

# Storage Ring Proton EDM Experiment

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# Storage Ring Proton EDM

- History of Storage Ring EDM
- Studies with Polarized Beams
- PEDM Ring Design
- Systematics
- Statistics
- Dark Matter/Dark Energy

# Idea came from BNL E821 muon $g - 2$ experiment

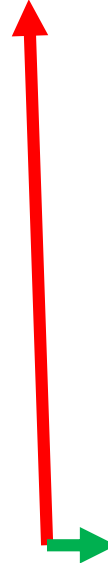
- **MDM** Magic  $\beta$   $a = G = (g - 2)/2$

$$\frac{d(\hat{\beta} \cdot \vec{S})}{dt} = \frac{e}{mc} \vec{S}_T \cdot \left[ a \hat{\beta} \times \vec{B} + \left( \frac{g\beta}{2} - \frac{1}{\beta} \right) \vec{E} \right]$$

- **EDM**

$$\frac{d\vec{S}}{dt} = d_\mu \vec{S} \times (\vec{E} + \vec{v} \times \vec{B})$$

- BNL E821:  $d_\mu < 1.9 \times 10^{-19} ecm$
- FNAL E989 should do ten times better.



# Yannis Semertzidis Idea 24 years ago

- MDM

$$\frac{d\hat{\beta} \cdot \vec{s}}{dt} = -\frac{e}{m} \vec{s}_p \cdot \left[ G\hat{\beta} \times \vec{B} + \left( \frac{g\beta}{2} - \frac{1}{\beta} \right) \frac{\vec{E}}{c} \right] = 0$$

- Deuteron has negative anomaly,
- Needs  $B$  and  $E$  bending.
- Proton has positive anomaly,
- Can have only  $E$  bending.
- E821 was our “day job”. Doing sredm “for the fun of it”.

1. Measure a change in the vertical polarization with a sensitivity of  $10^{-6}$ .  
Provide a continuous record with time.  
Reduce systematic errors to below the sensitivity limit.
2. Track the magnitude of the polarization with time.
3. Provide transverse (X) asymmetry data continuously for control.  
Operate at high efficiency.

Development proposal made to COSY-Jülich in 2007.  
Ring design was for 1 GeV/c deuteron beam (250 MeV)

Best scheme requires deuteron scattering from carbon.

Conduct study using as much existing equipment as possible.

(Begin studies of production/preservation of horizontal polarization.)

Deuteron and proton polarimeters are similar.

# Experiments with Polarized Beams

- We have done polarized beam experiments at:
- KVI, Groningen, Netherlands,
- COSY, Jülich, Germany,
- AGS, BNL, USA.

# Polarized Beam Experiments Publications

G. Guidoboni, E. Stephenson, A. Wronska <i>et al.</i> ,	<a href="#">Connection between zero chromaticity and long in-plane polarization lifetime in a magnetic storage ring</a>	Phys. Rev. ST Accel Beams 22, 024201 (2018)	
A. Saleev, N. Nikolaev, F. Rathmann <i>et al.</i> ,	<a href="#">Spin tune mapping as a novel tool to probe the spin dynamics in storage rings</a>	Phys. Rev. ST Accel. Beams 20, 072801 (2017)	
Nils Hempelmann	<a href="#">Phase Locking the Spin Precession in a Storage Ring</a>	Phys. Rev. Lett, 119, 014801 (2017)	
G. Guidoboni, E.J. Stephenson, <i>et al.</i>	<a href="#">How to reach a thousand-second in-plane polarization lifetime with 0.97-GeV/c deuterons in a storage ring</a>	Phys. Rev. Lett. 117, 054801 (2016)	
1 - 3	D. Eversmann <i>et al.</i>	<a href="#">New method for a continuous determination of the spin tune in storage rings and implications for precision experiments</a>	Phys. Rev. Lett. 115, 094801 (2015)
	Z. Bagdasarian <i>et al.</i>	<a href="#">Measuring the polarization of a rapidly precessing deuteron beam</a>	Phys. Rev. ST Accel. Beams 17, 052803 (2014)
	N. Brantjes <i>et al.</i>	<a href="#">Correcting Systematic Errors in High Sensitivity Deuteron Polarization Measurements</a>	Nucl. Inst. Meth. A664, 49 (2012)
P. Benati <i>et al.</i>	<a href="#">Synchrotron oscillation effects on an rf-solenoid spin resonance</a>	Phys. Rev. ST Accel. Beams 15, 124202 (2012)	

Y. Orlov  
Edm Note

FNAL  
E989

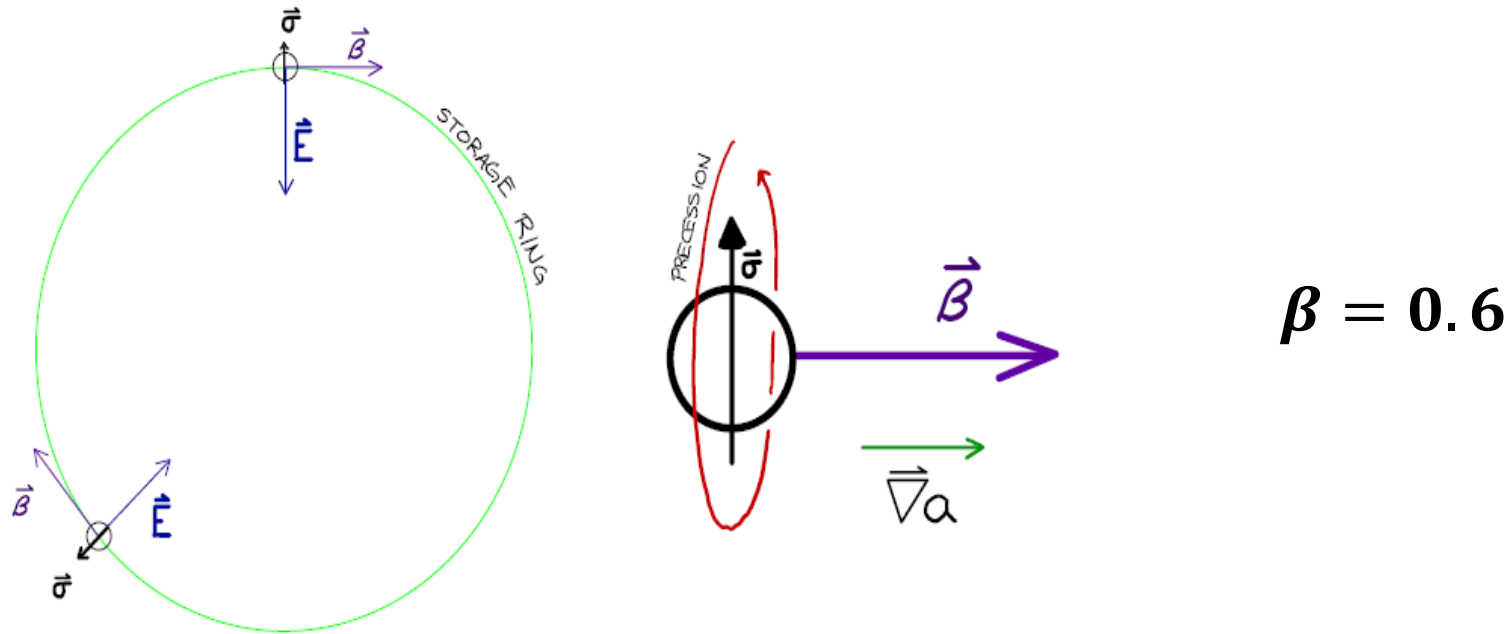
# Symmetries

$$\frac{d\vec{S}_y}{dt} = d_p \vec{S}_z \times (\vec{E}_x + \vec{v} \times \vec{B})$$

- Radial =  $x$ , vertical =  $y$ , longitudinal =  $z$ .
- Electric bending, with strong alternating gradient magnetic focusing.
- CW/CCW Injection.
- Store CW and CCW at the same time for protons.
- Chose protons over deuterons.
- Symmetric lattice.
- Longitudinal (sensitive to edm) and radial (not sensitive to edm) polarized bunches.



