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Leptonic Unitarity: Current & Future

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Based on [2008.01088] with Sebastian Ellis & Shirley Li

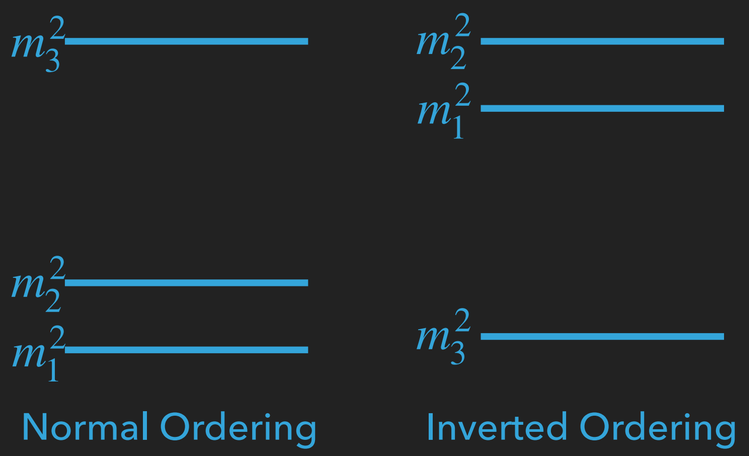
Outline

- What do we know about leptonic mixing when we assume unitarity?
  - How consistent is this assumption with experimental data?
  - Can we over-constrain this system?
  - How large is CP violation in the lepton sector?
- What about if we do not assume unitarity?
  - Different probes of unitarity.
  - Are the experimental data still consistent?
  - How will next-generation experiments probe unitarity?

Three-Neutrino Mixing with Unitarity

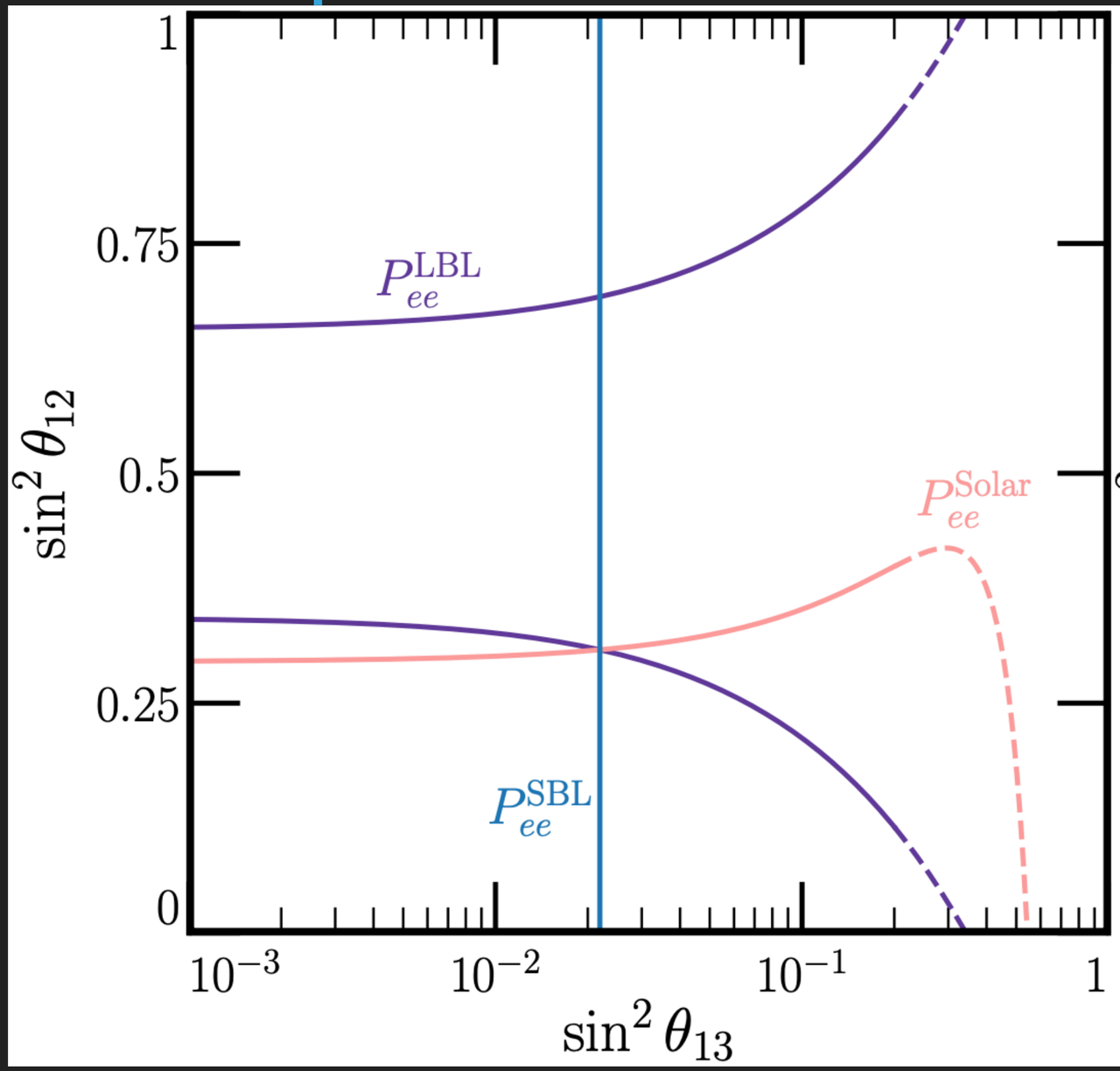
- Standard assumption for neutrino oscillations: three neutrinos exist, mix via a unitary, 3 x 3 matrix – the PMNS matrix. $\left(c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}\right)$

