Lattice QCD for EDMs: Current Status and Future Prospects

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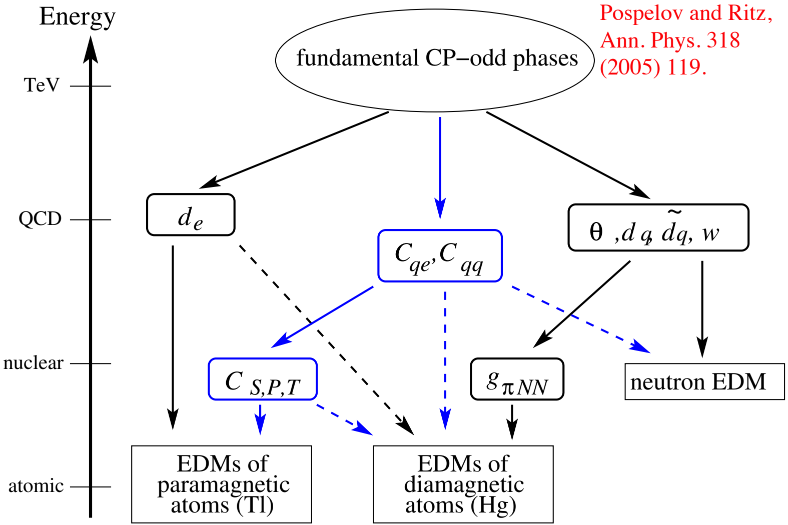
Hierarchy of EDM Scales



Pospelov and Ritz, Ann. Phys. 318 (2005) 119.

$$\mathcal{L}_{\rm CPV} = \mathcal{L}_{\rm CKM} + \mathcal{L}_{\bar\theta} + \mathcal{L}_{\rm BSM} \quad \longrightarrow \quad \mathcal{L}^{\rm eff}_{\rm CPV} \tag{1}$$

Effective CPV Lagrangian at Hadronic Scale

$$\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& - \frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& - \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}$$

- $\overline{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \nu/\Lambda_{BSM}^2$; effectively dim=6
- All terms up to $d = 6$ are leading order

Calculation of Neutron EDM d_n

$$d_n = \overline{\theta} \cdot C_\theta + d_q \cdot C_{\mathrm{qEDM}} + \tilde{d}_q \cdot C_{\mathrm{CEDM}} + \cdots$$

- SM and BSM theories
  $\longrightarrow$ Coefficients of the effective CPV Lagrangian ($\overline{\theta}, d_q, \tilde{d}_q, \ldots$)
- Lattice QCD
  $\longrightarrow$ Nucleon matrix elements in presence of CPV interactions ($C_\theta, C_{\mathrm{qEDM}}, C_{\mathrm{CEDM}}, \ldots$)

Physical Results from Simulations of Lattice QCD

- **Finite Lattice Spacing**
  - Simulations at finite lattice spacings $a \approx 0.045 - 0.15$ fm
  - $\Rightarrow$ Extrapolate to continuum limit, $a = 0$
- **Heavy $\rightarrow$ Physical Pion Mass**
  - Lattice simulation: Smaller quark mass $\longrightarrow$ Larger computational cost
  - Simulations increasingly being done at physical pion mass
- **Finite Volume**
  - Finite lattice volume effects small in most EDM calculations
- **Renormalization**
  - Lattice scheme $\longrightarrow$ continuum $\overline{\mathrm{MS}}$; involves complicated/divergent mixing
- **Removing Excited state contamination**
  - Lattice meson and nucleon interpolating operators also couple to excited states

Neutron EDM from Quark EDM term

$$
\begin{aligned}
\mathcal{L}_{\rm CPV}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\bar{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w\frac{g_s}{6}G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

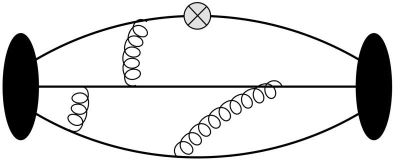
Quark EDM given by the Tensor Charge

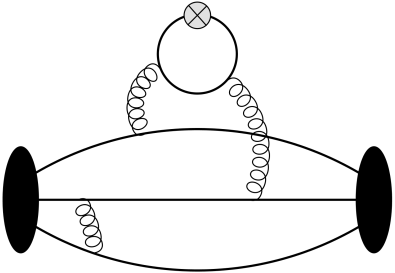
- Neutron EDM (d_N) from Quark EDMs can be written in tensor charges g_T

$$-\frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q \quad\longrightarrow\quad d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

$$\langle N|\overline{q}\sigma_{\mu\nu}q|N\rangle = g_T^q\overline{u}_N\sigma_{\mu\nu}u_N$$

