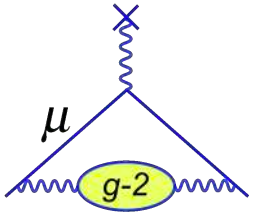
Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

The muon g-2 in the SM and its connection with EW precision physics

Alex Keshavarzi

Argonne HEP Division Seminar

2nd September 2020



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Theory initiative result released

4-year long international collaborative effort:

arXiv.org > hep-ph > arXiv:2006.04822

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High Energy Physics – Phenomenology

[Submitted on 8 Jun 2020]

The anomalous magnetic moment of the muon in the Standard Model

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News

Physicists publish worldwide consensus of muon magnetic moment calculation

June 11, 2020 | Jerald Pinson

Share Tweet Email

For decades, scientists studying the muon have been puzzled by a strange pattern in the way muons rotate in magnetic fields, one that left physicists wondering if it can be explained by the Standard Model — the best tool physicists have to understand the universe.

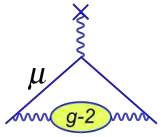
This week, an international team of more than 170 physicists published the most reliable prediction so far for the theoretical value of the muon's anomalous magnetic moment, which would account for its particular rotation, or precession. The magnetic moment of subatomic particles is generally expressed in terms of the dimensionless Landé factor, called g. While a number of international groups have worked separately on the calculation, this publication marks the first time the global theoretical physics community has come together to publish a consensus value for the muon's magnetic moment.

The result differs from the most recent experimental measurement, which was performed at Brookhaven National Laboratory in 2004, but not significantly enough to unambiguously answer this question.

Now the world awaits the result from Fermilab's current Muon g-2 experiment. In the upcoming months, physicists working on the experiment will unveil their preliminary measurement for the value. Depending on how much the Standard Model theoretical calculation differs from the upcoming experimental measurement, physicists may be one step closer to determining whether the muon's magnetic interactions are hinting at particles or forces that have yet to be discovered.

Theory Initiative Website:
https://muon-gm2-theory.illinois.edu/white-paper/

The anomalous magnetic moment



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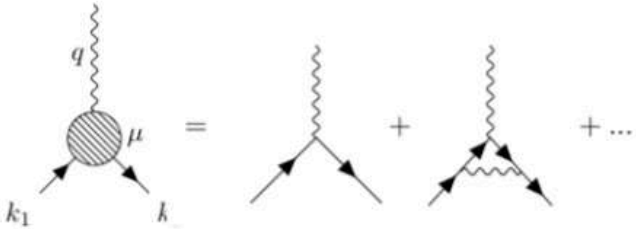
Magentic moment: $\vec{\mu} = -\frac{e}{2m} g \vec{S}$: $g = 2$ (Dirac) $\Rightarrow$ bare (no quantum effects)

$\Rightarrow$ Renormalise the QED vertex: $\Gamma^{\mu}_{\text{phys}}(k_1, k_2) =$



$\Rightarrow$ By virtue of Gordon decomposition: $\Gamma^{\mu}(k_1, k_2) = -ie\left[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2)\right]$

$\Rightarrow$ In NR limit, with $k_2 - k_1 = q \to 0$: $\left(\gamma^{\mu}(\partial_{\mu} - ieA_{\mu}) - m + \frac{eF_2(0)}{4m} F_{\mu\nu}\sigma^{\mu\nu}\right)\psi = 0$

$\to$ New $F_2(0)$ form factor term in Dirac equation from radiative corrections

$\Rightarrow$ Effective Hamiltonian: $H = \left[\frac{1}{2m}(\vec{p} - e\vec{A})^2 + eA^0 + \frac{e}{2m}(1 + F_2(0))\vec{\sigma}.\vec{B}\right]$

$\to$ Magnetic potential $U = -\vec{\mu} \cdot \vec{B} = \frac{e}{2m}(1 + F_2(0))\vec{\sigma}.\vec{B} = \frac{e}{2m} g \vec{S}.\vec{B} = \frac{e}{4m} g \vec{\sigma}.\vec{B}$

$\Rightarrow$ Implies $g = 2 + 2F_2(0)$ with quantum effects $\longrightarrow$ $a = F_2(0) = (g-2)/2$

The muon $g-2$

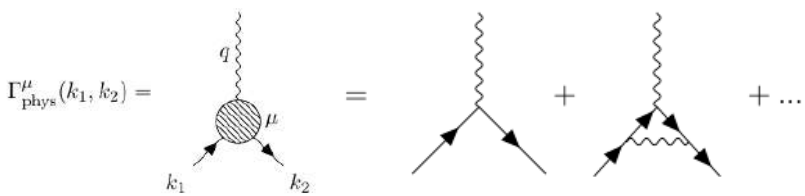
Magnetic moment: $\vec{\mu} = \frac{-e}{2m} g_\mu \vec{S}$: $g = 2$ (Dirac) $\rightarrow$ $g = 2 + 2a_\mu$ (+ radiative corrections)

- a_μ arises due to quantum corrections inherent from RQFTs (QED, SM)
- These effects manifest differently for the theoretical/experimental determination.

Theory, a_μ^{SM}

$\rightarrow$ 1948, Schwinger: $a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.001162$

$\rightarrow$ Corrections to the QED vertex function:

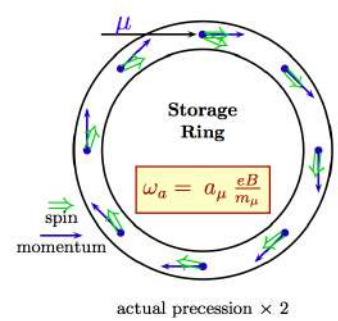


$\rightarrow$ Must determine all SM contributions to sufficient loop order

Experiment, a_μ^{exp}

$\rightarrow$ 1948, Kusch & Foley: $g_e = 2.00238$

$\rightarrow$ Anomalous precession frequency, ω_a

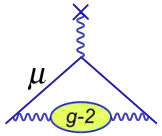


$$\overrightarrow{\omega_a} = \overrightarrow{\omega_s} - \overrightarrow{\omega_c}.$$
$$= -a_\mu \frac{Qe}{m} \vec{B}$$

Experimental measurements naturally contains all higher order effects

$\rightarrow$ Compare a_μ^{SM} and a_μ^{exp} to rigorously test SM

The muon $g-2$



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$\rightarrow$ Today, The Muon g-2 Theory Initiative

Experiment, a_μ^{exp}

$\rightarrow$ 2003, BNL measurement

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Theory Initiative Website:
https://muon-gm2-theory.illinois.edu/white-paper/

Community approved a_μ^{SM} vs. BNL a_μ^{exp} yields $\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (27.9 \pm 7.6)\times 10^{-10}$

$\rightarrow$ 3.7σ discrepancy hints at new physics beyond the SM.

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Experiment, a_μ^{exp}

$\rightarrow$ Today, Muon g-2 Experiment at Fermilab

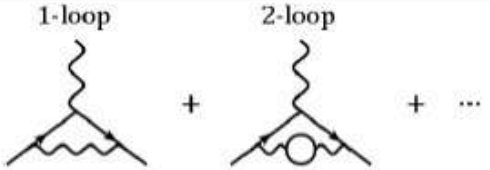
Aiming for x4 improvement in total uncertainty.

Analysis of Run-1 data being finalised for publication.
$\rightarrow$ Will confirm validity of Muon g-2 discrepancy and potential for BSM discovery.

The muon $g-2$ in the SM

$$a_\mu^{\text{Theory}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had, VP}} + a_\mu^{\text{had, LbL}} + a_\mu^{\text{BSM??}}$$

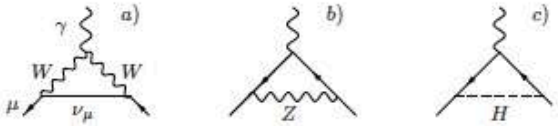
QED



Known to five-loop (12,672 diagrams)

99.99% of a_μ^{SM} ~0.001% of δa_μ^{SM}

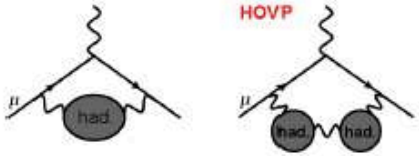
EW



Known to two-loop (with m_H known)

0.0001% of a_μ^{SM} ~0.2% of δa_μ^{SM}

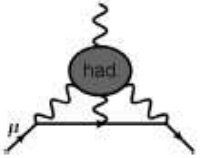
HVP



Non-perturbative (data input + lattice)

0.006% of a_μ^{SM} ~47% of δa_μ^{SM}

HLbL



Non-perturbative (data input + model/lattice)

0.0001% of a_μ^{SM} ~53% of δa_μ^{SM}

BSM

? ? ? ? ? ? ?

QED fully cross checked

Slide content by Thomas Teubner.

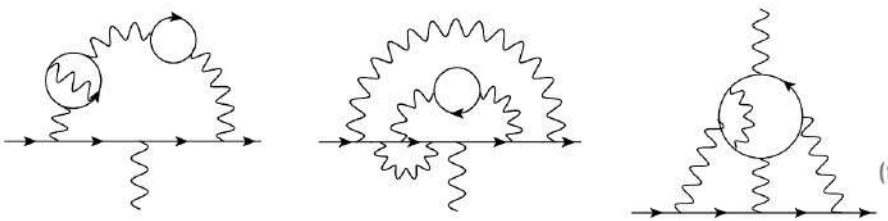
Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

... but 4-loop and 5-loop rely heavily on numerical integrations

Recently several independent checks of 4-loop and 5-loop diagrams:

Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:

all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., PRD 93 (2016) 053017)

4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, arXiv:1704.06996, also new numerical results by Volkov, 1705.05800

all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

$$a_\mu^{\mathrm{QED}}(\alpha(\mathrm{Cs})) = 116\,584\,718.931\,(104) \times 10^{-11}$$

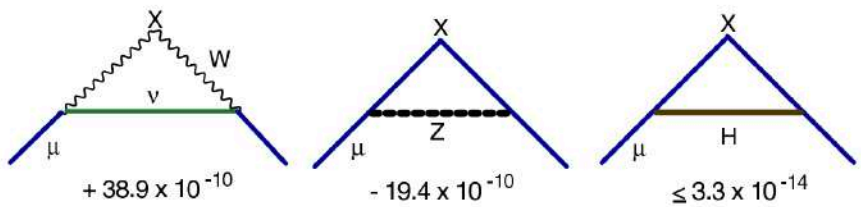
[T. Aoyma et al, 2012, 2019, Laporta 2017,...]

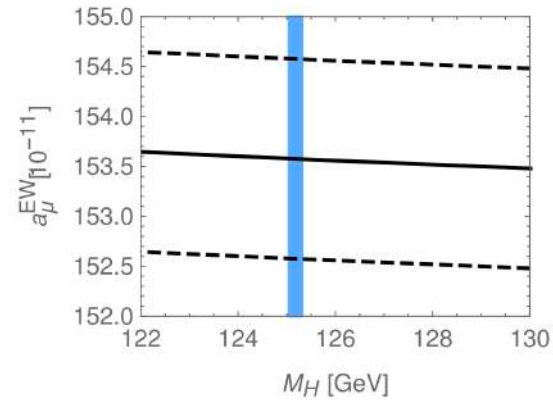
uncertainty dominated by $\mathcal{O}(\alpha^6)$ contributions

EW unchanged

Slide content by Thomas Teubner.

Electro-Weak 1-loop diagrams:





known to 2-loop (1650 diagrams, the first full EW 2-loop calculation):
Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2\ loop)} = (154\pm2)\times10^{-11}$

Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,
PRD 88 (2013) 053005

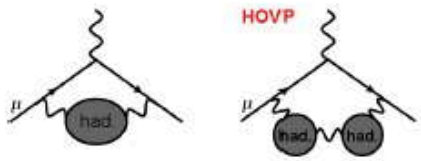
$a_\mu^{EW(1+2\ loop)} = (153.6\pm1.0)\times10^{-11}$ ✓

Uncertainty dominated by hadronic contributions.

compared with $a_\mu^{QED} = 116\ 584\ 718.951\ (80)\times10^{-11}$

Hadronic $g-2$

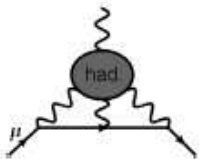
HVP



Non-perturbative
(data input + lattice)

0.006% of a_μ^{SM} ~47% of δa_μ^{SM}

HLbL

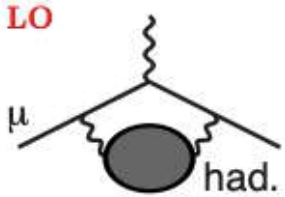


Non-perturbative
(data input + model/lattice)

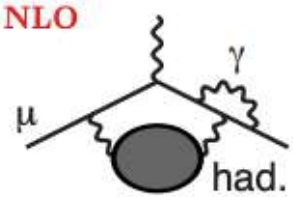
0.0001% of a_μ^{SM} ~53% of δa_μ^{SM}

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

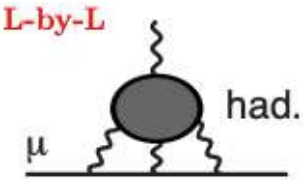
LO



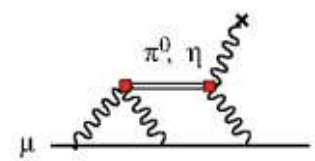
NLO



L-by-L



e.g.



HLbL: previously only estimated visa model calculations

→ Made use of e.g. form factor data & pQCD constraints

→ Very different to quantify and control systematics.

Updated `Glasgow consensus' result (Prades + deRafael + Vainshtein) gave:

$$a_\mu^{\text{HLbL}} = (98 \pm 24)\times 10^{-11}$$

Dispersive HLbL

Slide content by Aida El-Khadra.

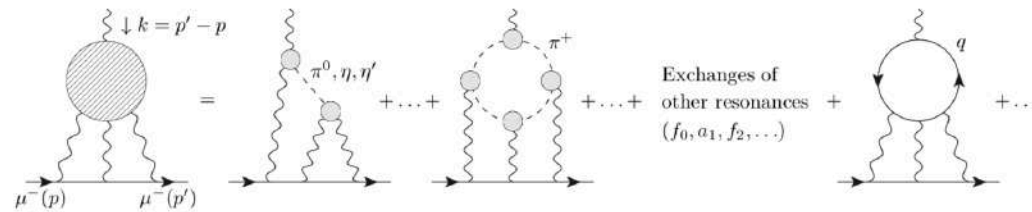
Had never been achieved before the theory initiative…

Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

✦ Target: ≲ 10% total error

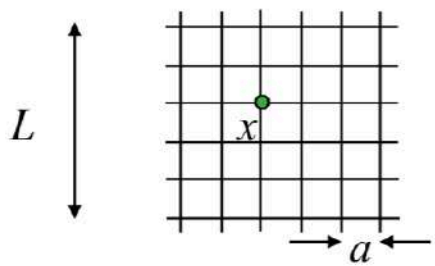


Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Lattice QCD

Slide content by Aida El-Khadra.

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$



- discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty, T > L$
- quark masses (m_f): $M_{H,\mathrm{lat}} = M_{H,\mathrm{exp}}$
  $m_f \to m_{f,\mathrm{phys}}$

  tune using hadron masses
  extrapolations/interpolations

m_{ud} m_s m_c m_b

Lattice HLbL

Slide content by Aida El-Khadra.

Had never been achieved before the theory initiative…

- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

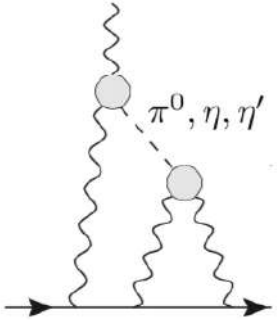
$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

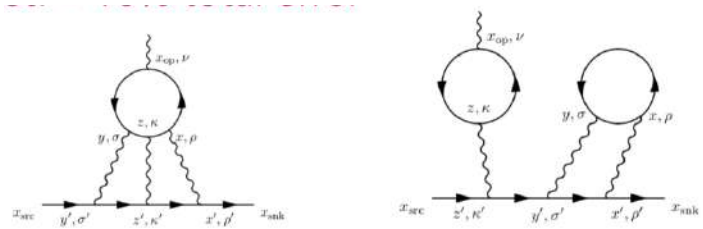
- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$





RBC/UKQCD [T. Blum et al, arXiv:1911.08123, PRL 2020]:

✦ Target: ≲ 10% total error

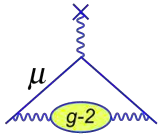
First complete LQCD calculation of connected and leading disconnected contribution with continuum and finite volume extrapolation

$$a_\mu^{\mathrm{HLbL}} = 7.87\,(3.06)\,(1.77) \times 10^{-10}$$

➠ a_μ^{HLbL} cannot "rescue" the SM

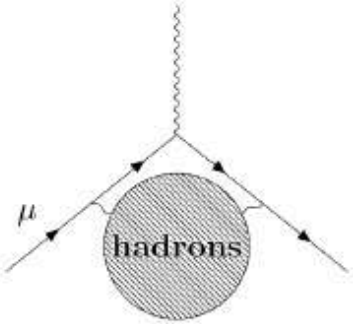
➠ combine disp and lattice HLbL for SM prediction

$a_\mu^{\text{had, VP}}$: theoretical setup

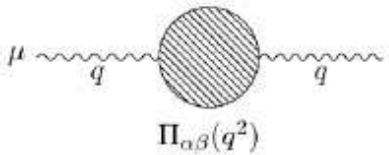


MANCHESTER 1824
The University of Manchester

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

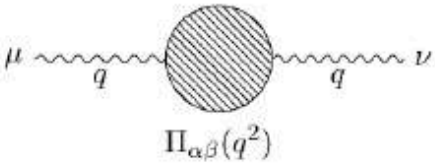


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2 - i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)}$$

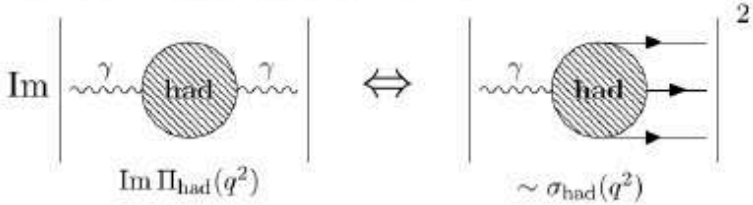
2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2 - i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s(s - q^2 - i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\text{had, LO VP}} = \frac{\alpha}{\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\mathrm{Im}\,\Pi_{\text{had}}(s)K(s)$

4) Utilise optical theorem:



5) Arrive at equation for $a_\mu^{\text{had, LO VP}}$:

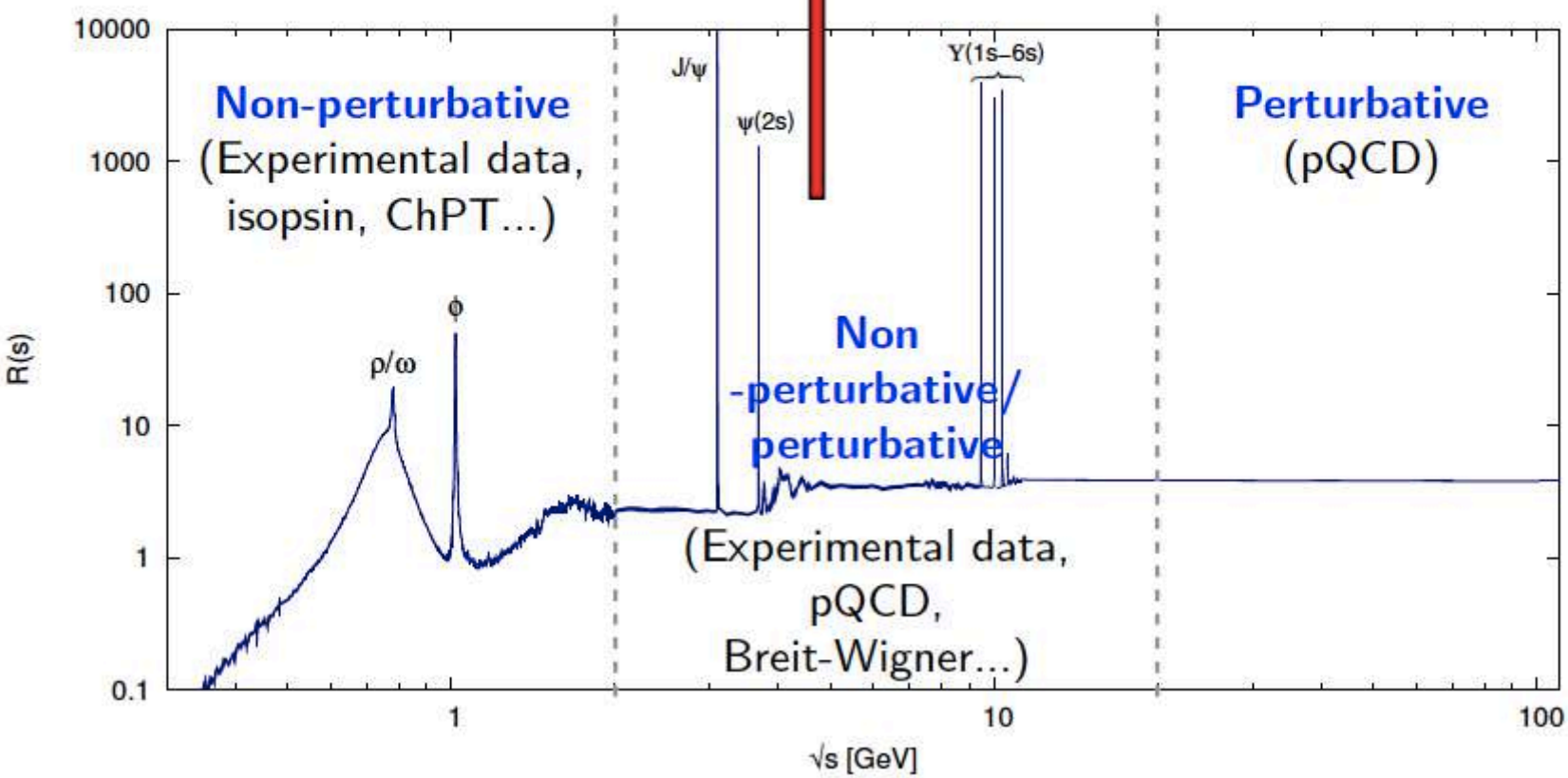
$$a_\mu^{\text{had, LO VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty}\mathrm{d}s\,\sigma^0_{\text{had},\gamma}(s)K(s)$$

$\sigma^0_{\text{had},\gamma}$ = bare cross section, FSR included

⇒ Similar dispersion integrals for NLO and NNLO HVP

Building the hadronic *R*-ratio

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Dispersive HVP

Slide content by Aida El-Khadra.

- Target: ~0.2% total error
- Dispersion relation + experimental data for $e^+e^- \to \text{hadrons}$ (and τ data)
  - current uncertainty ~0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,….
- Challenges:
  - below ~2 GeV: sum > 30 exclusive channels: 2π, 3π, 4π, 5π, 6π, $2K$, $2K\pi$, $2K2\pi$, $\eta\pi$,…. (use isospin relations for missing channels)
  - above ~1.8 GeV:
    inclusive, pQCD (away from flavor thresholds)
    + narrow resonances (J/ψ, Υ,..)
  - Combine data from different experiments/measurements:
    understanding correlations, sources of sys. error, tensions…
  - include FS radiative corrections

$a_\mu^{\text{had, VP}}$ from KNT

The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis

Alexander Keshavarzi[1], Daisuke Nomura[2,3] and Thomas Teubner[4]

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Abstract

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\text{had, VP}}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\text{had}}(M_Z^2)$, from the combination of $e^+e^- \to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \to$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and perturbative QCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g-2$ of the muon of $a_\mu^{\text{had, LO VP}} = (693.26 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\text{had, NLO VP}} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\text{SM}} = (11\,659\,182.04 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{\ \ 4}$, resulting in $\alpha^{\ \ 1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

← Major 2018 update to data combination methodology and data input.
Results for $a_\mu^{\text{had, VP}}$ and $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$.
Phys.Rev.D 97 (2018) 114025.

The $g-2$ of charged leptons, $\alpha(M_Z^2)$ and the hyperfine splitting of muonium

Alexander Keshavarzi[1,2], Daisuke Nomura[3] and Thomas Teubner[4]

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Abstract

Following updates in the compilation of $e^+e^- \to$ hadrons data, this work presents re-evaluations of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the electron (a_e), muon (a_μ) and tau lepton (a_τ), to the ground-state hyperfine splitting of muonium and also updates the hadronic contributions to the running of the QED coupling at the mass scale of the Z boson, $\alpha(M_Z^2)$. Combining the results for the hadronic vacuum polarisation contributions with recent updates for the hadronic light-by-light corrections, the electromagnetic and the weak contributions, the deviation between the measured value of a_μ and its Standard Model prediction amounts to $\Delta a_\mu = (28.02 \pm 7.37) \times 10^{-10}$, corresponding to a muon $g-2$ discrepancy of 3.8σ.

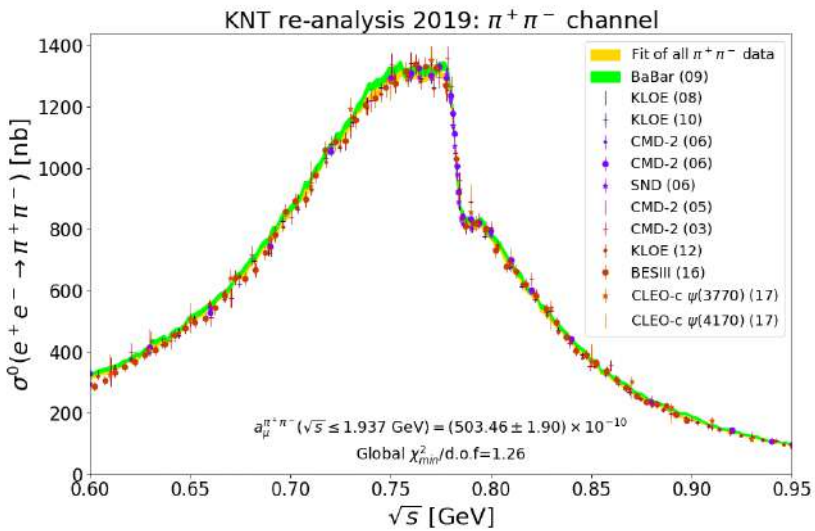
2019 data update and applications of data → compilation to other observables.
Results for a_e, a_μ, a_τ, $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ and $\Delta\nu_{\text{Mu}}^{\text{had, VP}}$.
Phys.Rev.D 101 (2020) 014029.

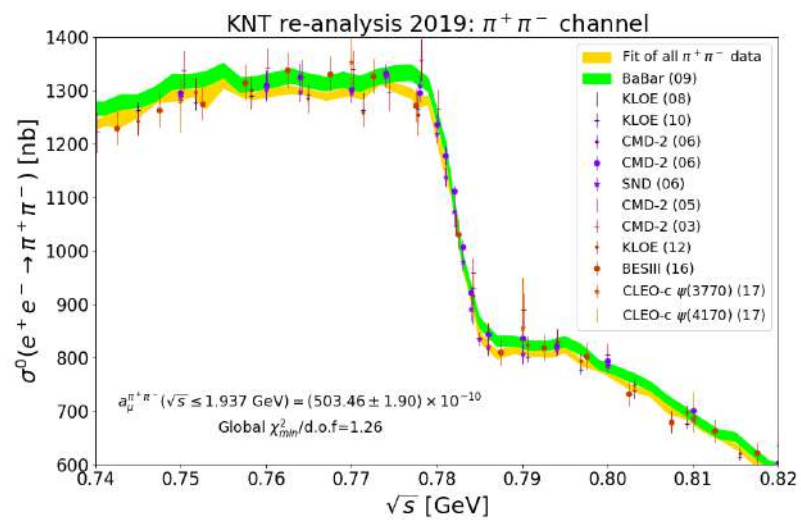
The $\pi^+\pi^-$ channel

Phys.Rev.D 101 (2020) 014029.

$\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LOVP}}$

→ Combines ~30 measurement totalling over 1000 data points





→ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data entirely dominant

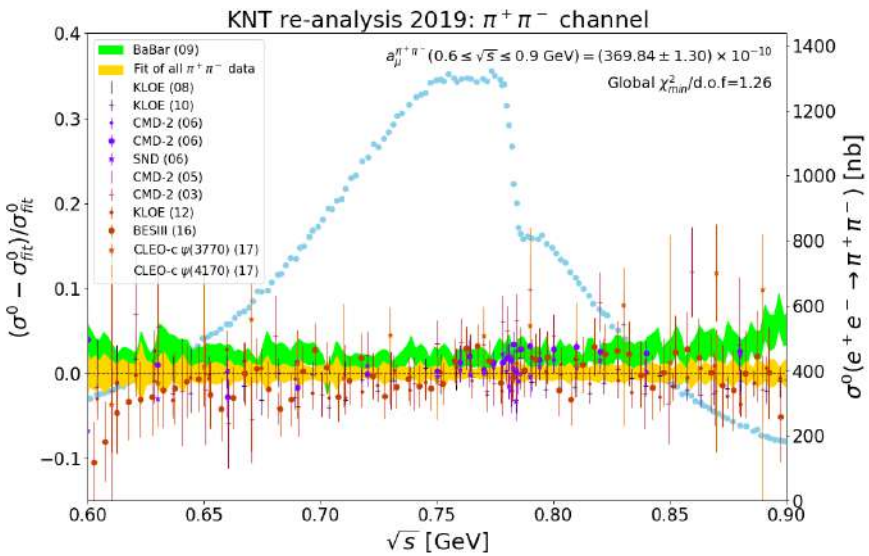
$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937\text{ GeV}] = 503.46 \pm 1.14_{stat} \pm 1.52_{sys} \pm 0.05_{vp} \pm 0.14_{fsr}$$
$$= 503.46 \pm 1.91_{tot}$$

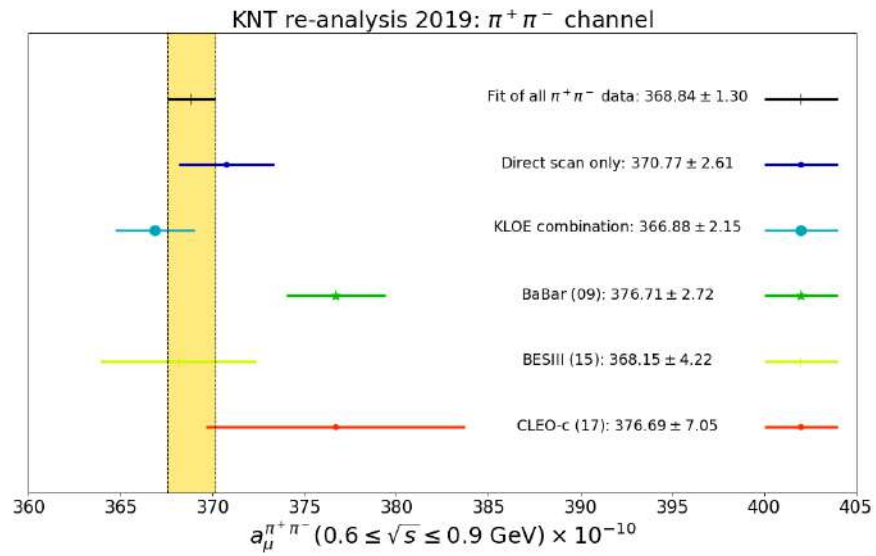
KNT18: $502.97 \pm 1.97_{tot}$

→ 14% local χ^2_{min}/d.o.f. error inflation due to tensions in clustered data

The $\pi^+\pi^-$ channel *Phys.Rev.D* 101 (2020) 014029.

Large difference between KNT vs. BaBar and KLOE vs. BaBar is still evident.





Compared to $a_\mu^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_\mu^{\pi^+\pi^-}$(BaBar data only) = 513.2 ± 3.8

Simple weighted average of all data → $a_\mu^{\pi^+\pi^-}$(weighted average) = 509.2 ± 2.9

(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

- Highlights the importance of incorporating available correlated uncertainties in fit.

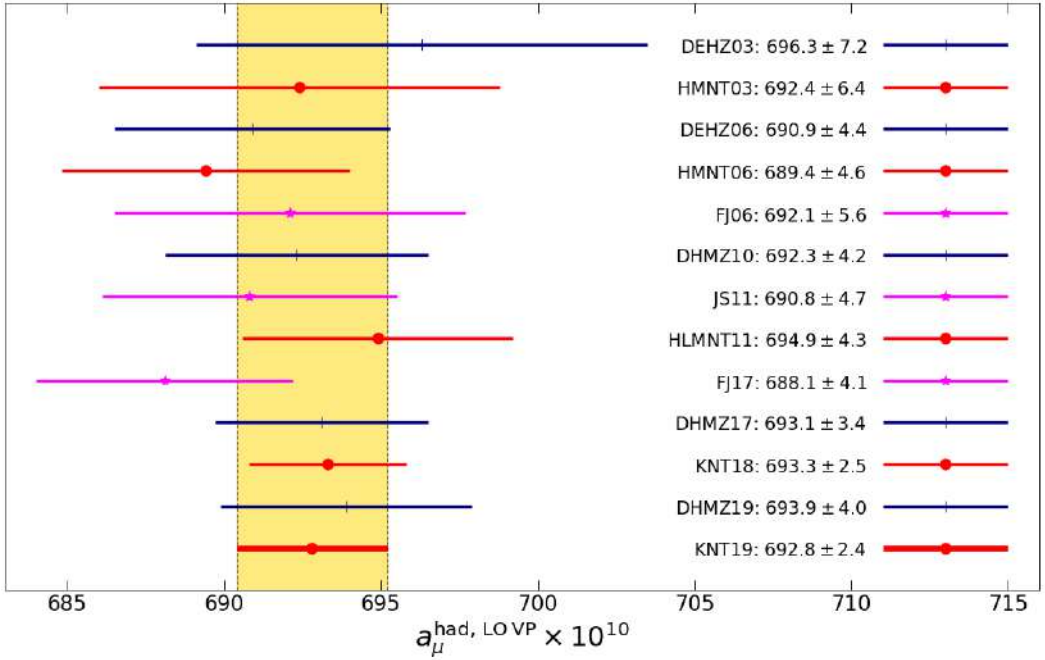
$a_\mu^{\text{had, LO VP}}$ from KNT

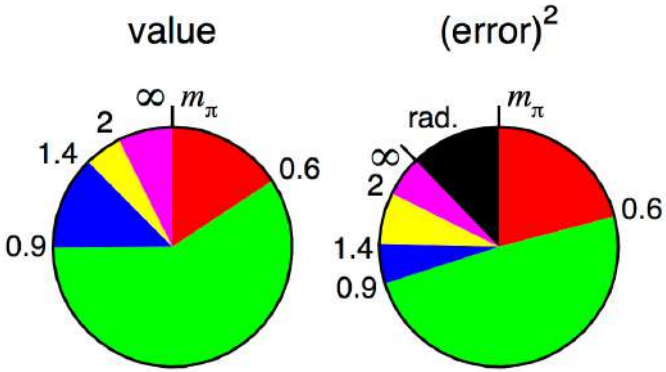
Phys.Rev.D 101 (2020) 014029.

KNT18: $a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46_{tot}$

$$a_\mu^{\text{had, LOVP}} = 693.84 \pm 1.19_{stat} \pm 1.96_{sys} \pm 0.22_{vp} \pm 0.71_{fsr}$$
$$= 693.84 \pm 2.29_{exp} \pm 0.74_{rad}$$
$$= 692.78 \pm 2.42_{tot}$$

- Precision better than 0.4% (uncertainties include all available correlations and χ^2 inflation)





- Clear $\pi^+\pi^-$ dominance

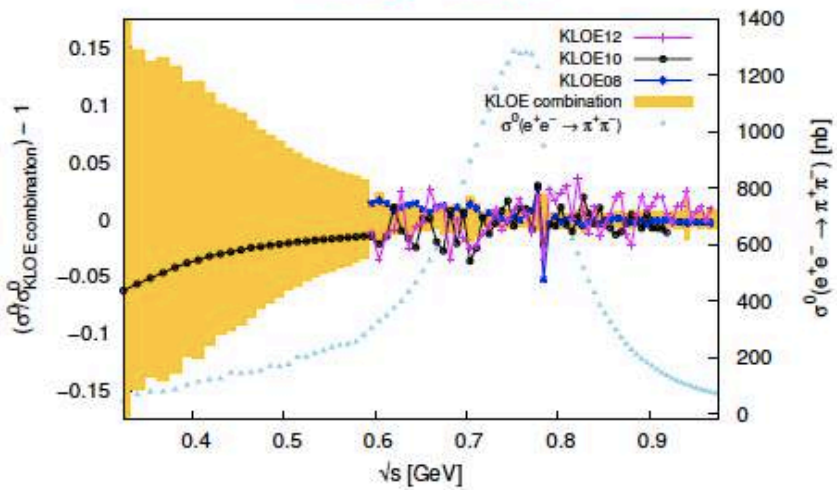
KNT vs. DHMZ: the use of correlations

Differences are dominated by the choices of how to use the correlations

⇒ Consider the combination of the KLOE $\pi^+\pi^-$ data sets:

KNT18

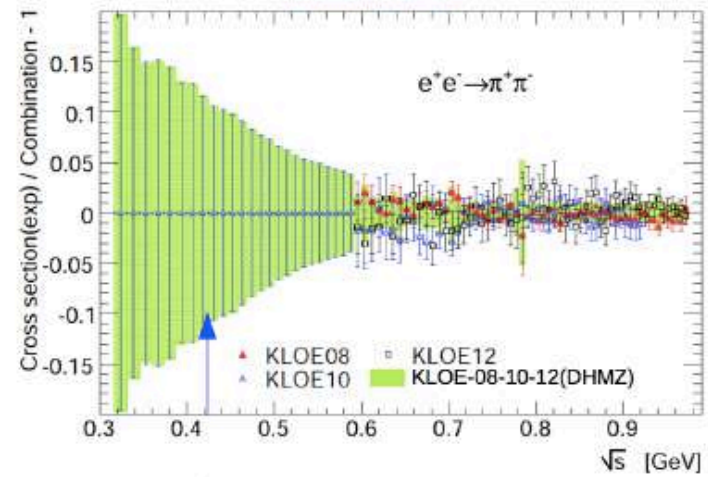
Use available uncertainty and correlation information to fully influence over entire energy range



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 503.74 \pm 1.96$$

DHMZ17

Only allow correlations to only influence the combination locally inside defined energy intervals



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 507.14 \pm 2.58$$

Take-home message: correlations are important and the choices of how to use them are not trivial

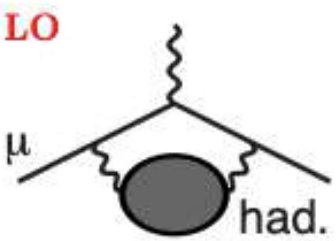
Data-driven HVP

Slide content by Aida El-Khadra.

