm}} + \frac{2e^2(\epsilon_2 + \epsilon_3)}{M_{atm}} \quad (19)$$

From the above equations we can see that only m_3 get correction at the first order level. Now, from the off-diagonal elements of Eq. (15), we get the following expressions.

$$\epsilon_1 = \sqrt{2}e^{i\delta_{CP}} s_{13}, \quad \epsilon_2 - \epsilon_3 = 2s \quad (20)$$

Second Order Correction

Expansion for m_ν^s and U_{PMNS} , up to second order in ϵ_i, r, s and s_{13} are given below

$$m_\nu^s = m_{\nu(0)}^s + m_{\nu(1)}^s + m_{\nu(2)}^s, \quad m_{\nu(2)}^s = \Delta m_D M_R^{-1} (\Delta m_D)^T, \quad (21)$$

$$U_{PMNS} = U_{TBM} + \Delta U + \Delta^2 U,$$

$$\Delta^2 U = \begin{pmatrix} -\frac{3r^2+4s^2}{4\sqrt{6}} & -\frac{s_{13}^2}{2\sqrt{3}} & 0 \\ \frac{\sqrt{2}(rs+s^2)+(r-2s)s_{13}e^{i\delta_{CP}}}{2\sqrt{3}} & -\frac{3r^2+4rs-8s^2-4\sqrt{2}(r+s)s_{13}e^{i\delta_{CP}}}{8\sqrt{3}} & -\frac{s_{13}^2}{2\sqrt{2}} \\ \frac{\sqrt{2}rs+(r+2s)s_{13}e^{i\delta_{CP}}}{2\sqrt{3}} & \frac{3r^2+4rs-4\sqrt{2}(r-s)s_{13}e^{i\delta_{CP}}}{8\sqrt{3}} & -\frac{2s^2+s_{13}^2}{2\sqrt{2}} \end{pmatrix}$$

$\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d$ can be computed up to second order in ϵ_i, r, s, s_{13} and $\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}$. Now, after using Eq.(20), the second order terms in $\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d$ will be simplified. These are given below.

$$\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d = \frac{1}{\sqrt{\Delta m_{atm}^2}} \begin{pmatrix} x''_{11} & x''_{12} & x''_{13} \\ x''_{12} & x''_{22} & x''_{23} \\ x''_{13} & x''_{23} & x''_{33} \end{pmatrix}, \quad (22)$$

$$x''_{11} = 0, \quad x''_{12} = \frac{a^2}{\sqrt{2}M_{sol}} (2\epsilon_4 - \epsilon_5 + \epsilon_6 - 3r),$$

$$x''_{13} = \frac{e^2}{\sqrt{3}M_{atm}} [s(3s - 2\sqrt{2}e^{i\delta_{CP}} s_{13}) + 2\epsilon_3(s - \sqrt{2}e^{i\delta_{CP}} s_{13})],$$

$$x''_{22} = \frac{2a^2}{M_{sol}} (\epsilon_4 + \epsilon_5 - \epsilon_6),$$

$$x''_{23} = \frac{\sqrt{3}a^2}{2M_{sol}} [\sqrt{2}(\epsilon_5 + \epsilon_6 + 2s) + 2e^{-i\delta_{CP}} s_{13}]$$

$$- \frac{e^2}{\sqrt{3}M_{atm}} [2\epsilon_3(\sqrt{2}s + e^{i\delta_{CP}} s_{13}) + s(3\sqrt{2}s + 2e^{i\delta_{CP}} s_{13})],$$

$$x''_{33} = \frac{2e^2}{M_{atm}} (\epsilon_3^2 + 2\epsilon_3s + 2s^2 + s_{13}^2)$$

Now, after equating the diagonal elements on both sides of Eq. (15), we get corrections up to second order to neutrino masses, which are given below.

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}} + \frac{2a^2}{M_{sol}} (\epsilon_4 + \epsilon_5 - \epsilon_6),$$

$$m_3 = \frac{2e^2}{M_{atm}} + \frac{4e^2}{M_{atm}} (\epsilon_3 + s) + \frac{2e^2}{M_{atm}} (s_{13}^2 + \epsilon_3^2 + 2\epsilon_3s + 2s^2) \quad (23)$$

After demanding that the off-diagonal elements of $\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^d$ should be zero, we get the following three relations.

$$2\epsilon_4 - \epsilon_5 + \epsilon_6 = 3r,$$

$$s(3s - 2\sqrt{2}e^{i\delta_{CP}} s_{13}) + 2\epsilon_3(s - \sqrt{2}e^{i\delta_{CP}} s_{13}) = 0,$$

$$\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} e^{i\phi} [\sqrt{2}(\epsilon_5 + \epsilon_6 + 2s) + 2e^{-i\delta_{CP}} s_{13}] - [2\epsilon_3(\sqrt{2}s + e^{i\delta_{CP}} s_{13}) + s(3\sqrt{2}s + 2e^{i\delta_{CP}} s_{13})] = 0. \quad (24)$$

Numerical Results

Since ϵ_i are complex, we have resolved them in to real and imaginary parts, whose expressions are given below.

$$\epsilon_i = \text{Re}(\epsilon_i) + i\text{Im}(\epsilon_i). \quad (25)$$

In order to be compatible with neutrino oscillation observables, we have obtained the allowed ranges for $\text{Re}(\epsilon_i)$ and $\text{Im}(\epsilon_i)$. These results are given in Table 1.