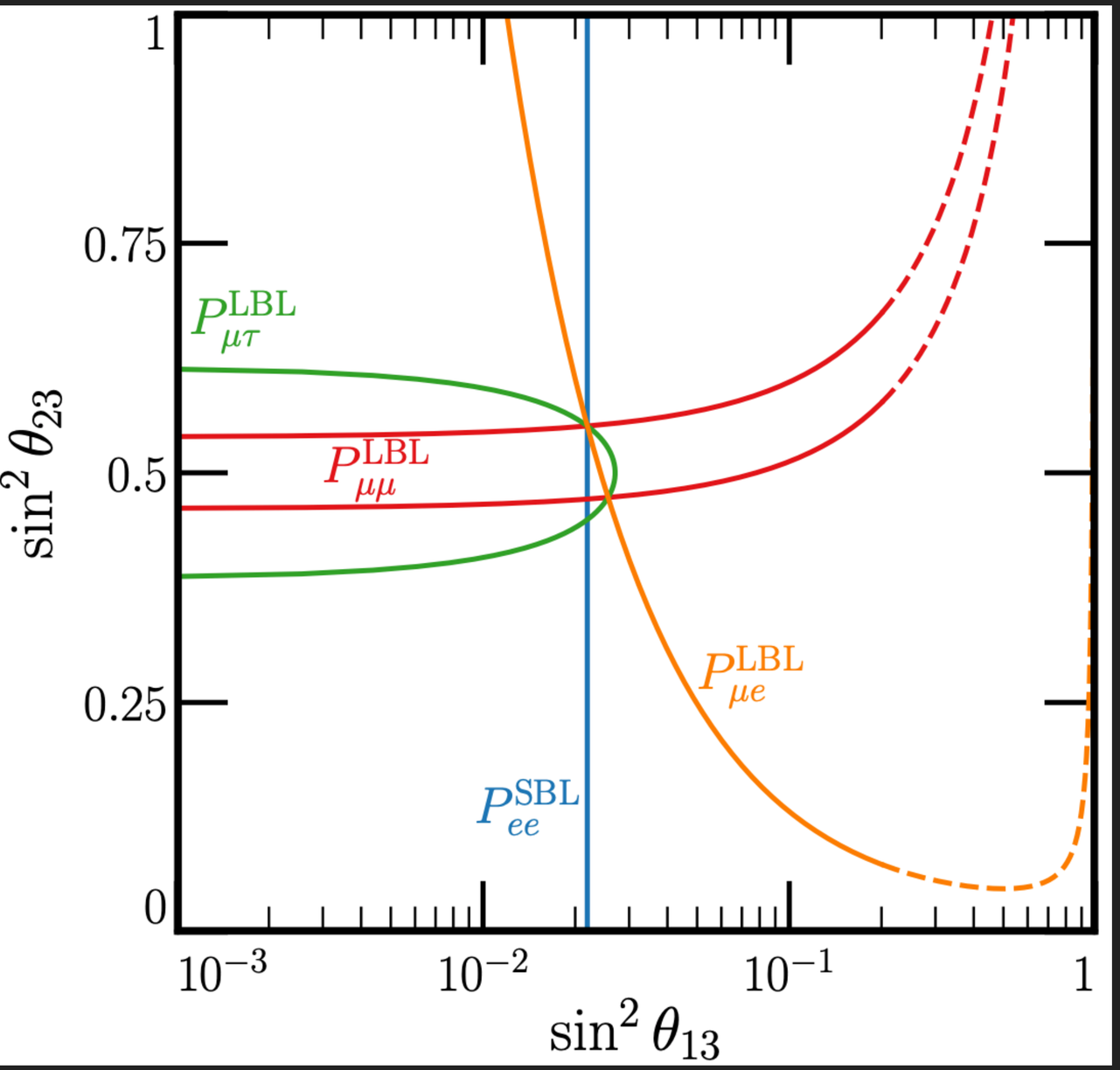
$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}$$



- Two non-zero mass-squared splittings relevant for three-neutrino mixing – different neutrino mass eigenstates have different energy and therefore time-evolve with a different relative phase as they propagate, leading to flavor oscillations.