First-time agreement between various groups…

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\mathrm{HVP,\,LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\mathrm{DV+QCD}}$	692.8(2.4)	1.2



+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:
- account for differences in results from the same experimental inputs
- include correlations between systematic errors

➠ $a_\mu^{\mathrm{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$

Lattice HVP

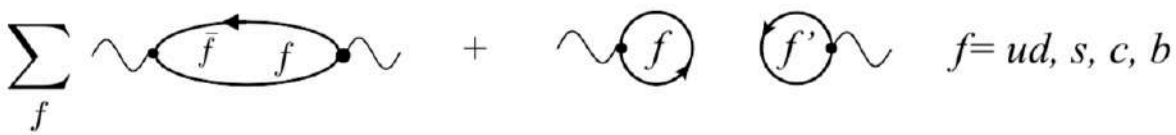
Slide content by Aida El-Khadra.

Calculate a_μ^{HVP} in Lattice QCD:

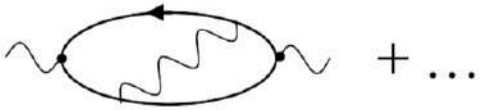
$$a_\mu^{\mathrm{HLO}} \equiv a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$



- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



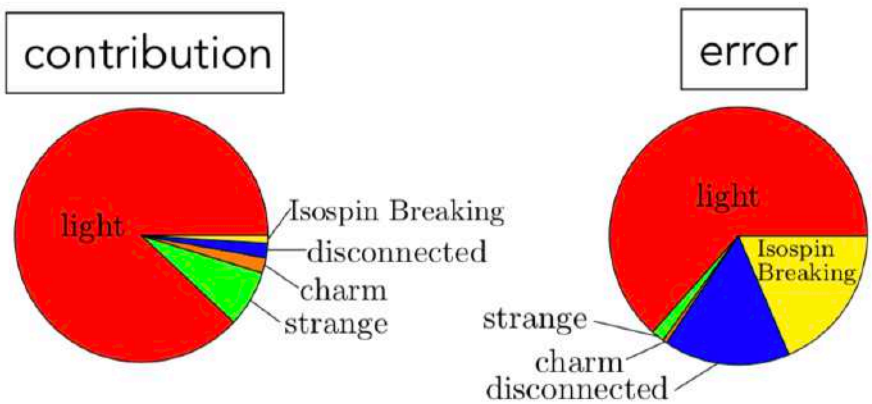
- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$

Lattice HVP

Slide content by Aida El-Khadra.

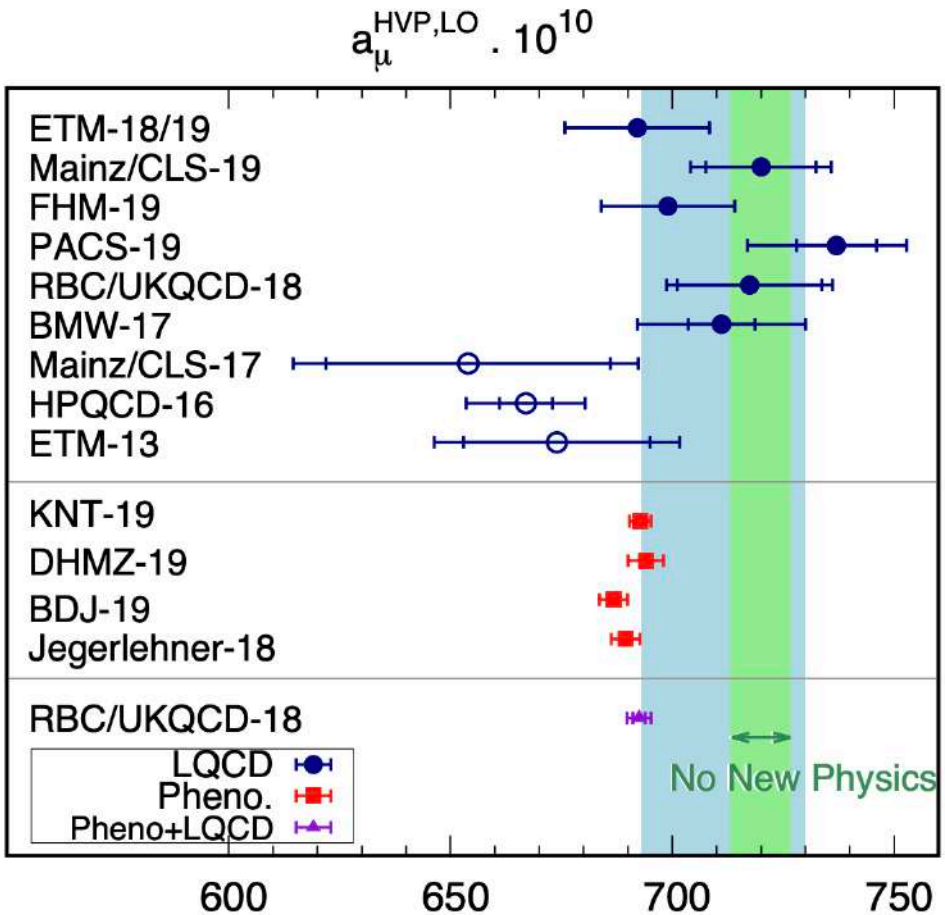
- Target: < 0.5% total error
- light-quark connected contribution, $a_{\mu,ud}^{\mathrm{HLO}}$:
  ~90% of total, with 1-3% error
- "heavy" flavor contributions, $a_{\mu,s}^{\mathrm{HLO}}$, $a_{\mu,c}^{\mathrm{HLO}}$, $a_{\mu,b}^{\mathrm{HLO}}$:
  ~8%, 2%, 0.05% of total a_{μ}^{HLO}, can be calculated with sufficient precision
- disc. contribution:
  ~2% of total a_{μ}^{HLO}, contributes ~0.3-1% error to a_{μ}^{HLO}
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  ~1% of total a_{μ}^{HLO}, contribute ~0.3-1% error

[V. Gülpers, adapted for WP from talk @ Lattice 2019, arXiv:2001.11898]



Lattice HVP

Slide content by Aida El-Khadra.



- The errors in (all but one of the) lattice QCD results are still large
- All results include contributions from connected ud, s, c, b + disconnected, QED + strong isospin breaking, and finite volume corrections.
- Lattice combination: included results shown with filled circles

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{-10}$$

Differences between lattice evaluations

Slide content by Aida El-Khadra.

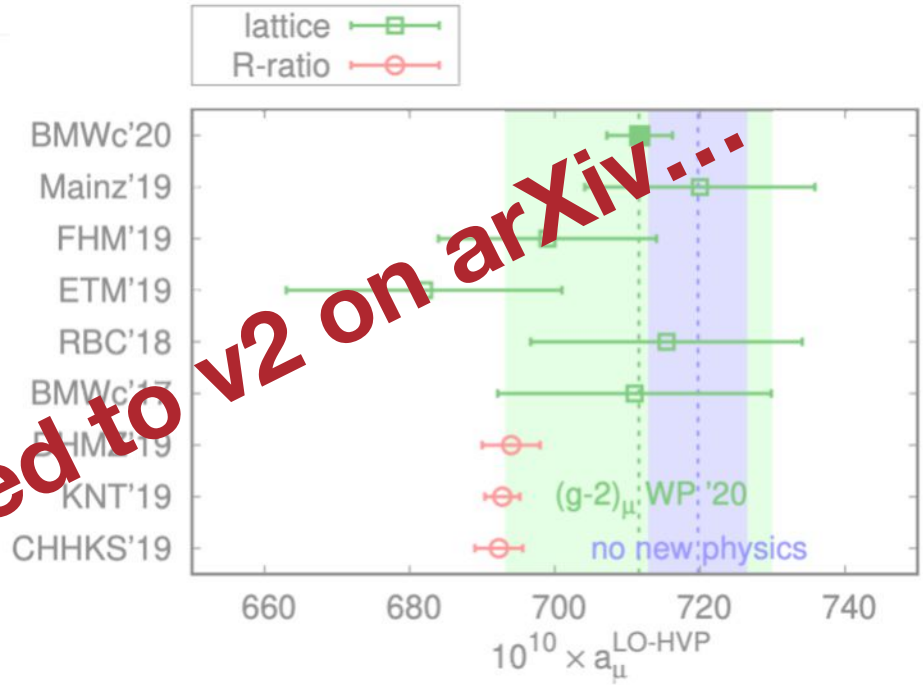
QED + Strong isospin breaking corrections

Collaboration	$\delta a_\mu^{\mathrm{HVP,\,LO}} \times 10^{10}$	Comments
ETM-19 [12]	7.1 (2.9)	SIB+QED, perturbative method: connected diagrams only
RBC/UKQCD-18 [11, 399]	9.5 (10.2)	SIB+QED, perturbative method: connected QED + SIB diagrams + leading disconnected QED diagram
FHM-17 [9]	9.5 (4.5)	Simulations with full-SIB for *ud*-conn: $m_d - m_u \neq 0$ while $\alpha = 0$.
BMW-17 [10]	7.8 (5.1)	pheno (non-lattice) estimate
CSSM/QCDSF/UKQCD Preliminary [433]	$\lesssim 1\% \times a_\mu^{\mathrm{HVP,\,LO}}$	Simulations with Full-QED for *ud*-conn: $\alpha \neq 0$ while $m_d - m_u = 0$. $M_\pi \sim 400$ MeV.

Lattice combination: Combine ETM-19 and RBC/UKQCD-18 assuming 100% correlation

➠ $\delta a_\mu^{\mathrm{HVP,LO}} = 7.2\ (3.4)\ \times 10^{-10}$

Lattice HVP from BMW



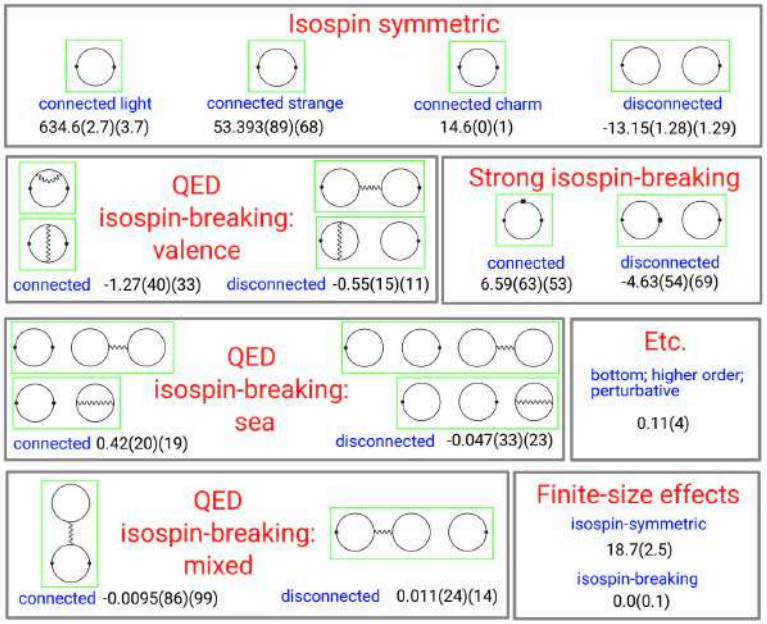
Recently updated to v2 on arXiv...

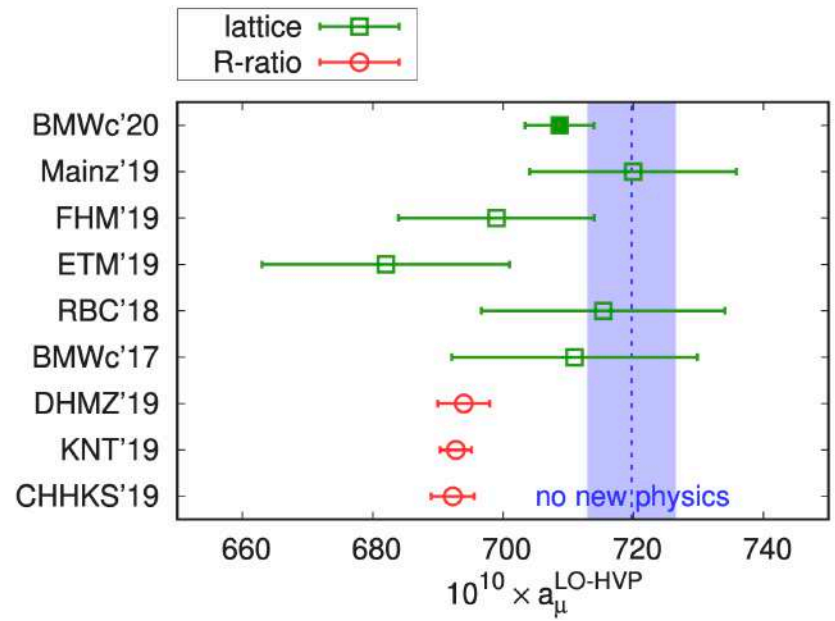
$$a_\mu^{\text{LO-HVP}} = 712.4(1.9)_{\text{stat}}(4.0)_{\text{syst}}\,[4.5]_{\text{tot}} \times 10^{-10}\ [0.6\%]$$

- Consistent with other lattice results
- Total uncertainty is $\sim \div 4$ ...
- ... and comparable to R-ratio
- Consistent w/ BNL experiment ("no new physics" scenario) !
- 3.1σ larger than DHMZ'19, 3.9σ than KNT'19 ?

Lattice HVP from BMW

arXiv:2002.12347





$$10^{10} \times a_\mu^{\text{LO-HVP}} = 708.7(2.8)_{\text{stat}}(4.5)_{\text{sys}}[5.3]_{\text{tot}}$$

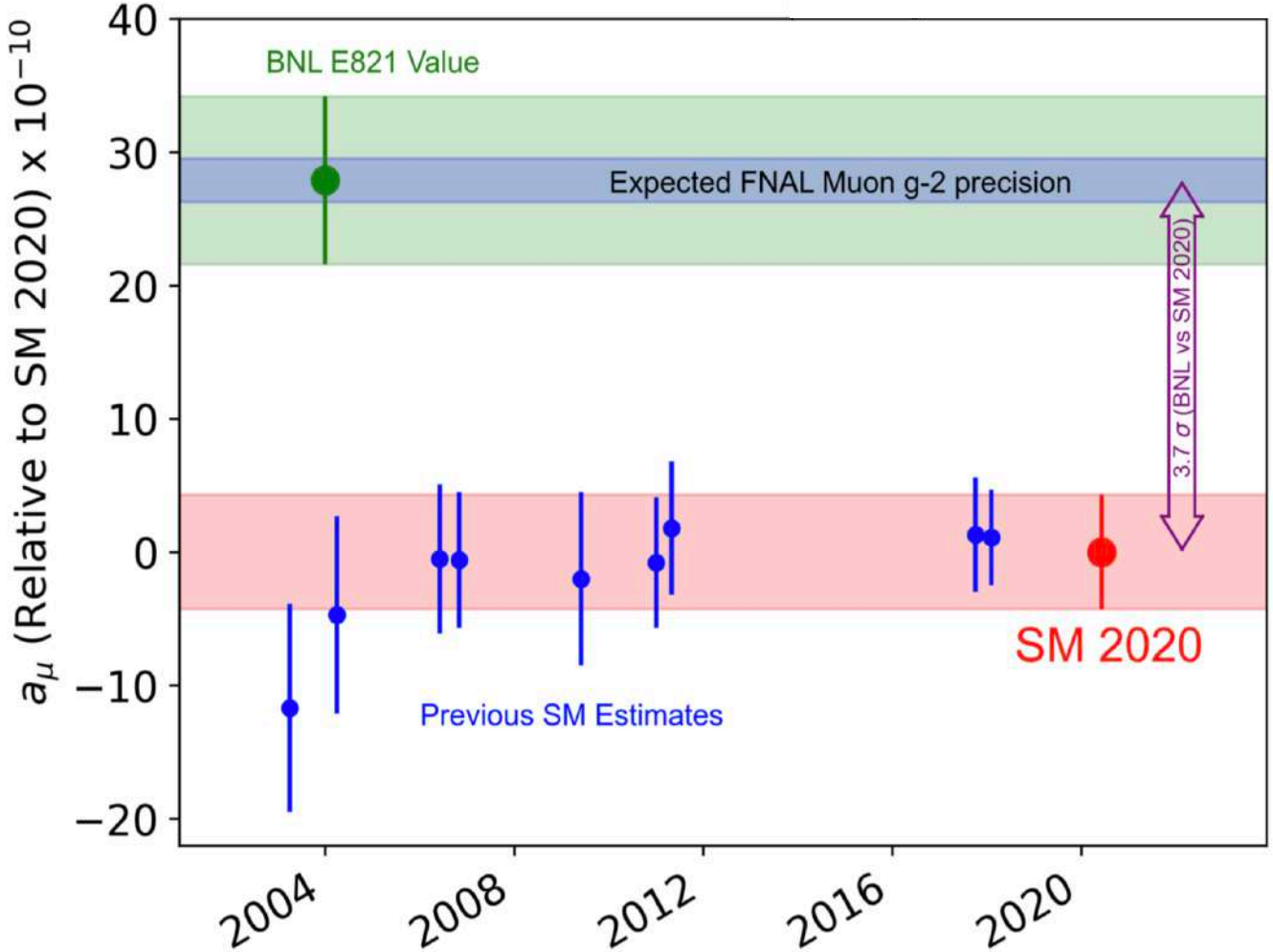
- Consistent with other lattice results
- Total uncertainty is $\sim \div 4$ . . .
- . . . and comparable to R-ratio
- Consistent w/ BNL experiment ("no new physics" scenario) !
- 2.2σ larger than DHMZ'19, 2.7σ than KNT'19 ?

Currently being scrutinised by theory initiative for white paper round 2...

Theory initiative result released

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, $udsc$)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, uds)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

Theory initiative result released



Without theory improvements:

$$\Delta a_\mu \sim 6\sigma$$

With theory improvements:

$$\Delta a_\mu > 10\sigma$$

What's next for the theory initiative?

To make the most out of the Fermilab and J-PARC experiments, theoretical SM predictions must be improved to stay commensurate with experimental uncertainty.

→ Muon g-2 Theory Initiative accelerated progress.
→ ongoing cross checks/tests and comparisons of different methods.
→ Plan to publish updated SM predictions ahead of each new major experimental update.
→ Improvements to SM evaluations from:
- Better experimental inputs for data-driven HVP.
- More experimental measurements for dispersive HLbL evaluations.
- Improved lattice QCD+QED calculations for HVP and HLbL.

→ Summer 2020 meeting at KEK postponed due to COVID-19.
→ Looking towards next year for the next theory initiative meeting.

The connection with EW precision physics

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

The muon g-2 and the bounds on the Higgs boson mass

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W.J. Marciano
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A. Sirlin
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After a brief review of the muon $g-2$ status, we analyze the possibility that the present discrepancy between experiment and the Standard Model (SM) prediction may be due to hypothetical errors in the determination of the hadronic leading-order contribution to the latter. In particular, we show how an increase of the hadro-production cross section in low-energy e^+e^- collisions could bridge the muon $g-2$ discrepancy, leading however to a decrease on the electroweak upper bound on M_H, the SM Higgs boson mass. That bound is currently $M_H \lesssim 150$ GeV (95%CL) based on the preliminary top quark mass $M_t = 172.6(1.4)$ GeV and the recent determination $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02768(22)$, while the direct-search lower bound is $M_H > 114.4$ GeV (95%CL). By means of a detailed analysis we conclude that this solution of the muon $g-2$ discrepancy is unlikely in view of current experimental error estimates. However, if this turns out to be the solution, the 95%CL upper bound on M_H is reduced to about 130 GeV which, in conjunction with the experimental lower bound, leaves a narrow window for the mass of this fundamental particle.