- $d_q \propto m_q$ in many models $\Rightarrow$ Precision determination of $g_T^{s,N}$ is important
- Requires computationally very expensive quark-line disconnected diagrams





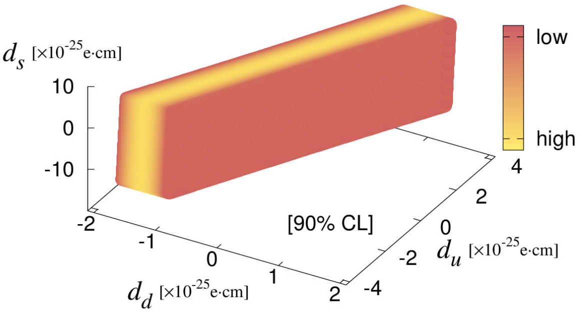
qEDM: Current Status

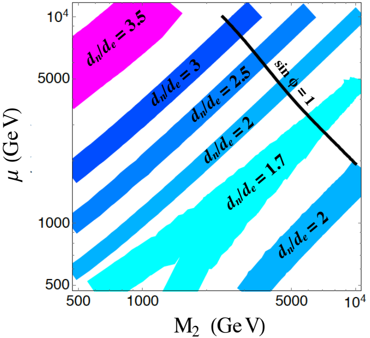
FLAG2019

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^u	g_T^d
PNDME 18B	2+1+1	★ ‡	★	★	★	★	$0.784(28)(10)^{\#}$	$-0.204(11)(10)^{\#}$
PNDME 16	2+1+1	○ ‡	★	★	★	★	$0.792(42)^{\#\&}$	$-0.194(14)^{\#\&}$
PNDME 15	2+1+1	○ ‡	★	★	★	★	$0.774(66)^{\#}$	$-0.233(28)^{\#}$
JLQCD 18	2+1	■	○	○	★	★	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	2	■	○	○	★	★	0.782(16)(2)(13)	−0.219(10)(2)(13)

Collaboration	N_f	a	m_π	FV	Z	ESC	g_T^s
PNDME 18B	2+1+1	★ ‡	★	★	★	★	$-0.0027(16)^{\#}$
PNDME 15	2+1+1	○ ‡	★	★	★	★	$0.008(9)^{\#}$
JLQCD 18	2+1	■	○	○	★	★	−0.012(16)(8)
ETM 17	2	■	○	○	★	★	−0.00319(69)(2)(22)

Constraints on BSM from qEDM and Future Prospects





[Bhattacharya, *et al.* (2015), Gupta, *et al.* (2018)]

Future Prospects:

- Results from multiple collaborations with control over $a \to 0$ extrapolation
- Improved precision on g_T^s and g_T^c
- Need large computations and algorithm development

Neutron EDM from QCD θ-term

$$\mathcal{L}_{\rm CPV}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\overline{\theta} G\tilde{G} \quad \text{dim=4 QCD } \theta\text{-term}$$

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma\cdot F)\gamma_5 q \quad \text{dim=5 Quark EDM (qEDM)}$$

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q \quad \text{dim=5 Quark Chromo EDM (CEDM)}$$

$$+ d_w \frac{g_s}{6} G\tilde{G}G \quad \text{dim=6 Weinberg's 3g operator}$$

$$+ \sum_i C_i^{(4q)} O_i^{(4q)} \quad \text{dim=6 Four-quark operators}$$

QCD θ-term

$$S = S_{QCD} + i\theta Q, \qquad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

- Three different approaches
  - External electric field method: $\langle N\overline{N}\rangle_\theta(\vec{\mathcal{E}}, t) = \langle N(t)\overline{N}(0)e^{i\theta Q}\rangle_{\vec{\mathcal{E}}}$

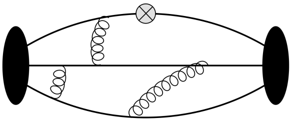
    Aoki and Gocksch (1989), Aoki, Gocksch, Manohar, and Sharpe (1990), CP-PACS Collaboration (2006), Abramczyk, *et al.* (2017)
  - Simulation with imaginary θ: $\theta = i\tilde{\theta}, \quad S_\theta^q = \tilde{\theta}\frac{m_l m_s}{2m_s + m_l}\sum_x \overline{q}\gamma_5 q$

