The Violation of Equivalence Principle and Four Neutrino Oscillations in Long Baseline Neutrinos

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1 Introduction

- The energies of the neutrinos would experience a gravitational redshift while traversing through a gravitational field, $E' = \sqrt{g_{00}}E \rightarrow E' = (1-\phi)E$, where $\phi(=\frac{GM}{R})$ is the gravitational potential.
- In case the principle of equivalence is even very weakly violated, different types of neutrinos would couple to gravity with different strengths and these energy shifts would be different for different types of neutrinos. This will induce a phase difference between two types of neutrinos during its propagation.
- In this work we study the violation of equivalence principle (VEP) with 4 (3 active + 1 sterile) flavour neutrinos in the context of a Long Baseline (LBL) neutrino experiment. We calculate the number of right sign and wrong sing muon yields at the LBL neutrino experiment and their variations in case equivalence principle is violated. Thus we address the VEP effect in the case of 4-flavour neutrinos.
- Keeping this in view we consider the neutrinos coming from a neutrino fcatory (such as CERN) and we estimate the possible neutrino induced muon yield considering an LBL neutrino experiment with the end detector to be a iron calorimeter (ICAL) (such as the one proposed for India-Neutrino Based Observatory (INO)) for 4-flavour scenario. The baseline length L is calculated to be 7359 Km.

2 Formalism

The evolution equation in flavour basis due to the presence of the gravitational field is written as

$$i\frac{d}{dt}|\nu_\alpha\rangle = H'|\nu_\alpha\rangle \ , \tag{1}$$

where $H' = U'_{(4\times4)}H_G U'^{\dagger}_{(4\times4)}$ and for 4-flavour scenario $H_G = \mathrm{diag}(E_{G1}, E_{G2}, E_{G3}, E_{G4})$. If the equivalence principle is indeed violated, all the gravitational energy eigenvalues will induce phase differences to neutrino eigenstates and therefore we have

$$H_G = \mathrm{diag}\left((1-\phi\alpha_1)E, (1-\phi\alpha_2)E, (1-\phi\alpha_3)E, (1-\phi\alpha_4)E\right)$$

with $\phi\alpha_i = \frac{G_iM}{R} = \frac{GM}{R}\alpha_i$. In this case, the phase differences can be expressed as

$$\Delta E_{ij,G} = \frac{GM}{R}\Delta\alpha_{ij}E = \frac{GM}{R}(\alpha_i - \alpha_j)E = \phi\Delta\alpha_{ij}E = \Delta f_{ij}E \ , \tag{2}$$

where $\Delta f_{ij} = \frac{GM}{R}\Delta\alpha_{ij} = \Delta\alpha_{ij}\phi; i,j = 1,2,3,4$. After substracting $(1-\phi\alpha_1)E$ term from all the diagonal elements of H_G, we have $H_G = \mathrm{diag}(0, \Delta f_{21}E, \Delta f_{31}E, \Delta f_{41}E)$. The effective Hamiltonian of the system including both gravity effect and matter effect is given by

$$\begin{aligned} H'' &= H + H' + V \\ &= U_{(4\times4)}H_d U^{\dagger}_{(4\times4)} + U'_{(4\times4)}H_G U'^{\dagger}_{(4\times4)} + V \ . \end{aligned} \tag{3}$$

where $V = \mathrm{diag}(V_{CC}, 0, 0, -V_{NC})$ with $V_{CC} = \sqrt{2}G_F N_e$ and $V_{NC} = \frac{G_F N_n}{\sqrt{2}}$. We assume $U_{(4\times4)} = U'_{(4\times4)} = U$ and according to this assumption the effective Hamiltonian takes the form

$$\begin{aligned} H'' &= U(H_d + H_G)U^{\dagger} + V \\ &= U(\mathrm{diag}(0, \frac{\Delta m^2_{21}}{2E}, \frac{\Delta m^2_{31}}{2E}, \frac{\Delta m^2_{41}}{2E}) \\ &\quad + \mathrm{diag}(0, \Delta f_{21}E, \Delta f_{31}E, \Delta f_{41}E))U^{\dagger} + V \ . \end{aligned} \tag{4}$$