The muon g-2 and $\Delta\alpha$ connection

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Hadronic vacuum polarization: $(g-2)_\mu$ versus global electroweak fits

Andreas Crivellin,[1,2,*] Martin Hoferichter,[3,†] Claudio Andrea Manzari,[1,2,‡] and Marc Montull[1,2,§]
[1] *Paul Scherrer Institut, CH–5232 Villigen PSI, Switzerland*
[2] *Physik-Institut, Universität Zürich, Winterthurerstrasse 190, CH–8057 Zürich, Switzerland*
[3] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH–3012 Bern, Switzerland*

Hadronic vacuum polarization (HVP) is not only a critical part of the Standard Model (SM) prediction for the anomalous magnetic moment of the muon $(g-2)_\mu$, but also a crucial ingredient for global fits to electroweak (EW) precision observables due to its contribution to the running of the fine-structure constant encoded in $\Delta\alpha^{(5)}_{\rm had}$. We find that with modern EW precision data, including the measurement of the Higgs mass, the global fit alone provides a competitive, independent determination of $\Delta\alpha^{(5)}_{\rm had}|_{\rm EW} = 270.2(3.0)\times 10^{-4}$. This value actually lies below the range derived from $e^+e^- \to$ hadrons cross-section data, and thus goes into the opposite direction as would be required if a change in HVP were to bring the SM prediction for $(g-2)_\mu$ into agreement with the Brookhaven measurement. Depending on the energy where the bulk of the changes in the cross section occurs, reconciling experiment and SM prediction for $(g-2)_\mu$ by adjusting HVP would thus not necessarily weaken the case for physics beyond the SM (BSM), but to some extent shift it from $(g-2)_\mu$ to the EW fit. We briefly explore some options of BSM scenarios that could conceivably explain the ensuing tension.

On Constraints Between $\Delta\alpha_{\rm had}(M_Z^2)$ and $(g_\mu-2)_{\rm HVP}$

Eduardo de Rafael

Aix-Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France

Impact of correlations between a_μ and $\alpha_{\rm QED}$ on the EW fit

Bogdan Malaescu[1,a] and Matthias Schott[2,b]
[1] LPNHE, Sorbonne Université, Université de Paris, CNRS/IN2P3, Paris, France
[2] Institute of Physics, Johannes Gutenberg University, Mainz, Germany

Abstract. We study the potential impact on the electroweak (EW) fits due to the tensions between the current determinations of the hadronic vacuum polarisation (HVP) contributions to the anomalous magnetic moment of the muon (a_μ), based on either phenomenological dispersion integrals using measured hadronic spectra or on Lattice QCD calculations. The impact of the current tension between the experimental measurement of a_μ and the total theoretical prediction based on the phenomenological calculations of the HVP are also studied. The correlations between the uncertainties of the theoretical predictions of a_μ and of the running of $\alpha_{\rm QED}$ are taken into account in the studies. We conclude that the impact on the EW fit can be large in improbable scenarios involving global shifts of the full HVP contribution, while it is much smaller if the shift is restricted to a lower mass range and/or if the shift in $\alpha_{\rm QED}$ is obtained from that in a_μ through appropriate use of the correlations. Indeed, the latter scenarios only imply at most a 2.6/16 increase in the χ^2/n.d.f. of the EW fits and relatively small changes for the resulting fit parameter values.

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Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

The muon g-2 and the bounds on the Higgs boson mass

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2008

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The muon g-2 and $\Delta\alpha$ connection

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2020 update

Hadronic vacuum polarization: $(g-2)_\mu$ versus global electroweak fits

Andreas Crivellin,[1,2,*] Martin Hoferichter,[3,†] Claudio Andrea Manzari,[1,2,‡] and Marc Montull[1,2,§]
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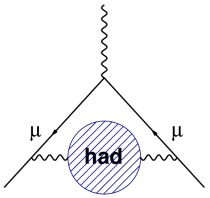
New similar analyses in 2020

(All analyses come to largely the same conclusions)

Hadronic cross section data

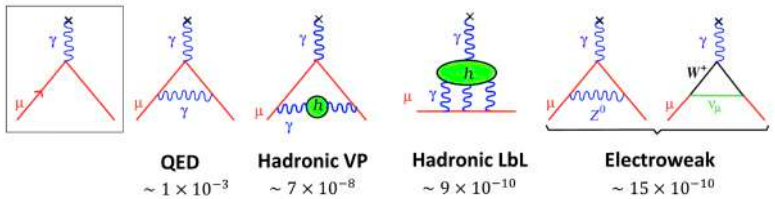
Experimentally measured hadronic cross section: σ_{had}

Muon g-2:
hadronic vacuum polarisation contribution



$$a_\mu^{\text{had, VP}} = \frac{1}{4\pi^3}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\, K(s)$$

… sum with other SM contributions…



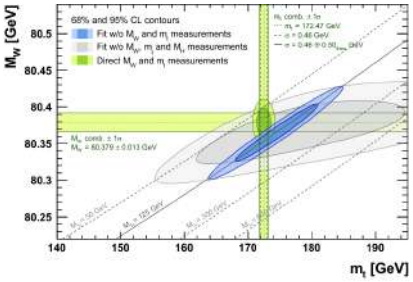
→ Determines a_μ^{SM} and $\Delta a_\mu = 3.7\sigma$

Increase cross section so that $\Delta a_\mu = 0$?
→ Solves muon g-2 discrepancy

Running QED coupling:
hadronic contribution to running

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = \frac{q^2}{4\pi\alpha^2}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\,\frac{q^2}{(q^2 - s)}$$

… evaluate at $q^2 = M_Z^2$ and input into **global EW fit**…



→ Predicts M_W, M_H, $\sin^2\theta_{eff}^{lep}$ and more…

Increase cross section so that $\Delta a_\mu = 0$?
→ What happens to precision EW parameters?

Muon g-2 and $\Delta\alpha$

Slide content by Massimo Passera.

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$$a_\mu^{HLO} \rightarrow \quad a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \; s_u < M_Z^2,$$

$$\Delta\alpha_{had}^{(5)} \rightarrow \quad b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ϵ>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Note the very different energy-dependent weighting of the integrands…

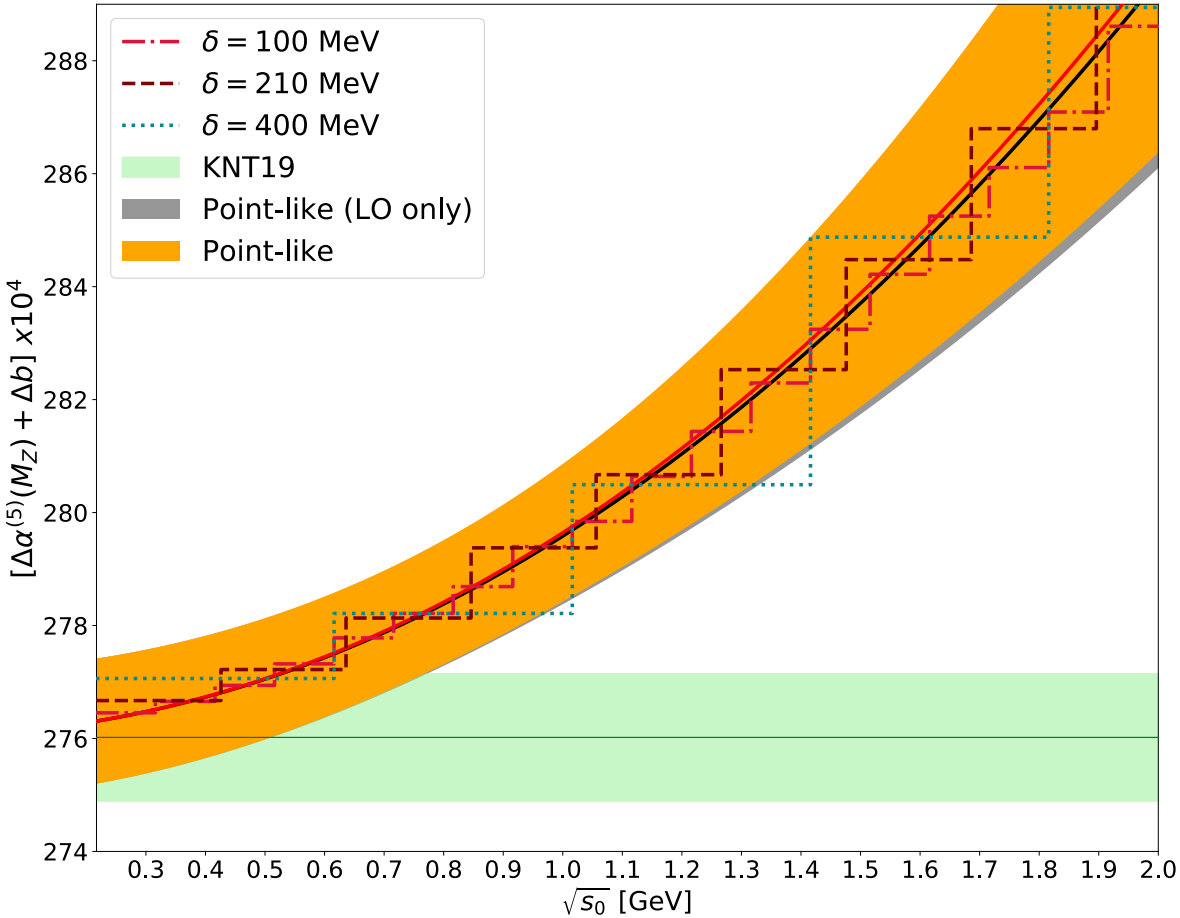
Use precise and up-to-date compilation of total hadronic cross section from KNT

Keshavarzi, Nomura and Teubner, Phys.Rev.D 101 (2020) 014029, arXiv:1911.00367

The muon g-2 in the SM

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.

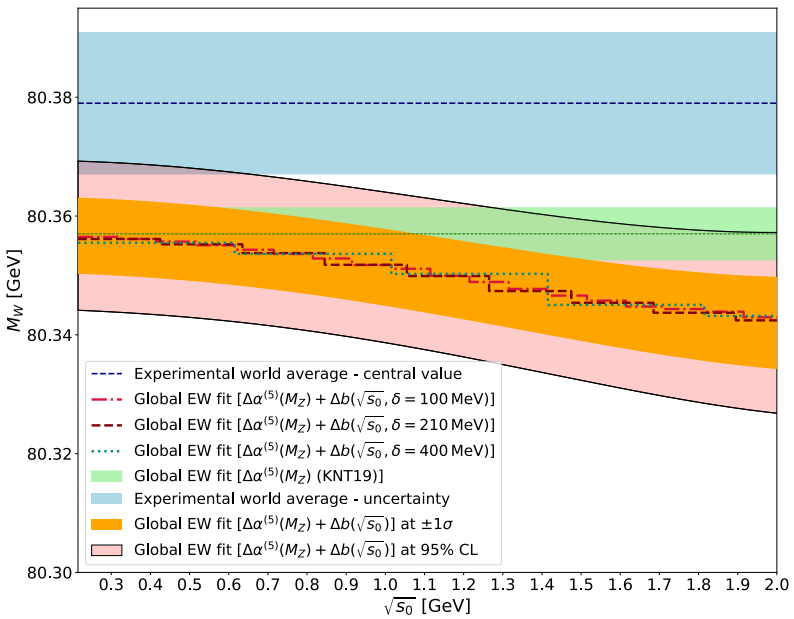


The muon g-2 in the SM

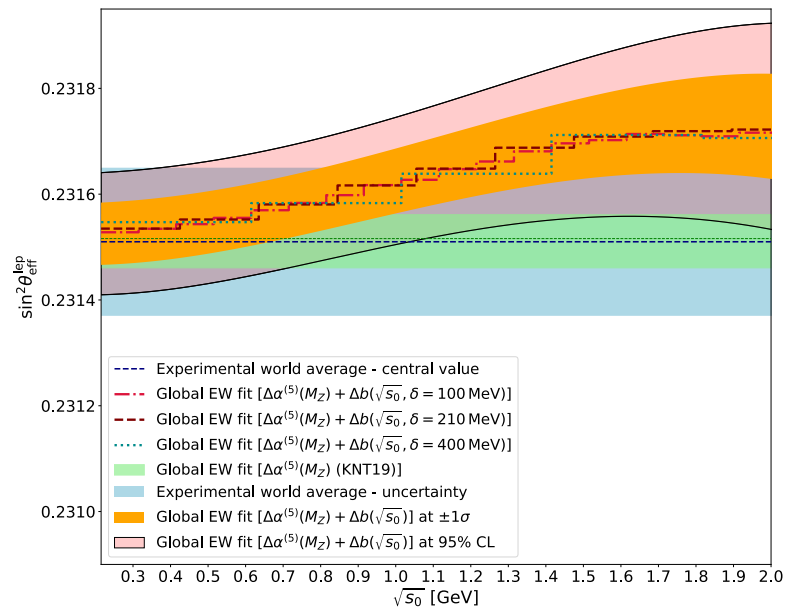
Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.
- Input new values of $\Delta\alpha$ into Gfitter to predict EW observables.
- Analysis greatly constrained from precise EW observables measurements and comprehensive hadronic cross section data.

M_W



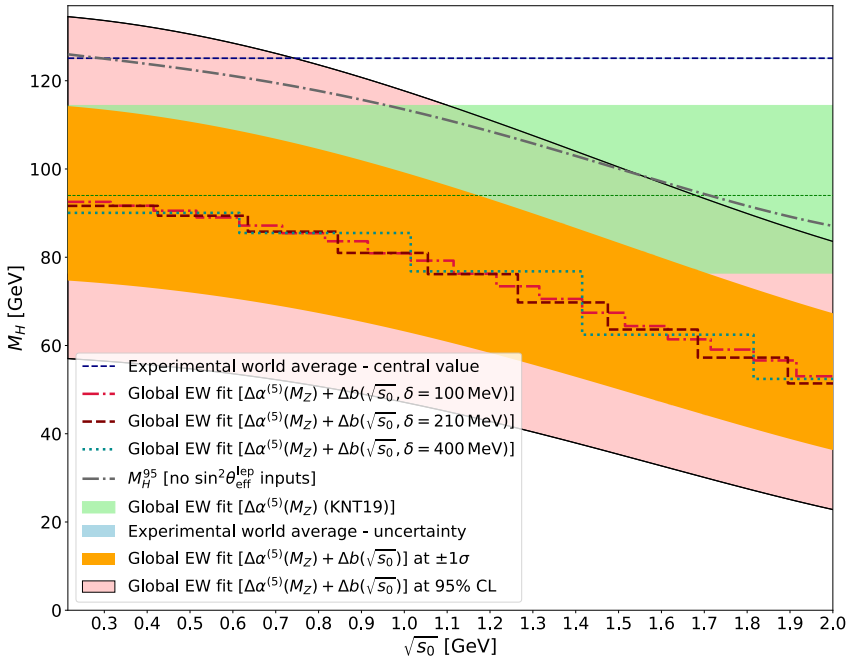
$\sin^2\theta_{eff}^{lep}$

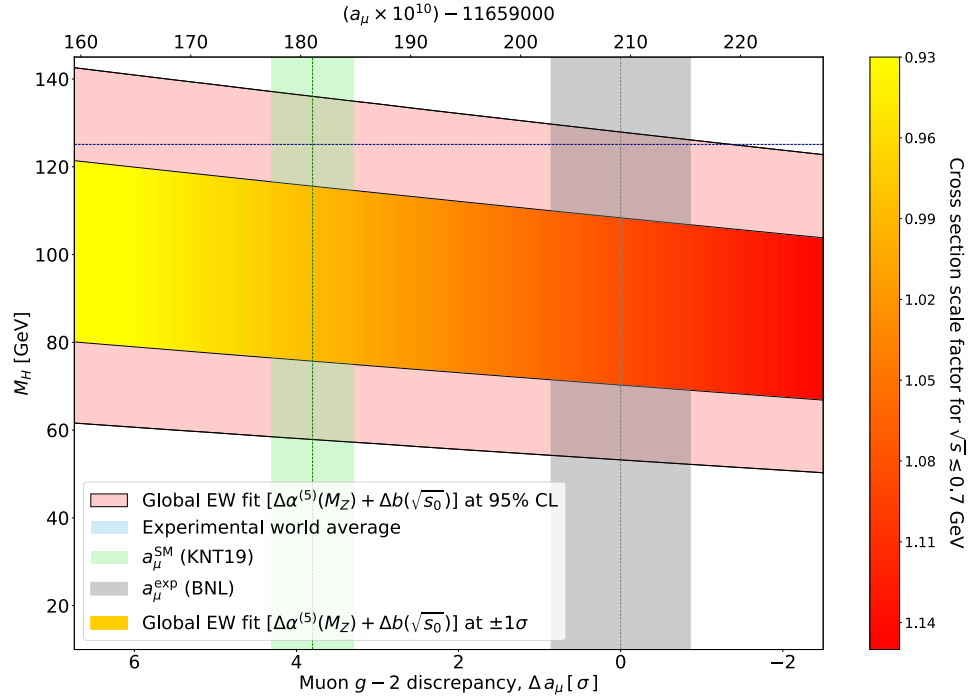


Bounds from the Higgs mass

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

Shifts in σ(s) needed to bridge Δa_μ are found to be excluded above $\sqrt{s} > 0.7$ GeV at the 95%CL.