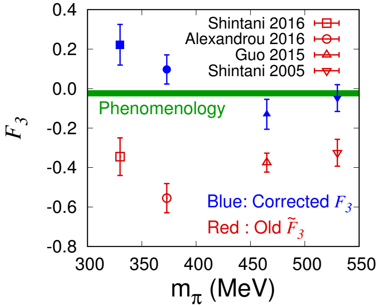
    Horsley, *et al.*, (2008), Guo, *et al.* (2015)
  - Expansion in small θ:

$$\begin{aligned}\langle O(x)\rangle_\theta &= \frac{1}{Z_\theta}\int d[U, q, \overline{q}]O(x)e^{-S_{QCD} - i\theta Q} \\ &= \langle O(x)\rangle_{\theta=0} - i\theta\langle O(x)Q\rangle_{\theta=0} + \mathcal{O}(\theta^2)\end{aligned}$$

Shintani, *et al.*, (2005), Berruto, Blum, Orginos, and Soni (2006) Shindler, T. Luu, J. de Vries (2015), Shintani, Blum, Izubuchi, and Soni (2016), Alexandrou, *et al.*, (2016), Abramczyk, *et al.* (2017), Dragos, *et al.* (2019)

Form Factors and states in theory with P and CP violation



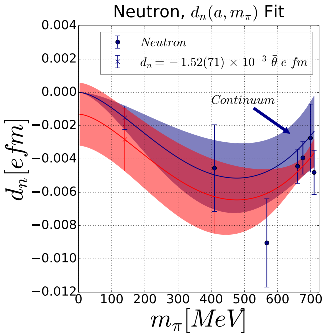


- In *'simulation with imaginary θ'* and *'expansion in θ'* approaches, neutron EDM $d_n = |e| F_3(Q^2 = 0) \; / \; 2M_N$ is extracted from vector current

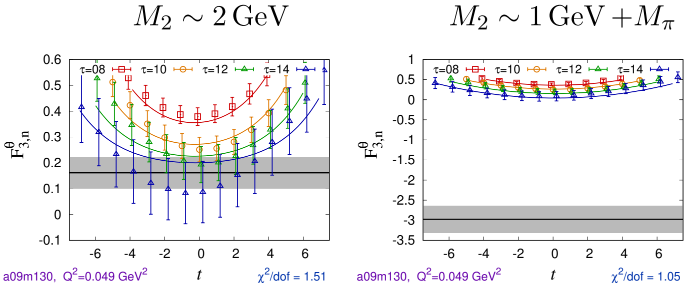
$$\langle N | V_\mu(q) | N \rangle_{\mathrm{CPV}} = \overline{u}_N \left[F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu - \frac{F_3(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu\gamma_5 \right] u_N(p)$$

- CPV $\rightarrow \gamma_4$ no longer the parity operator for neutron state [Abramczyk, *et al.*, 2017]
  F_3 in the naive decomposition is not the correct CP-odd form factor
- With this correction, previous lattice results moved close to zero

Physical Pion Mass Simulations are Expensive



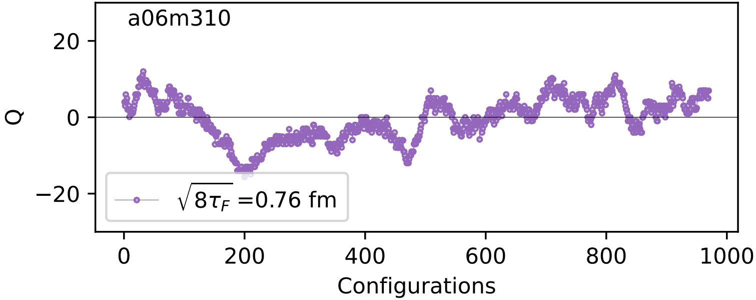
Dragos, *et al.* (2019)



Bhattacharya, *et al.*, (2020)

- Very few calculations are carried at $m_\pi < 300\text{MeV}$; all with a poor signal
- The most recent result from multiple a with large pion mass $m_\pi > 400\text{MeV}$ show marginal signal $d_n = -1.52(71) \times 10^{-3}\ \bar{\theta}\ e \cdot fm$ [Dragos, *et al.* (2019)]
- Significant contamination possible from $N\pi$ as lowest excited-state at physical m_π Calculations with high statistics needed [Bhattacharya, *et al.*, (2020)]

