Towards global fits of three-dimensional hadron structure

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Challenges for global fits of GPDs

Natural to try to extend successful PDF global fitting framework to GPDs

But

- limited experimental data, although JLab 12GeV will help, e.g.:
    - deeply virtual Compton scattering (E12-06-114 [C])
    - time-like Compton scattering (E12-10-006A [A], E12-12-001 [B])
    - deep exclusive meson electroproduction (E12-10-006C [A])
    - deeply virtual meson production (E12-06-108 [B])
    - …
- challenging experimental interpretation, e.g.:
    - separating Bethe-Heitler process from DVCS signatures
    - DVCS on neutrons require nuclear targets/assumption of isospin symmetry
    - multiple GPDs contribute to individual structure functions
    - Bjorken-x dependence not directly accessible
    - …
- challenging theoretical framework
    - complicated Wilson coefficients
    - higher twist and target mass effects
    - …

Challenges for global fits of GPDs

Potential for significant impact

But significant theoretical and computational challenges

How can the lattice help?

First community report:
Constantinou et al., 2006.08636

Many new and old approaches to x-dependent hadron structure

Liu & Dong, PRL 72 (1994) 1790
Detmold & Lin, PRD 73 (2006) 014501
Braun & Müller, EPJC 55 (2008) 349
Ma & Qiu, PRD 98 (2018) 074021
Chambers et al., PRL 118 (2017) 242001

$$h^{(0)}_{j/H}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle H(P)|\overline{\psi}(n) W(n,0) \Gamma^\mu_j \psi(0)|H(P)\rangle$$

$$W(n(u),0) = \mathcal{P} \exp\Big[- i g_0 \int_0^u \mathrm{d}v \frac{\mathrm{d}y^\mu}{\mathrm{d}v} A^a_\mu(y(v)) T^a \Big]$$

Radyushkin, PRD 96 (2017) 034025
Musch et al., PRD 83 (2011) 094507

$$f^{(0)}_{j/H}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \langle H(P) \, | \overline{\psi}(0, \omega^-, \mathbf{0}_{\mathrm{T}}) W(\omega^-, 0) \Gamma_j \psi(0) | \, H(P) \rangle$$

$$\boxed{\text{PDFs}}$$

$$\xi^- \uparrow \; n^2 = 0$$

$$h^{(0)}_{j/H}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle H(P) | \overline{\psi}(n) W(n, 0) \Gamma^\mu_j \psi(0) | H(P) \rangle$$

$$f^{(0)}_{j/H}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \langle H(P) \,|\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\Gamma_j \psi(0)|\, H(P)\rangle$$

$\boxed{\text{PDFs}}$

ξ^- | $n^2 = 0$

$$h^{(0)}_{j/H}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle H(P)|\overline{\psi}(n) W(n,0)\Gamma^\mu_j \psi(0)|H(P)\rangle$$

P^z $\qquad$ ζ

$$n^2 = -z^2$$

$\boxed{\text{quasi PDFs}}$ $\qquad$ $\boxed{\text{pseudo PDFs}}$

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \langle H(P) \,|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T}) W(z,0)\Gamma_j \psi(0)|\, H(P)\rangle$$