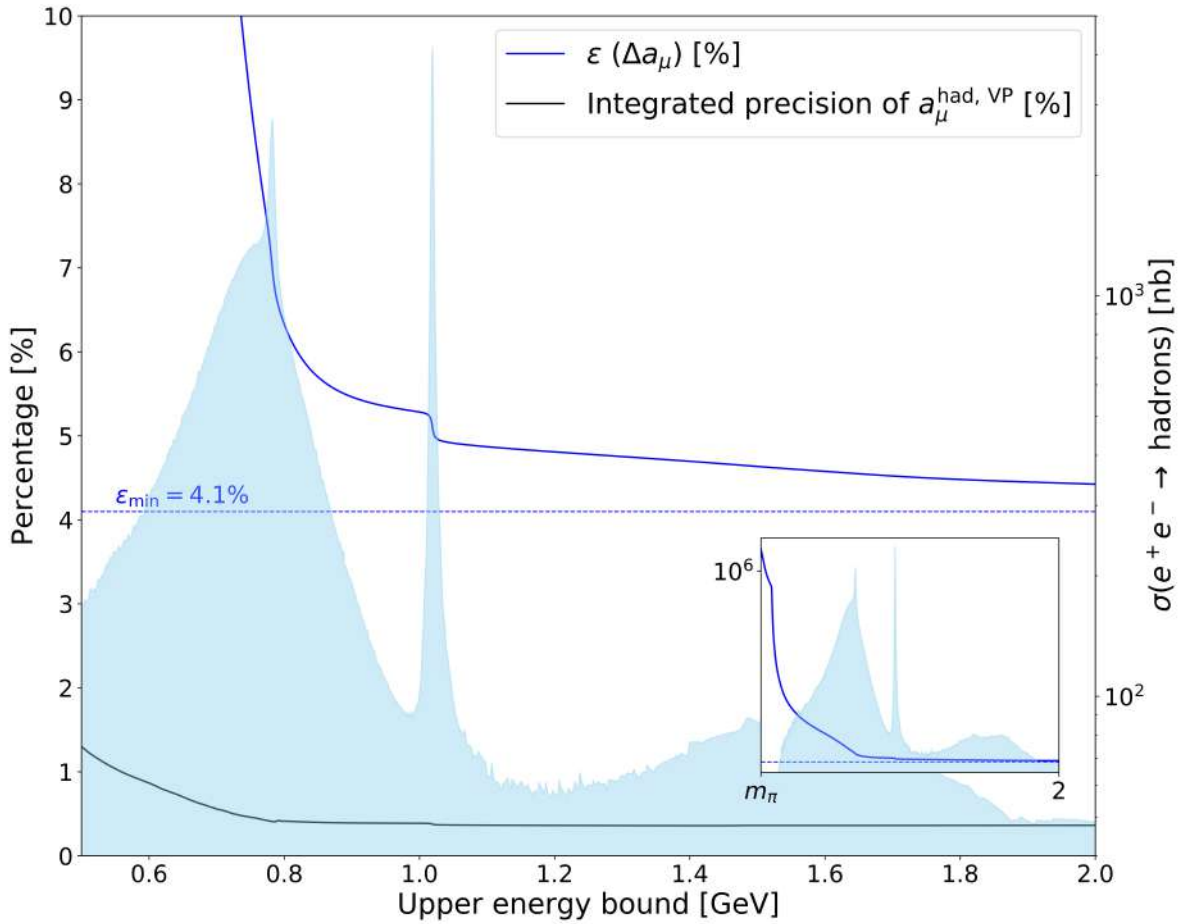


So, from EW sector, shifts to σ(s) to bridge g-2 discrepancy and BMW are allowed below 0.7 GeV…?

→ But, how realistic are the required shifts in σ(s)?

How realistic are the required shifts in σ(s)?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

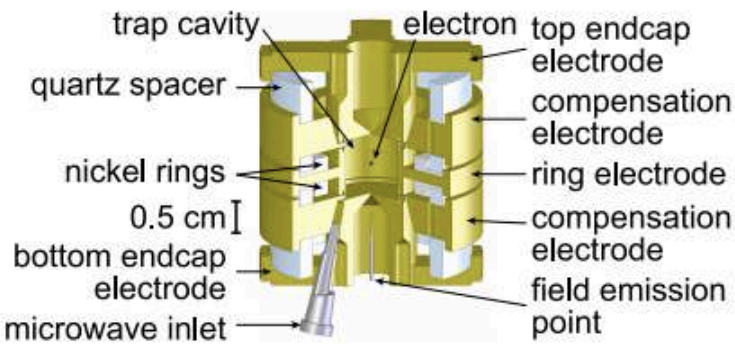


Size of missed contributions would need to be implausibly large given the robust status of the hadronic cross section measurements.

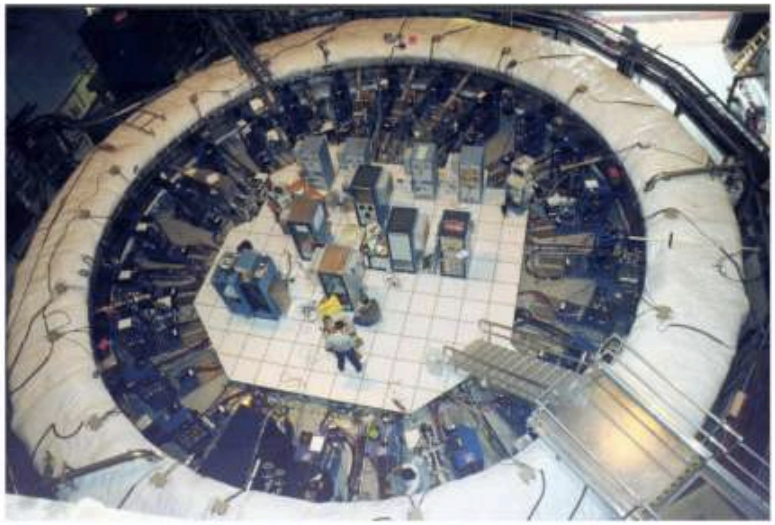
Magnetic moments: a_e vs. a_μ

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]
Bennet et al., PRD 73(2006)072003



one electron quantum cyclotron

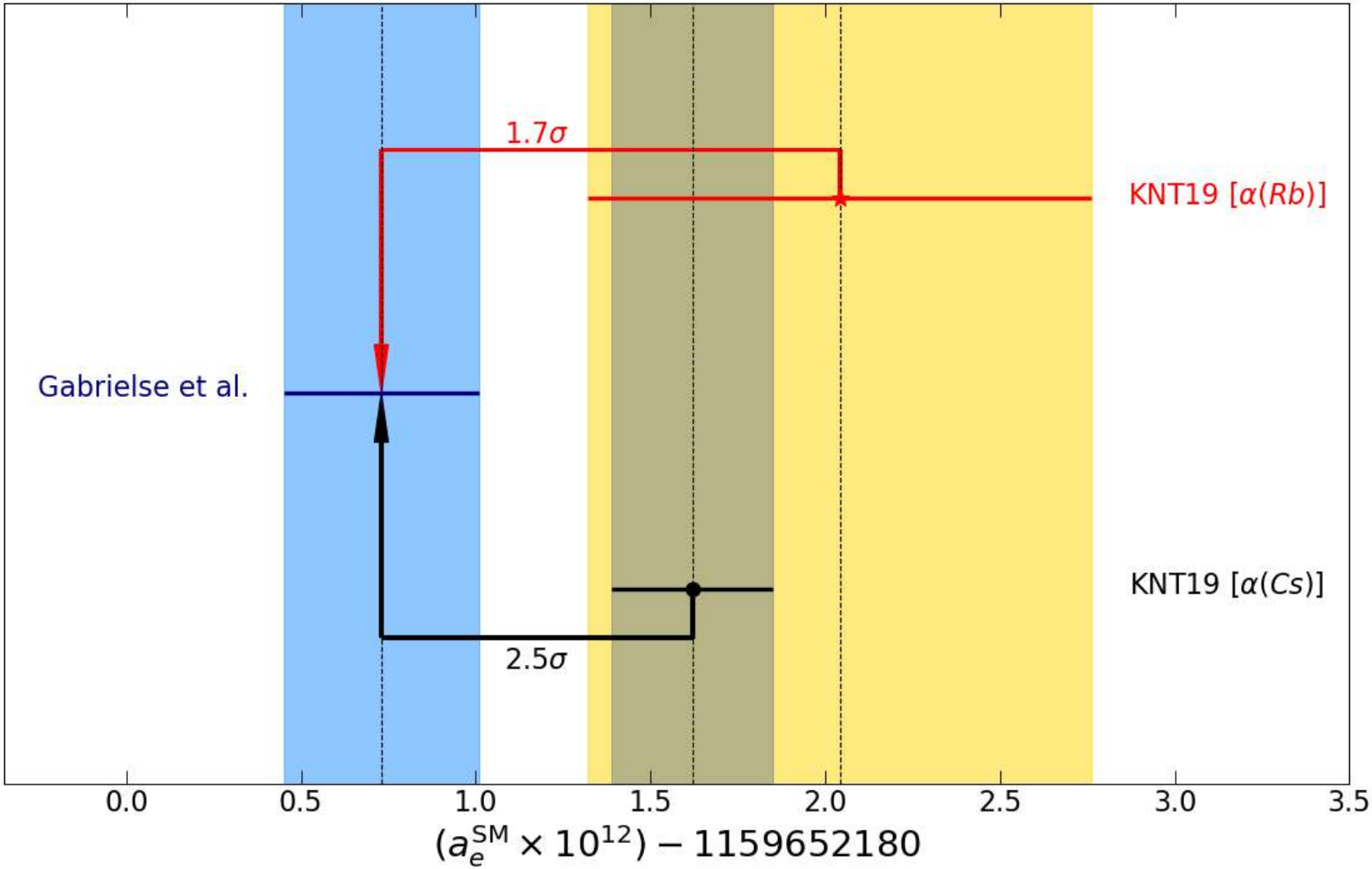


- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales
- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\rm NP} \sim C\, m_\ell^2/\Lambda_{\rm NP}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, but a_e provides precise determination of α

The electron $g-2$

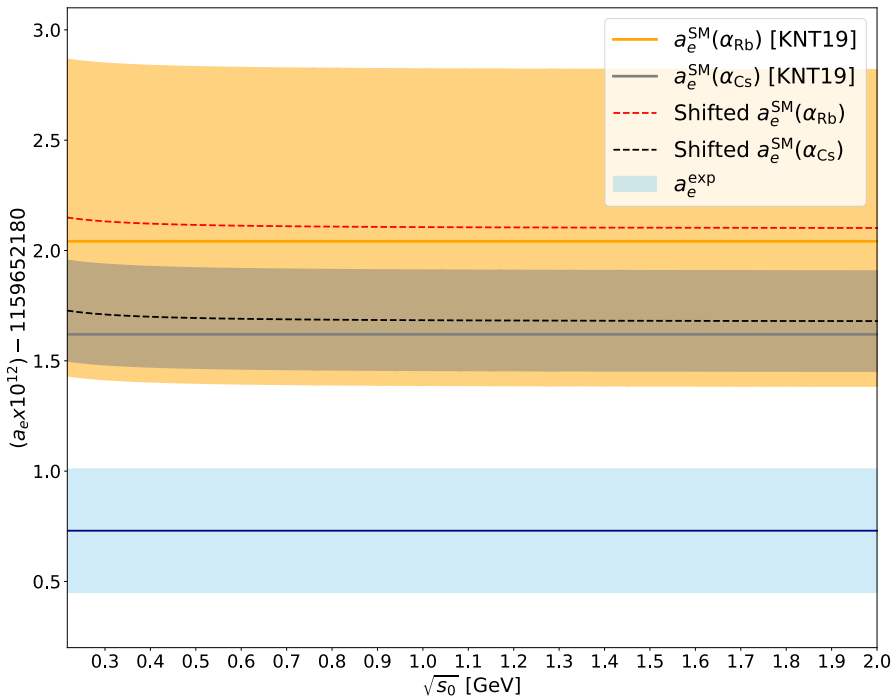
Phys.Rev.D 101 (2020) 014029.

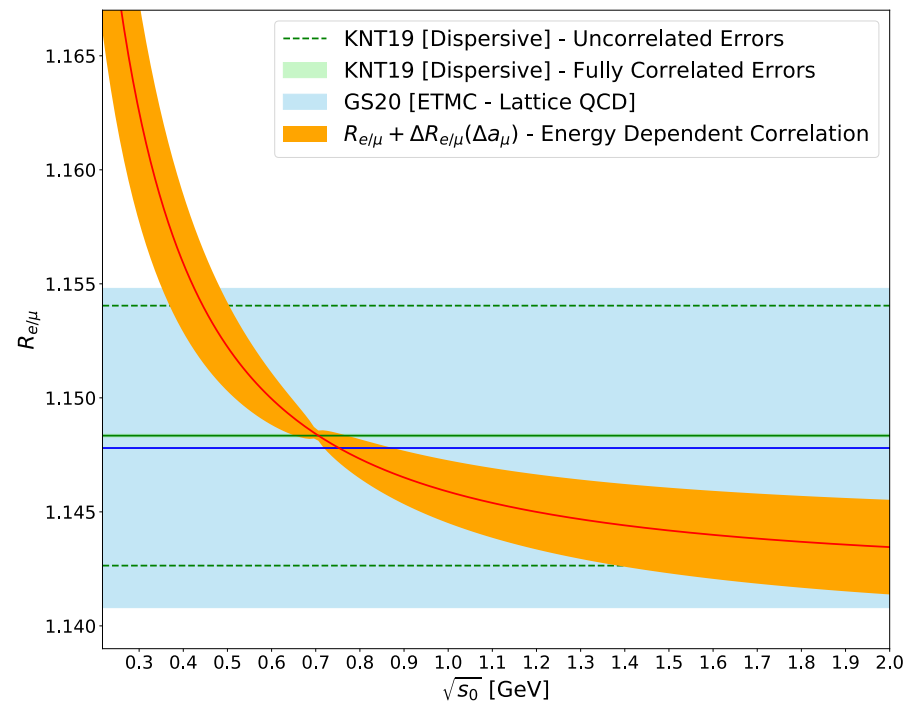


What do this shifts do the electron g-2?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

a_e^{SM} vs. a_e^{exp} yields $\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = (-0.89 \pm 0.36)\times 10^{-12}$ $[2.5\sigma]$





Shifts at low-energy that are “allowed” in muon g-2 invoke additional tension for electron g-2.

Conclusions

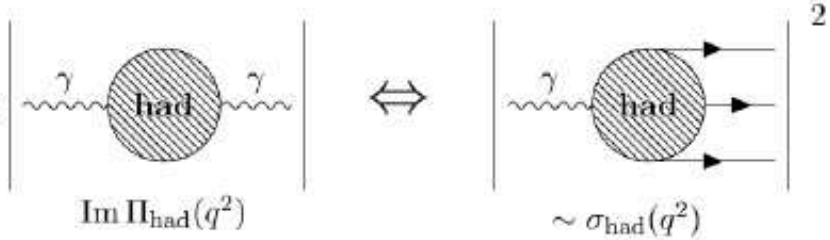
- Community-approved theory result from Muon g-2 theory initiative now released.
- HLbL from models, dispersive approaches and lattice all in agreement
  → cannot be source of g-2 discrepancy.
- Recommended HVP value is taken from merging of data-driven dispersive results.
- Errors of recommended lattice HVP values are too large to make definitive statements.
- BMW lattice 2020 result has 0.6% errors and agrees with BNL
  → not so significant after update and currently being checked by theory initiative.
- Connection between g-2 and $\Delta\alpha$ allows impact of g-2 discrepancy on EW fit to be explored.
- EW study excludes shifts to hadronic cross section above 0.7 GeV to bridge muon g-2 discrepancy.
- Required shifts to hadronic cross section below 0.7 GeV are implausibly large.
- These low energy shifts also worsen tension in electron g-2.

Thank you.

Backups

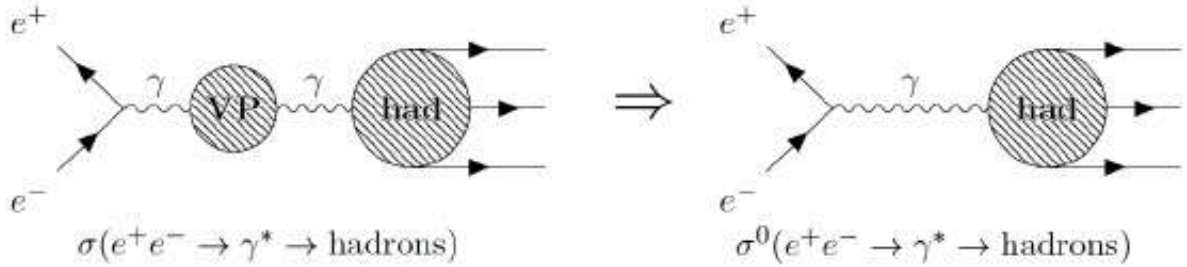
$\sigma^0_{\text{had},\,\gamma}$: VP corrections

⇒ Reconsider the optical theorem: Im



⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\text{had, VP}}$

→ Must subtract VP:



⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

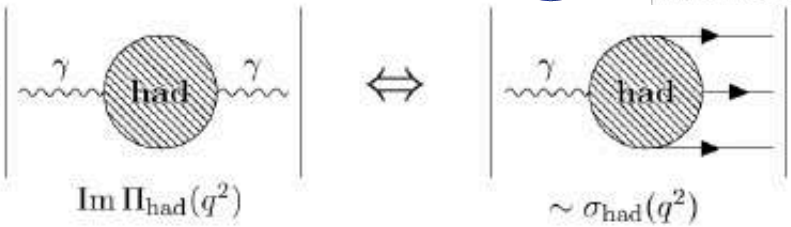
→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\text{had}}(s) = \sigma_{\text{had}}(s)|1 - \Pi(s)|^2$

⇒ If correcting data, apply corresponding radiative correction uncertainty

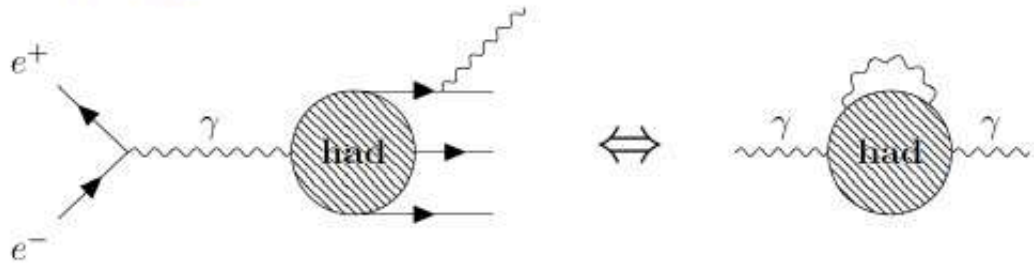
→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

$\sigma^0_{\text{had},\,\gamma}$: FSR corrections

⇒ Reconsider the optical theorem: Im



⇒ Photon FSR formally higher order corrections to $a_\mu^{\text{had, VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Building the hadronic *R*-ratio

$m_\pi \leq \sqrt{s} \leq 2$ GeV

- Input experimental hadronic cross section data*
- Combine all available data in exclusive hadronic final states ($\pi^+\pi^-, K^+K^-, \ldots$)
- Sum ~ 35 exclusive channels
- Detailed data analysis
- Robust treatment of experimental errors
- Estimate missing data input (isospin relations, ChPT...)

$2 \leq \sqrt{s} \leq 11.2$ GeV

- Can use experimental inclusive R data* or pQCD
- Must use data at quark flavour thresholds
- Combine all available R data
- Robust treatment of experimental errors
- Include narrow resonances

$11.2 \leq \sqrt{s} < \infty$ GeV

- Calculate R using pQCD (`rhad`)

*σ_{had} experiments

- KLOE
- BaBar
- SND
- CMD-(2/3)
- KEDR
- BESIII

Question: for reliable precision, how are data correlated and how should those correlations be implemented?

Data combination: setup

⇒ Re-bin data into *clusters*

→ Scan cluster sizes for preferred solution (error, χ^2, check by sight...)

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Use all information provided
- Use correlations to full capacity

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

⇒ When combining data...

→ ...how to best combine large amounts of data from different experiments

→ ...the correct implementation of correlated uncertainties (statistical and systematic)

→ ...finding a solution that is free from bias

d'Agostini bias [Nucl.Instrum.Meth. A346 (1994) 306-311]

$$x_1 = 0.9 \pm \delta x_1 \qquad C_{\text{sys}} = \begin{pmatrix} p^2 x_1^2 & p^2 x_1 x_2 \\ p^2 x_2 x_1 & p^2 x_2^2 \end{pmatrix} \qquad \Rightarrow \bar{x} \simeq 0.98 \text{ (systematic bias)}$$

$$x_2 = 1.1 \pm \delta x_2$$

(Normalisation uncertainties defined by data)

Effect worsened with full, iterative data combination

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

⇒ When combining data...

