# Simulation of Third-Integer Resonant Extraction of 800 MeV Protons in the Delivery Ring

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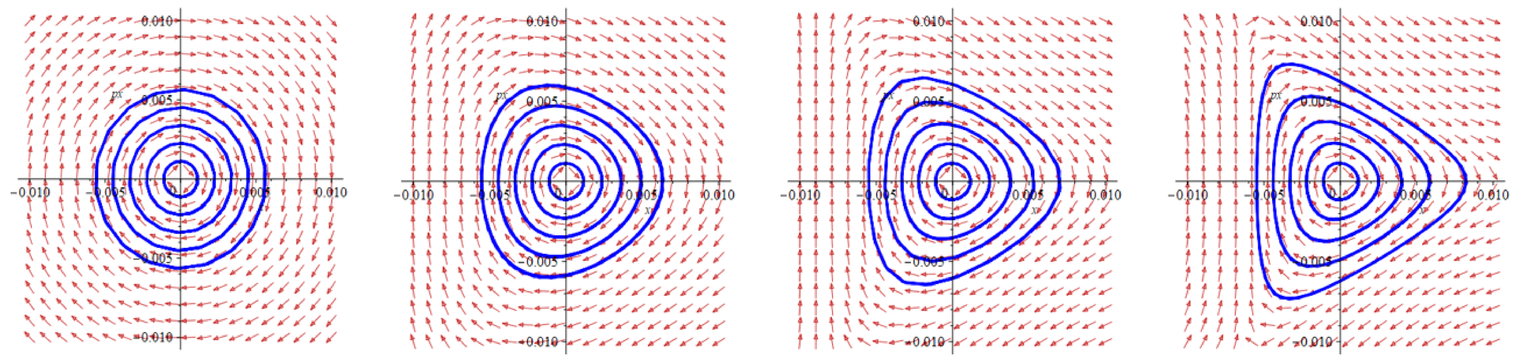
## Third-Integer Extraction Overview

Third-Integer Resonant Extraction is a technique to slowly extract particles from a ring over many turns. Delivery Ring uses a fractional machine tune close to 1/3 or 2/3, so that

$$\upsilon_x = k \pm \frac{1}{3} + \delta.$$

For small δ, sextupole magnets drive resonances around the ring, which distort normalized phase space from a circle into a triangle.

