Overview $g-2$ theory

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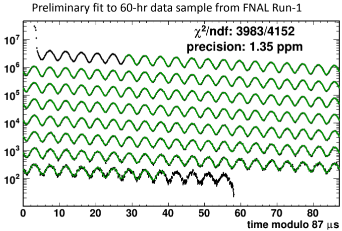
Snowmass Virtual Workshop on

Electric and Magnetic Dipole Moments



The Standard Model prediction for $a_\ell = (g-2)_\ell/2$

- Experiment for a_e Talk by Gabrielse and a_μ Talks by Mibe, Bhattacharya
- Here: overview of theory status (Standard Model)

$$a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{EW}} + a_\ell^{\text{had}} \qquad \ell = e, \mu$$

- Connection to EDMs

$\hookrightarrow$ a_ℓ and d_ℓ real and imaginary part of the same Wilson coefficient

QED: mass-independent terms

$$a_\ell^{\text{QED}} = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{j=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^j A_i^{(2j)}$$

- **Mass-independent** term A_1 universal

$$A_1^{(2)} = 0.5$$

$$A_1^{(4)} = -0.328478965579193784582\ldots$$

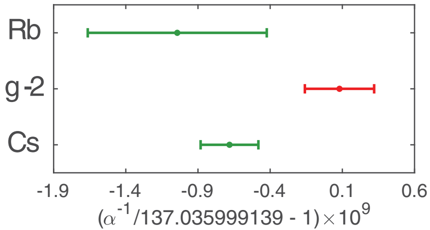
$$A_1^{(6)} = 1.181241456587200\ldots$$

$$A_1^{(8)} = -1.912245764926445574\ldots$$ Laporta 2017

$$A_1^{(10)} = 6.737(159)$$ Aoyama, Kinoshita, Nio 2019

- 4.8σ discrepancy between $A_1^{(10)}$[no lepton loops] $= 7.668(159)$ Aoyama, Kinoshita, Nio 2019 and $A_1^{(10)}$[no lepton loops] $= 6.793(90)$ Volkov 2019

QED: electron



- SM prediction Aoyama, Kinoshita, Nio 2019 with α from Cs measurement Berkeley 2018

$$a_e^{\text{SM}} = 1\,159\,652\,181.606(229)_{\alpha(\text{Cs})}(11)_{\text{5-loop QED}}(12)_{\text{had}} \times 10^{-12}$$

$\hookrightarrow$ discrepancy in 5-loop coefficient not yet relevant, electroweak and hadronic effects well under control

- Experiment Harvard 2008 lower by 2.4σ

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(28) \times 10^{-12}$$

- Improved Rb measurement Paris 2020 currently under review

QED: muon

- Large contribution from **mass-dependent** terms due to $\log m_\mu/m_e$ enhancement

$$A_2^{(12)}\left(\frac{m_\mu}{m_e}\right) \sim A_2^{(6)}\left(\frac{m_\mu}{m_e};\text{LbL}\right) \times \left\{\frac{2}{3}\log\left(\frac{m_\mu}{m_e}\right) - \frac{5}{9}\right\}^3 \times 10 \sim 5400$$

$$A_2^{(12)}\left(\frac{m_\mu}{m_e}\right) \times \left(\frac{\alpha}{\pi}\right)^6 \sim 0.8 \times 10^{-12}$$

$\hookrightarrow$ much bigger than uncertainty in mass-independent 5-loop coefficient

- With α from Cs measurement or a_e

$$a_\mu^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931(7)(17)(6)(100)(23)[104] \times 10^{-11}$$

$$a_\mu^{\text{QED}}(\alpha(a_e)) = 116\,584\,718.842(7)(17)(6)(100)(28)[106] \times 10^{-11}$$

$\hookrightarrow$ dominant uncertainty from 6-loop QED, $(g-2)_e$ tension irrelevant for $(g-2)_\mu$

- Electroweak contribution Gnendiger et al. 2013

$$a_\mu^{\text{EW}} = (194.8 - 41.2) \times 10^{-11} = 153.6(1.0) \times 10^{-11}$$

- Remaining uncertainty dominated by $q = u, d, s$ loops

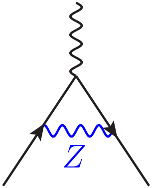
  $\hookrightarrow$ nonperturbative effects Czarnecki, Marciano, Vainshtein 2003

