Time evolution of lepton number of Majorana neutrinos: the Schrödinger picture

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Motivation

In our past work we studied the time evolution of lepton number (L) of Majorana neutrinos by constructing the Heisenberg operator for it . The initial state is a neutrino state with a definite lepton number but is not a mass eigenstate. The expectation value of L oscillates. we study the same phenomena by adopting the Schrödinger represetation.

Questions:

1) Time evolution of the state will be clarified
2) The understanding of the difference of two vacua one which the operator with a definite lepton number operates and the other on which the massive Majorana operator acts on.

Goals

Starting with an initial state with a definite lepton number and let it evolve with respect to time and construct the state at arbitrary time. With the state, one can compute the expectation value of lepton number.

$$< \Psi(0)|L_H(t)|\Psi(0) >=< \Psi(t)|L_S|\Psi(t) >$$

One can also check the previous result in Heisenberg rep..

Method

(1) Relation bet. neutrino and anti-neutrinos creation and annhilation opperators, (a(p), b(p)) and massive Majorana particles with different helicities (h) (aM(p, h=+1), aM(p, h=-1)) is obtained in the form Bogoliubov transformation.

(2) Rewrite the lepton number operator which is originally written with a and b in terms of operators of massive Majorana neutrinos.

$$L = \int'_{p\in A} \frac{d^3p}{(2\pi)^3 2|p|}\{(a^\dagger(p)a(p) - b^\dagger(p)b(p)) + (a^\dagger(-p)a(-p) - b^\dagger(-p)b(-p))\}$$

$$\begin{aligned} L = & \int'_{p\in A} \frac{d^3p}{(2\pi)^3}\frac{p}{2E_p^2}(a_M^\dagger(p,-)a_M(p,-) + a_M^\dagger(-p,-)a_M(-p,-) - a_M^\dagger(p,+)a_M(p,+) - a_M^\dagger(-p,+)a_M(-p,+)) \\ & + \int'_{p\in A} \frac{d^3p}{(2\pi)^3}\frac{im}{2E_p^2}\{a_M^\dagger(p,-)a_M^\dagger(-p,-) - a_M(-p,-)a_M(p,-) - a_M^\dagger(p,+)a_M^\dagger(-p,+) + a_M(-p,+)a_M(p,+)\} \end{aligned}$$

$$|0\rangle = \prod'_{p\subset A} (\cos^2\phi_p|0_M\rangle_p - \sin^2\phi_p|4_M\rangle_p + i\sin\phi_p\cos\phi_p|2_M\rangle_p)$$

The vacuum with 0 lepton number is expressed by the state in which one and two pairs of the Majorana neutrinos are condensed.

Bogoliubov transformation.

$$\begin{aligned} T_p(\phi_p) &= \mathrm{Exp}[\, i\,(g_+(p) + g_-(p))\phi_p] \\ g_\lambda(p) &= \alpha_M(p,\lambda)\alpha_M(-p,\lambda) + \alpha_M^\dagger(-p,\lambda)\,\alpha_M^\dagger(p,\lambda) \\ \alpha_M(p,\lambda) &= \frac{a_M(p,\lambda)}{\sqrt{2E_p(2\pi)^3\delta^{(3)}(0)}} \qquad \text{(Dimensionless operators)} \end{aligned}$$

$$\begin{aligned} B_{M\lambda}^\dagger(p) &= \alpha_M^\dagger(-p,\lambda)\,\alpha_M^\dagger(p,\lambda) \\ \cos\phi_p &= \frac{\sqrt{1+v_p}}{\sqrt{2}} \\ \sin\phi_p &= \frac{\sqrt{1-v_p}}{\sqrt{2}} \end{aligned}$$

Results

The state at time t is derived.