→ ...how to best combine large amounts of data from different experiments

→ ...the correct implementation of correlated uncertainties (statistical and systematic)

→ ...finding a solution that is free from bias

Fixed matrix method [R. D. Ball et al. [NNPDF Collaboration], JHEP 1005 (2010) 075.]

$$x_1 = 0.9 \pm \delta x_1 \qquad C_{\text{sys}} = \begin{pmatrix} p^2\bar{x}^2 & p^2\bar{x}^2 \\ p^2\bar{x}^2 & p^2\bar{x}^2 \end{pmatrix} \qquad \Rightarrow \bar{x} = 1.00 \text{ (systematic bias)}$$

$$x_2 = 1.1 \pm \delta x_2$$

(Normalisation uncertainties defined by estimator)

Redefinition repeated at each stage of iterative data combination

Linear χ^2 minimisation

⇒ Clusters are defined to have linear cross section

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

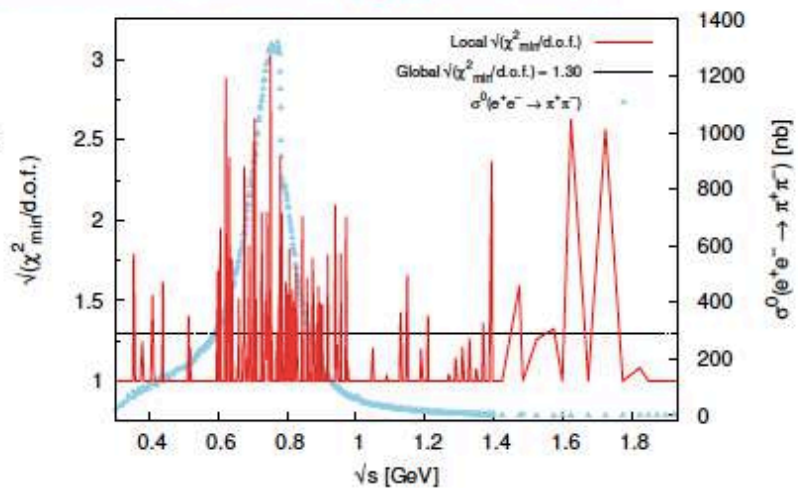
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} (R_i^{(m)} - \mathcal{R}_m^i) \mathbf{C}^{-1}(i^{(m)}, j^{(n)}) (R_j^{(n)} - \mathcal{R}_n^j)$$

⇒ Through correlations and linearisation, result is the minimised solution of all available uncertainty information

→ ... through a method that has been shown to avoid d'Agostini bias

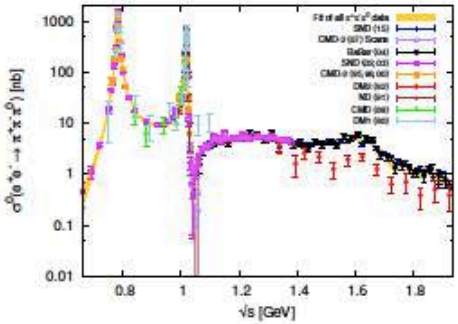
⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

→ ... and any data tensions are reflected in a local and global $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation



Other notable channels

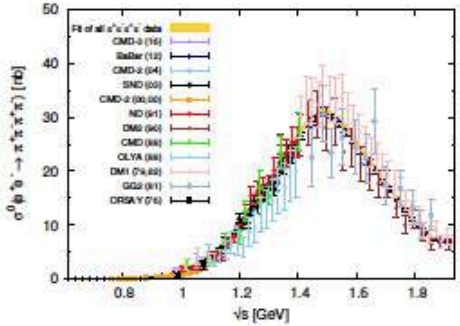
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

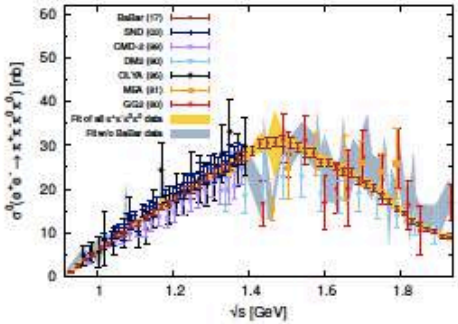
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

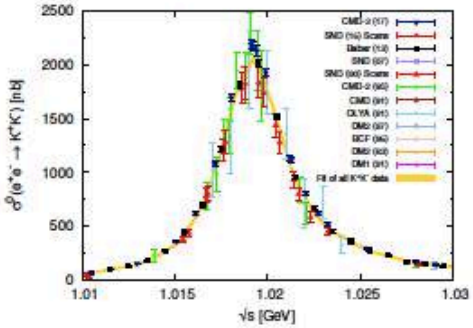
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

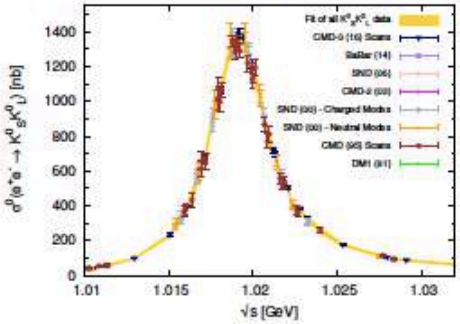
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

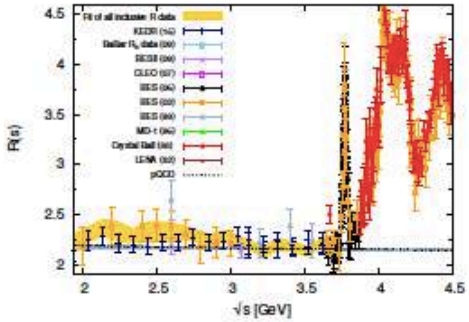
$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)

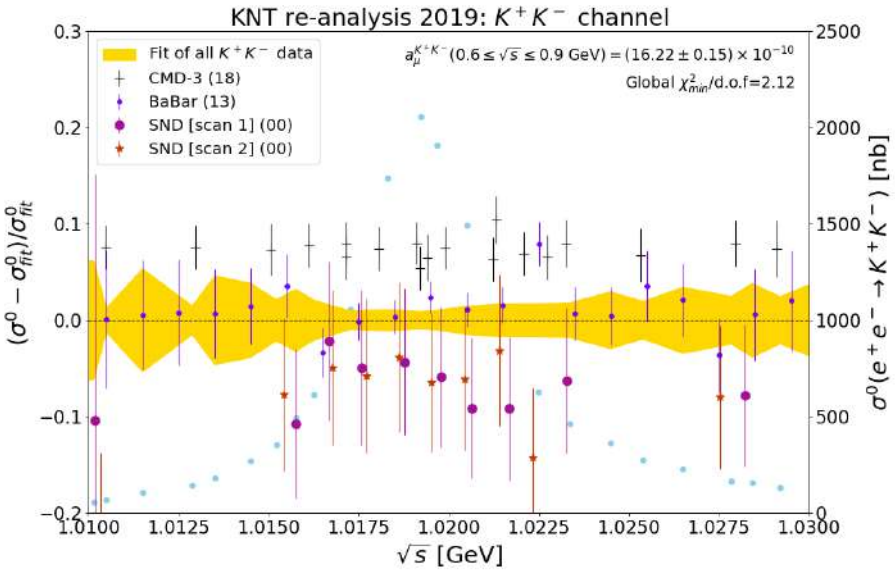


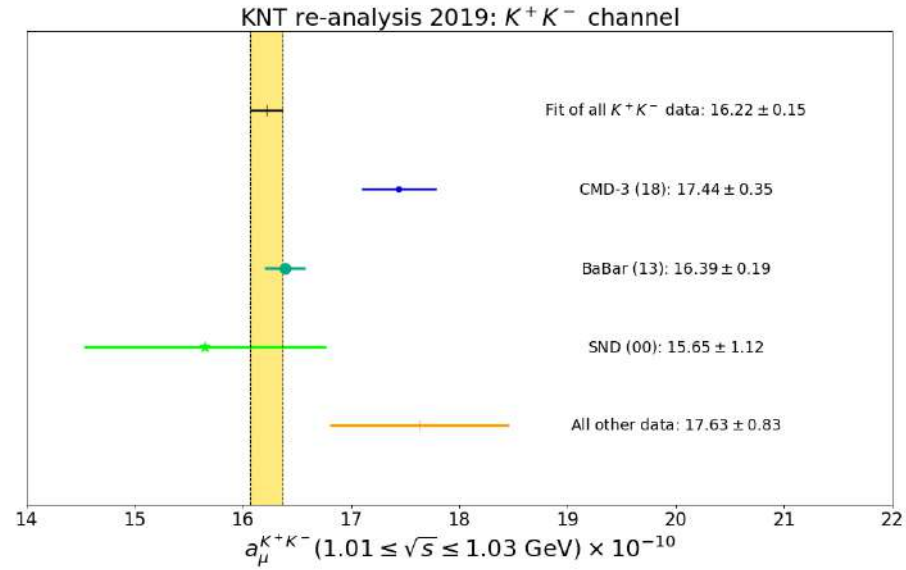
HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

Highlight: data tensions in K^+K^- channel

Notable tension also now exists on ϕ in K^+K^- channel:





Most recent CMD-3 data is higher than BaBar data…

BaBar was already relatively high

$$a_\mu^{K^+K^-}[0.9875 \le \sqrt{s} \le 1.937\ \text{GeV}] = 23.03 \pm 0.08_{stat} \pm 0.20_{sys} \pm 0.03_{vp} \pm 0.00_{fsr}$$
$$= 23.03 \pm 0.22_{tot}$$

- Tensions results in 20 % local χ^2_{min}/d.o.f. error inflation

$KK\pi, KK\pi\pi$ & isospin

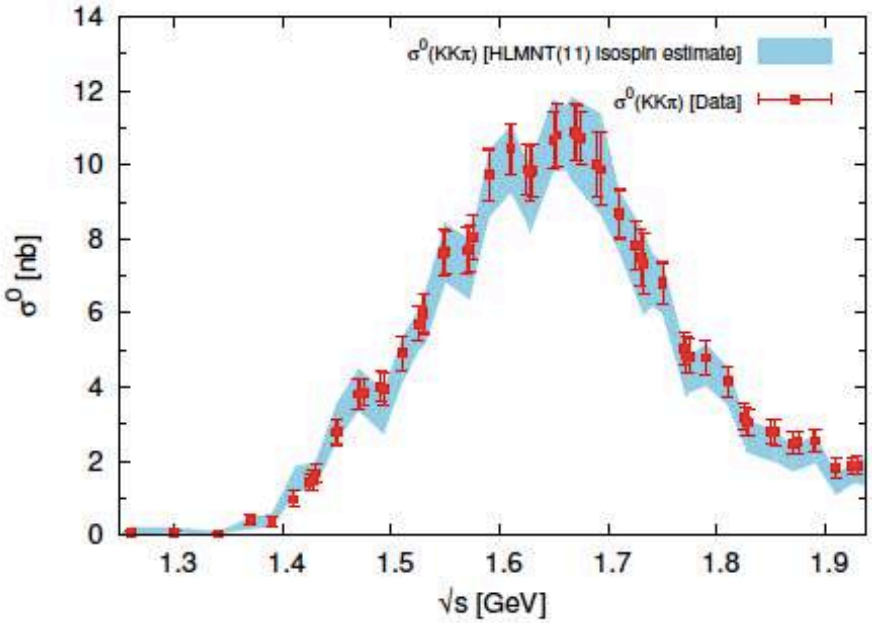
⇒ New data for $KK\pi$ and $KK\pi\pi$
removes reliance on isopsin (only $K_S^0 \cong K_L^0$)

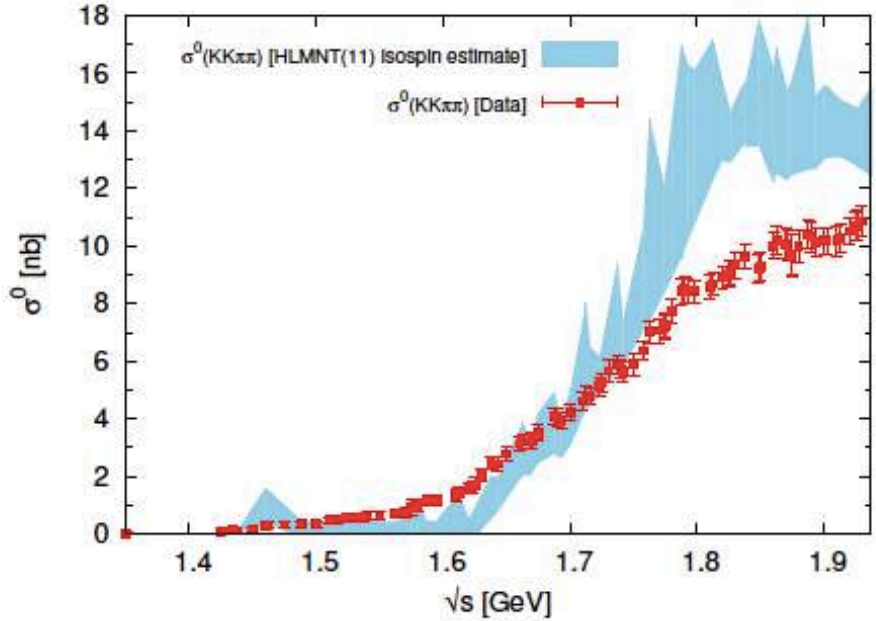
$KK\pi$

$K_S^0 K_L^0 \pi^0$ [Phys.Rev. D95 (2017), 052001, arXiv:1711.07143]

$KK\pi\pi$

$K_S^0 K_L^0 \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002]
$K_S^0 K_S^0 \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002],
$K_S^0 K_L^0 \pi^0\pi^0$ [Phys.Rev. D95 (2017), 052001]
$K_S^0 K^\pm \pi^\mp \pi^0$ [Phys.Rev. D95 (2017), 092005]



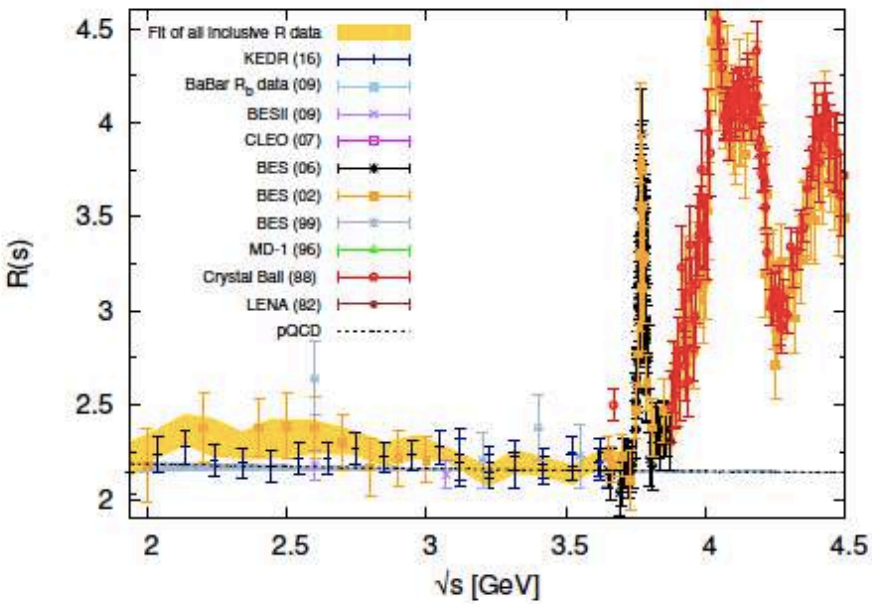


HLMNT11: 2.65 ± 0.14
KNT18: 2.71 ± 0.12

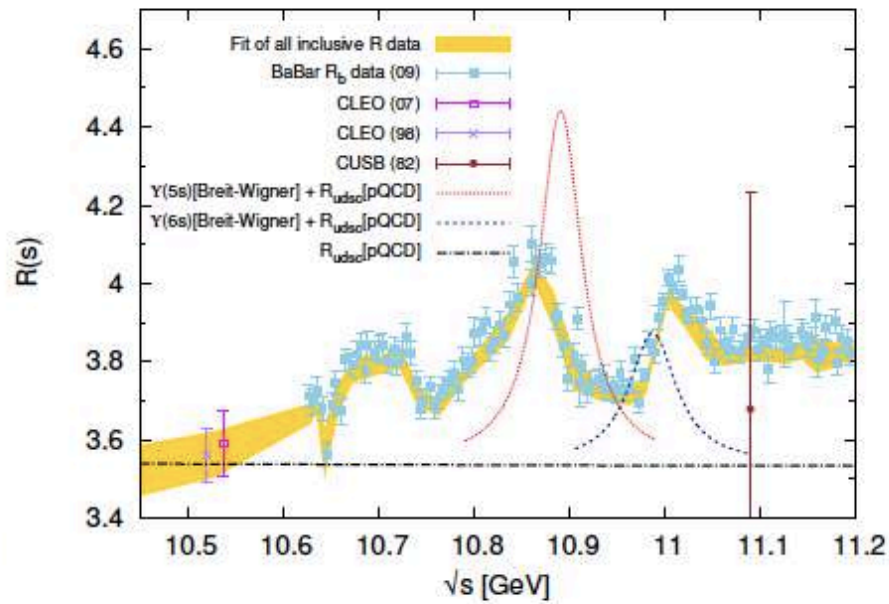
HLMNT11: 2.51 ± 0.35
KNT18: 1.93 ± 0.08

Inclusive channel

⇒ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

⟹ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

Exclusive/inclusive transition point

⇒ New KEDR data allow reconsideration of exclusive/inclusive transition point

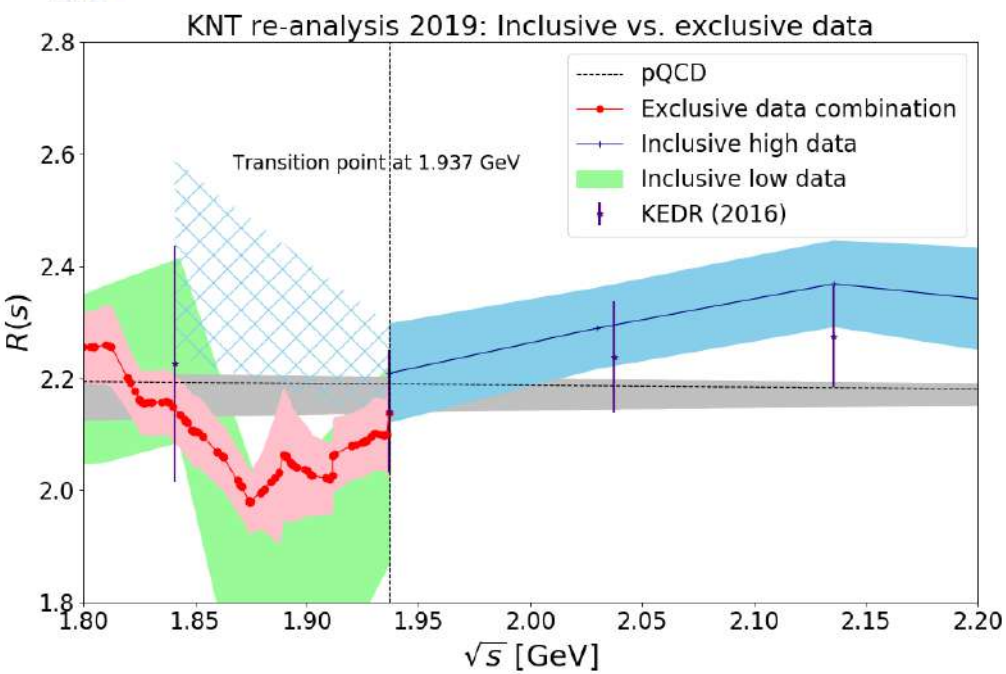
→ KNT18 aim to avoid use of pQCD and keep a data-driven analysis