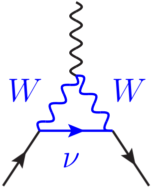
- Two-loop calculation recently revisited without asymptotic expansion Ishikawa, Nakazawa, Yasui 2019

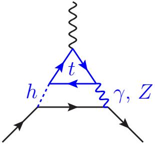
$$a_\mu^{\text{EW}} = 152.9(1.0) \times 10^{-11}$$

- 3-loop corrections?
  - 3-loop RG estimate accidentally cancels in scheme chosen by Gnendiger et al. 2013, with an error of 0.2×10^{-11}
  - α_s corrections to t-loop should scale as

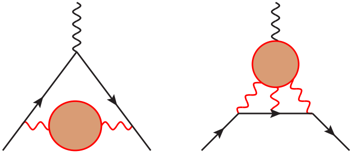
$$a_\mu^{t\text{-loop}}\Big|_{2\text{-loop}} \times \frac{\alpha_s}{\pi} \sim 0.3 \times 10^{-11}$$







Hadronic effects



- **Hadronic vacuum polarization**: need hadronic two-point function

$$\Pi_{\mu\nu} = \langle 0|T\{j_\mu j_\nu\}|0\rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

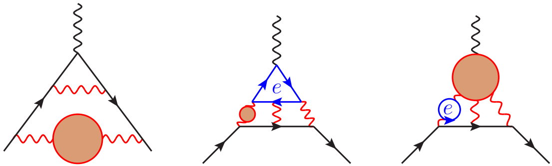
$$\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_\mu j_\nu j_\lambda j_\sigma\}|0\rangle$$

- In the following: status of the hadronic contributions

The Muon $g-2$ Theory Initiative

- Formed in 2017, series of workshops since (last one at the INT in Sep 2019)
  https://indico.fnal.gov/event/21626/ https://muon-gm2-theory.illinois.edu/
- Map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results
- White paper with recommended values 2006.04822, accepted for publication in Phys. Rept.

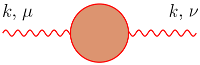
Higher-order hadronic effects



- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014

Hadronic vacuum polarization from e^+e^- data

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)\Pi\left(k^2\right)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi\left(k^2\right) - \Pi(0) = \frac{k^2}{\pi}\int\limits_{4M_\pi^2}^{\infty} \text{d}s \frac{\text{Im}\,\Pi(s)}{s\left(s-k^2\right)}$$

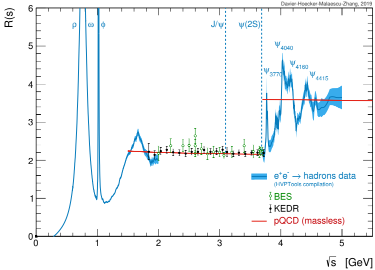
  - **Unitarity**

$$\text{Im}\,\Pi(s) = \frac{s}{4\pi\alpha}\sigma_{\text{tot}}\left(e^+e^- \to \text{hadrons}\right) = \frac{\alpha}{3}R(s)$$

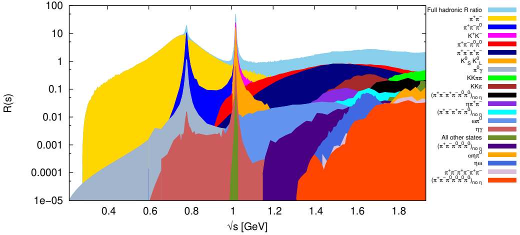
- Master formula

$$a_\mu^{\text{HVP LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R(s)$$

Hadronic vacuum polarization from e^+e^- data



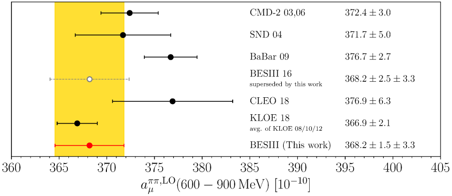
Davier, Hoecker, Malaescu, Zhang 2019



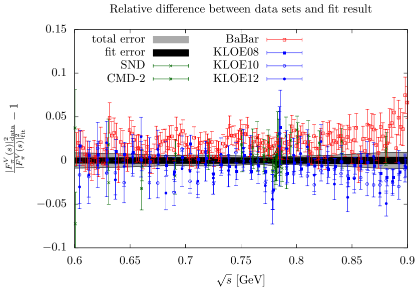
Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - Up to about 2 GeV: sum of exclusive channels
  - Above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