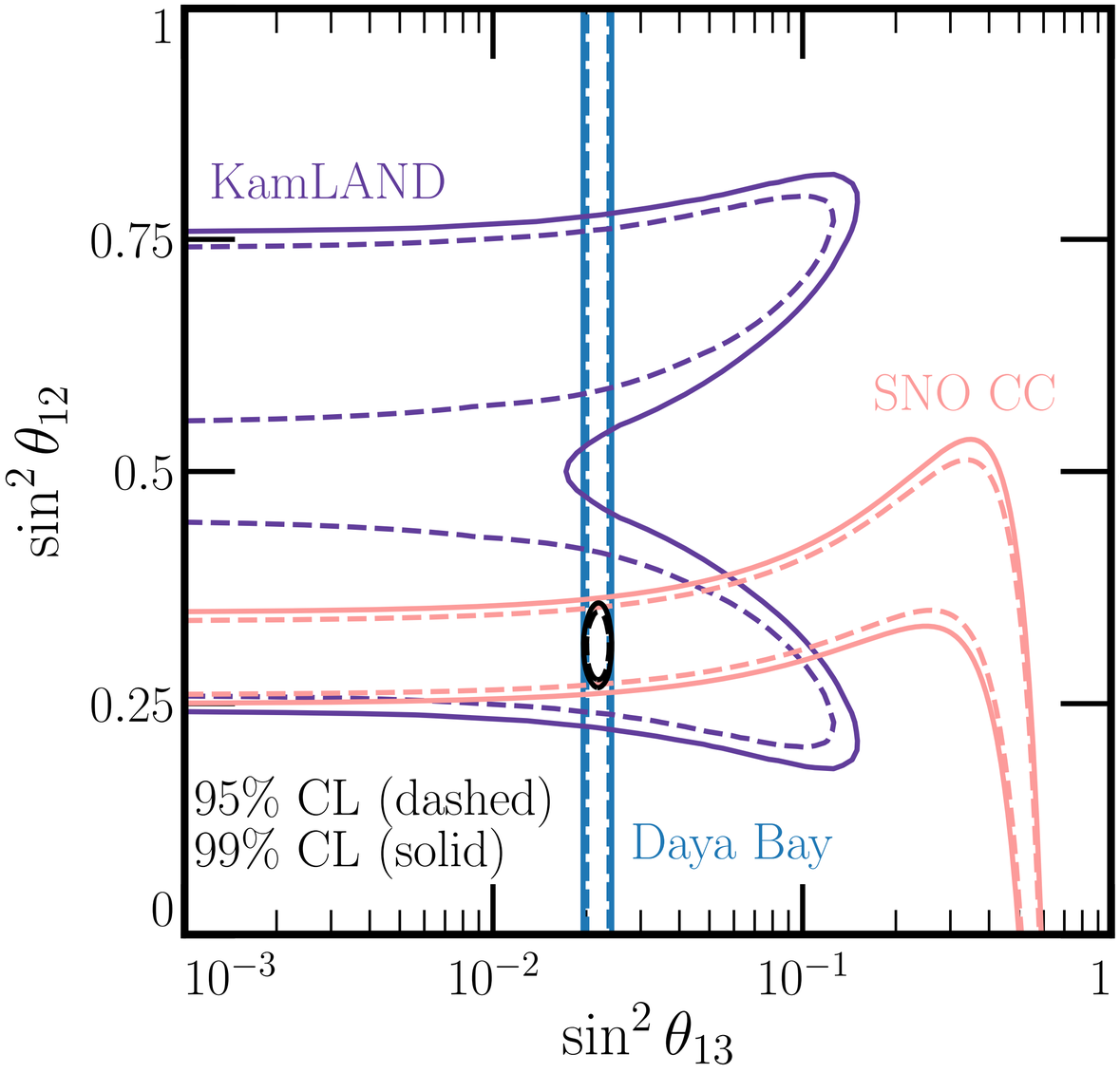
Perfect Amplitude Measurements

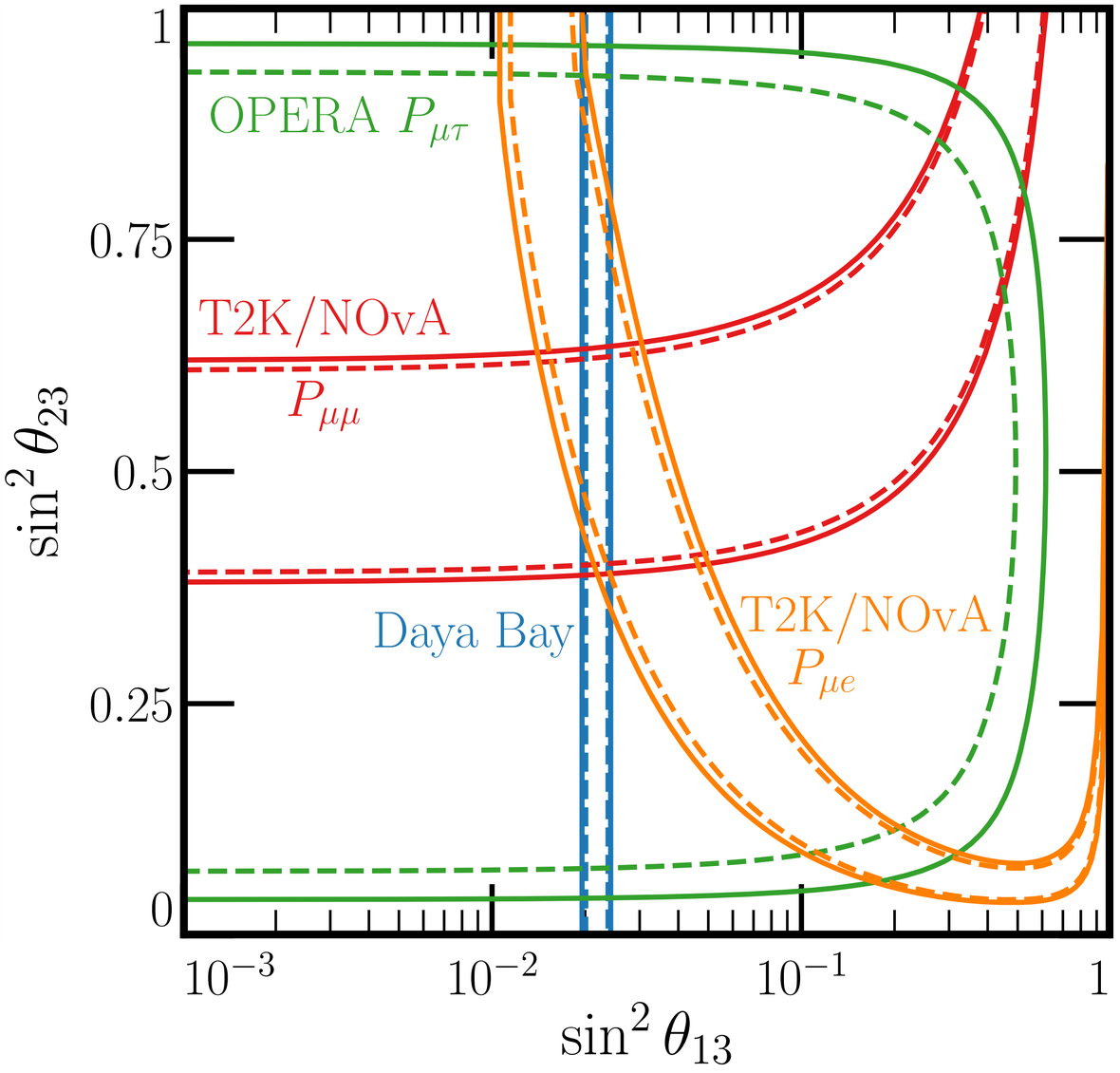




Ellis, KJK, Li, [2008.01088]

How does this compare with actual precision?





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What happens if we abandon the Unitarity assumption?

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}$$

- Using the PMNS Parameterization automatically implies unitarity. In order to perform an analysis that does not make this implicit assumption, we need a new parameterization.

$$U_{\mathrm{LMM}} = \begin{pmatrix} |U_{e1}| & |U_{e2}|\, e^{i\phi_{e2}} & |U_{e3}|\, e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|\, e^{i\phi_{\tau 2}} & |U_{\tau 3}|\, e^{i\phi_{\tau 3}} \end{pmatrix}$$

- 13 parameters (instead of 4). Can be thought of as the 4 original parameters plus nine conditions to be met if U is unitary.

In abandoning the Unitary Assumption, we introduced a new parameterization:

$$U_{\mathrm{LMM}} = \begin{pmatrix} |U_{e1}| & |U_{e2}|\, e^{i\phi_{e2}} & |U_{e3}|\, e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|\, e^{i\phi_{\tau 2}} & |U_{\tau 3}|\, e^{i\phi_{\tau 3}} \end{pmatrix}$$