We neglect the terms Δm^2_{21} and Δf_{21} by assuming that both the neutrino mass eigenstates $|\nu_1\rangle, |\nu_2\rangle$ and gravity eigenstates $|\nu_{G1}\rangle, |\nu_{G2}\rangle$ are very close to each other. Thus the above Eq. (5) can be written as

$$\begin{aligned} H'' &= U\,\mathrm{diag}(0, 0, \frac{\Delta m^2_{31}}{2E} + \Delta f_{31}E, \frac{\Delta m^2_{41}}{2E} + \Delta f_{41}E)U^{\dagger} + V \\ &= U\,\mathrm{diag}(0, 0, \frac{\Delta\mu^2_{31}}{2E}, \frac{\Delta\mu^2_{41}}{2E})U^{\dagger} + V \ . \end{aligned} \tag{5}$$

In the above,

$$\begin{aligned} \frac{\Delta\mu^2_{31}}{2E} &= \frac{\Delta m^2_{31}}{2E} + \Delta f_{31}E \ , \\ \frac{\Delta\mu^2_{41}}{2E} &= \frac{\Delta m^2_{41}}{2E} + \Delta f_{41}E \ . \end{aligned} \tag{6}$$

Therefore, the oscillation probability for a neutrino $|\nu_\alpha\rangle$ having flavour α oscillate to a neutrino $|\nu_\beta\rangle$ of flavour β is given by the expression

$$P_{\nu_\alpha\rightarrow\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U^m_{\alpha i}U^m_{\beta i}U^m_{\alpha j}U^m_{\beta j}\sin^2\left(\frac{\pi L}{\lambda_{ij}}\right) \ , \tag{7}$$

where $\lambda_{ij} = \frac{2\pi}{E'_j - E'_i} = \frac{2\pi}{\Delta E'_{ij}}$. For baseline L = 7359 Km, the matter induced oscillations are effective for three Earth layers as shown in Fig. 1.

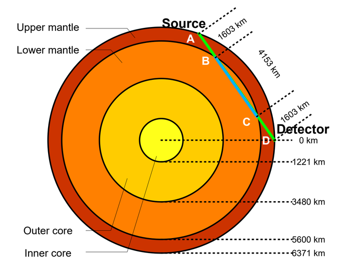

Figure 1: . Different layers of earth and projected travel path of neutrinos for long basline neutrino detector placed at 7359 km from source.

The density of the upper (up) and the lower $(lower)$ mantle are $\rho_{up} = 3.9$ gm/cc and $\rho_{low} = 4.5$ gm/cc [Phys. Earth Planet. Interiors **25**, 297-356 (1981)] respectively. The probability amplitude is computed as

$$A_{ll'} = \sum_{k,k',k'',\alpha,\beta} A_{lk}A^{up}_{kk}(d)A_{k\alpha}A_{\alpha k'}A^{low}_{k'k'}(D)A_{k'\beta}A_{\beta k''}A^{up}_{k''k''}(d)A_{k''l'} \ , \tag{8}$$

where $l, l', \alpha, \beta = e, \mu, \tau, s$; $k, k', k'' = 1, 2, 3, 4$.

3 Calculation and Results

We have chosen two values for Δm^2_{41} namely, $\Delta m^2_{41} = 1\times10^{-3}$ eV^2 and $\Delta m^2_{41} = 3\times10^{-3}$ eV^2 and we consider $\Delta f_{31} = 5\times10^{-27}$ as earlier studies of VEP with IceCube neutrino data by Esmaili *et. al* set a stringent bound on $\Delta f_{31} \leq 7\times10^{-27}$ [Phys. Rev. D **89**, no.11, 113003 (2014)]. We use $\Delta m^2_{21} = 7.53\times10^{-5}\mathrm{eV}^{-2}$, $\Delta m^2_{31} = 2.5\times10^{-3}\mathrm{eV}^{-2}$ (the best fit values).

Set	θ_{14}	θ_{24}	θ_{34}	θ_{12}	θ_{23}	θ_{13}
1	3.6°	4.0°	18.48°	33.96°	48.3°	8.61°
2	2.5°	10.0°	30.0°	33.96°	48.3°	8.61°

Table 1: Chosen 4-flavour mixing angle parameter sets and the best fit values of the 3 active neutrino mixing angle.