Hadronic vacuum polarization from e^+e^- data: 2π channel



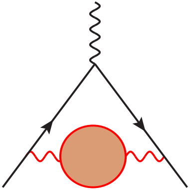
BESIII 2009.05011

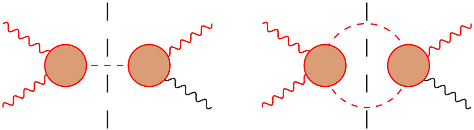


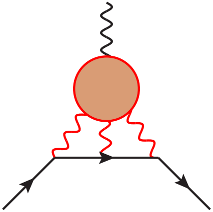
Colangelo, MH, Stoffer 2018

- Tension between KLOE and BaBar data:
  - Cross checks from **analyticity** and **unitarity** of the pion form factor
  - Affects the combination of data sets: different results depending on methodology
  - For white paper: adopt a **conservative merging procedure** that accounts for the 2π tension
- Our final recommendation: $a_\mu^{\text{HVP LO}}(e^+e^-) = 693.1(4.0) \times 10^{-10}$

- In the past: hadronic models, inspired by various QCD limits, but error estimates difficult
- Dispersive approach: use again **analyticity**, **unitarity**, **crossing**, and **gauge invariance** for data-driven approach Colangelo, MH, Procura, Stoffer 2014, . . .
- For simplest intermediate states: relation to $\pi^0 \to \gamma^*\gamma^*$ **transition form factor** and $\gamma^*\gamma^* \to \pi\pi$ **partial waves**







HLbL scattering: white paper

- Reference points:

$$a_\mu^{\text{HLbL}}\Big|_{\text{“Glasgow consensus” 2009}} = 105(26) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}\Big|_{\text{Jegerlehner, Nyffeler 2009}} = 116(39) \times 10^{-11}$$

- Strategy in the white paper
  - Take well-controlled results for the low-energy contributions
  - Combine errors in quadrature
  - Take best guesses for medium-range and short-distance matching
  - Add these errors linearly, since errors hard to disentangle at the moment
- **Recommended value**

$$a_\mu^{\text{HLbL}}\,(\text{phenomenology}) = 92(19) \times 10^{-11}$$

- **Lattice QCD**: first complete calculation RBC/UKQCD 2019

$$a_\mu^{\text{HLbL}}\,(\text{lattice}, uds) = 79(35) \times 10^{-11}$$

$\hookrightarrow$ can combine with phenomenological value

The anomalous magnetic moment of the muon in the Standard Model

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

Hadronic vacuum polarization: lattice QCD

- **White paper average**: $a_\mu^{\text{HVP LO}}$ (lattice) $= 711.6(18.4) \times 10^{-10}$

  $\hookrightarrow$ large uncertainty, consistent with both e^+e^- data and "no new physics"
- Does not include $a_\mu^{\text{HVP LO}}$ (BMWc) $= 708.7(5.3) \times 10^{-10}$ 2002.12347, first lattice result at $< 1\%$ precision

  $\hookrightarrow$ 2.3σ above e^+e^-, 1.5σ below "no new physics"
- How to resolve this?
  - Scrutiny by other lattice collaborations ongoing focused (virtual) workshop on lattice HVP (instigated by the Muon $g-2$ Theory Initiative) in November
  - Need to know at which energies the changes to the e^+e^- cross section occur

    $\hookrightarrow$ 2002.12347 points to low energies below 2 GeV
  - Would require changes to 2π cross section much bigger than the KLOE/BaBar tension
  - **New 2π data**: SND (under review), CMD3 (forthcoming), BaBar (reanalysis on larger data set), Belle II, BESIII
  - **MUonE project**: extract space-like HVP from μe scattering

Connection to EDMs

- **Effective dipole operators** $\mathcal{H}_{\text{eff}} = c_R^{\ell_f \ell_i}\, \bar{\ell}_f \sigma_{\mu\nu} P_R \ell_i F^{\mu\nu} + \text{h.c.}$

$$a_\ell = -\frac{4m_\ell}{e}\,\text{Re}\, c_R^{\ell\ell} \qquad d_\ell = -2\,\text{Im}\, c_R^{\ell\ell} \qquad \text{Br}[\mu \to e\gamma] = \frac{m_\mu^3}{4\pi\,\Gamma_\mu}\left(|c_R^{e\mu}|^2 + |c_R^{\mu e}|^2\right)$$