# P. Graham et al. Dark matter, dark energy



$$\beta = 0.6$$

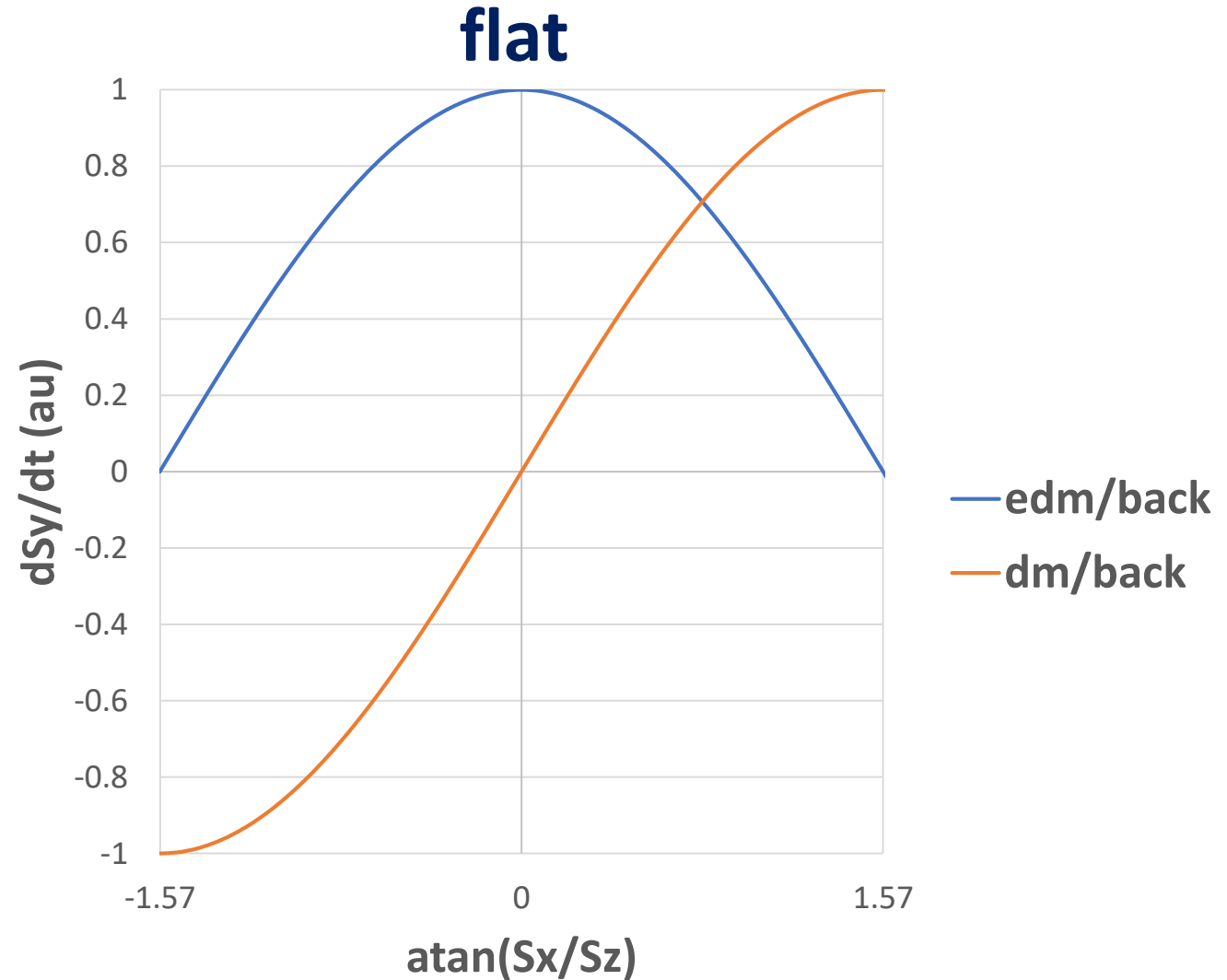
Figure 1: A sketch of the geometry for this storage ring proposal (left figure) and the directions of the proton's spin  $\vec{\sigma}$ , velocity  $\vec{B}$  and precession, as well as the axion field gradient seen by the proton (right figure). The proton's spin must be oriented radially and will then precess around its velocity (out of the plane of the ring).

# P. Graham et al. Dark matter, dark energy

Inspired by these designs, we evaluate the feasibility of storage rings as a way to search for ultra-light dark matter and dark energy. We will show that storage rings can have sensitivities comparable to atomic co-magnetometer techniques for pseudo-scalar interactions. For vector backgrounds, due to the relativistic nature of the beam, these rings have enhanced sensitivity to magnetic dipole interactions and can thus distinguish between electric and magnetic dipole interactions. Storage ring techniques are thus complementary to atomic co-magnetometer searches — the combination of both techniques can be used to extract the underlying nature of any new physics discovered in such experiments. The rest of this paper is organized as follows:

# Longitudinal (edm) and radial (DM/DE) polarized bunches.

- Longitudinal spin: edm and edm-like background is max.
- Radial spin: DM and DM-like background is max.
- DM-like background can not imitate edm – wrong symmetry.

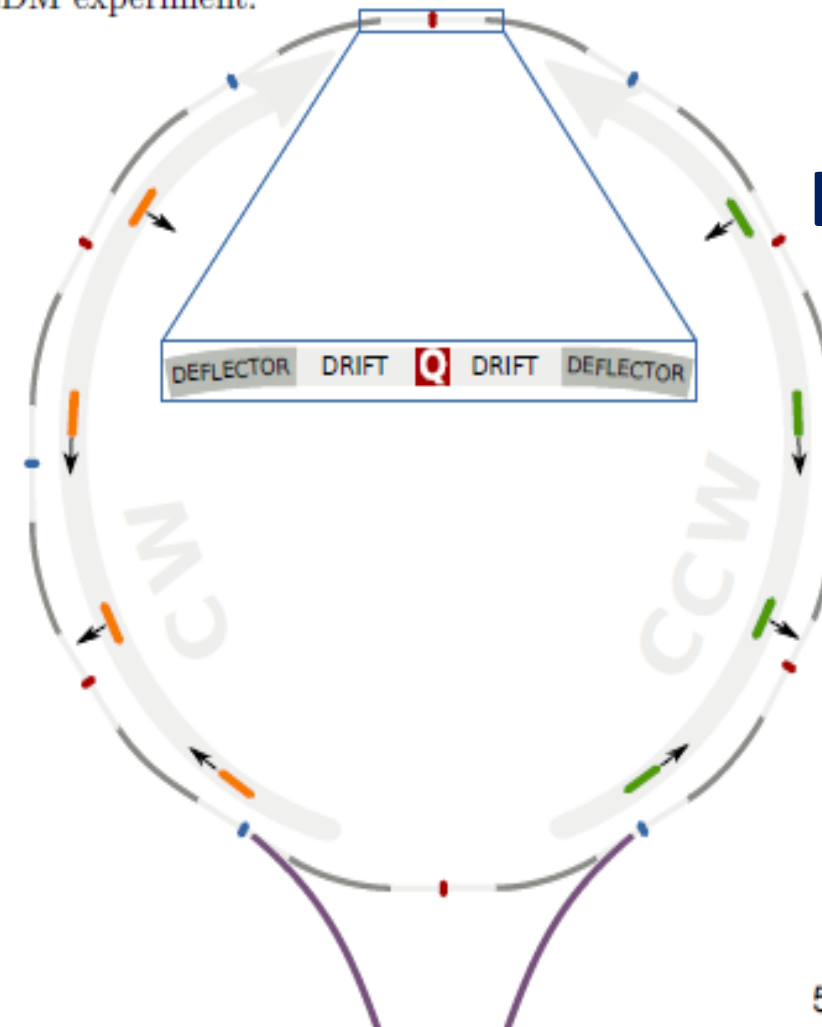


# EDM, Dark Matter/Dark Energy, with the hybrid, symmetric ring lattice

arXiv:2005.11867v1 [hep-ph] 25 May 2020

Table 1: The lattice parameters for the storage-ring proton EDM experiment.

Parameter	Magnitude	Description
$p_0$	<u>0.71 GeV/c</u>	<u>Magic momentum</u>
$\beta$	<u>0.59</u>	<u><math>= v/c</math>, the particle speed</u>
$R_0$	95.5 m	Deflector radius
$C$	<u>800 m</u>	<u>Ring circumference</u>
$f_c$	0.22 MHz	Cyclotron frequency
$f_x$	0.51 MHz	Horizontal betatron frequency
$Q_x$	2.3	Horizontal betatron tune
$f_y$	0.49 MHz	Vertical betatron frequency
$Q_y$	2.2	Vertical betatron tune
$E_0$	<u>4.4 MV/m</u>	<u>Deflector electric field</u>
$k$	0.2 T/m	Quadrupole strength
$L_{quad}$	40 cm	Quadrupole length
$L_{str}$	4.6 m	Straight section length (incl. quad.)
$N$	48	Number of cells



**CW,CCW,  
Long, Radial**

Stray Magnetic Field < 0.1mG

Flux gates and trim coils

# Discuss the two most difficult systematic effects:

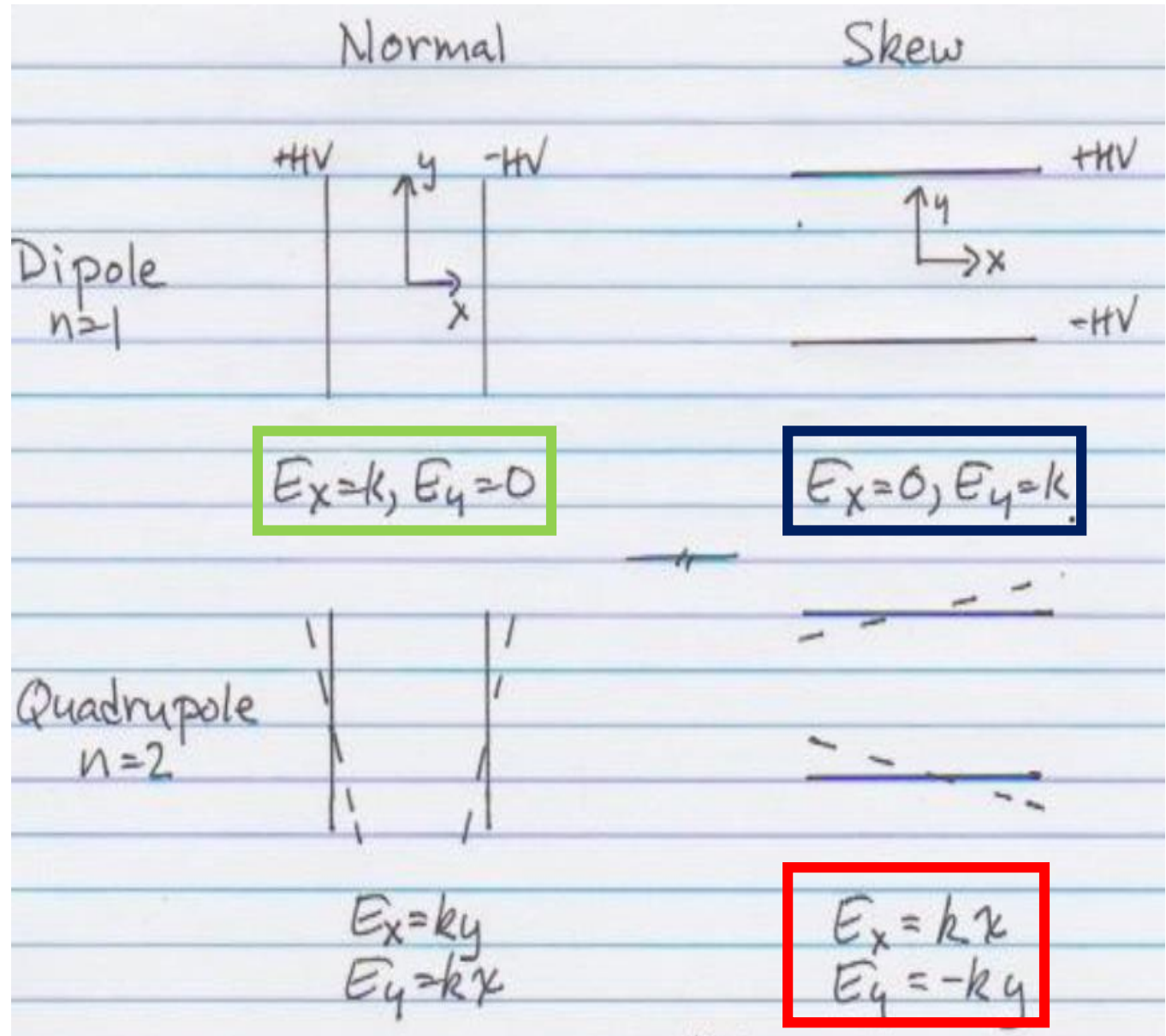
- EDM-like systematic effect.
- Electric quadrupole multipole effect.
- DM-like effect.
- Vertical velocity effect.