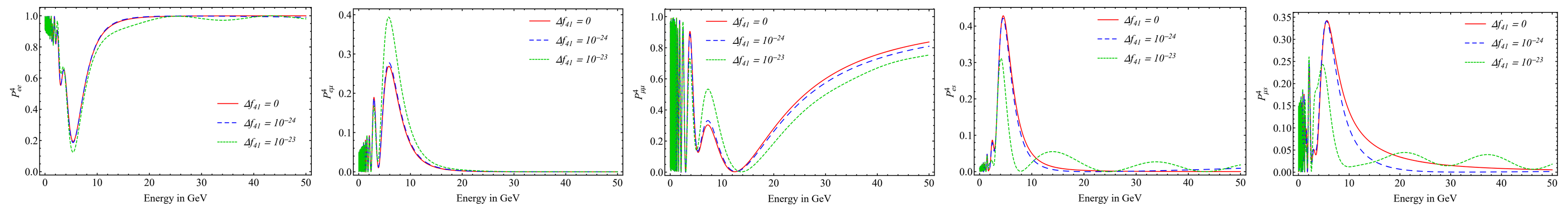

Figure 2: . Neutrino oscillation probabilities in matter for a fixed value of $\Delta m^2_{41} = 1\times10^{-3}$ eV^2, $\Delta f_{31} = 5\times10^{-27}$ and for Set-1 with baseline length L = 7000 km.

3.1 Effect of Gravity Induced Oscillation on a LBL Neutrino Experiment

$$\begin{aligned} \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \ , \\ \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \ . \end{aligned} \tag{9}$$

- The $\nu_\mu(\bar{\nu}_\mu)$ will produce $\mu^-(\mu^+)$ at the detector by charged current (CC) interaction with the detector material.
- If it is pure μ^- at the source then only ν_μ beam will propagate along the baseline and at the detector μ^- will produce $\Rightarrow$ right sign muon $\Longrightarrow$ disappearence channel ($\nu_\mu \rightarrow \nu_x, x \neq \mu$).
- If the detector detects a μ^+, then it must be that $\bar{\nu}_\mu$ reaches the detector and $\bar{\nu}_\mu$ can only be created through the oscillation $\bar{\nu}_e \rightarrow \bar{\nu}_\mu \Rightarrow$ wrong sign muon $\Longrightarrow$ appearence channel.

Δm^2_{41} in eV^2	Δf_{41}	Right sign μ	Wrong sign μ
1×10^{-3}	0	3115191.8	6031.1
	10^{-24}	2973398.1	5920.1
	10^{-23}	2662702.8	6332.4
3×10^{-3}	0	3006509.5	8349.9
	10^{-24}	2896293.3	8465.6
	10^{-23}	2654969.2	7687.9

Table 2: The right sign μ yield and the wrong sign μ yield in the presence of gravity induced 4-flavour oscillations in matter for Set-1 and for the fixed values of $\Delta f_{31} = 5\times10^{-27}$. The muon injection energy is fixed at 50 GeV.

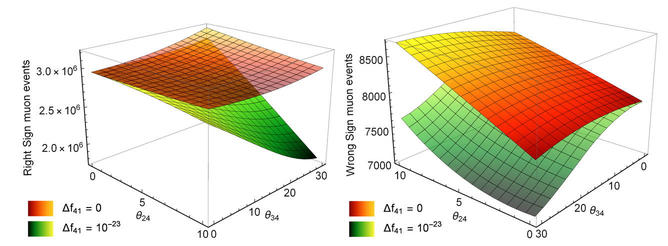

Figure 3: . (a) Right sign and (b) wrong sign muon yield variations with θ_{24} and θ_{34} for fixed $\theta_{14} = 2.5^0$, $\Delta f_{31} = 5\times10^{-27}$ and two chosen values of $\Delta f_{41} = 0, 10^{-23}$.

4 Summary

We observe that significant deviations in oscillation probabilities $P_{\nu_\alpha\rightarrow\nu_\beta}$ occur with the changes in Δf_{41} indicating that even a very weak violation of the equivalence principle will affect the oscillation probabilities over a chosen representative baseline of 7000 km.

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