→ Disagreement between sum of exclusive states and inclusive data/pQCD

→ New $\pi^+\pi^-\pi^0\pi^0$ data result in reduction of the cross section

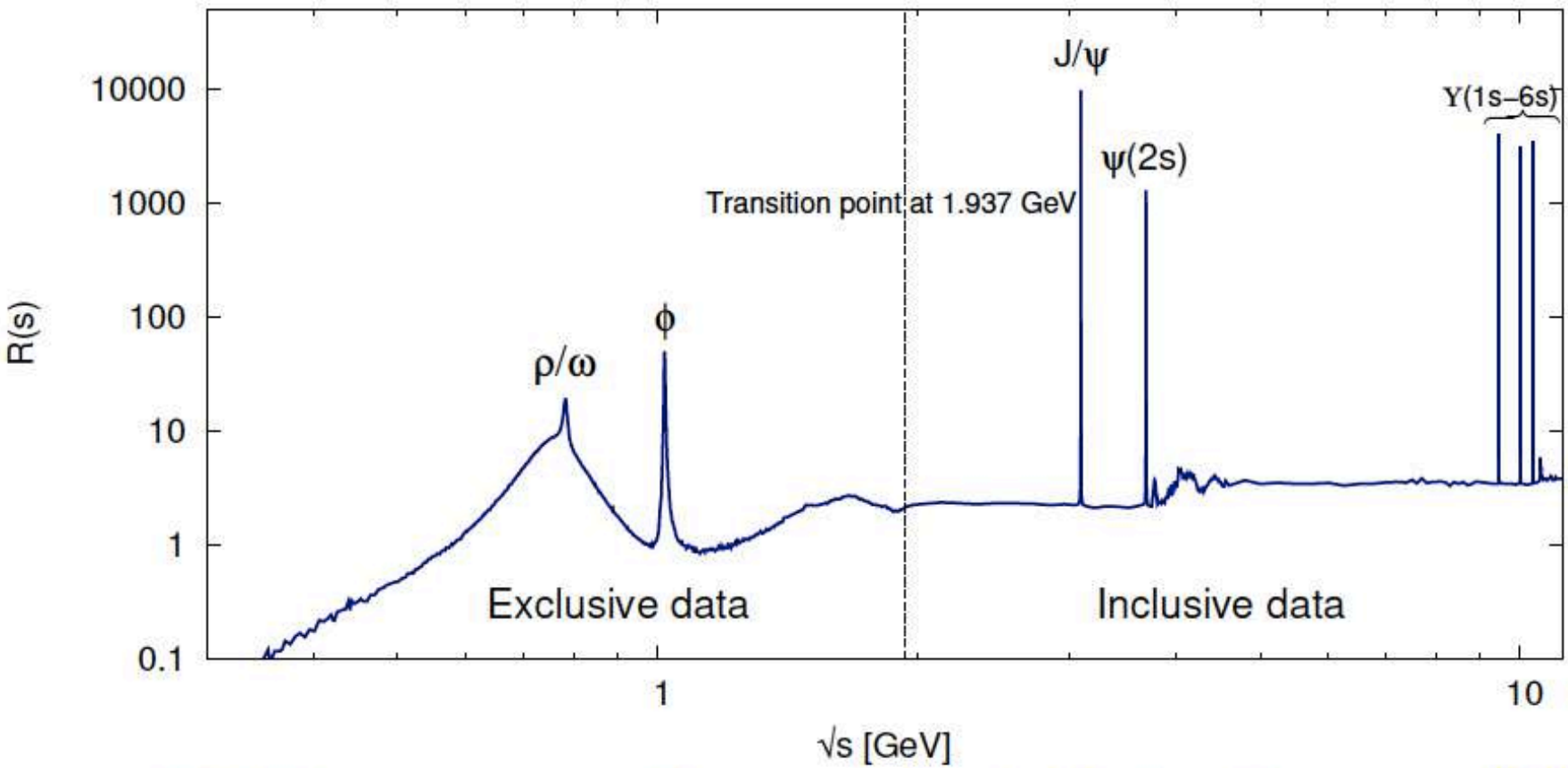
→ Previous transition point at 2 GeV no longer the preferred choice

→ More natural choice for this transition point at 1.937 GeV



Input	$a_\mu^{\text{had, LO VP}}[1.841 \leq \sqrt{s} \leq 2.00\ \text{GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive (< 1.937 GeV) + inclusive (> 1.937 GeV)	6.23 ± 0.13

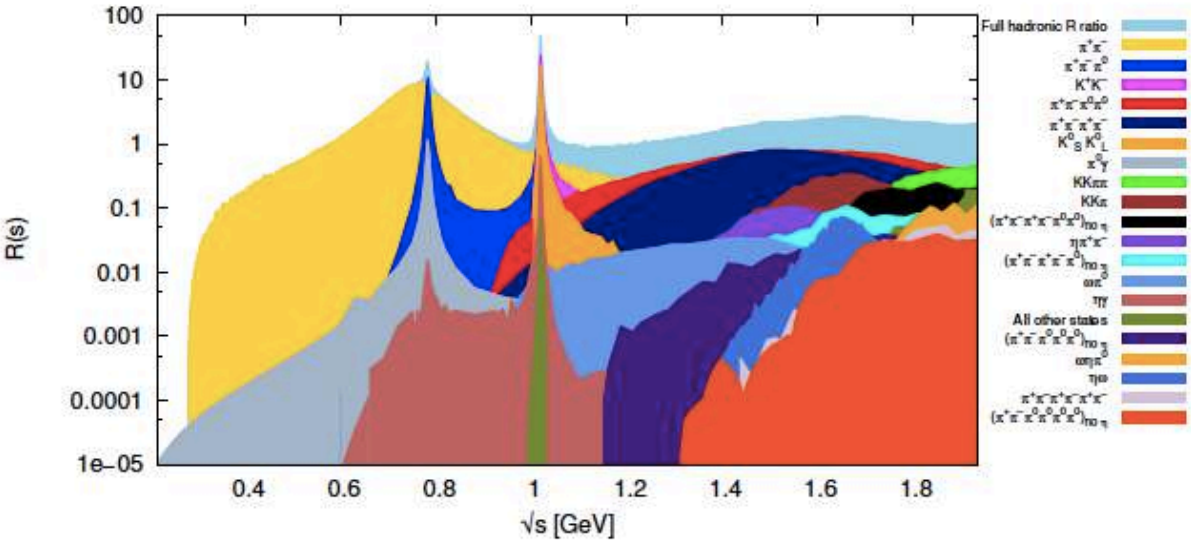
$R(s)$ for $m_\pi \leq \sqrt{s} \leq 11.2$ GeV



⇒ Full KNT18 compilation data set for hadronic R-ratio now available...

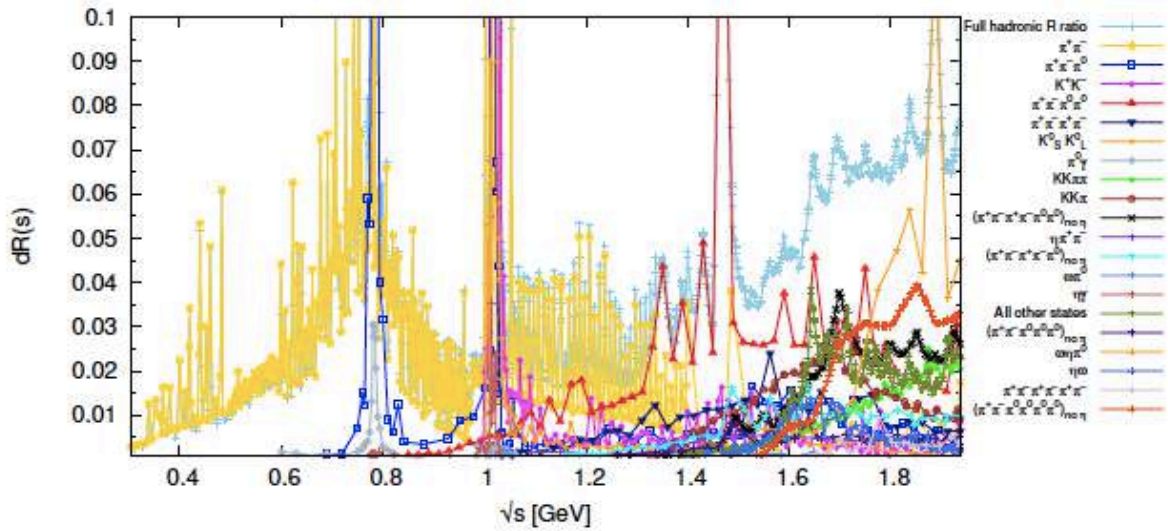
⟹ **...complete with full covariance matrix**

Contributions below 2 GeV



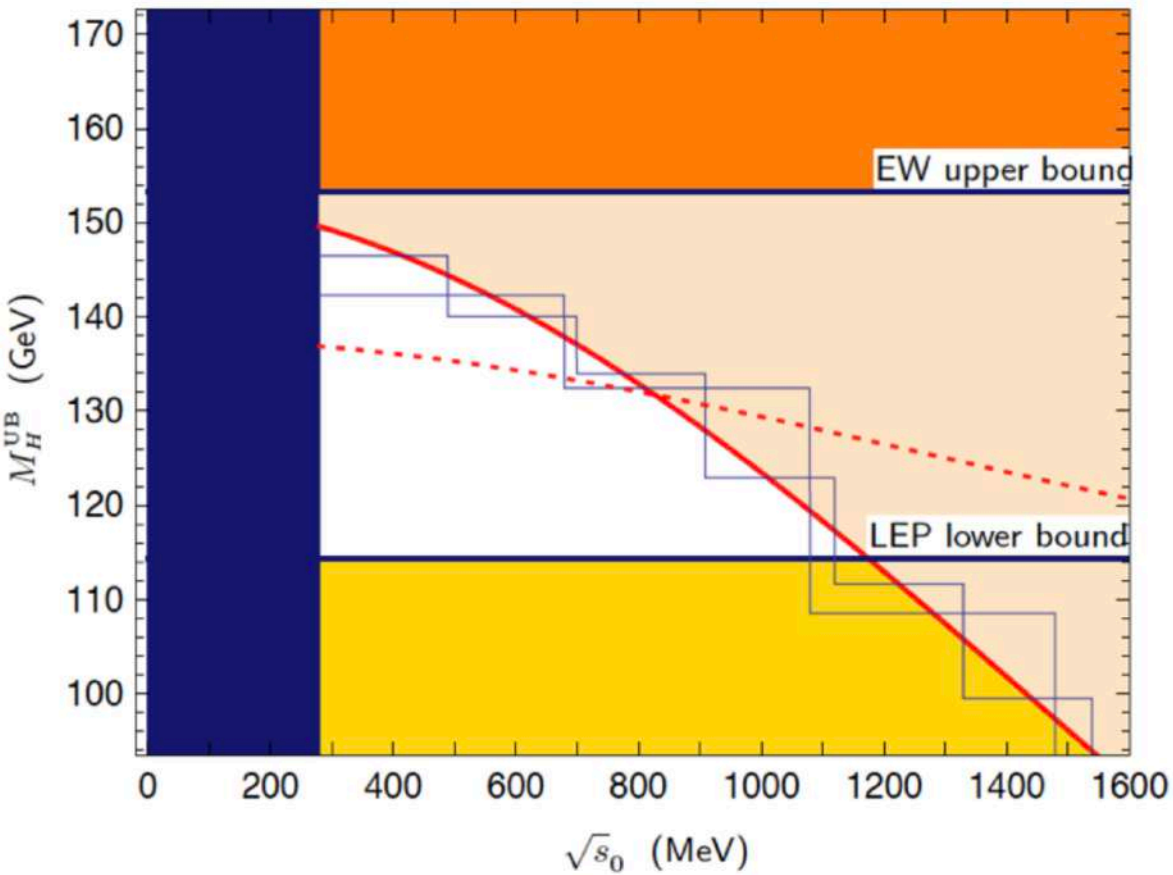
→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

→ Large improvement to cross section and uncertainty from new 4π data



The Muon g-2 and the bounds on the Higgs boson mass

Marciano, Passera and Sirlin (2008)

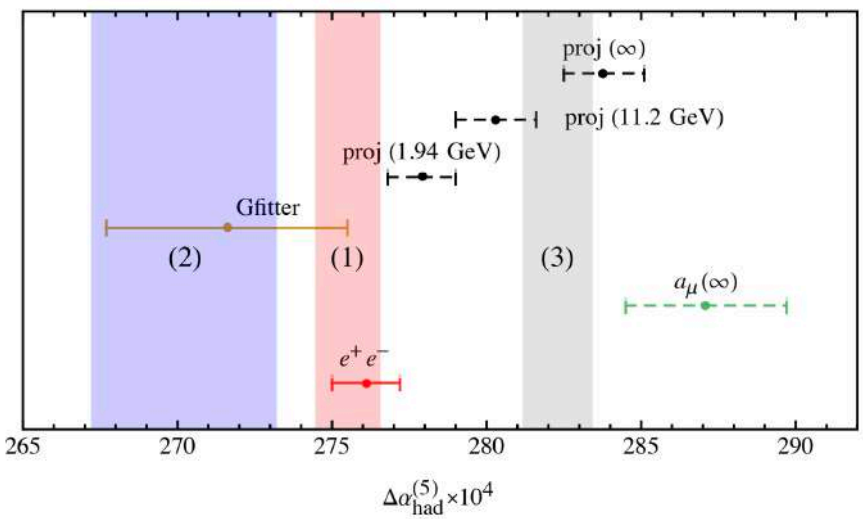


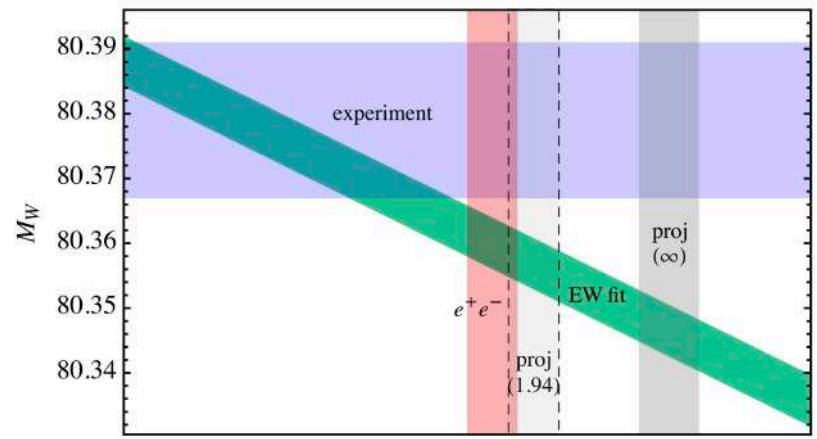
"… if the hadronic cross section is shifted up in energy regions centred above ~ 1.2 GeV to bridge the muon g–2 discrepancy, the Higgs mass upper bound becomes inconsistent with the LEP lower limit."

Hadronic vacuum polarization: $(g-2)_\mu$ versus global electroweak fits

Crivellin, Hoferichter, Manzari, Montull, arXiv:2003.04886 (2020).

- Using global EW fit (HEPFitter) results in a tension with the BMW result.
- A significant shift in HVP exacerbates tensions within the EW fit → inconsistent W mass prediction.
- Does not weaken the case for BSM physics , but to some extent shifts it from (g – 2)μ to the EW fit.





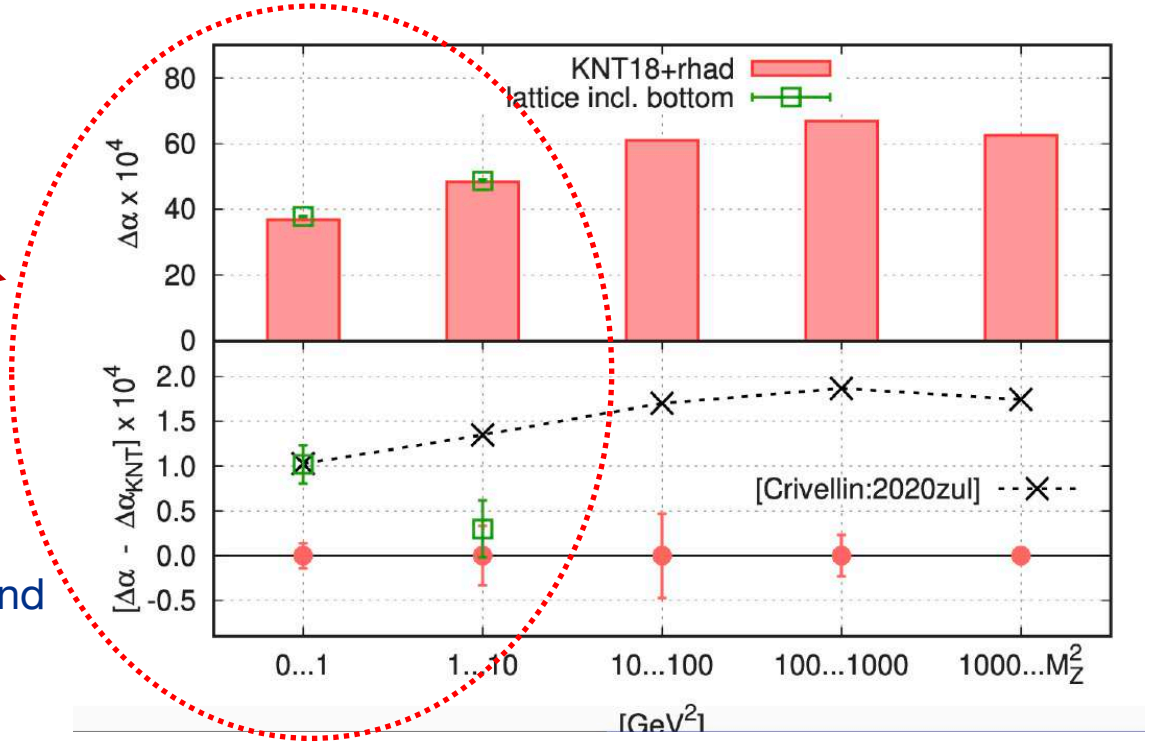
- Results cannot rule out BMW.
- Analysis uses strong assumptions → energy independent shifts of the cross section.
- Incorporating energy-dependence at low energies is crucial.

Energy dependence of BMW

Slide content by Laurent Lellouch.

- Crivellin et al '20, most aggressive scenario: our results suggest a 4.2σ overshoot in $\Delta^{(5)}_{\text{had}}\alpha(M_Z^2)$ compared to result of fit to EWPO
- Assume same 2.8% relative deviation from R-ratio as we find in $a_\mu^{\text{LO-HVP}}$
- Hypothesis is not consistent w/ BMWc '17 nor new preliminary result

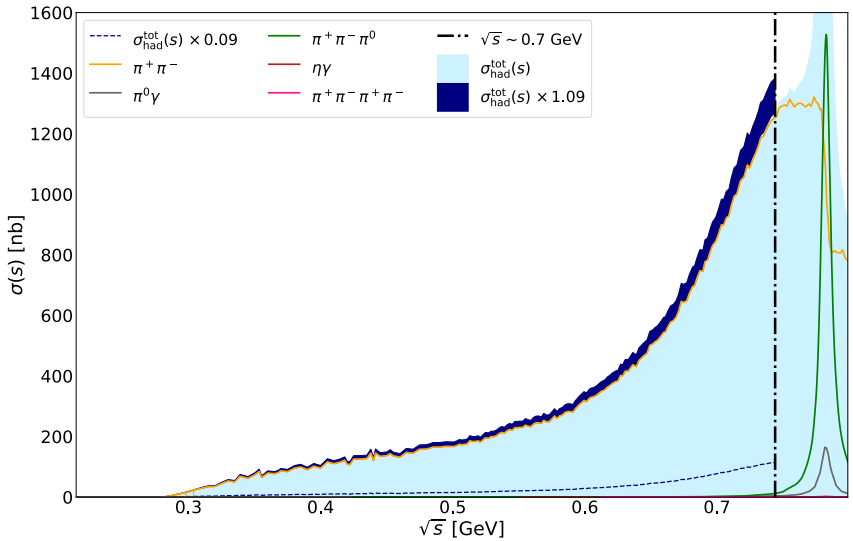
Differences between BMW and KNT are contained within low-energy, non-perturbative hadronic domain.