# EDM like systematic

- For a storage ring with dipole electric bending field, magnetic quadrupole focusing, and a stray radial magnetic field:
- Stray radial magnetic field gives a distortion of the ideal orbit, in such a way that the total radial magnetic field (stray plus focusing) = 0 on the closed orbit.
- Self fixing!
- Problem comes if we have an unwanted electric quadrupole multipole:
- $E_y = -k_2 y$ , then the radial magnetic field is not zero on the closed orbit.

$$E_y = -k_2 y$$

- Normal dipole.
- What we want for bending.
- Skew dipole.
- Plates are rotated by  $\vartheta$ .
- Spin effect canceled by CW/CCW symmetry.
- Skew quadrupole.
- Skew dipole plus misaligned by angle  $\alpha$ .  $\propto \vartheta \alpha$ .
- Ronald Reagan: Trust but verify.



# Closed Orbit

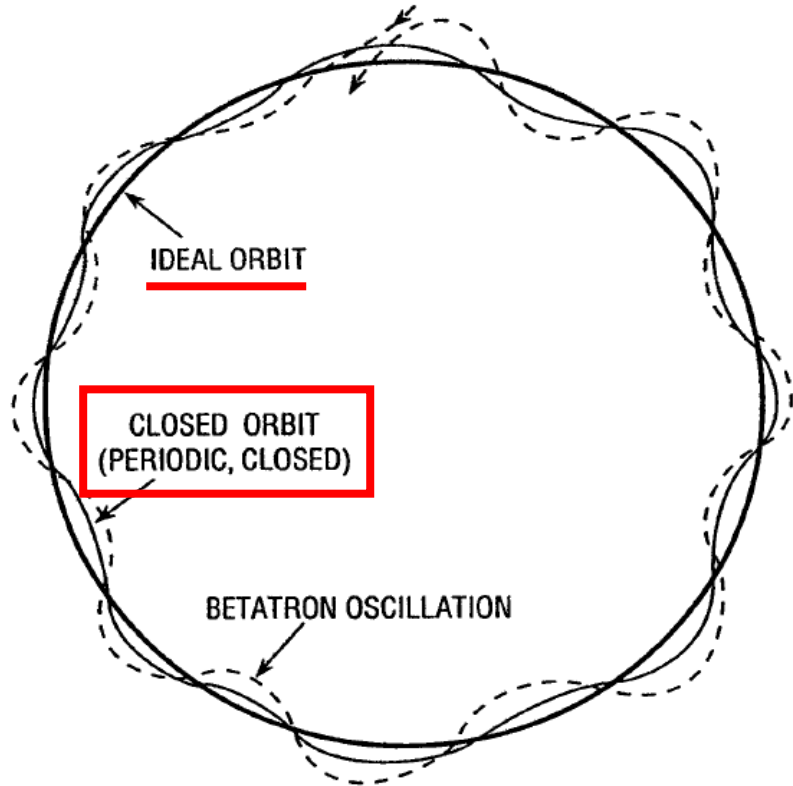


Fig. 2.1 The ideal orbit and closed orbit close after one turn. The betatron oscillation is quasiperiodic and always open.

## 3.2 Orbit Distortion due to Field Error

In the presence of a field error  $\Delta B$  in the dipoles, the actual closed orbit deviates from the ideal orbit,

$$\frac{d^2x}{ds^2} + k(s)x = -\frac{\Delta B}{B\rho} \equiv F(s). \quad (3.2)$$

Since the field error is in the dipoles,  $\Delta B$  depends on  $s$ , but not on  $x$ . The Courant-Snyder transformation yields

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = \nu^2\beta^{3/2}F(\phi). \quad (3.3)$$

The Green function method gives the periodic solution

$$\eta(\phi) = \frac{\nu}{2 \sin(\pi\nu)} \int_{\phi}^{\phi+2\pi} \beta^{3/2} F(\phi') \cos[\nu(\pi + \phi - \phi')] d\phi' \quad (3.4)$$

as long as  $\nu \neq \text{integer}$ . We see that if  $\nu = \text{integer}$ , then the closed orbit  $\rightarrow \infty$ , i.e. even a small field error  $\Delta B$  will cause a large closed orbit distortion. Since small errors  $\Delta B$  are always present, this makes it undesirable to choose a tune close to an integer.



# Electric quadrupole multipole

- Create a large radial magnetic field with trim coils.
- $\approx 10^2 \times$  larger than stray radial magnetic fields.
- See how much  $dS_y/dt$  changes.
- Adjust trim electric quads to get acceptable  $dS_y/dt$  .

# Electric Quadrupole

- Muon g-2 ring approximation:

- $$y_{CO} \approx \sum_{N=0}^{\infty} \frac{R_0}{(N^2 - v_y^2)} \frac{[B_{xNs} \sin(N\vartheta) + B_{xNc} \cos(N\vartheta)]}{B_0}$$

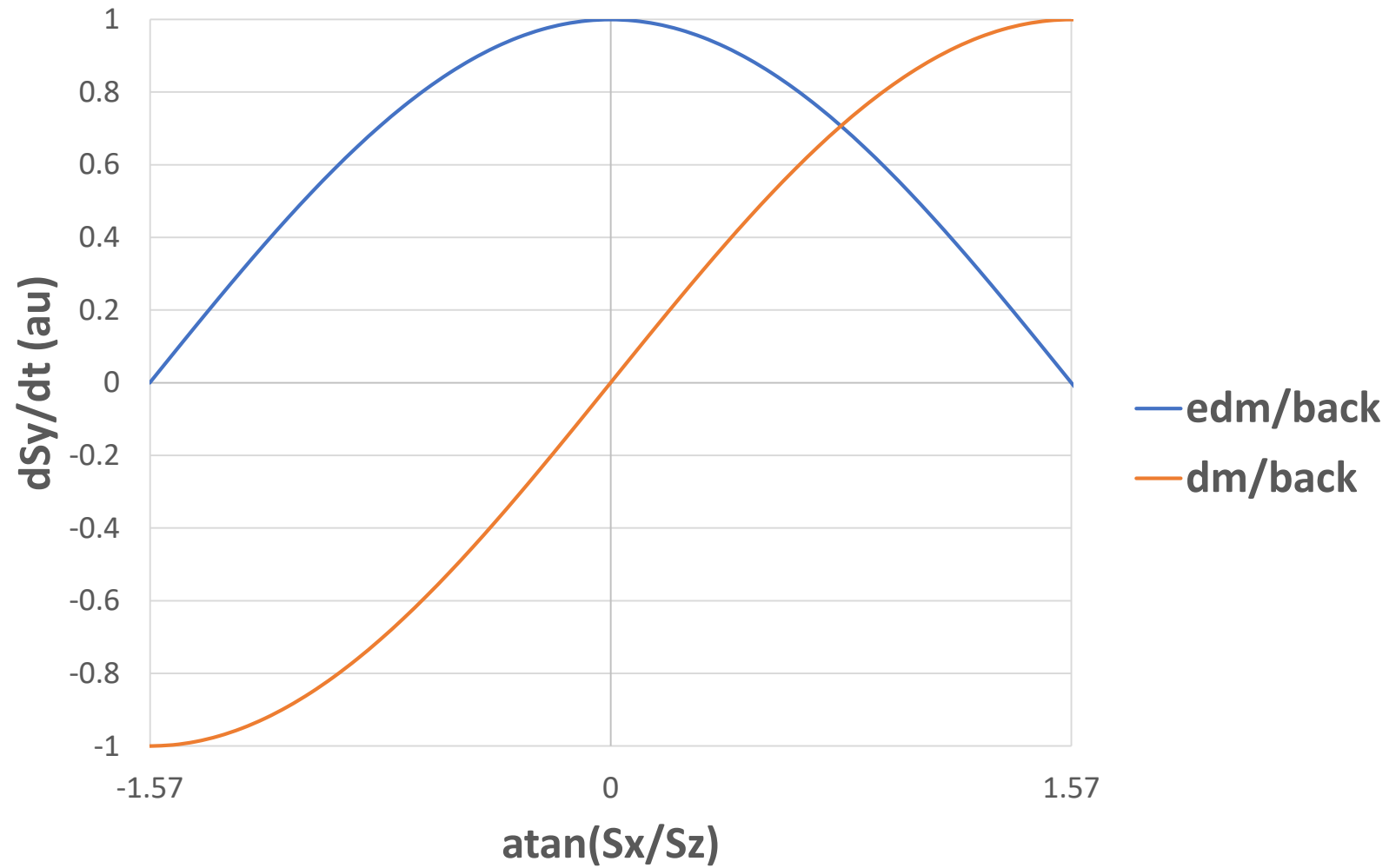
- $$E_y = -k_{2E} y_{CO} =$$

- $$\sum_{N=0}^{\infty} \frac{R_0}{(N^2 - v_y^2)} \frac{[k_{2ENs} B_{xNs} \sin^2(N\vartheta) + k_{2ENc} B_{xNc} \cos^2(N\vartheta)]}{B_0}$$

- Do each Fourier term, i.e.,  $N = 0$ ,  $N = 1 \sin \vartheta$ ,  $N = 1 \cos \vartheta$

- $v_y = 2.3$ . High  $N$  terms drop off as  $1/N^2$ .

