Extending subtraction schemes to N^3LO

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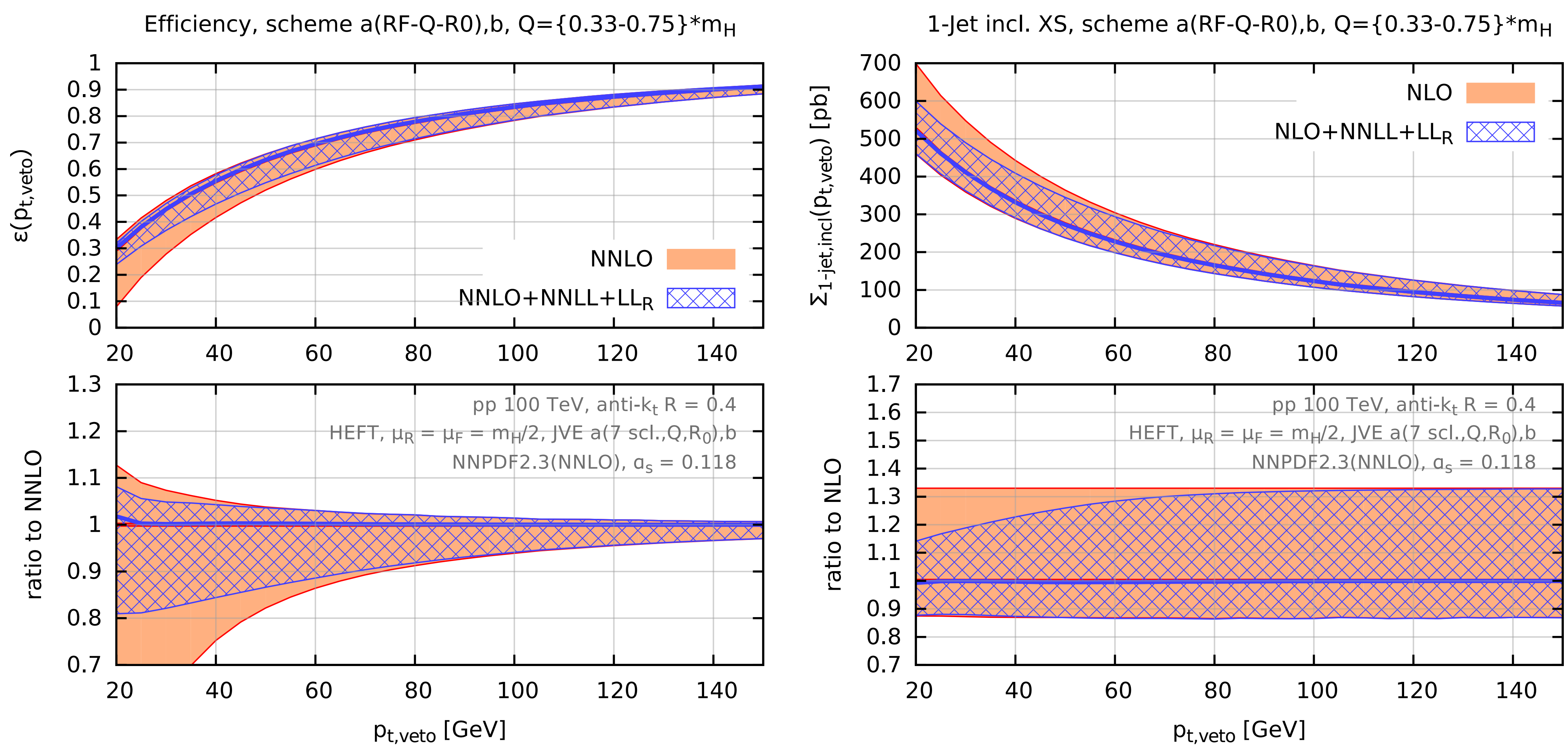
Why N^{3}LO?

LHC: fixed-order calculation essential for precision@colliders*

- a <u>clean</u>, <u>systematically improvable</u> [*see later*] framework for <u>first principle</u> calculations → solid, unambiguous predictions → new physics from small deviations

- applicable whenever we are far from soft/collinear regions → likely to play a crucial role for high energy colliders (*less issues with acceptances etc*)



Higgs jet veto efficiency at 100 TeV
[FCC report (2016)]

NB: this does not mean that f.o. is the <u>only</u> important ingredient

Why N^{3}LO?