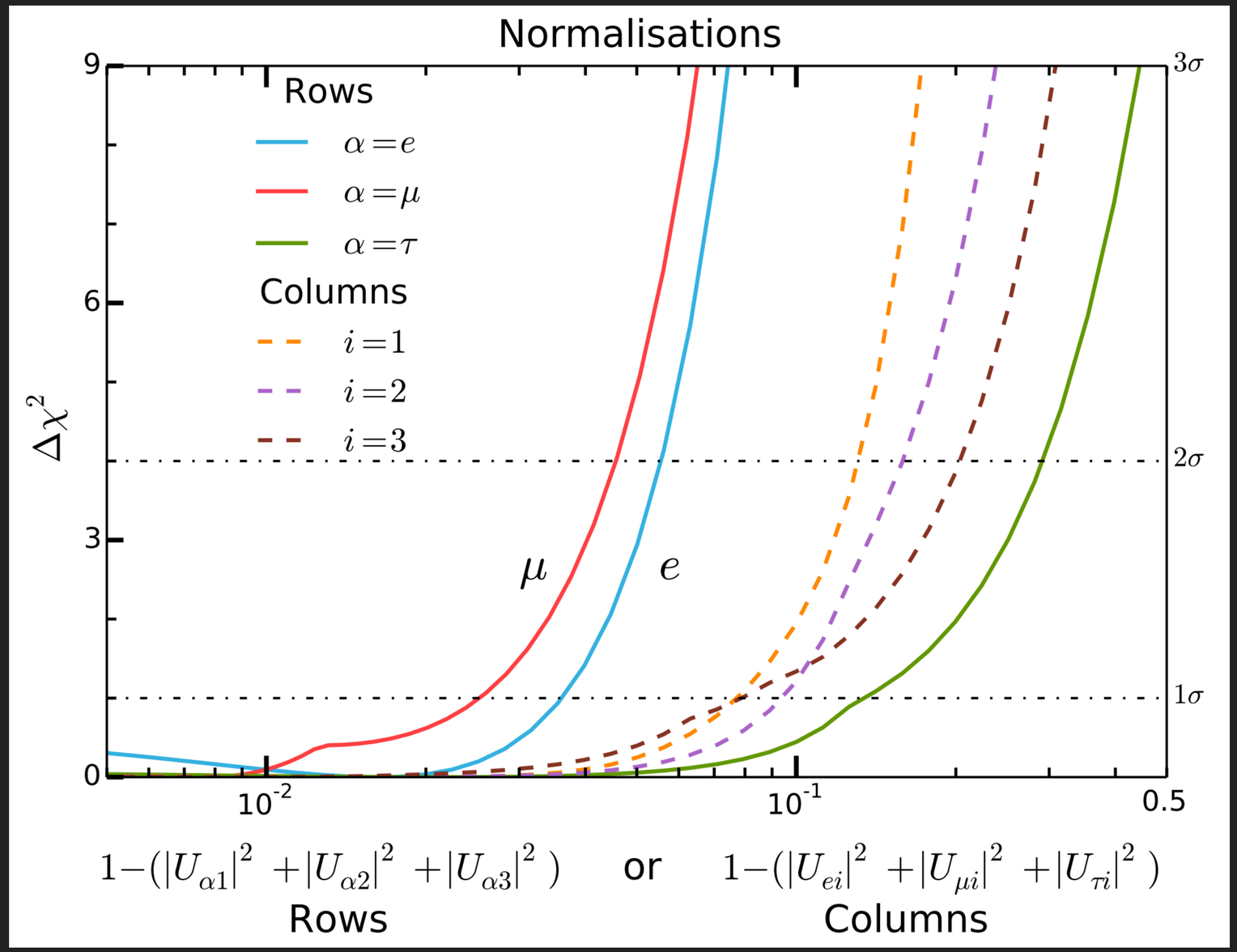
- If U is unitary, then certain conditions must be met on this matrix:

$$N_\alpha \equiv \sum_{i=1}^{3} \left| U_{\alpha i} \right|^2 = 1 \qquad t_{\alpha\beta} \equiv \sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^{*} = 0$$

$$N_k \equiv \sum_{\alpha = e,\mu,\tau} \left| U_{\alpha k} \right|^2 = 1 \qquad t_{kl} \equiv \sum_{\alpha = e,\mu,\tau} U_{\alpha k} U_{\alpha l}^{*} = 0$$

Constraints on Unitarity Normalizations circa 2015

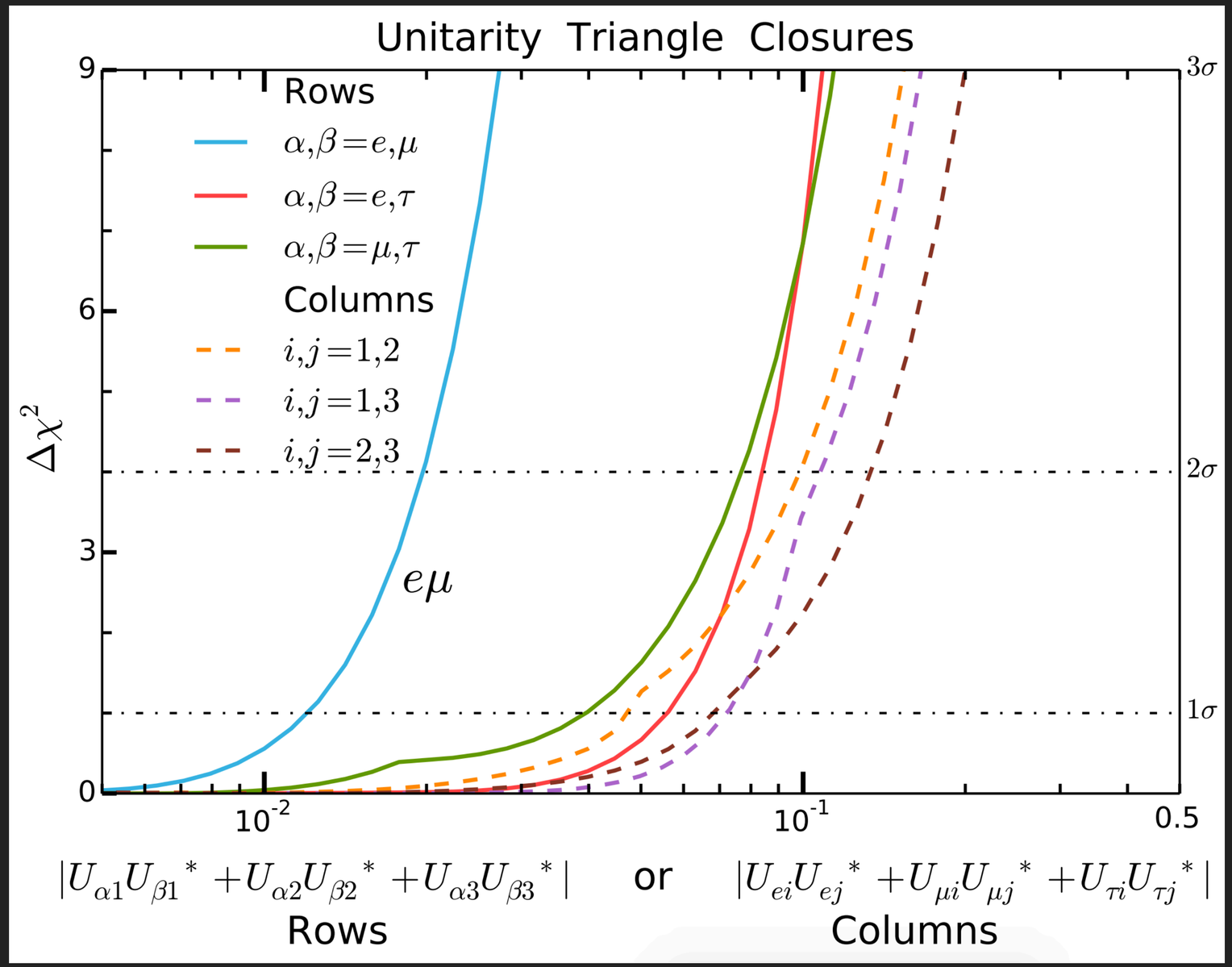
Parke and Ross-Lonergan, [1508.05095]



Except for more up-to-date T2K/ NOvA results, very similar approach to the one we take.