# DM-like background



# Four Straight Sections – Minimum Number

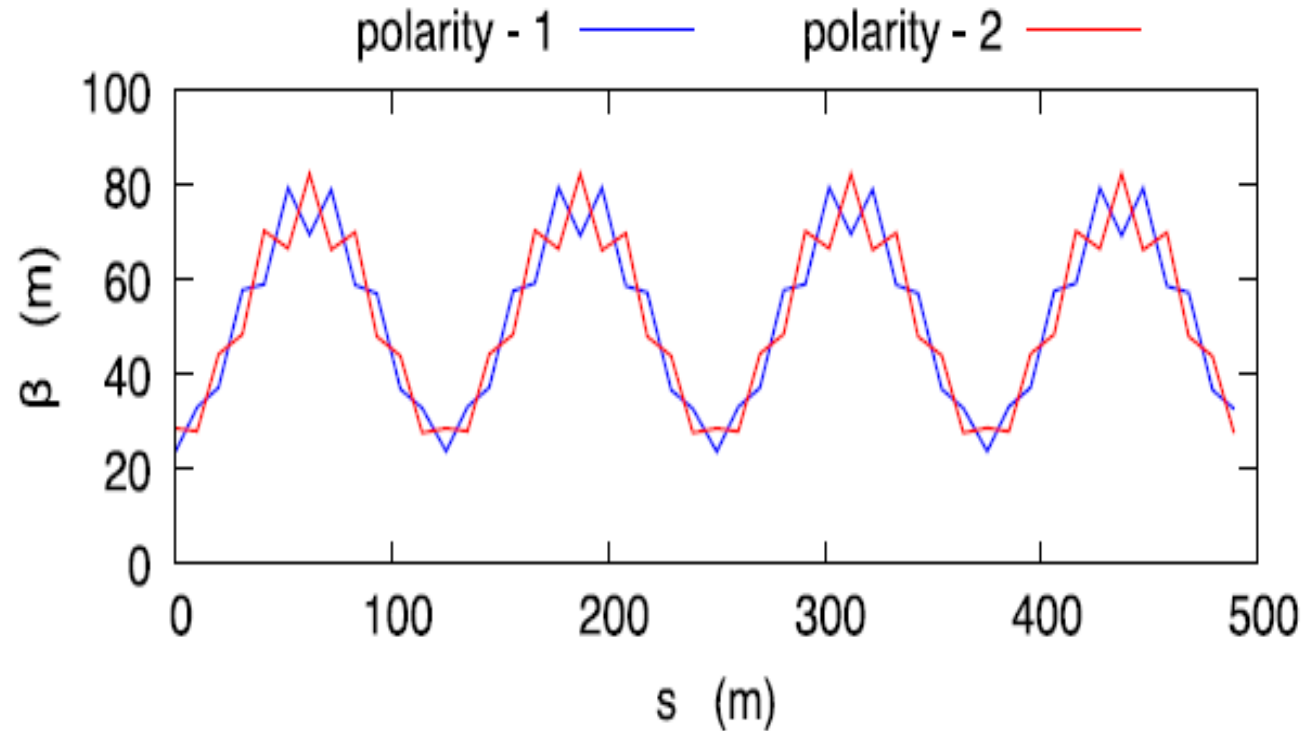
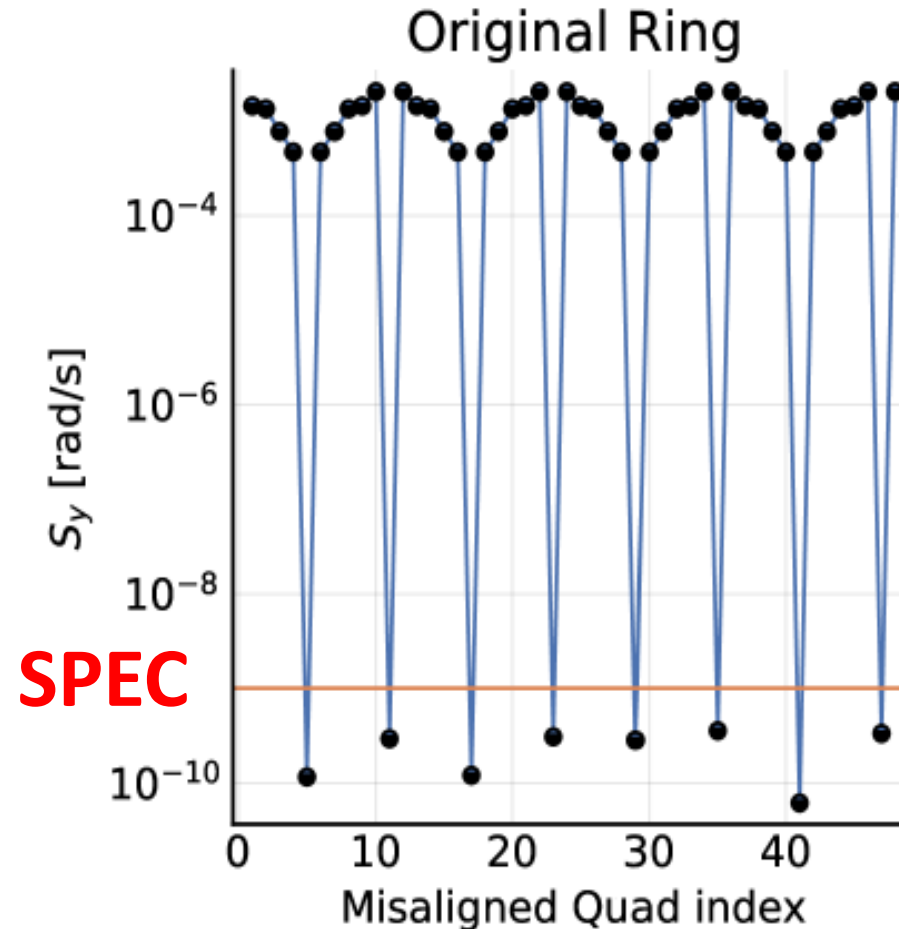


FIG. 3. The horizontal beta-function values around the ring for CW and CCW operations.

# Scale of the problem in the 4-fold ring

- Radially polarized beam
- Misalign one quadrupole at a time by  $1\ \mu m$
- 8 islands of high symmetry



# 48 cells – more symmetry

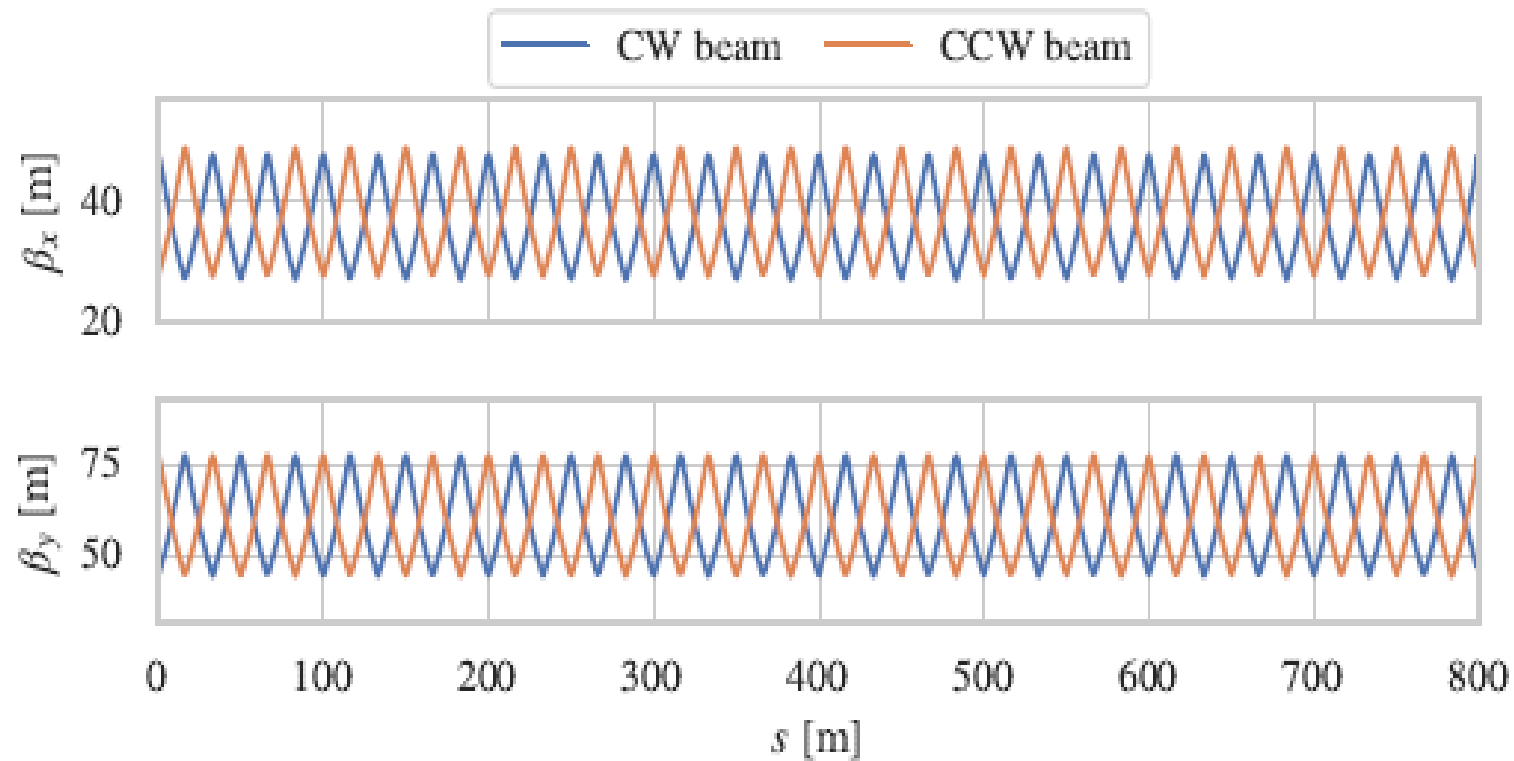


Figure 4: Beta functions along the ring. The results were obtained via numerical tracking.