1) Precision

$\alpha_s \sim 0.1, \quad \alpha_s C_A \sim 0.3 \quad \longrightarrow \quad$ few percent $\Leftrightarrow$ N$^{(3)}$LO

``Few percent"

- Experimentally achievable, at least for standard candles (V, H, top…)
- $\Lambda_{NP}/Q^2 \sim 1\%$ → sensitive to new physics at the (multi)-TeV scale, may not be accessible directly
- $\alpha \sim 0.01$ → study EW theory at the quantum level, without worrying about QCD contaminations

Why N^{3}LO?

1) Precision: some caveats

[Mistlberger, QCD@LHC2016]

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

$48.58\,\text{pb} =$	$16.00\,\text{pb}$	$(+32.9\%)$	(LO, rEFT)
	$+\,20.84\,\text{pb}$	$(+42.9\%)$	(NLO, rEFT)
	$-\ \ 2.05\,\text{pb}$	(-4.2%)	$((t, b, c)$, exact NLO)
	$+\ \ 9.56\,\text{pb}$	$(+19.7\%)$	(NNLO, rEFT)
	$+\ \ 0.34\,\text{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)
	$+\ \ 2.40\,\text{pb}$	$(+4.9\%)$	(EW, QCD-EW)
	$+\ \ 1.49\,\text{pb}$	$(+3.1\%)$	(N^{3}LO, rEFT)

ggH

At the few-percent level: many non-trivial subtle effects

Why N³LO?

1) Precision: some caveats

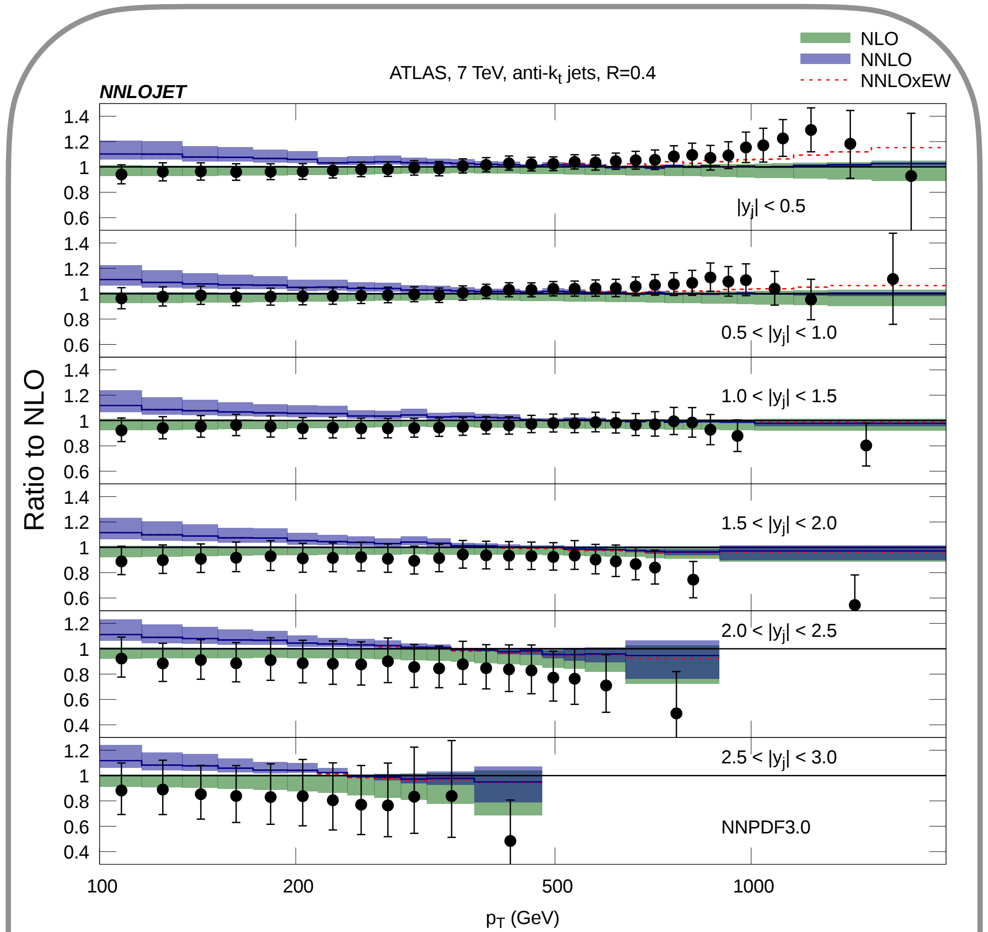
$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2 f(x_1) f(x_2) \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J \left[1 + \mathcal{O}\left(\frac{\Lambda^n_{\mathrm{QCD}}}{Q^n}\right)\right]$$

In many cases, few-percent could be a theoretical limit (given our current understanding of QCD)

- Q ~ EW scale, linear power corrections (e.g. jets) → percent
- At least a partial understanding would be as important as developing higher order calculations (already now, e.g. *top mass measurements*)

Why N³LO?