Constraints on Unitarity Closures circa 2015

Parke and Ross-Lonergan, [1508.05095]



- Except for more up-to-date T2K/NOvA results, very similar approach to the one we take.
- Row Triangle Closures are constrained strongly by sterile neutrino searches.
- Column Triangle Closures are constrained indirectly by Cauchy-Schwarz Inequalities.

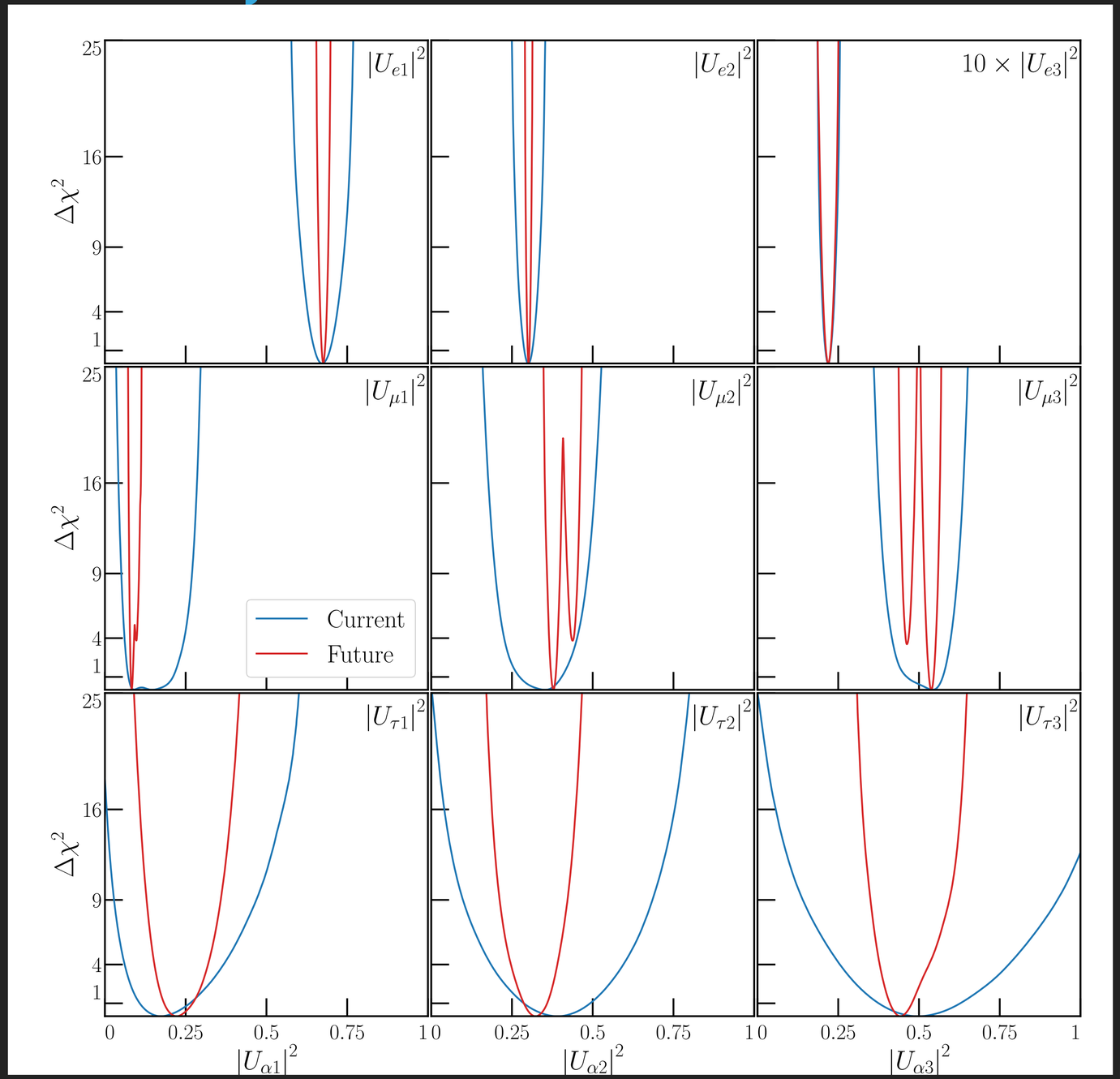
Theoretical Assumption & Cauchy–Schwarz Inequalities

- If U is not unitary, it is reasonable to ask why.
  - One straightforward explanation – heavier sterile neutrinos exist and our 3x3 mixing matrix is a subset of a larger, unitary matrix.
  - Small deviations have to do with small mixing between the light, mostly-active neutrinos and the heavy, mostly-sterile ones.
- If U is such a submatrix, further restrictions apply:

$$\left|t_{\alpha\beta}\right|^2 \leq \left(1 - N_\alpha\right)\left(1 - N_\beta\right)$$