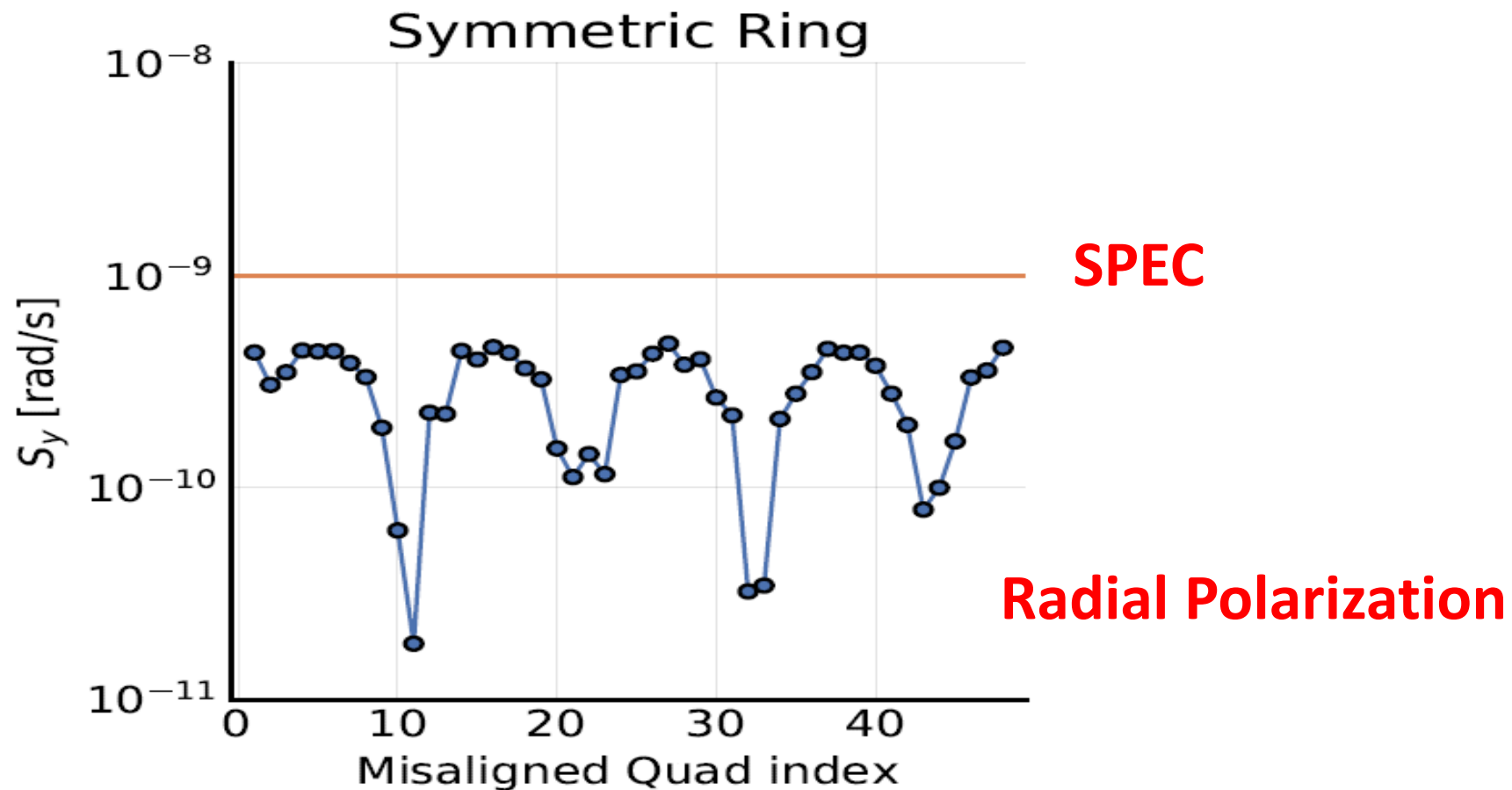


Figure 8: Vertical spin precession rate vs. index of the 1  $\mu\text{m}$  misaligned quad along the azimuth. Irregularities of the points are due to the inability to determine the exact precession rate from the simulation results. Hence, the points only show upper limit of the possible vertical precession rate, actual rates could be lower. The orange line corresponds to the target EDM sensitivity

# Vertical Velocity

- Vertical velocity  $\beta_y$  combined with  $E_x$ :

$$\left( \alpha \vec{S} \times (\vec{\beta} \times \vec{E})_s \right)_y = \alpha S_x \beta_y E_x$$

- Intuitively  $\langle \beta_y \rangle \equiv 0$  by definition
- However,

$$\langle \beta_y \rangle_{\text{straight}} + \langle \beta_y \rangle_{\text{bending}} \equiv 0$$

- Hence,

$$\langle \beta_y \times E_x \rangle \neq 0$$



# Beam Based Alignment/Spin Based Alignment

- BBA developed in the 1990s for ILC, NLC, ...
- They needed the beam to go through the center of hundreds of quads in LINAC to  $1\mu m$  in order to get the correct final focus.
- In order to avoid using spin based alignment, need  $0.01mm$  BBA.
- Can't use spin based alignment for DM/DE.
- RHIC achieved  $0.1 mm$  BBA [1].
- This was plenty good enough for RHIC.

1. BNL CA Dept. Note 83395.

# Beam Based Alignment/Spin Based Alignment

- We will have:
- SQUID BPM resolution:  $0.01\mu m$  [1].
- BMP at every quad location.
- AC modulation of closed orbit, instead of DC.
- Correct higher order multipoles in quads with correction windings.
- After BBA, use transverse polarization, and adjust with spin based alignment.

1. S. Hacıomeroglu, ICHEP 2018, Proc. Sci. (2018).

# Statistics

- $10^{11}$  polarized protons per fill from AGS.
- Storing protons in a ring is easy.
- Excellent polarimeter analyzing power.
- $10^{-29} \text{ ecm} / 10^7 \text{ s}$

# P Magic Energy: 232 MeV

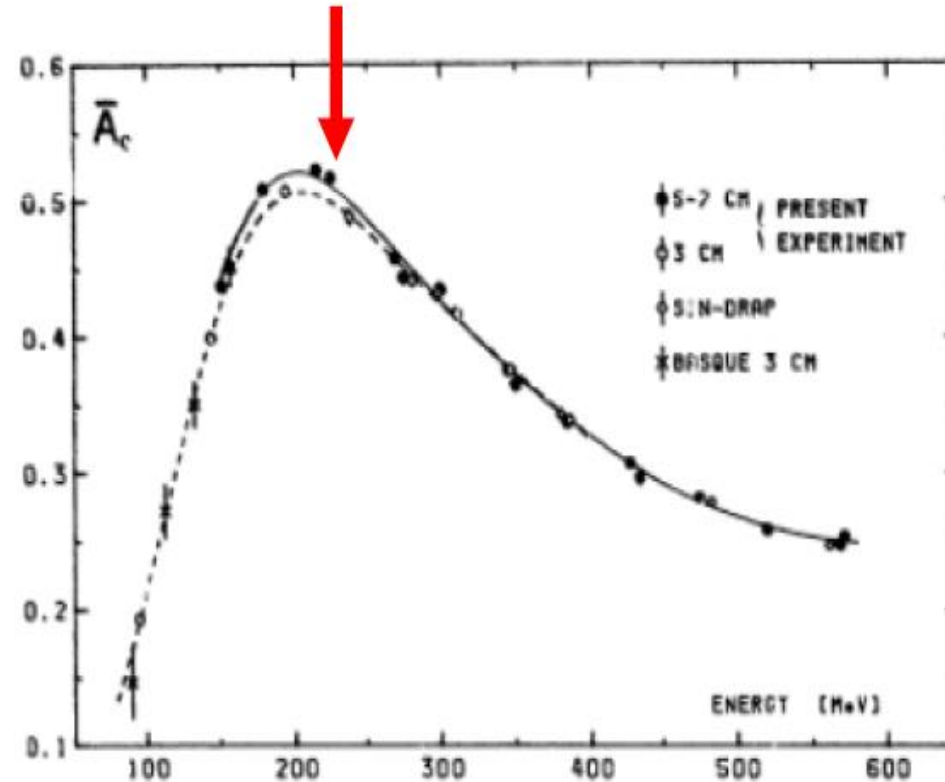
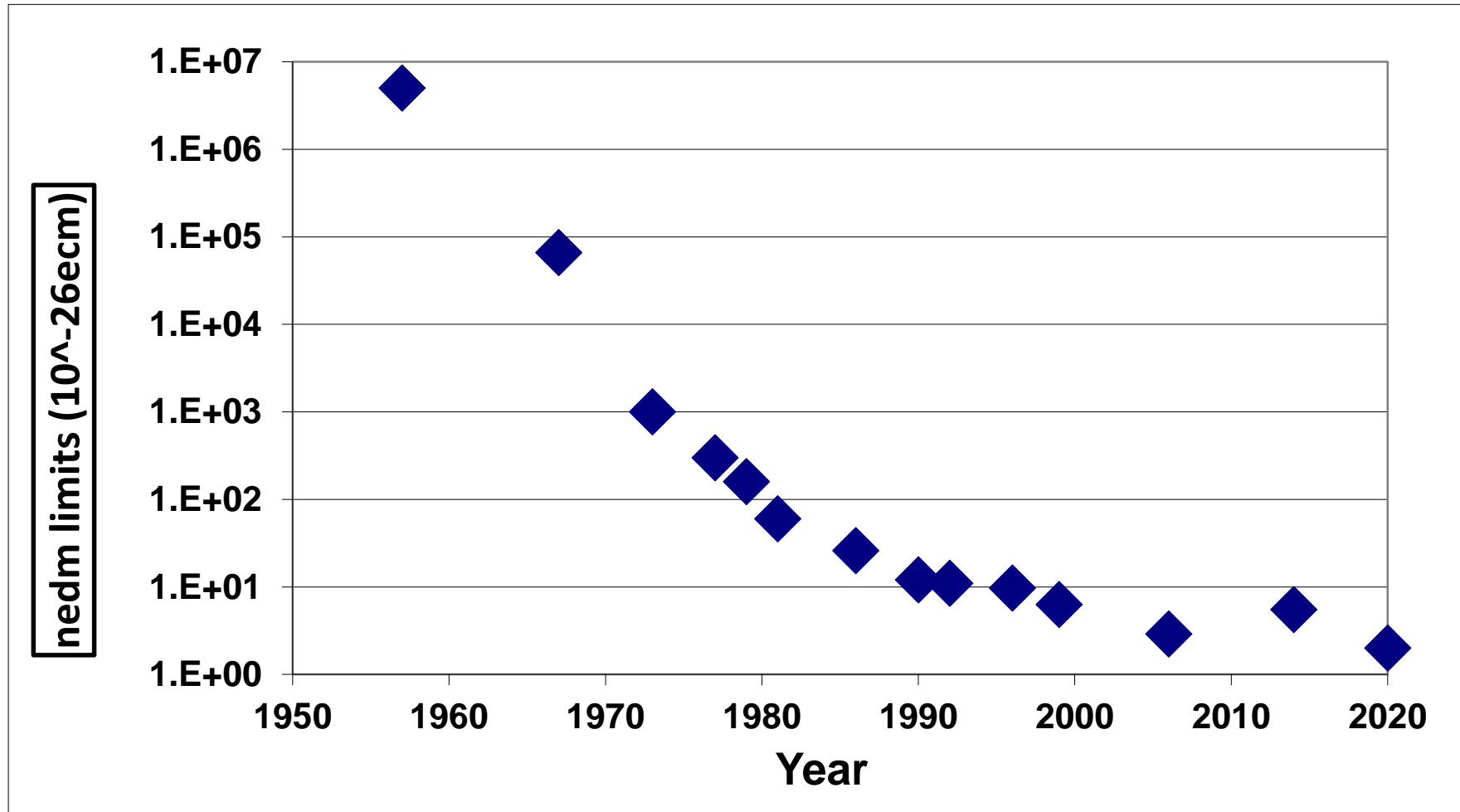