	$\text{Re}(\epsilon_1)$	$\text{Im}(\epsilon_1)$	$\text{Re}(\epsilon_2)$	$\text{Im}(\epsilon_2), \text{Im}(\epsilon_3)$	$\text{Re}(\epsilon_3)$
ϕ	ϵ_4	$\text{Re}(\epsilon_5)$	$\text{Im}(\epsilon_5)$	$\text{Re}(\epsilon_6)$	$\text{Im}(\epsilon_6)$
0	0.1	(-0.084, 0.462)	(-0.119, 0.101)	(-0.375, 0.168)	(-0.119, 0.101)
0	-0.1	(-0.282, 0.26)	(-0.119, 0.101)	(-0.175, 0.367)	(-0.119, 0.101)
0	0.1i	(-0.182, 0.362)	(-0.019, 0.199)	(-0.275, 0.267)	(-0.219, 0.001)
0	-0.1i	(-0.182, 0.362)	(-0.219, 0.001)	(-0.275, 0.267)	(-0.019, 0.199)

Table 1: Allowed ranges for the real and imaginary parts of the ϵ_i parameters.

The maximum values of $|\text{Re}(\epsilon_5)|$ and $|\text{Re}(\epsilon_6)|$ can be around 0.4 depending on ϵ_4 and ϕ values. For this reason we have computed allowed values for neutrino mixing angles and δ_{CP} by restricting $|\text{Re}(\epsilon_5)|$ and $|\text{Re}(\epsilon_6)|$ to be less than 0.23 for the case of $\phi = 0$ and $\epsilon_4 = 0.1$.