Long Autocorrelations in Fine Lattice Simulations



- Simulations on small a lattices required to reduce discretization artifact
- Autocorrelation in opological charge Q increase as $a \to 0$

QCD θ-term: Future Prospects

With significant increase in computational resources we will perform

- Simulations at physical pion mass
- Simulations with high statistics (long autocorrelation lengths)
- New algorithms for lattice generation at $a \lesssim 0.6\ \mathrm{fm}$

Neutron EDM from quark Chromo-EDM (CEDM)

$$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+d_w\frac{g_s}{6}G\tilde{G}G$$ dim=6 Weinberg's 3g operator

$$+\sum_i C_i^{(4q)}O_i^{(4q)}$$ dim=6 Four-quark operators

Lattice QCD approaches for CEDM

$$S = S_{QCD} + S_{CEDM}; \qquad S_{CEDM} = \frac{g_s}{2} \sum_{q=u,d,s} \tilde{d}_q \int d^4x \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three different approaches developed
  - Schwinger source method [Bhattacharya, *et al.* (2016)]:

$$D_{clov} \to D_{clov} + \frac{i}{2}\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

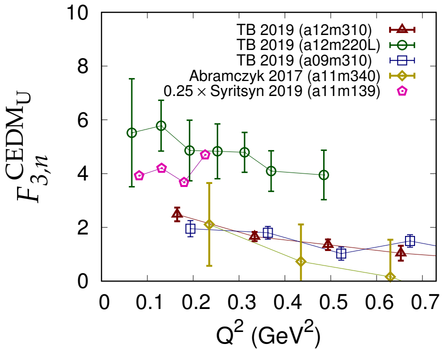
  - Expansion in $\tilde{d}_q$ [Abramczyk, *et al.* (2017)]:

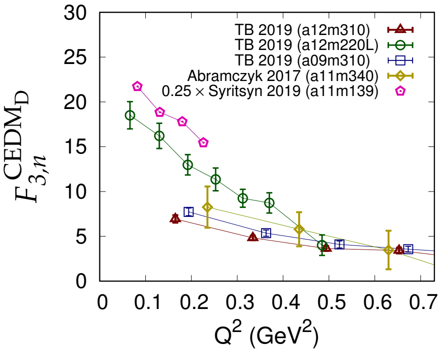
$$\langle N V_\mu \overline{N} \rangle_{CEDM} = \langle N V_\mu \overline{N} \rangle + \tilde{d}_q \langle N V_\mu \overline{N} \sum_x O_{CEDM} \rangle + \mathcal{O}(\tilde{d}_q^2)$$

  - External electric field method [Abramczyk, *et al.* (2017)]:

$$\langle N\overline{N} \rangle_{CEDM}(\vec{\mathcal{E}}, t) = \langle N(t)\overline{N}(0) O_{CEDM} \rangle_{\vec{\mathcal{E}}}$$

Current Status





- Two groups presented CEDM data using different approaches
- Results seem to have better statistical signal than QCD θ-term
- Current calculations are without renormalization
  - RI-MOM schemes for CEDM with one-loop conversion factors to $\overline{\mathrm{MS}}$ is available
  - Mixing structure is complicated and involves mixing with lower dimensional operator

[Bhattacharya, *et al.* (2015), Constantinou, *et al.*(2015)]

Renormalization using Gradient Flow

Gradient flow [Lüscher and Weisz (2011)]:

$$\partial_t B_\mu(t) = D_\nu G_{\nu\mu}, \qquad B_\mu(x, t=0) = A_\mu(x),$$
$$\partial_t \chi(t) = \Delta^2 \chi, \qquad \chi(x, t=0) = \psi(x)$$

- Smear (flow) gluon and quark fields along the gradient of an action to a fixed physical size (sets ultraviolet cutoff of the theory)
- The flowed operators have finite matrix elements except for an universal Z_ψ $\longrightarrow$ Allow us to take continuum limit without power-divergent subtractions
- Mixing and connection to $\overline{\mathrm{MS}}$: simpler perturbative calculation in continuum
- Calculations for CPV ops underway [Rizik, Monahan, and Shindler (2020)]

CEDM: Future Prospects

- Renormalization and operator mixing essential; gradient flow scheme seems promising
- Need algorithm developments for large scale simulations at physical pion mass and smaller lattice spacing
- Machine Learning methods could reduce computational cost

  [Yoon, Bhattacharya, and Gupta (2019)]