2) Multi-Parton Dynamics

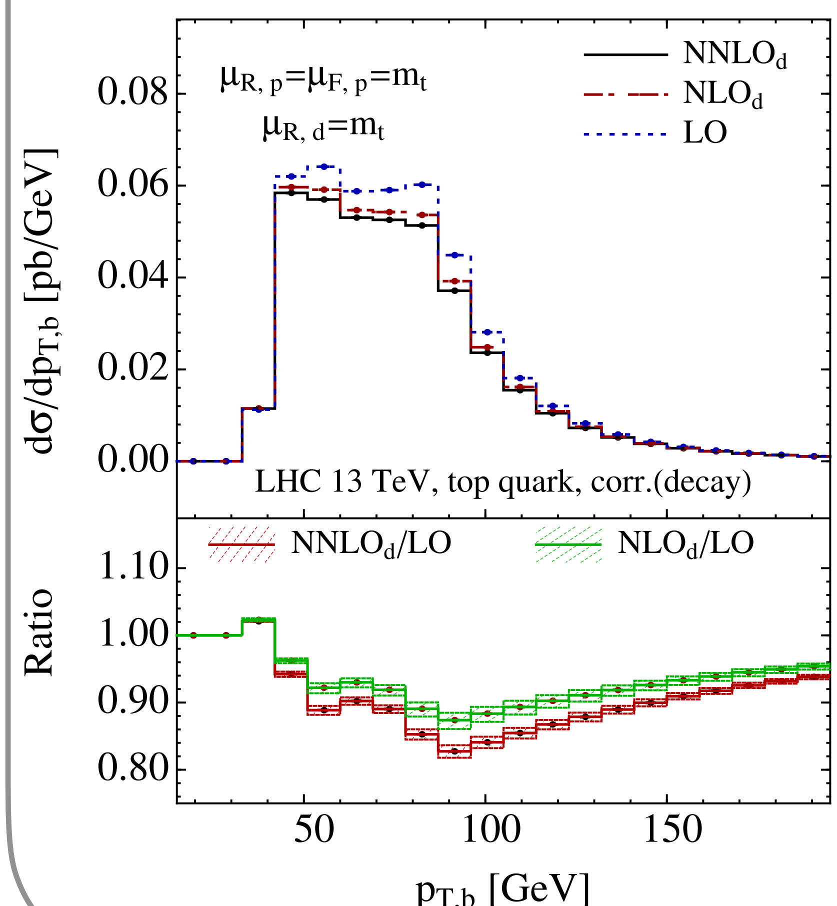


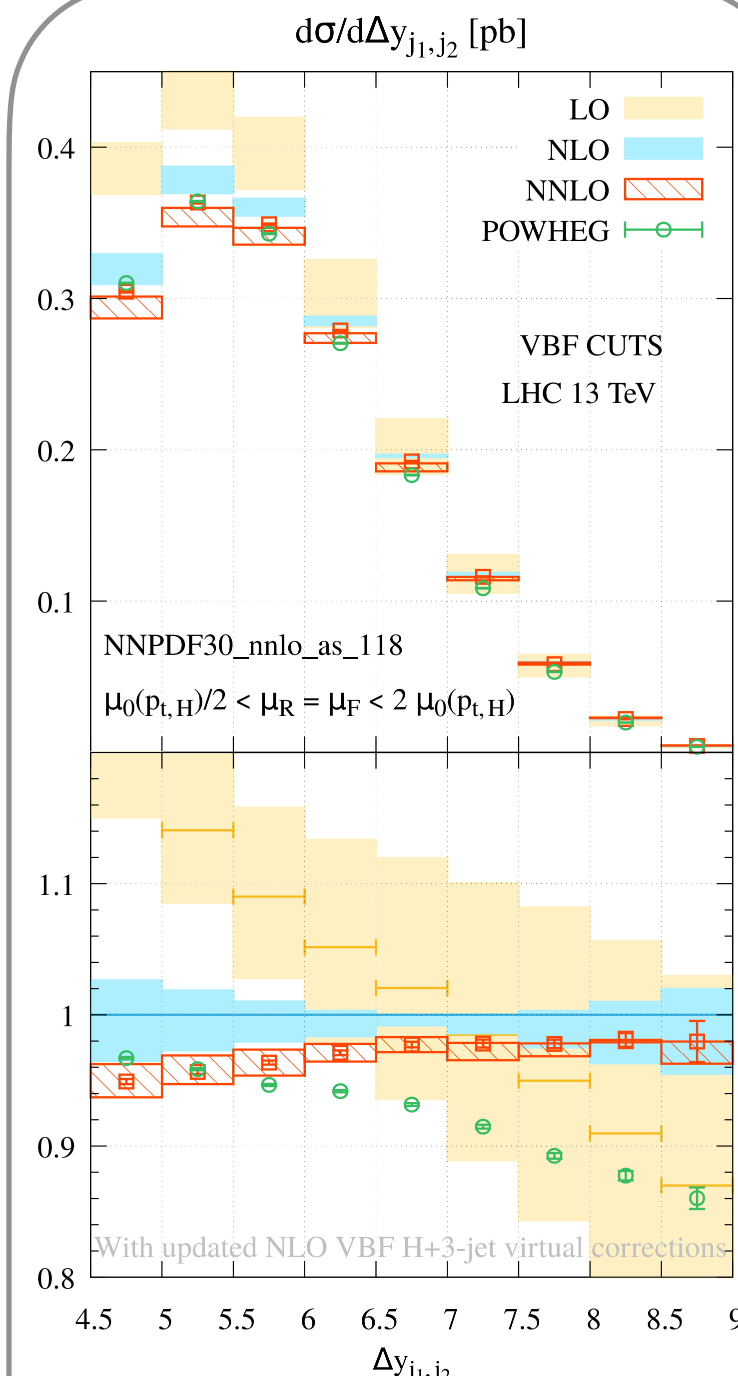
Dijet

[Currie et al., NNLOJET (2016-…); Czakon, van Hameren, Mitov, Poncelet (2020)]

Single-top

[Berger, Gao, Yuan, Zhu (2016-…)]



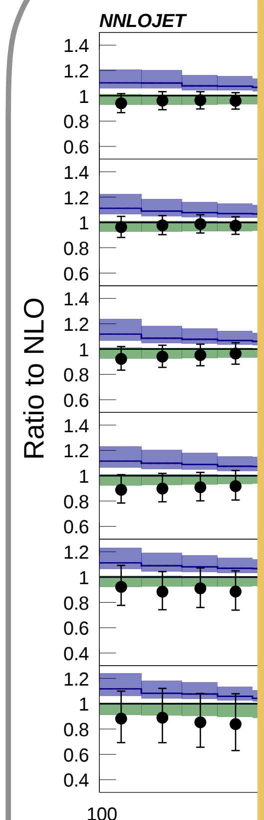


VBF

[Cacciari et al. (2015); Cruz-Martinez, Gehrmann, Glover, Huss (2-18)]

New handles on non-trivial multi-particle dynamics, complementary to standard ones

2)



[Currie
Czakon
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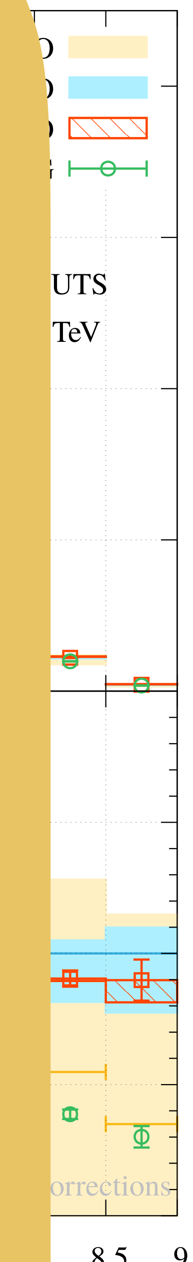
- Good phenomenological reasons to explore N^{3}LO, at least for standard candles

- If result not (purely) result-driven (→ *brute force*): interesting opportunity to explore soft/collinear structure of gauge theories