Fig. 4. Angle-averaged effective analyzing power. Curves show our fits. Points are the data included in the fits. Errors are statistical only

Fig.4. The angle averaged effective analyzing power as a function of the proton kinetic energy. The magic momentum of 0.7GeV/c corresponds to 232MeV.

PEDM limited by statistics at  $10^{-29} ecm$ .



**Limited by statistics.**

# LOI

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# Summary

- Exciting sredm physics.
- CP violation beyond the SM and/or QCD CP violation, i.e., Peccei-Quinn symmetry axion physics.
- EDM has a spin flip, so proportion to mass.
- Physics is complementary to electron, neutron, Hg, etc., searches.
- PEDM systematics are OK. E821 and E989 found new beam/spin dynamics systematics at the level of sensitivity. Need statistics.
- DM/DE [1] systematics are worse than edm systematics, but if we “only” get a factor of  $10^2$  improvement, for example, that would be wonderful!

1. P.W. Graham, et. al., *Storage Ring Probes of Dark Matter and Dark Energy*, arXiv:2005.11867 (2020).

Extra

# Storage Ring Probes of Dark Matter and Dark Energy

Peter W. Graham<sup>1</sup>, Selcuk Haciomeroglu<sup>2</sup>, David E. Kaplan<sup>3</sup>, Zhanibek Omarov<sup>4</sup>, Surjeet Rajendran<sup>3</sup>, and Yannis K. Semertzidis<sup>2,4</sup>

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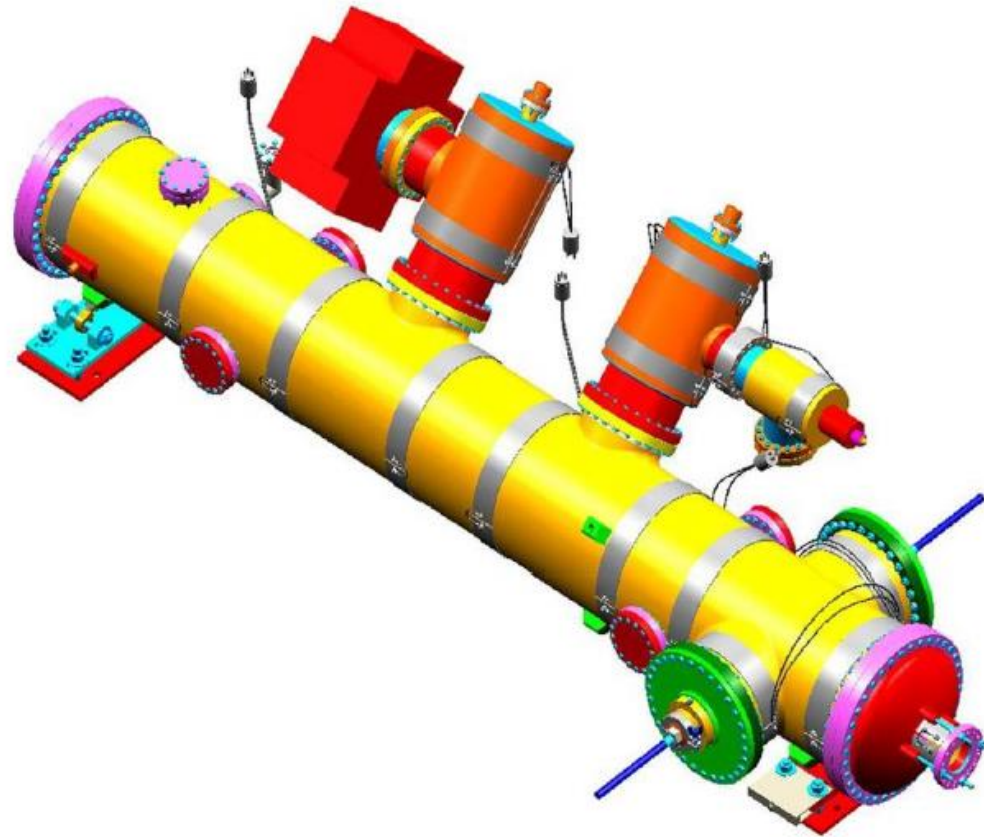
May 26, 2020

## Abstract

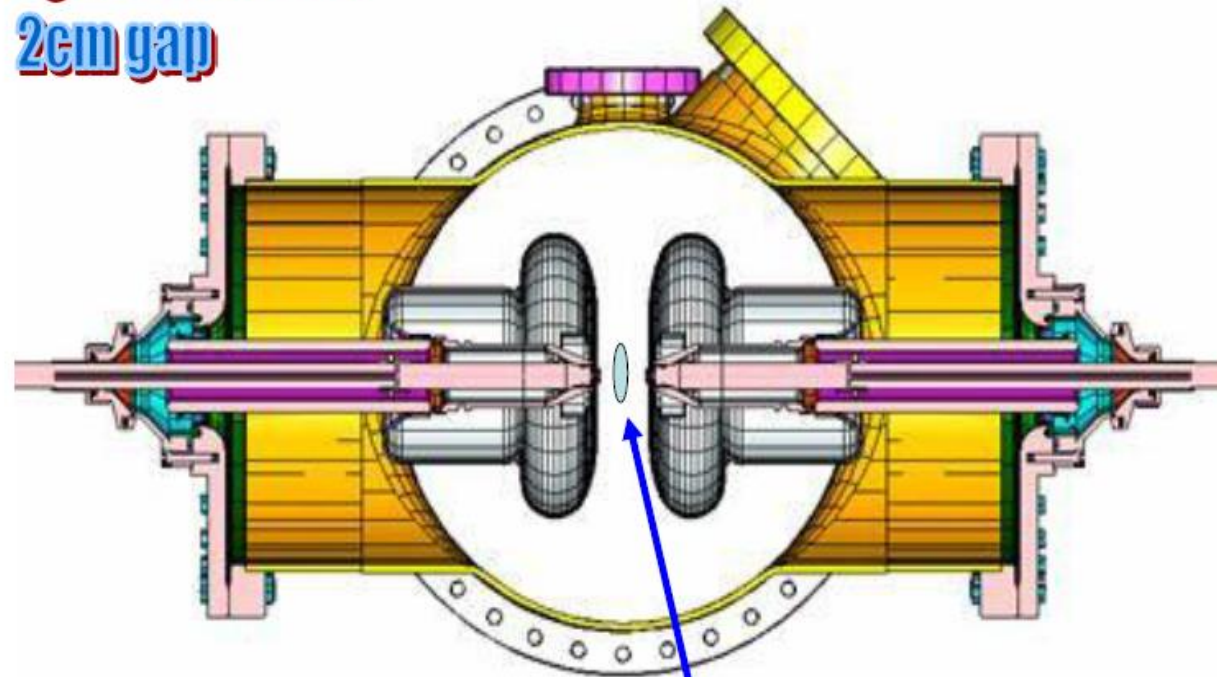
We show that proton storage ring experiments designed to search for proton electric dipole moments can also be used to look for the nearly dc spin precession induced by dark energy and ultra-light dark matter. These experiments are sensitive to both axion-like and vector fields. Current technology permits probes of these phenomena up to three orders of magnitude beyond astrophysical limits. The relativistic boost of the protons in these rings allows this scheme to have sensitivities comparable to atomic co-magnetometer experiments that can also probe similar phenomena. These complementary approaches can be used to extract the micro-physics of a signal, allowing us to distinguish between pseudo-scalar, magnetic and electric dipole moment interactions.