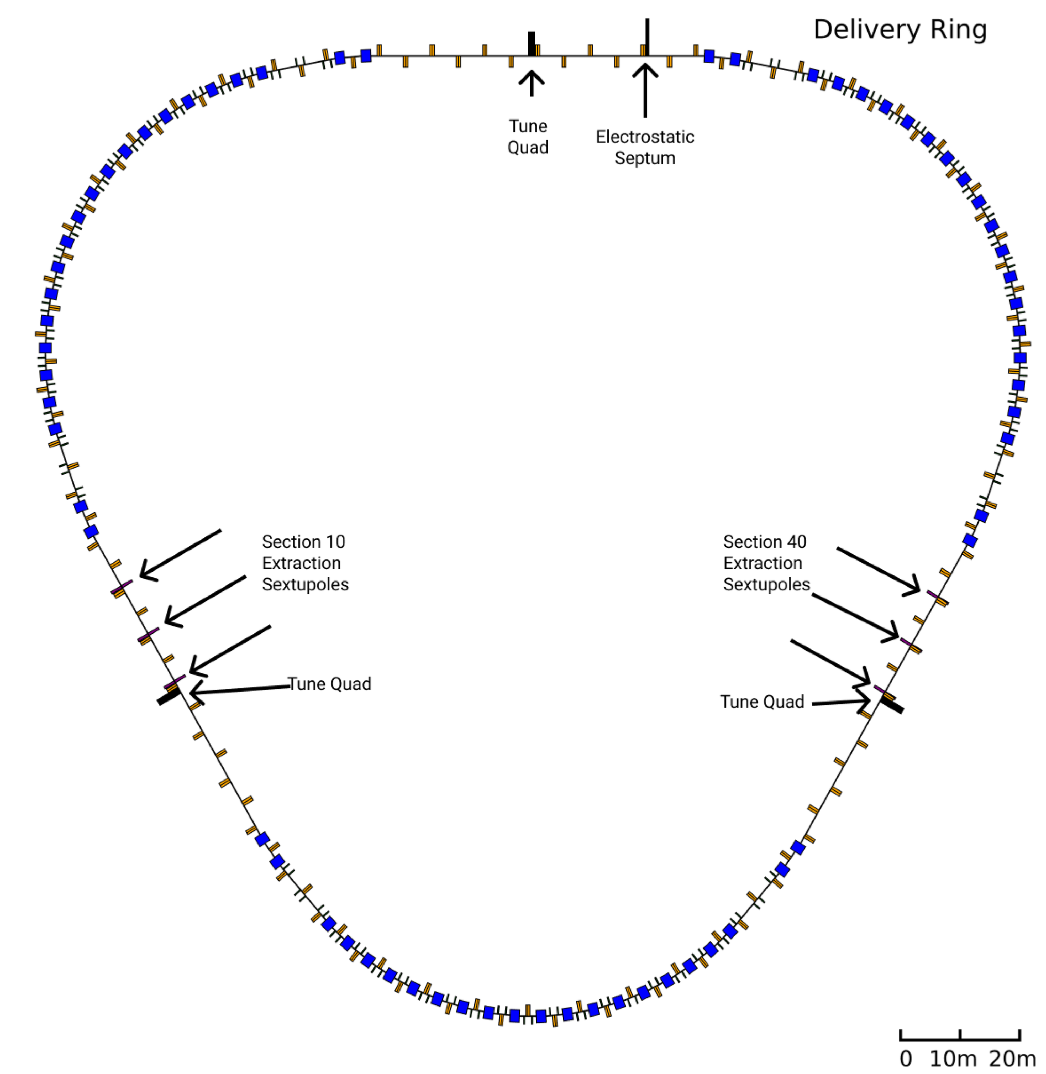
A family of 0th harmonic quadrupoles drive the machine tune towards the third-integer. As $\delta \to 0$, the size of stable phase space shrinks, and particles are squeezed out of the machine. Unstable particles travel along the Hamiltonian separatrices, which are the edges of the triangle.



Circular normalized phase space distorting into a triangle. Pictures left to right give contour lines for increasing s.

## Delivery Ring

The Delivery Ring lattice has 57 focusing-defocusing (FODO) cells in three-fold symmetry. Extraction sextupoles generate transverse resonances around the ring. Tune quadrupoles bring $\delta \to 0$. Electrostatic septum catches spilled particles It has zero dispersion in the straight sections so there is no chromatic effect from the extraction sextupoles.




Delivery Ring Lattice. Extraction sextupoles, tune quadrupoles, and the electrostatic septum are marked with arrows. Blue boxes about the line are dipoles and orange boxes pointing outwards and inwards are focusing and defocusing quadrupoles, respectively.