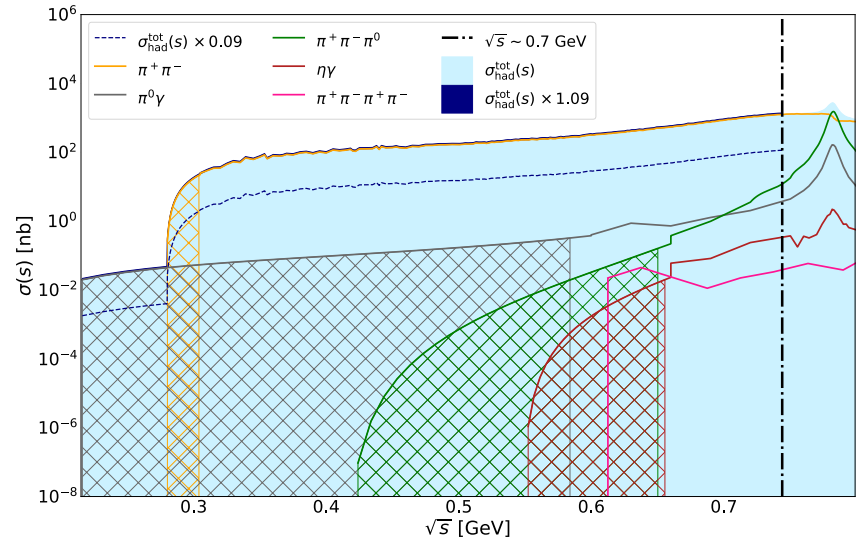


Note: Comparison is between time-like $\Delta\alpha$ from e^+e^- data and space-like $\Delta\alpha$ from lattice in Euclidean space-time.

What about estimated threshold contributions?

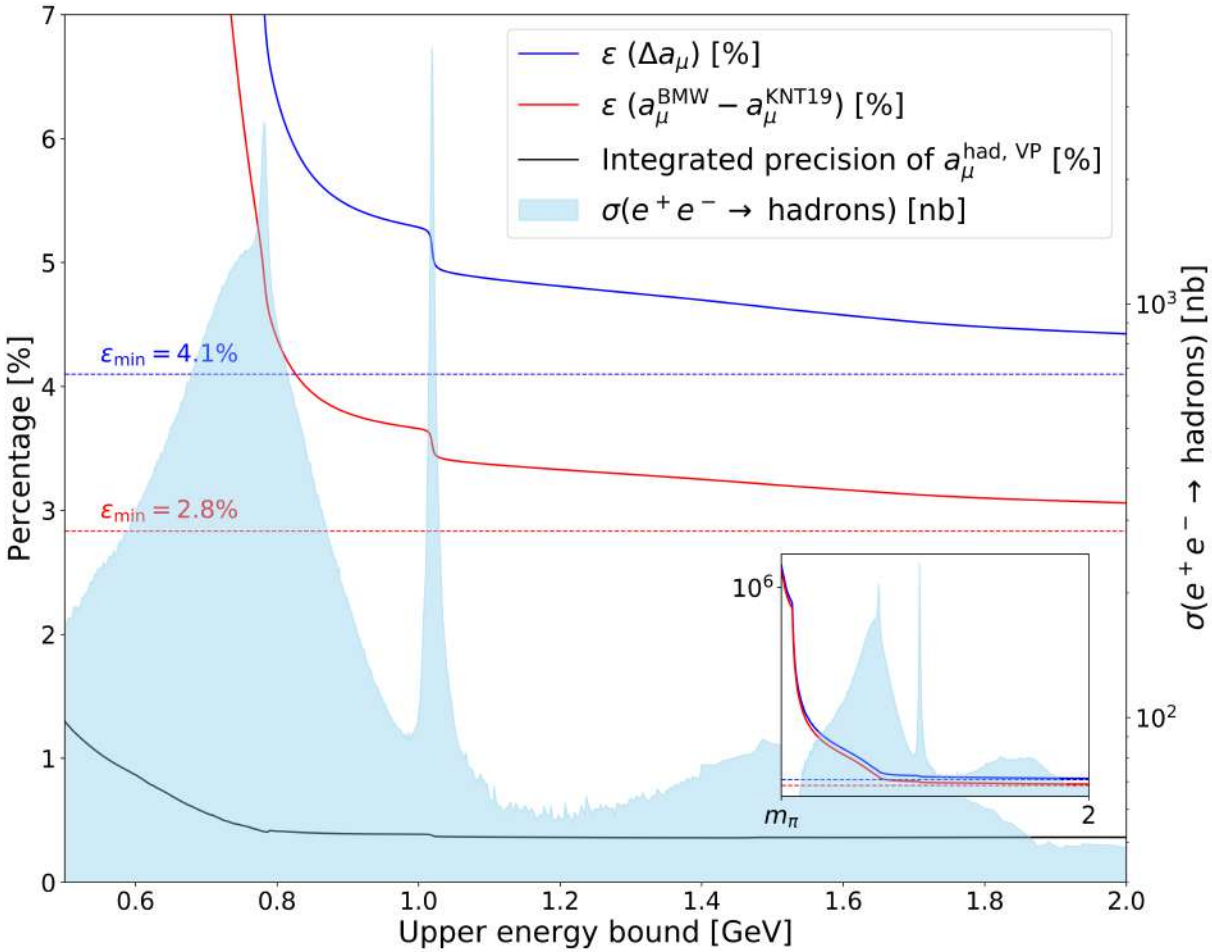
Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002





How realistic are the required shifts in σ(s)?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

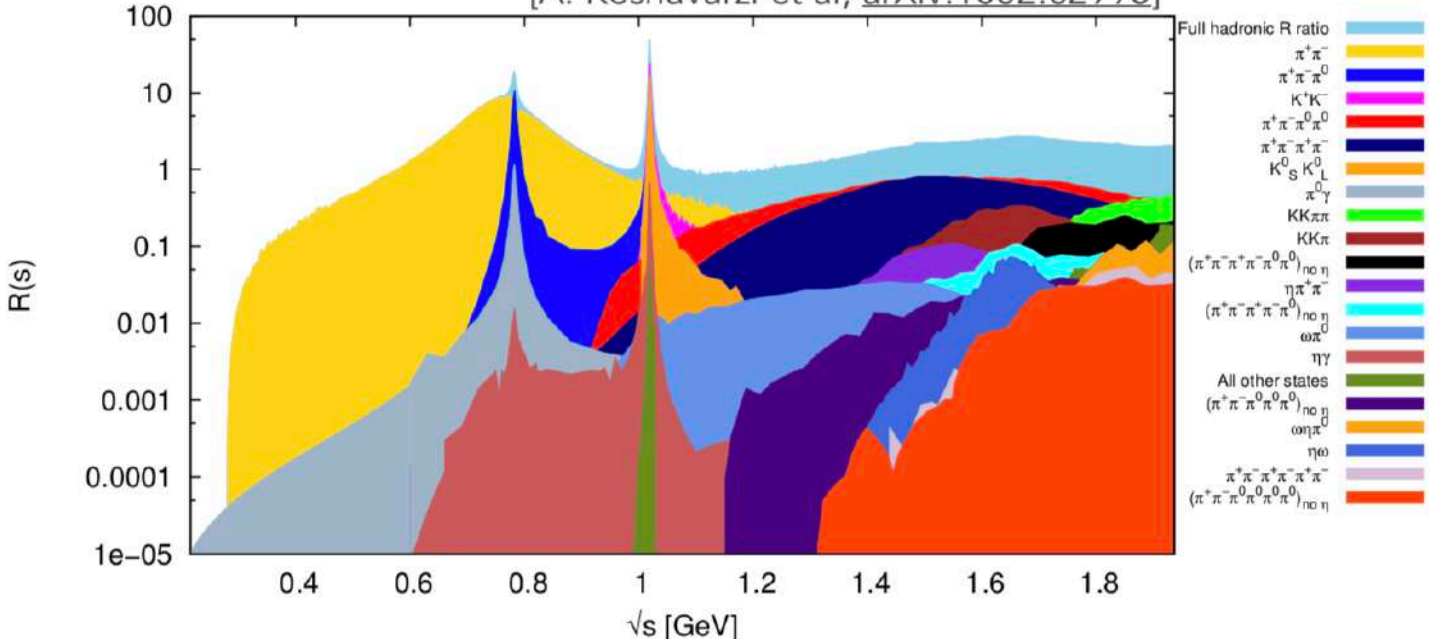


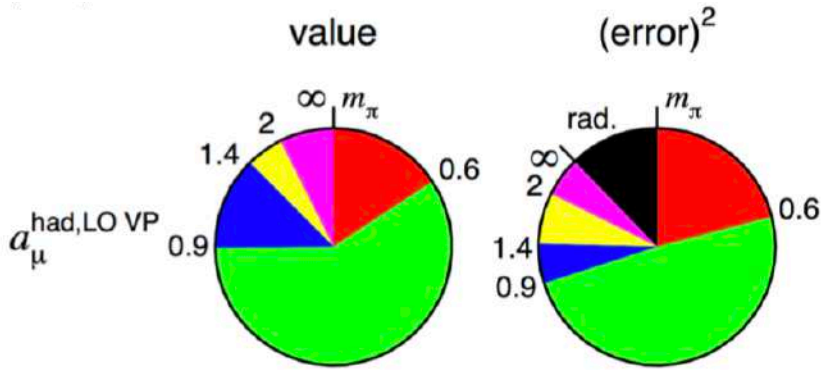
Size of missed contributions would need to be implausibly large given the robust status of the hadronic cross section measurements.

Dispersive HVP

Slide content by Aida El-Khadra.

[A. Keshavarzi et al, arXiv:1802.02995]

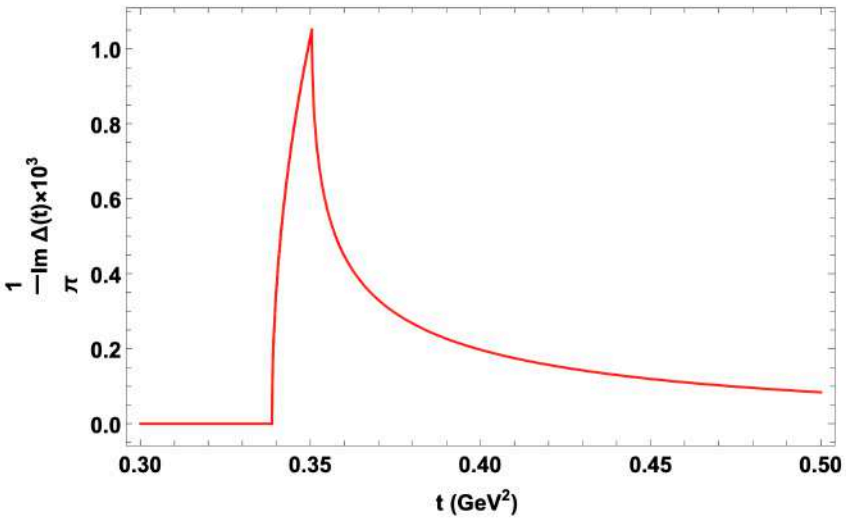




On constraints between $\Delta\alpha$ and g-2

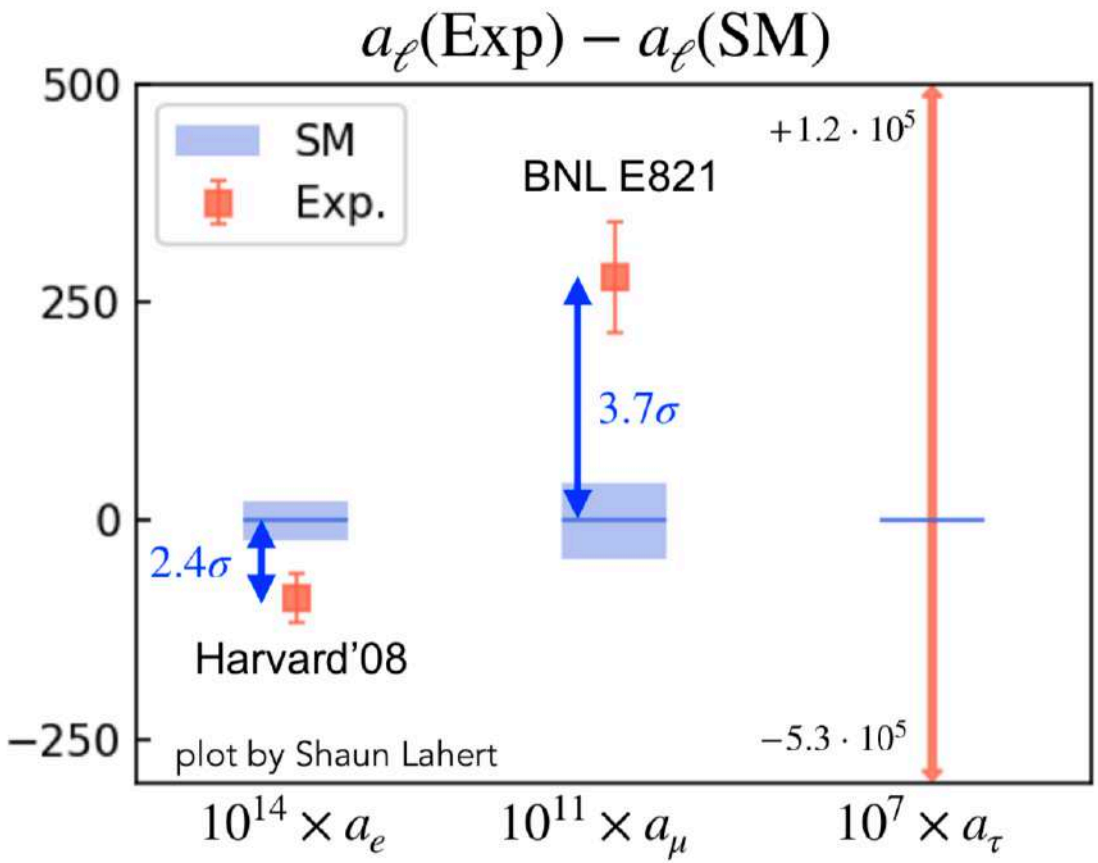
de Rafael, arXiv:2006.13880 (2020).

- Claim that it is possible to construct spectral functions which reproduce both BMW and KNT, inducing only a very small change in $\Delta\alpha$.
- Constrain the contribution to the hadronic running that can be related to a particular dimension-6 operator (or, equivalently, to the first moment of the HVP function).
- Implication is there is a bound that restricts all effects in the hadronic running to the small piece that is related to this first moment (the first term in Eq. (6) of the attached notes). This is not the case.
- Needs to go past just first moment.
- Additionally, these shifts in the cross section at such low energies will have to be enormous.
- But, provides interesting comparison between time-like and space-like $\Delta\alpha$.



Lepton moments summary

Slide content by Aida El-Khadra.



Sensitivity to heavy new physics:

$$a_\ell^{\text{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Ongoing experimental programs for improved measurements of α

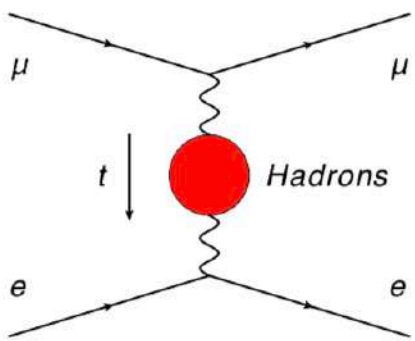
[S. Guellati-Khelifa (Paris), Z. Pagel (Berkeley) @ INT workshop]

MUonE

Slide content by Massimo Passera.

μ-e elastic scattering to measure a_μ^{HVP}

M. Passera @ HVP KEK 2018 [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\mathrm{had}}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024

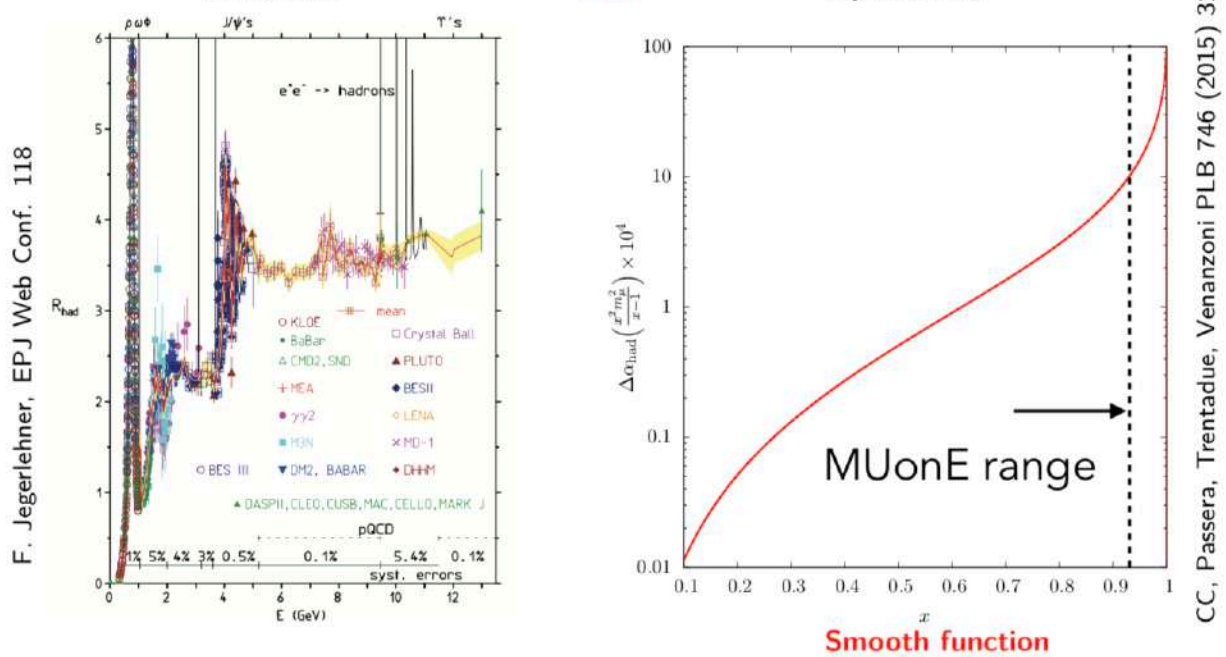
MUonE

Slide content by Massimo Passera.

μ-e elastic scattering to measure a_μ^{HVP}

C. Carloni @ g-2 INT workshop [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]

Time-like ↦ **Space-like**



- requires calculations of radiative corrections [M. Fael @ g-2 INT workshop]
- complement region not accessible to experiment with LQCD calculation [M. Marinkovic @ g-2 INT workshop]