$$\left|t_{kl}\right|^2 \leq \left(1 - N_k\right)\left(1 - N_l\right)$$

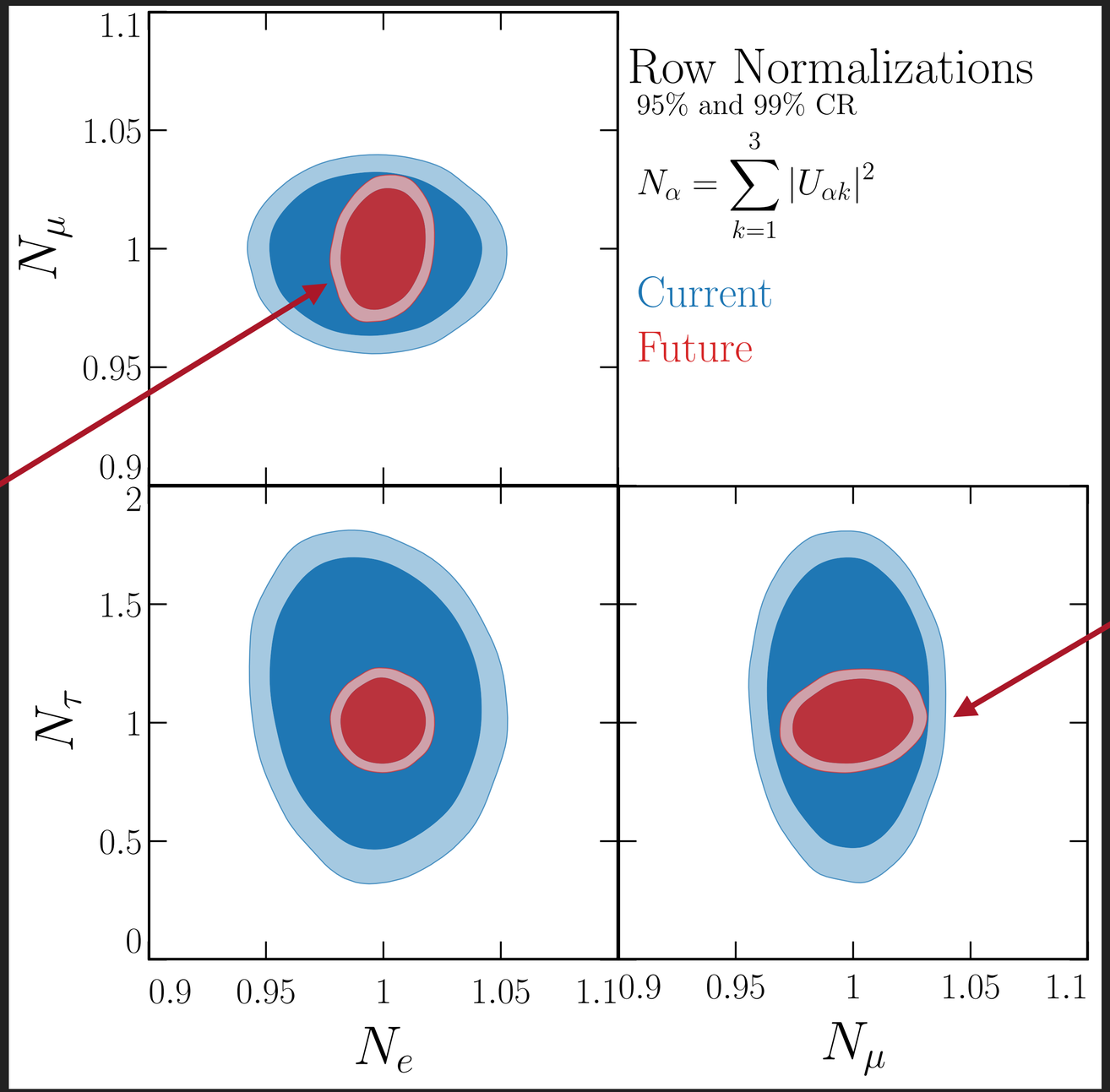
Agnostic to why U isn't Unitary, how well do we know the elements' magnitudes?



Ellis, KJK, Li, [2008.01088]

Without such an assumption, how well can we constrain the Normalizations/Closures?

Factor of ~2 increase in precision in measuring the electron row normalization, mostly thanks to JUNO.

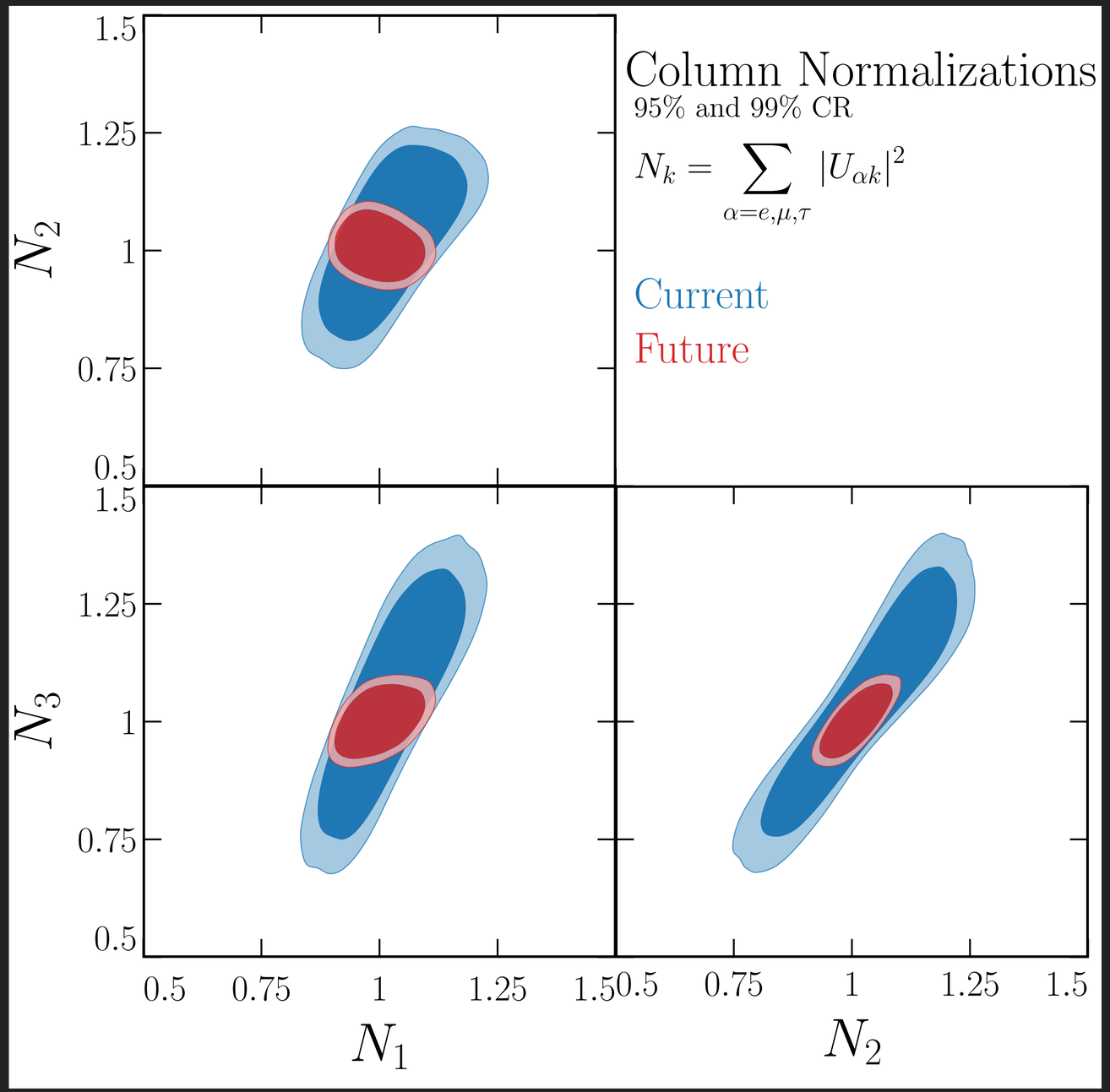


Factor of > 2 increase in precision in measuring the tau row normalization, mostly thanks to IceCube, some assistance from DUNE.