Neutron EDM from Weinberg's ggg and Various Four-quark Ops

$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\overline{\theta}G\tilde{G}$	dim=4 QCD θ-term
$-\frac{i}{2}\sum_{q=u,d,s} d_q\overline{q}(\sigma\cdot F)\gamma_5 q$	dim=5 Quark EDM (qEDM)
$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\overline{q}(\sigma\cdot G)\gamma_5 q$	dim=5 Quark Chromo EDM (CEDM)
$+\, d_w\frac{g_s}{6}G\tilde{G}G$	dim=6 Weinberg's 3g operator
$+\sum_i C_i^{(4q)}O_i^{(4q)}$	dim=6 Four-quark operators

Weinberg's 3g Op: Current Status and Future Prospects

$$\mathcal{L}_{W_{ggg}} = \frac{1}{6} d_w g_s G\tilde{G}G$$

- Calculation is almost the same as for the QCD θ-term
- No publications yet, only a few preliminary studies

  [Yoon, Bhattacharya, Cirigliano, and Gupta (2019)]
- Signal is noisier than QCD θ-term
- Suffers from the long autocorrelations on $a \lesssim 0.06$ fm lattices
- Requires solving operator renormalization and mixing
  - RI-MOM scheme and its perturbative conversion to $\overline{\mathrm{MS}}$ is available

    [Cirigliano, Mereghetti, and Stoffer (2020)]
  - Gradient flow scheme is a favored option to address divergent mixing structure

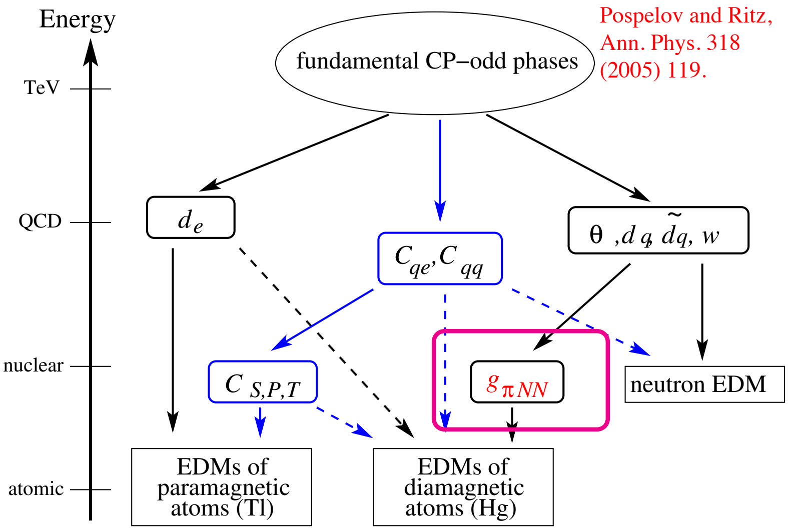
    [Rizik, Monahan, and Shindler (2020)]

Four-quark operators: Current Status and Future Prospects

$$\mathcal{L}_{4q} = \sum_i C_{ij}^{(4q)} (\bar{\psi}_i \psi_i)(\bar{\psi}_j i\gamma_5 \psi_j) + \cdots$$

- No lattice QCD study has been started
- Calculation expected to be statistically noisy and computationally expensive
- Will be included in a long range (5–10y) plan

Lattice Calculations for $g_{\pi NN}$



$g_{\pi NN}$: Current Status and Future Prospects

$$\mathcal{L}_{\pi NN}^{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N - \frac{\bar{g}_1}{2F_\pi}\pi_0\bar{N}N - \frac{\bar{g}_2}{2F_\pi}\pi_0\bar{N}\tau^3 N + \cdots$$

- Chiral symmetry relations + nucleon σ-term & mass splittings $\longrightarrow g_{\pi NN}$
  [Vries, Mereghetti, Seng, and Walker-Loud (2017)]
- No direct lattice calculation of $g_{\pi NN}$ published yet, but
- Can be calculated from $\langle N|A_\mu(q)|N\rangle_{\mathrm{CPV}}$ following the same methodology used for neutron EDM ($\langle N|V_\mu(q)|N\rangle_{\mathrm{CPV}}$)

Conclusion

- Over the next 10 years, lattice QCD community will calculate matrix elements for neutron EDM and pion-nucleon coupling
- The matrix elements will play a crucial role in using experimental results to constrain BSM theories
- Significant progress has been made in formulating methodology and problems and carrying out preliminary studies
- Large computational resources needed
- Inputs from effective field theory will guide ananlyses and the continuum/chiral/excited-state fits and extrapolations