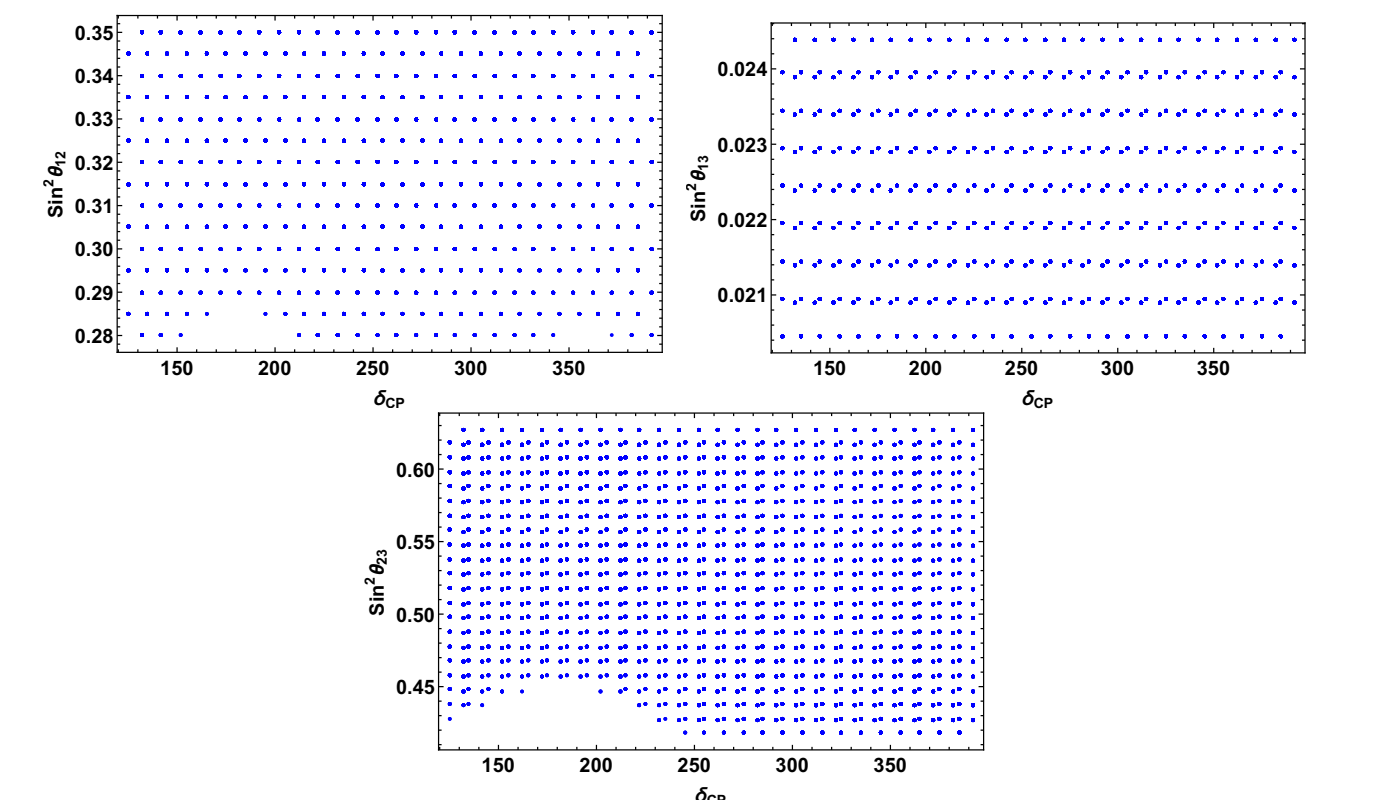


Figure 1: Allowed regions in neutrino mixing angles and δ_{CP} by demanding $|\text{Re}(\epsilon_5)|$ and $|\text{Re}(\epsilon_6)|$ to be less than 0.23, for the case of $\phi = 0$ and $\epsilon_4 = 0.1$. δ_{CP} is expressed in degrees.

A model for our Dirac Mass mixing

Here we construct a model in order to justify our Dirac mass matrix and also to explain the smallness of ϵ_i . In Table 2, charges assignments of the fields, which are relevant to neutrino sector, are given.

	ϕ_a	ϕ_s	ϕ'_a	ϕ'_s	ξ	χ_a	χ_s	ν_R^{atm}	ν_R^{sol}	L	H
$SU(3)$	3	3	3	3	1	1	1	1	3	1	1
Z_3	ω	ω^2	ω	ω^2	1	ω^2	ω	ω^2	ω	1	1
Z'_3	ω^2	ω^2	ω	ω	ω	ω	ω	ω	ω	1	1

Table 2: Charge assignments of the relevant fields under the flavor symmetry $SU(3) \times Z_3 \times Z'_3$ are given. Here, $\omega = e^{2\pi i/3}$. For other details, see the text.

With these charge assignments, the leading terms in the Lagrangian are

$$\mathcal{L} = \frac{\phi_a}{M_P} \bar{L}_R^{atm} H + \frac{\phi_s}{M_P} \bar{L}_R^{sol} H + \frac{\xi}{M_P M_P} \bar{L}_R^{atm} H + \frac{\xi}{M_P M_P} \bar{L}_R^{sol} H + \frac{\chi_a}{2} (\bar{\nu}_R^{atm})^c \nu_R^{atm} + \frac{\chi_s}{2} (\bar{\nu}_R^{sol})^c \nu_R^{sol} + h.c. \quad (26)$$

Here, $M_P \sim 2 \times 10^{18}$ GeV is the reduced Planck scale, which is the cutoff scale for this model. In order to explain the structure of Dirac mass matrix of CSD, we assume that these vevs to have the following pattern

$$\frac{\langle \phi_a \rangle}{M_P} = y_a \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{\langle \phi_s \rangle}{M_P} = y_s \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (27)$$

Here, y_a, y_s are dimensionless quantities. The vevs of ϕ'_a, ϕ'_s, ξ give sub-leading contribution to Yukawa couplings for neutrinos. Here, we need not assume any pattern for the vevs of ϕ'_a, ϕ'_s . Hence, after writing $\frac{\langle \xi \rangle}{M_P} = \epsilon$, we can have

$$\frac{\langle \xi \rangle \langle \phi'_a \rangle}{M_P M_P} = y_a \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \epsilon = y_a \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}, \quad \frac{\langle \xi \rangle \langle \phi'_s \rangle}{M_P M_P} = y_s \begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix} \epsilon = y_s \begin{pmatrix} \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix} \quad (28)$$

Here, y_i, y'_i , where $i = 1, \dots, 3$, are $\mathcal{O}(1)$ parameters. The vevs of χ_a and χ_s generate Majorana masses. Scalar vevs are found to be $\langle \chi_a \rangle, \langle \chi_s \rangle \sim 1$ TeV, $\langle \xi \rangle \sim 10^{17}$ GeV, $\langle \phi_a \rangle, \langle \phi_s \rangle, \langle \phi'_a \rangle, \langle \phi'_s \rangle \sim 10^{12}$ GeV.

There is a large hierarchy among these vevs. We can achieve this hierarchy, in this model, by appropriately fixing the relevant parameters in the scalar potential among the above mentioned scalar fields.

Conclusion

- In this work we have attempted to explain the neutrino mixing in order to explain neutrino oscillation data.
- Here we have considered a model where we have modified the Yukawa couplings of CSD model by introducing small ϵ_i parameters.
- Thereafter we followed an approximation procedure in order to diagonalize the seesaw formula and we have computed expressions up to second order level to neutrino mass and mixing angles in terms of small ϵ_i parameters.
- Using these expressions we have demonstrated that neutrino mixing can deviate from TBM pattern by choosing ϵ_i parameters.
- Finally we have constructed a model in order to justify the neutrino Yukawa coupling structure of our model.

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