Ellis, KJK, Li, [2008.01088]

Without such an assumption, how well can we constrain the Normalizations/Closures?

Significant improvement for each column – mostly driven by better measurement of the tau element from IceCube and DUNE.



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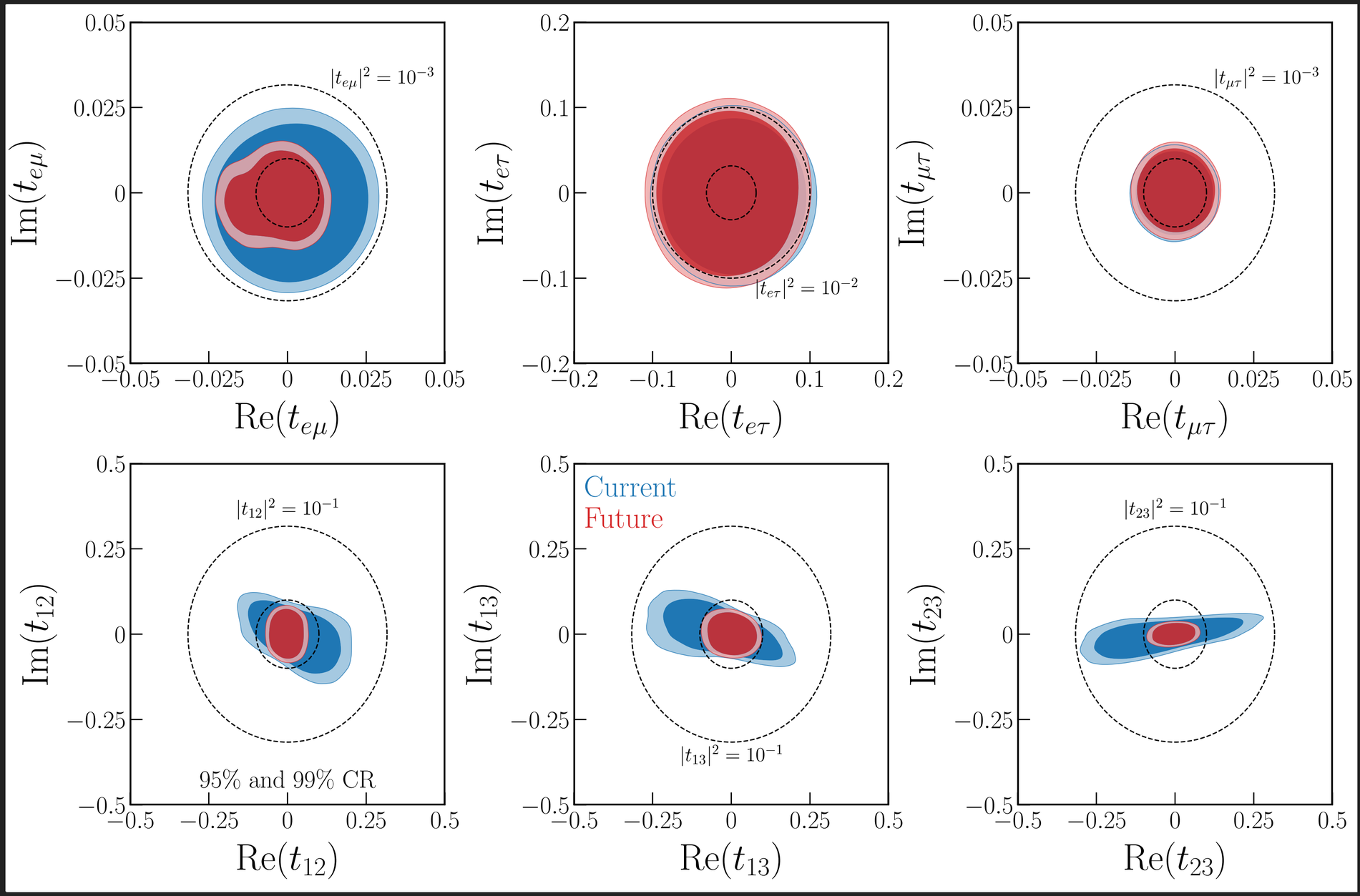


Ellis, KJK, Li, [2008.01088]

We had been agnostic to why U wasn't unitary until now

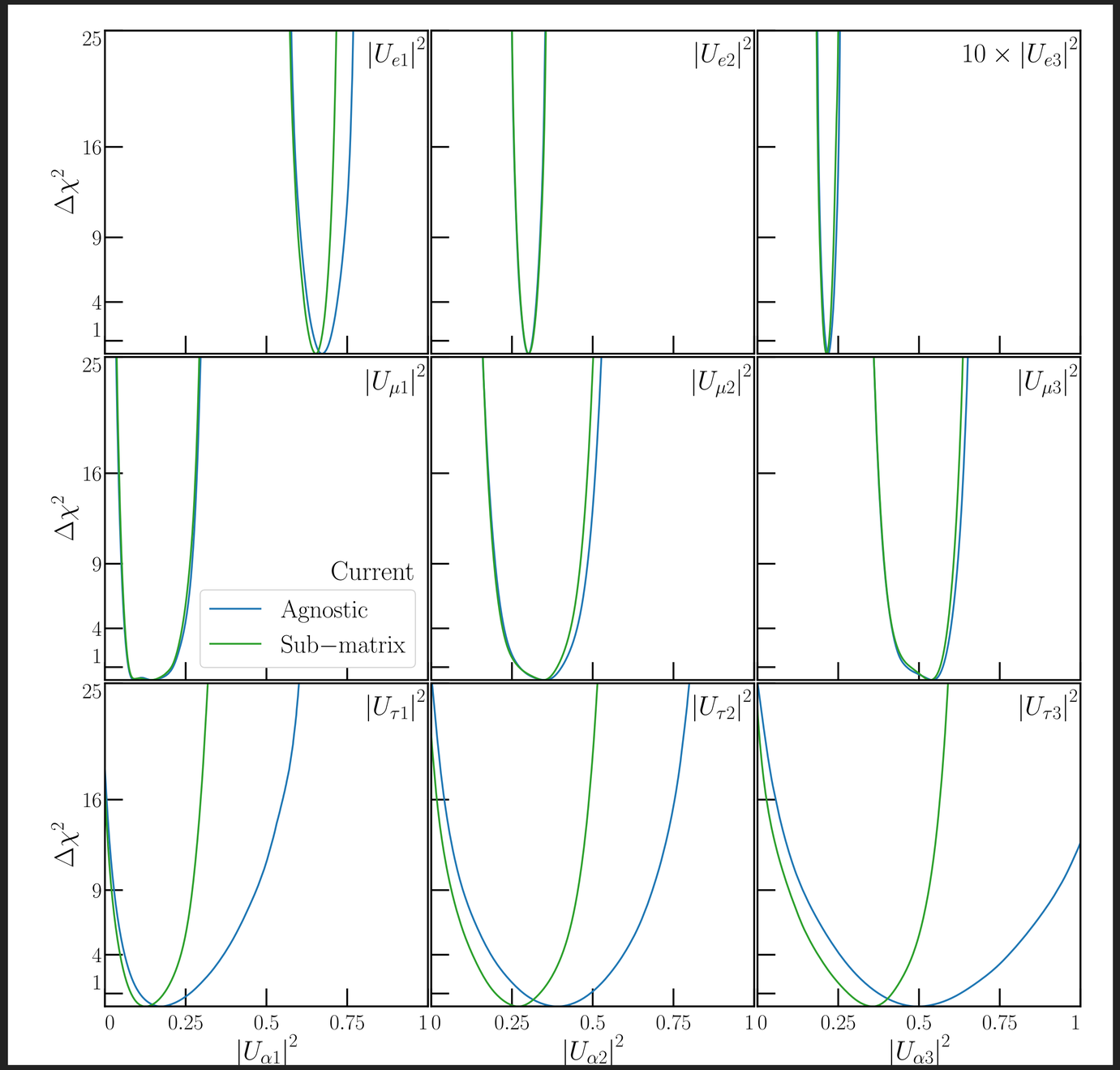
- If U is not unitary because additional sterile neutrinos exist, or the 3x3 matrix is a sub matrix of a larger, unitary matrix:

$$\left|t_{\alpha\beta}\right|^2 \leq \left(1 - N_\alpha\right)\left(1 - N_\beta\right)$$

$$\left|t_{kl}\right|^2 \leq \left(1 - N_k\right)\left(1 - N_l\right)$$

How does this Sub-matrix Assumption Change our Results?

Ellis, KJK, Li, [2008.01088]



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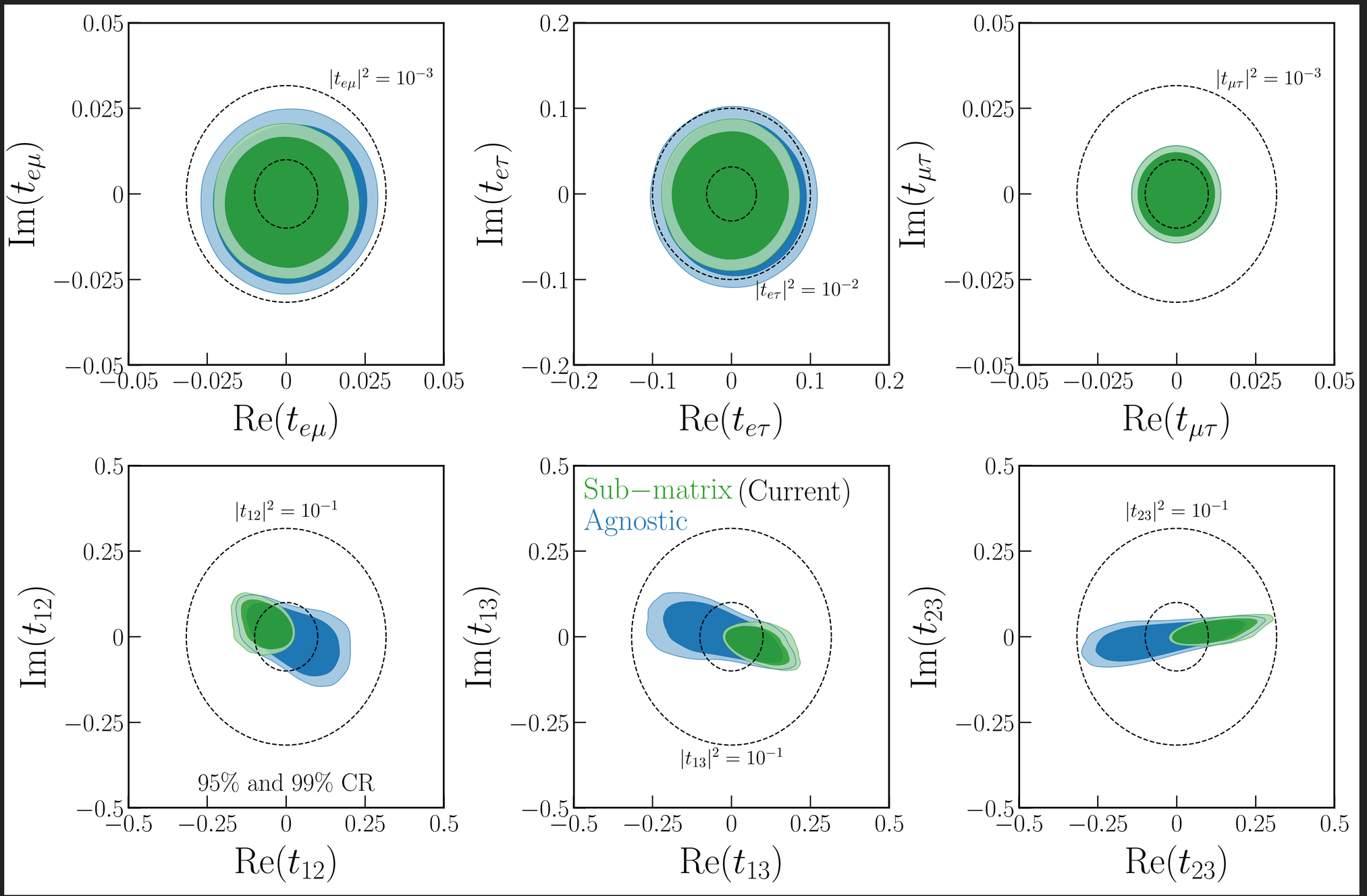

Conclusions

- Dividing up measurements of mixing angles by the channels they observe can serve as a consistency check of the three-neutrino hypothesis.
- Current oscillation data can constrain the unitarity of the Leptonic Mixing Matrix at the few percent level.
- Future data will improve these constraints by ~factors of 2 in the next decade.
- Assumptions about the origin of non-unitarity can lead to even stronger, theoretically motivated, constraints.

Thank you!