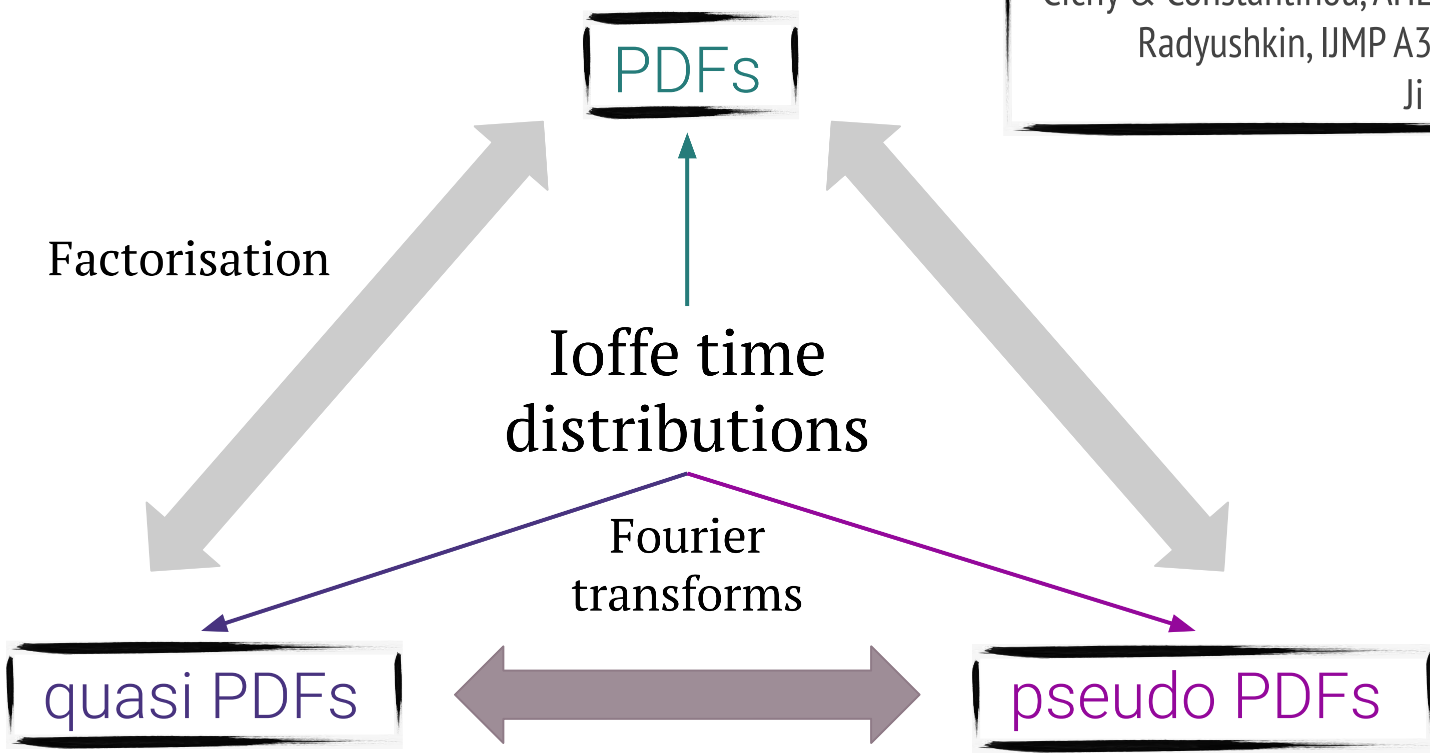
Ji, PRL 110 (2013) 262002

$$\widetilde{p}^{(0)}_{j/H}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \langle H(P) \,|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T}) W(z,0)\Gamma_j \psi(0)|\, H(P)\rangle$$

Radyushkin, PRD 96 (2017) 034025

Collinear structure from lattice QCD

Reviews in:
CJM, PoS(LATTICE2018) 1811.00678
Zhao, IJMP A33 (2019) 1830033
Cichy & Constantinou, AHEP (2019) 3036904
Radyushkin, IJMP A35 (2020) 2030002
Ji et al., 2004.03543



For an explicit example: all relations worked out in detail at one loop in scalar field theory
Giani, Del Debbio & CJM, 2007.02131 [to appear in JHEP]

Collinear structure from lattice QCD

Collinear structure (PDFs and DAs):

- significant effort from multiple collaborations
- preliminary calculations now well-established
- effort largely focussed on quantifying and reducing systematic uncertainties

Formally intractable ``inverse problem”

- reconstructing Fourier transform from limited and finite lattice data
- natural to move away from “first principles calculation of PDFs” view to a “global fitting” framework
- treat lattice results as data for global fits
- impact is greatest where experimental data is least

Ioffe time fitting framework

Lattice data can be incorporated in various ways:

- constraints imposed via Mellin moments (nucleon charges)

First application [transversity]:
Lin et al., PRL 120 (2018) 152502

- PDFs themselves

First community white paper:
Lin et al., PPNP 100 (2018) 107

- Ioffe-time distributions (matrix elements)

Advocated in:
CJM, PoS(LATTICE2018) 1811.00678
Karpie et al., JHEP 04 (2019) 057
Cichy, Del Debbio & Giani, JHEP 10 (2019) 137
Giani, Del Debbio & CJM, 2007.02131

In the spirit of "factorizable matrix elements":
Ma & Qiu, PRD 98 (2018) 074021

Natural to combine "Ioffe-time fitting" framework with GPDs

- ``conceptually straightforward'' for lattice calculations

$$\langle H(P)|\overline{\psi}(n)W(n,0)\Gamma_j^{\mu}\psi(0)|H(P)\rangle \to \langle H(P')|\overline{\psi}(n)W(n,0)\Gamma_j^{\mu}\psi(0)|H(P)\rangle$$

BUT

Both quasi and pseudo distribution approaches require large momenta

- particularly challenging signal-to-noise issue for baryons
- exacerbated at nonzero momentum transfer
- Breit frame is computationally expensive

Data analysis is a headache

- requires a coordinated and systematic approach

Global fitting framework yet to be established

- reliable parametrizations/models required

First steps towards first principles' calculations

First community report:
Constantinou et al., 2006.08636

Form factors widely studied on the lattice

- good agreement between calculations
- precision as good as (or better than) experimental data
- strange form factors particularly promising

Moments of GPDs [generalized form factors]

- new field of study for lattice QCD

Alexandrou et al., PRD 101 (2019) 034519
Bali et al., PRD 100 (2019) 014507

GPDs themselves

- formalism developed for quasi and pseudo distribution approaches

Ji et al., PRD 92 (2015) 014039
Radyushkin, PRD 100 (2019) 116011

- first results for isovector nucleon and pion GPDs

Alexandrou et al., 1910.13229
Chen et al., NPB 952 (2020) 114940

First community report:
Constantinou et al., 2006.08636

First steps towards global fits

1. What quantities are most tractable on the lattice?

   [generally speaking: isovector (valence) quark distributions]

   a. what kinematic regions are accessible, and with what precision?

2. What quantities are most desirable?
   a. what prospects, if any, for coordinated fits in the near future?
   b. how do these depend on reliable parametrizations or model Ansatze?
   c. what kinematic regions would be useful, and with what precision?

3. What quantities will be feasible on a (say) ten year timescale at a precision that can guide the EIC program?
   a. what will be essential, and what desirable?

4. **How can we work together to make this happen?**

Thank you

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