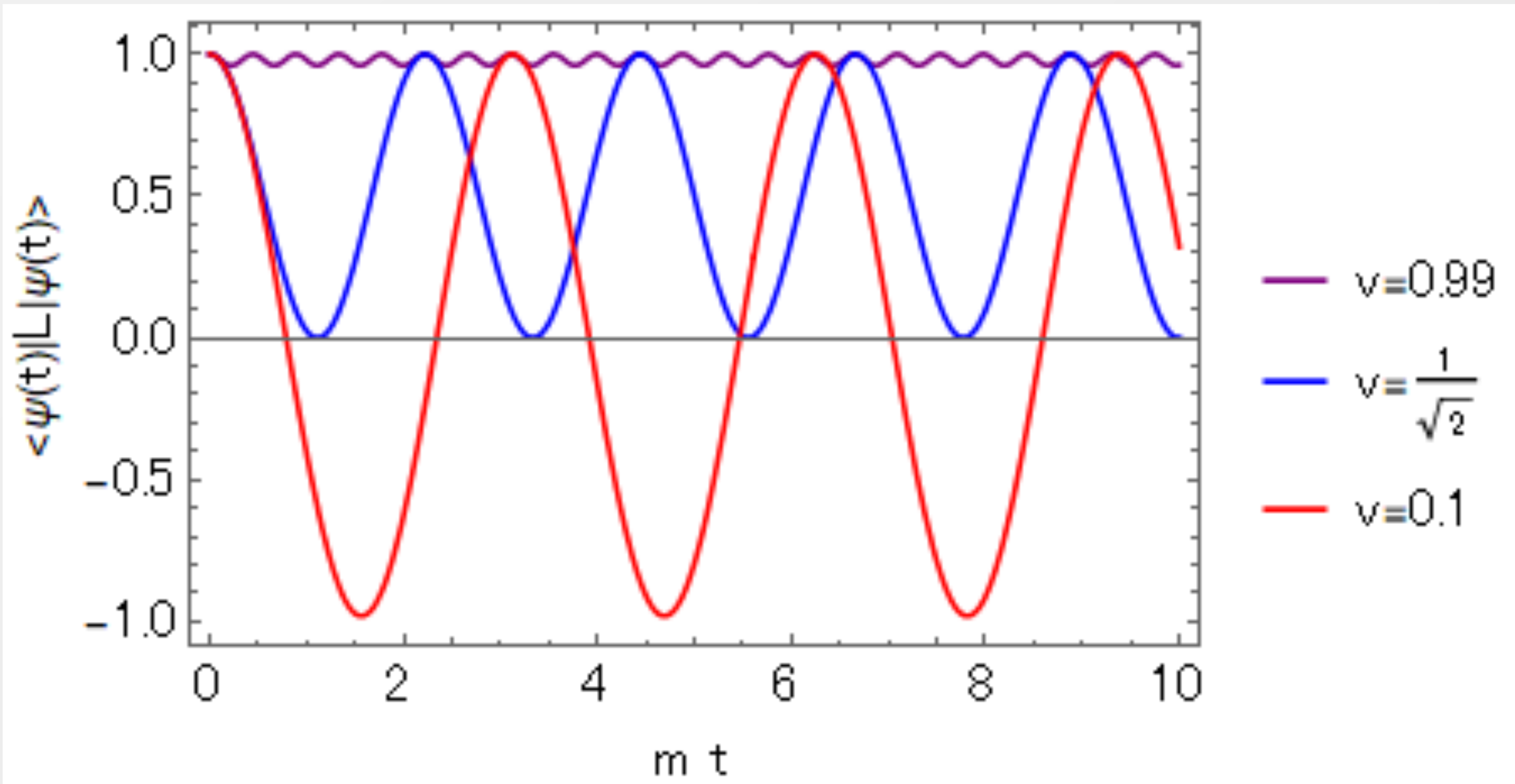
$$|\psi(t)\rangle_q = N_q\sqrt{\cos 2\phi_q}\, e^{-\mathrm{i}E_q t}\{\cos\phi_q + i\sin\phi_q\, e^{-i2E_q t}B_{M+}^\dagger(q)\}a_M^\dagger(q,-)|0_M\rangle_q$$

$$\hat{L} = \int'_{p\in A} \frac{Vd^3p}{(2\pi)^3}\hat{l}(p)$$

The Fock state with a momentum q is a superposition of a massive Majorana state and a state with a Majorana particle accomodated with a pair of Majorana particles. The mixing of the latter state is maximum for the velocity zero limit.

$$\langle\psi(t)|\hat{l}_q|\psi(t)\rangle = v^2 + (1-v^2)\cos\left[\frac{2mt}{\sqrt{1-v^2}}\right]$$

Results of the time evolution of L is the same as that obtained by Heisenberg picture.



References: ※ A. S. Adam, etc.; PTEP, ptab025, arXiv:2101.07751 [hep-ph] https://doi.org/10.1093/ptep/ptab025