# Extras 3m long Tevatron pbar-p Separator

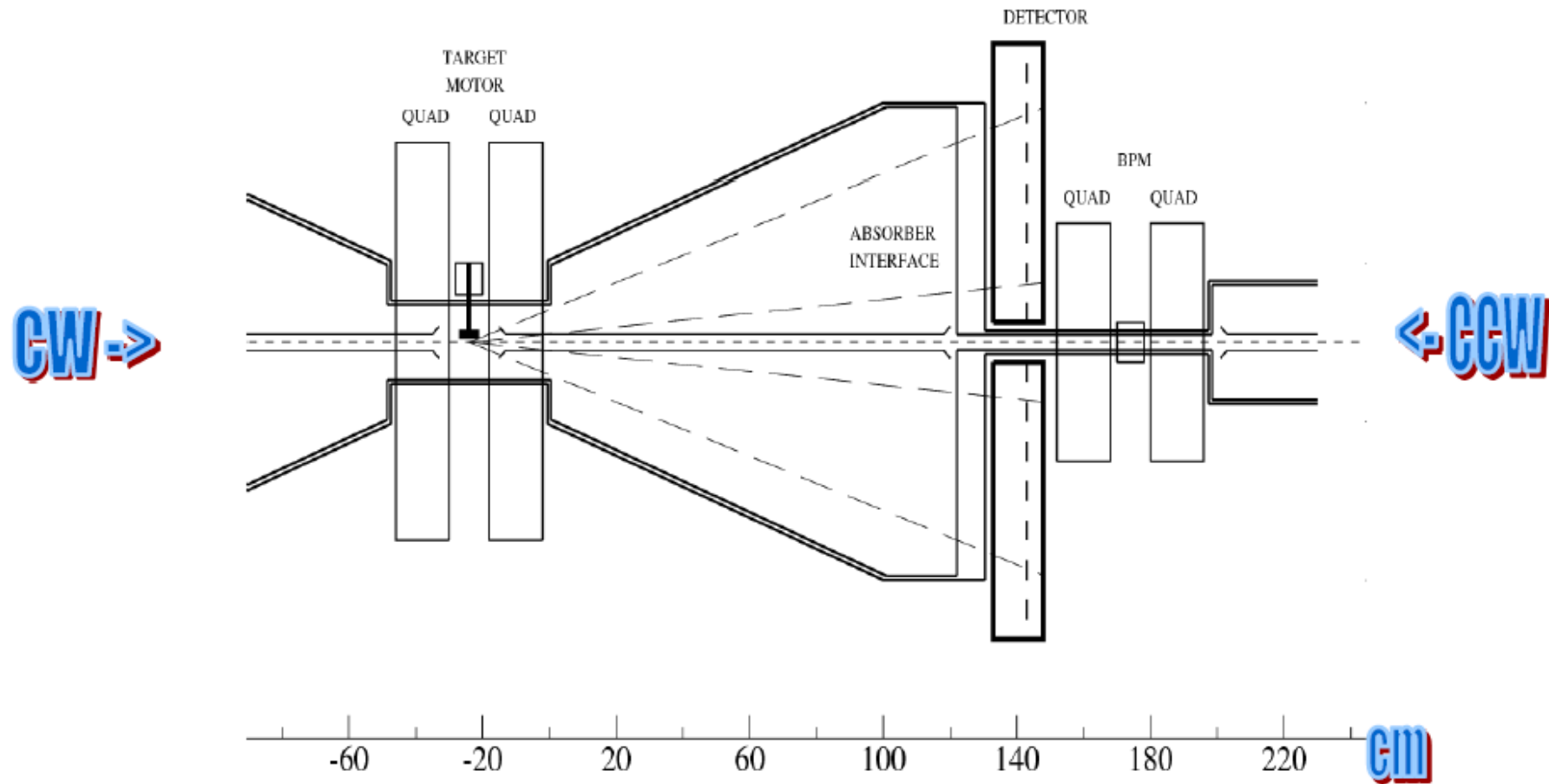


**0.2m high electrodes**  
**2cm gap**



**CW/CCW Beams**

# 1/2 of PEDM Polarimeter



It turns out that the vertical electric field dipole multipole gets canceled CW/CCW [1]. Briefly, the radial focusing magnetic field flips sign CW/CCW:

$$\langle F_y \rangle = e \langle E_y + vB_x \rangle = 0 \quad (2)$$

$$\langle B_x \rangle = -\langle \frac{E_y}{v} \rangle = \langle k_{2B}y \rangle \quad (3)$$

In equ. 3,  $k_{2B}$  gives the strength of the magnetic focusing quadrupole = 0.2T/m.

$$y_{CO} = -\frac{\langle E_y \rangle}{k_{2B} v} \quad (4)$$

The MDM spin precession flips sign CW/CCW:

$$\frac{d\hat{s}}{dt} = \frac{e}{mc} \hat{s} \times \left\{ \left( \frac{g}{2} - 1 + \frac{1}{\gamma} \right) \vec{B} - \left( \frac{g}{2} - 1 \right) \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left( \frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right\} \quad (5)$$

The EDM spin precession from the bending electric field doesn't flip sign CW/CCW:

$$\frac{d\vec{s}}{dt} = d_p \vec{s} \times (\vec{E} + \vec{v} \times \vec{B}) \quad (6)$$

The accuracy limitation of BBA on RHIC is about 0.1 mm because of the following factors:

1. BPM accuracy is temperature dependent. Each  $\Delta 10^\circ\text{F}$  temperature change results in  $\sim \Delta 1\text{ns}$  change in trigger time which adds an uncertainty of  $\sim \pm 0.1\text{mm}$  to  $\sim 1\text{mm}$  in BPM reading. BPM resolution is  $\sim 0.01\text{mm}$  at a constant temperature;
2. The physical misalignment of quadrupoles relative to each other is  $\sim \pm 0.06\text{mm}$ ;
3. BPM position with respect to the outside fiducials is  $\sim \pm 0.13\text{mm}$ ;
4. Unknown beam positions and angles at the first corrector (beam angle at the triplet). It was estimated to have an effect  $\sim \pm 0.1\text{mm}$ ;
5. Repeatability under the same machine settings is found by experiment to be  $\sim \pm 0.1\text{mm}$ .

# CW/CCW

- The major conceptual and technological strengths of the srEDM method render it ready for technical evaluation. Its critical conceptual strength is the realization that a ring with purely electric bending sections and alternate magnetic focusing (a hybrid-ring lattice) permits simultaneous clock-wise (CW) and counter-clock-wise (CCW) storage, thus *eliminating first order systematic error sources*, i.e., out-of-plane electric fields, *as well as the need to significantly shield the ring from external magnetic fields*.

## Resolution and systematic limitations in beam-based alignment

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(Received 23 March 2000; published 25 May 2000)

Beam-based alignment of quadrupoles by variation of quadrupole strength is a widely used technique in accelerators today. We describe the dominant systematic limitation of such algorithms, which arises from the change in the center position of the quadrupole as the strength is varied, and derive expressions for the resulting error. In addition, we derive an expression for the statistical resolution of such techniques in a periodic transport line, given knowledge of the line's transport matrices, the resolution of the beam position monitor system, and the details of the strength variation procedure. These results are applied to the Next Linear Collider main linear accelerator, an 11 km accelerator containing 750 quadrupoles and 5 000 accelerator structures. We find that, in principle, a statistical resolution of  $1\ \mu\text{m}$  is easily achievable, but the systematic error due to variation of the magnetic centers could be several times larger.

<b>Effect</b>	<b>Remediation</b>
Radial B-field.	Magnetic focusing.
Unwanted vertical forces when other than magnetic focusing is present.	Vary the magnetic focusing strength and fit for the DC offset in the vertical precession rate.[14]
Dipole vertical E-fields.	Cancel exactly with CW and CCW beam storage.
Quadrupole E-field in the electric bending sections.	Probe it by locally splitting the counter-rotating beams and cancel it with trim E-fields. Finally, keep the counter-rotating beams at the same position to S-BPM resolution.
Corrugated (non-planar) orbit.	Minimize effect with symmetric lattice design. Finally, keep the CR beams at same position, at the electric field bending sections, using beam-based alignment.
Longitudinal B-field.	Small effect.
Geometrical phase effect due to lattice elements imperfections.	Equivalent to a spin resonance due to lattice elements imperfections. Magnetic quadrupoles: beam-based alignment to $1\mu\text{m}$ rms. E-field sections: Absolute beam position monitors to $<0.1\text{mm}$ .
Geometrical phase effect due to external magnetic fields.	Equivalent to a spin resonance due to external magnetic interference coupled with electric field bending section misplacement.[24, 26] When the local spin effects are kept below 1nT B-field equivalent, the effect is negligible even for one directional (CW or CCW only) storage.
RF cavity vertical and horizontal misalignment.	Vary the longitudinal lattice impedance to probe the effect of the cavity's vertical and horizontal angular misalignments. The vertical and horizontal offsets are much smaller effects.



# BEAM BASED ALIGNMENT

## Derivation of formula for orbit change

$$\Delta x(s) = \left( \frac{\Delta k \cdot x(\bar{s}) l}{B\rho} \right) \left( \frac{1}{1 - k \frac{l\beta(\bar{s})}{2B\rho \tan \pi\nu}} \right) \frac{\sqrt{\beta(s)}\sqrt{\beta(\bar{s})}}{2 \sin \pi\nu} \cos(\phi(s) - \phi(\bar{s}) - \pi\nu)$$

- $\Delta x$  = orbit change
- $s$  = measurement position
- $\bar{s}$  = position of quadrupole
- $\Delta k$  = change of quadrupole strength
- $x(\bar{s})$  = position of beam inside the quadrupole
- $\beta$  = beta function
- $\nu$  = tune
- $\phi$  = betatron phase
- $k$  = quadrupole strength
- $l$  = length of quadrupole
- $B\rho$  = magnetic rigidity of the beam

## Resolution and systematic limitations in beam-based alignment

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Beam-based alignment of quadrupoles by variation of quadrupole strength is a widely used technique in accelerators today. We describe the dominant systematic limitation of such algorithms, which arises from the change in the center position of the quadrupole as the strength is varied, and derive expressions for the resulting error. In addition, we derive an expression for the statistical resolution of such techniques in a periodic transport line, given knowledge of the line's transport matrices, the resolution of the beam position monitor system, and the details of the strength variation procedure. These results are applied to the Next Linear Collider main linear accelerator, an 11 km accelerator containing 750 quadrupoles and 5 000 accelerator structures. We find that, in principle, a statistical resolution of  $1 \mu\text{m}$  is easily achievable, but the systematic error due to variation of the magnetic centers could be several times larger.