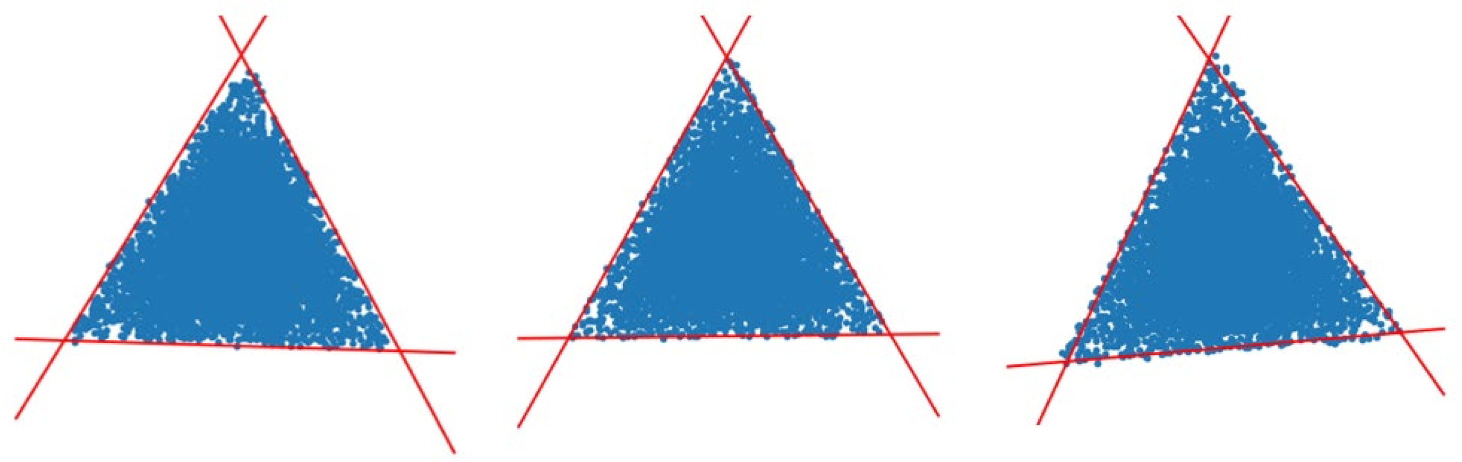
## Sextupole Parameters

Sextupole resonances depend on both field strength and positioning of sextupoles around the lattice. Two orthogonal families with reinforcing contributions are made by approximately 60° phase advance between the same family and an odd multiple of 30° between sextupoles of different families.

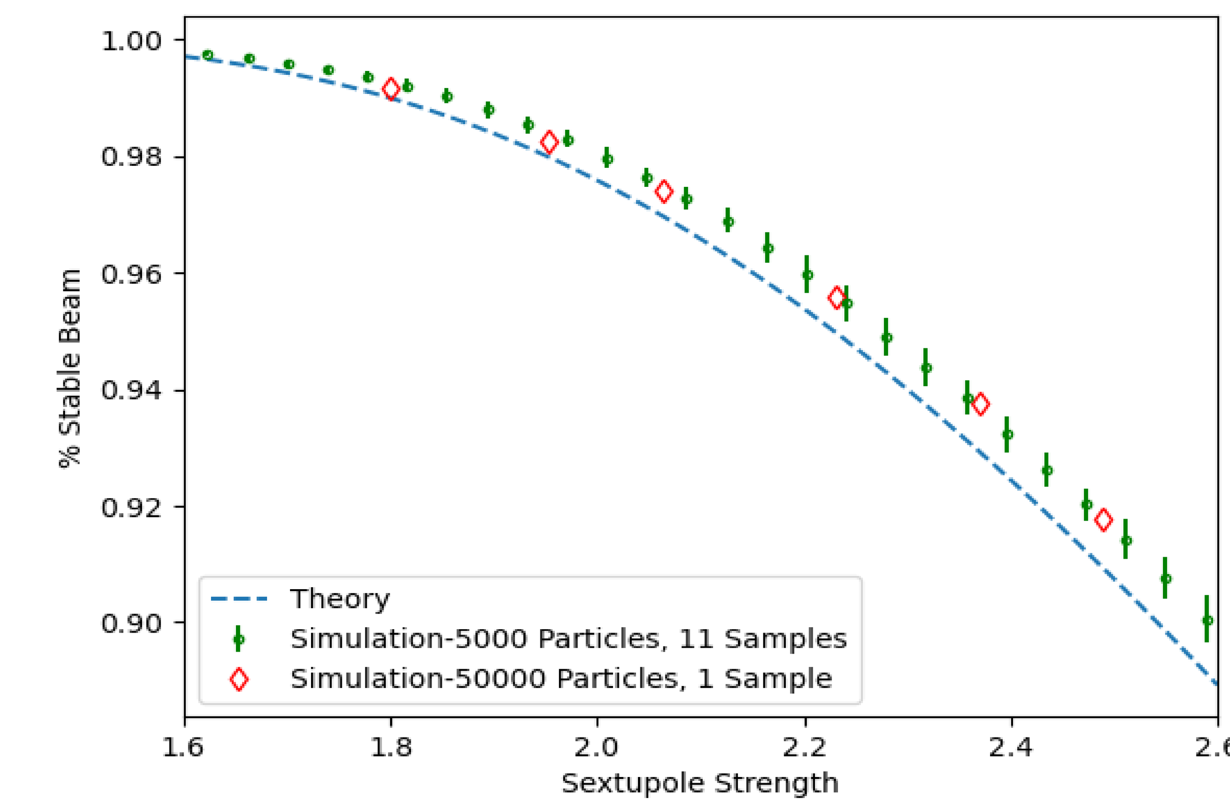
Strength combination of the two families by:

$$s_1 = s\sin(\zeta)$$
$$s_2 = s\cos(\zeta)$$

Gives rotation ζ of the phase space triangle about equilibrium.



Rotation of phase space for $\zeta = 80, 90$ and $100$ degrees respectively. Approximate separatrices are shown.




Study of particle losses with total sextupole strength.

Sextupole strength was empirically chosen from 95% emittance, which was roughly $s = 2.28m^{-\frac{1}{2}}$.

## Septum Placement

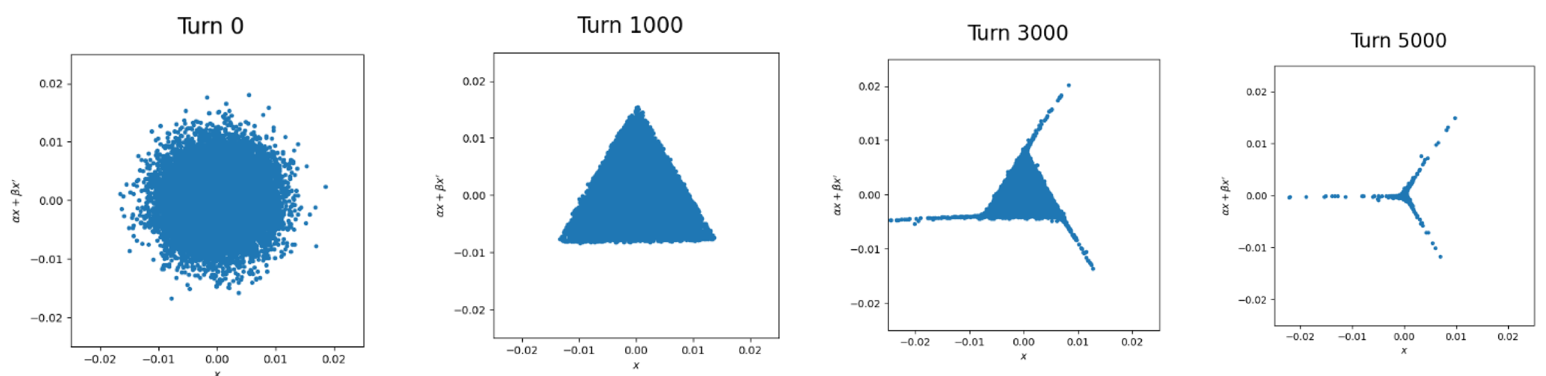
The electrostatic septum catches particles in between the wire plane and the conducting plate to give them an electric field kick. Septum offset from the reference trajectory should be maximized to increase efficiency. Offset is limited by step size (growth in amplitude of transverse oscillation over 3 turns) being equal to the septum aperture.

$$\frac{\Delta x}{\sqrt{\beta}} = \frac{apert.}{\sqrt{\beta}} = \left(9\pi s\frac{x_s}{\sqrt{\beta}}\right)\left(1 - 9\pi s\frac{x_s}{\sqrt{\beta}}\right)^{-1}\frac{x_s}{\sqrt{\beta}}$$

Septum was placed at $18mm$ for $12mm$ step size where the maximum theoretical placement was $20.3mm$ for $14mm$ aperture width.

## Simulation

Extraction was simulated over 6500 turns with 25000 particles, as shown in the snapshots below.




Snapshots of extraction simulation.