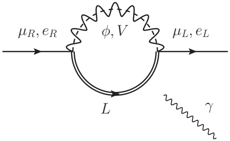
$\hookrightarrow$ in general only one power in m_ℓ for a_ℓ

- Consequences
  - Phase of c_R^{ee} much better constrained than phase of $c_R^{\mu\mu}$

$$\left|\frac{\text{Im}\, c_R^{ee}}{\text{Re}\, c_R^{ee}}\right| \lesssim 6 \times 10^{-7} \qquad \left|\frac{\text{Im}\, c_R^{\mu\mu}}{\text{Re}\, c_R^{\mu\mu}}\right| \lesssim 600$$

  - For models that fulfill $c_R^{e\mu} = \sqrt{c_R^{ee} c_R^{\mu\mu}}$

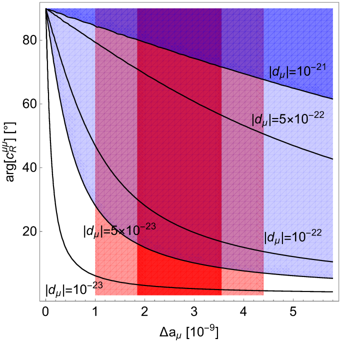
$$\text{Br}[\mu \to e\gamma] = \frac{\alpha m_\mu^2}{16 m_e \Gamma_\mu} |\Delta a_\mu \Delta a_e| \sim 8 \times 10^{-5}$$

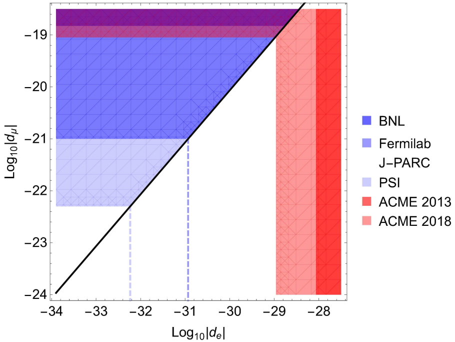


$\hookrightarrow$ violates **MEG bound** $\text{Br}[\mu \to e\gamma] < 4.2 \times 10^{-13}$ by 8 orders of magnitude!

  - For $\mathcal{O}(1)$ phase: $|d_\mu| = \mathcal{O}(10^{-22}\, e\,\text{cm})$ Talk by Price

Future experimental sensitivity to muon EDM

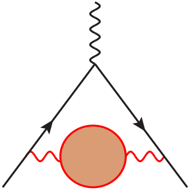


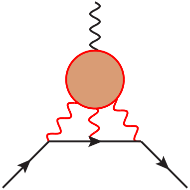


- Current limit E821: $|d_\mu| < 1.5 \times 10^{-19}\, e\,\mathrm{cm}$
- Fermilab/J-PARC $(g-2)_\mu$ experiments will be sensitive to $|d_\mu| \sim 10^{-21}\, e\,\mathrm{cm}$
- Proposal for a dedicated muon EDM experiment at PSI, could reach $|d_\mu| \sim 5 \times 10^{-23}\, e\,\mathrm{cm}$

Conclusions

- QED and EW contribution well under control
- **Hadronic vacuum polarization**
  - Presently largest systematic uncertainty in $\pi\pi$ channel
  - Comparison with lattice QCD just beginning
  - New data upcoming: SND, CMD-3, BaBar, Belle II, BESIII
- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible (π^0 and leading $\pi\pi$ effects done)
  - Evaluation of subleading terms and improved lattice-QCD calculations in progress
- Current theory matches expected experimental precision after first E989 release white paper
- **Electron $g-2$**: hadronic effects under control for (at least) another order of magnitude





Indirect limit on muon EDM

- Minimal flavor violation

$$|d_\mu^{\text{MFV}}| = \frac{m_\mu}{m_e}|d_e| < 2.3 \times 10^{-27}\, e\,\text{cm}$$

- **Direct limit** E821

$$|d_\mu| < 1.5 \times 10^{-19}\, e\,\text{cm} \qquad 90\%\ \text{C.L.}$$

- **Indirect limit** from electron EDM ACME 2018

$$|d_\mu| \leq \left[\left(\frac{15}{4}\zeta(3) - \frac{31}{12}\right)\frac{m_e}{m_\mu}\left(\frac{\alpha}{\pi}\right)^3\right]^{-1}|d_e|$$
$$\leq 0.9 \times 10^{-19}\, e\,\text{cm} \qquad 90\%\ \text{C.L.}$$