### G. $K$ modulation

Finally, another approach that is worth mentioning is the strength modulation technique. In  $K$  modulation, the strength of a quadrupole is varied harmonically at a frequency that is low compared to the betatron and synchrotron frequencies of a storage ring; this generates oscillations in the closed orbit at the same frequency. Because  $K$  modulation causes narrow band orbit oscillations, it is possible to use lock-in amplifiers and other techniques to vastly improve the signal-to-noise performance of the measurement [12]. This technique has been used in a number of storage rings where it is not possible to make large changes to the quadrupole strengths because of the

# Field Gradients OK for E989

## Eric Metodiev

Thomas-BMT equation generalized  
to electric dipole moments and field gradients

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An expression is presented for the relativistic equations of motion, including field gradients, of a particle and its spin with electric and magnetic dipole moments aligned along the spin axis. An electromagnetic duality transformation is used to generalize a Thomas-BMT equation with gradient terms. Corrections to particle dynamics in storage rings for precision ( $g - 2$ ) and electric dipole moment measurements are calculated, and applications to precision particle tracking programs are considered.

### I. INTRODUCTION

A detailed knowledge of the spin dynamics of particles with non-zero electric dipole moments (EDMs) and magnetic dipole moments (MDMs) is necessary for precision EDM and ( $g - 2$ ) measurements using spin precession in storage rings [1–5]. The Thomas-Bargmann-Michel-Telegdi (T-BMT) equation [6, 7] governs the classical spin dynamics of a particle with a non-zero MDM in electric and magnetic fields, neglecting field gradients. Recently, derivations have been presented which generalize the T-BMT equation to include a non-zero particle EDM based on duality transformations [8, 9] and explicit relativistic constructions [10].

The spin equation of motion of a particle with a non-zero MDM including first order field gradients has also been established [11]. By making use of an electromagnetic duality transformation on these equations of motion, a generalization of the T-BMT equation for non-zero particle EDMs and MDMs and first order field gradients is determined. The corrections to the spin and particle equations of motion are then studied. We find that typical experimental methods in storage ring EDM methods are robust to higher order than previously demonstrated. Higher-order corrections to the dynamical equations used in many precision particle tracking programs [12, 13] are also presented.

### II. DYNAMICS WITH A NON-ZERO MAGNETIC DIPOLE MOMENT

R.H. Good [11] determined the classical equations of motion for a particle and its spin with a MDM  $\mu$  along its spin direction to first order in the field gradients. For a particle of mass  $m$ , electric charge  $e$ , spin angular momentum  $\hbar$ , and velocity  $\mathbf{v} = c\boldsymbol{\beta}$ , he derived that:

$$m\mathbf{c}\frac{d(\gamma\boldsymbol{\beta})}{dt} = e\mathbf{E} + e\mathbf{c}\boldsymbol{\beta} \times \mathbf{B} + \frac{\mu\gamma}{c\hbar}[\nabla + \boldsymbol{\beta} \times (\boldsymbol{\beta} \times \nabla) + \frac{1}{c}\boldsymbol{\beta}\partial_t]\mathbf{s} \cdot \mathbf{R} \quad (1)$$

and

$$\frac{d\mathbf{s}}{dt} = \frac{\mu}{c\hbar}\frac{1}{\gamma}\mathbf{s} \times \mathbf{R} - \frac{c}{mc}\mathbf{s} \times \mathbf{N} + \frac{\mu}{\hbar mc^2}\frac{1}{\gamma+1}\mathbf{s} \times (\boldsymbol{\beta} \times \nabla)[\mathbf{s} \cdot \mathbf{R}], \quad (2)$$

where:

$$\begin{aligned} \mathbf{M} &= e\mathbf{B} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta} \times \mathbf{E} \\ \mathbf{N} &= \frac{\gamma}{\gamma+1}(\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}) \times \boldsymbol{\beta} \\ \mathbf{R} &= \mathbf{M} + \gamma\mathbf{N}. \end{aligned} \quad (3)$$

A non-zero quadrupole moment  $q$  is also considered in the solution, but we omit it here for clarity. This method can also be used to extend the T-BMT equation to electric quadrupole moments. We define the EDM  $\mathbf{d}$  and MDM  $\boldsymbol{\mu}$  in terms of the rest frame spin  $\mathbf{s}$  as:

$$\mathbf{d} = \frac{q}{2}\frac{e}{mc}\mathbf{s}, \quad \boldsymbol{\mu} = \frac{g}{2}\frac{e}{m}\mathbf{s}, \quad (4)$$

where these relations define  $g$  and  $\eta$ .

We can also write the Equation 1 as an equation for  $\boldsymbol{\beta}$  in the form:

$$\gamma mc\frac{d\boldsymbol{\beta}}{dt} = e\mathbf{E} + e\mathbf{c}\boldsymbol{\beta} \times \mathbf{B} - e\boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E}) + \frac{\mu\gamma}{c\hbar}[\nabla + \boldsymbol{\beta} \times (\boldsymbol{\beta} \times \nabla) + \frac{1}{c}\boldsymbol{\beta}\partial_t]\mathbf{s} \cdot \mathbf{R}. \quad (5)$$

To write the equations in covariant form, we use the four-velocity  $w^\nu$  and define the spin 4-pseudovector which takes the form  $a^\mu = (0, \mathbf{s})$  in the particle rest frame. Further we use the electromagnetic field strength tensor  $F^{\nu\sigma}$  and its dual  $F^{*\nu\sigma}$ . In these terms, the equations become:

$$m\frac{dw_\nu}{d\tau} = eF^{\nu\rho}u_\rho + \frac{\mu}{\hbar c}u_\sigma\left(\partial_\nu + \frac{1}{c^2}u_\alpha u^\alpha\partial_\nu\right)F^{*\sigma\rho}a_\rho \quad (6)$$

$$\frac{da_\nu}{d\tau} = \frac{\mu}{\hbar}F_{\nu\sigma}a^\sigma - \frac{1}{c^2}\left(\frac{\mu}{\hbar} - \frac{c}{m}\right)u_\alpha u_\nu F^{\mu\sigma}a_\sigma + \frac{\mu}{\hbar mc^2}u_\alpha u_\nu a^\alpha\partial_\nu F^{*\mu\sigma}a_\sigma. \quad (7)$$

arXiv:1507.04440v1 [physics.acc-ph] 16 Jul 2015

1. Z. Omarov, et. al., *Systematic error analysis of the Symmetric Hybrid ring design in the storage-ring proton electric dipole moment experiment*, arXiv:2007.10332 (2020)
2. S. Haciomeroglu, *Real time sextupole tuning for a long in-plane polarization at storage rings*, NIM A 982, 164550 (2020)
3. P.W. Graham, et. al., *Storage Ring Probes of Dark Matter and Dark Energy*, arXiv:2005.11867 (2020)
4. F. Mueller et al., *Measurement of deuteron carbon vector analyzing powers in the kinetic energy range 170-380 MeV*, arXiv:2003.07566 (2020)
5. S. Haciomeroglu, Y.K. Semertzidis, *Hybrid ring design in the storage-ring proton electric dipole moment experiment*, PRAB 22, 034001 (2019)
6. S.P. Chang, et. al., *Axion-like dark matter search using the storage ring EDM method*, PRD 99, 083002 (2019)
7. S. Haciomeroglu, et.al., *Magnetic field effects on the proton EDM in a continuous all-electric storage ring*, NIM A 927, 262-266 (2019)
8. S. Haciomeroglu, et. al., *SQUID-based beam position monitor*, ICHEP2018 Proceedings (2018)
9. G. Guidoboni, et al., *Connection between zero chromaticity and long in-plane polarization lifetime in a magnetic storage ring*, Phys. Rev. Accelerators and Beams 22, 024201 (2018)
10. N. Hempelmann et al., *Phase Measurement for Driven Spin Oscillations in a Storage Ring* arXiv:1801.03445 (2018)
11. A.J. Silenko, *Berry phases in an electric-dipole-moment experiment in an all-electric storage ring*, arXiv:1710.01609 (2017)
12. A.J. Silenko, *General description of spin motion in storage rings in presence of oscillating horizontal fields*, arXiv:1706.0206 (2017)

13. S. Haciomeroglu, Y. Semertzidis, *Systematic errors related to quadrupole misplacement in an all-electric storage ring for proton EDM experiment*, arXiv:1709.01208 (2017)
14. A. Saleev, et al., *Spin tune mapping as a novel tool to probe the spin dynamics in storage rings*, Phys. Rev. ST Accel. Beams 20, 072801 (2017)
15. Nils Hempelmann, *Phase Locking the Spin Precession in a Storage Ring*, PRL, 119, 014801 (2017)
16. V. Anastassopoulos, et. al., *A storage ring experiment to detect a proton electric dipole moment*, RSI 87, 115116 (2016)
17. G. Guidoboni, et al., *How to reach a thousand-second in-plane polarization lifetime with 0.97-GeV/c deuterons in a storage ring*, Phys. Rev. Lett. 117, 054801 (2016)
18. E. Methodiev, et. al., *Analytical benchmarks for precision particle tracking in electric and magnetic rings*, NIM A 797, 311 (2015)
19. D. Eversmann et al., *New method for a continuous determination of the spin tune in storage rings and implications for precision experiments*, Phys. Rev. Lett. 115, 094801 (2015)
20. S. Haciomeroglu, Y. K. Semertzidis, *Results of precision particle simulations in an all-electric ring lattice using fourth-order Runge-Kutta integration*, NIM A 743, 96 (2014)
21. Z. Bagdasarian et al., *Measuring the polarization of a rapidly precessing deuteron beam*, Phys. Rev. ST Accel. Beams 17, 052803 (2014)
22. Frank Rathmann, et. al., *The search for electric dipole moments of light ions in storage rings*, J. Phys.: Conf. Ser. 447 012011
23. P. Benati et al., *Synchrotron oscillation effects on an rf-solenoid spin resonance*, Phys. Rev. ST Accel. Beams 15, 124202 (2012)