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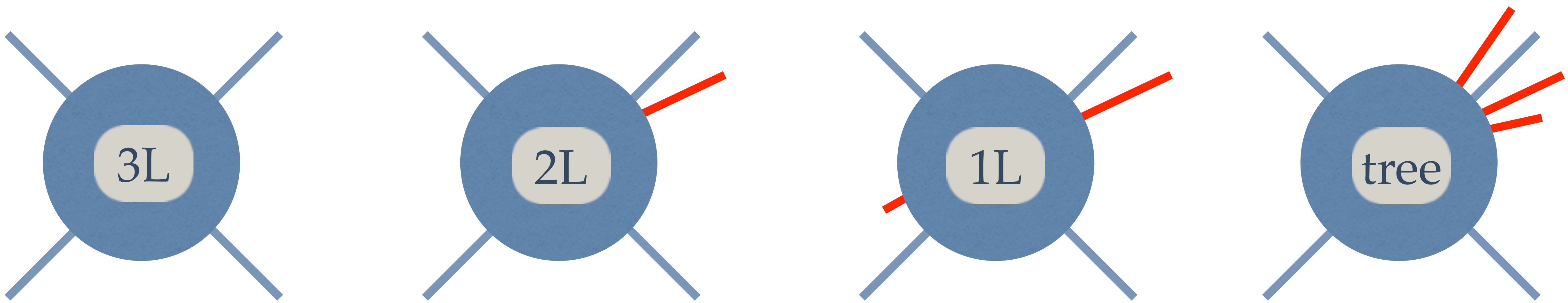
N^3LO: how to get there



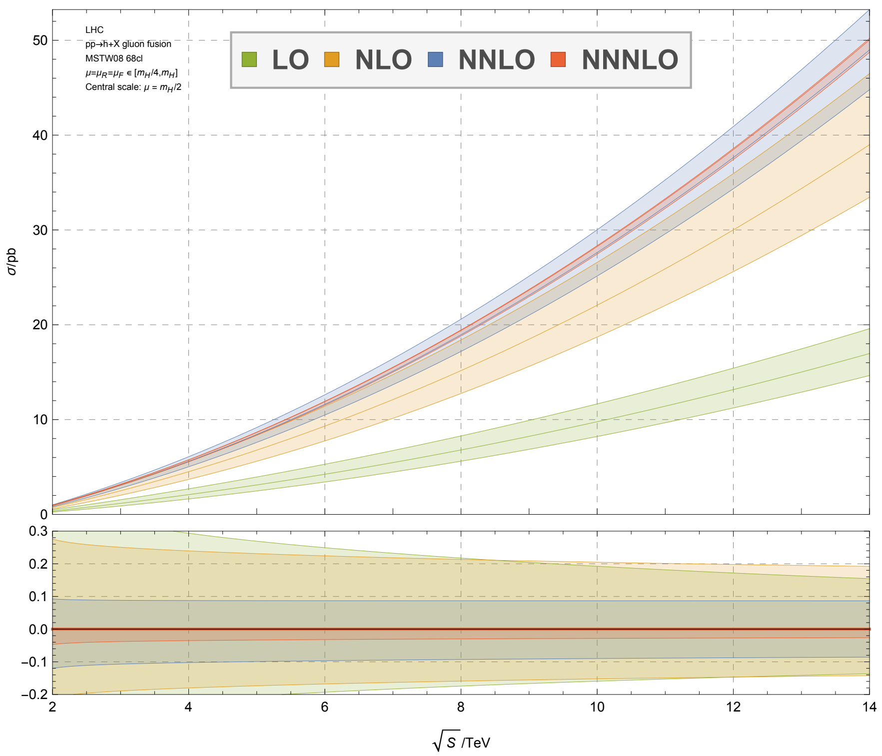
$\sim 1/\epsilon^6 \ldots$ $\quad \sim \int d\phi_1 1/\epsilon^4 \ldots$ $\quad \sim \int d\phi_{1,2} 1/\epsilon^2 \ldots$ $\quad \sim \int d\phi_{1,2,3}\, T$

Two main issues

- Multi-loop amplitudes (*especially in unresolved configurations*) → see Harald's talk
- How to properly extract (and regulate) IR from loop integrals. Highly non-trivial for fully exclusive calculations → focus here. An optimal solution at NNLO is not yet available (*my opinion*), we can get results thanks to large computing resources

N^3LO right now: fully inclusive

gg→H



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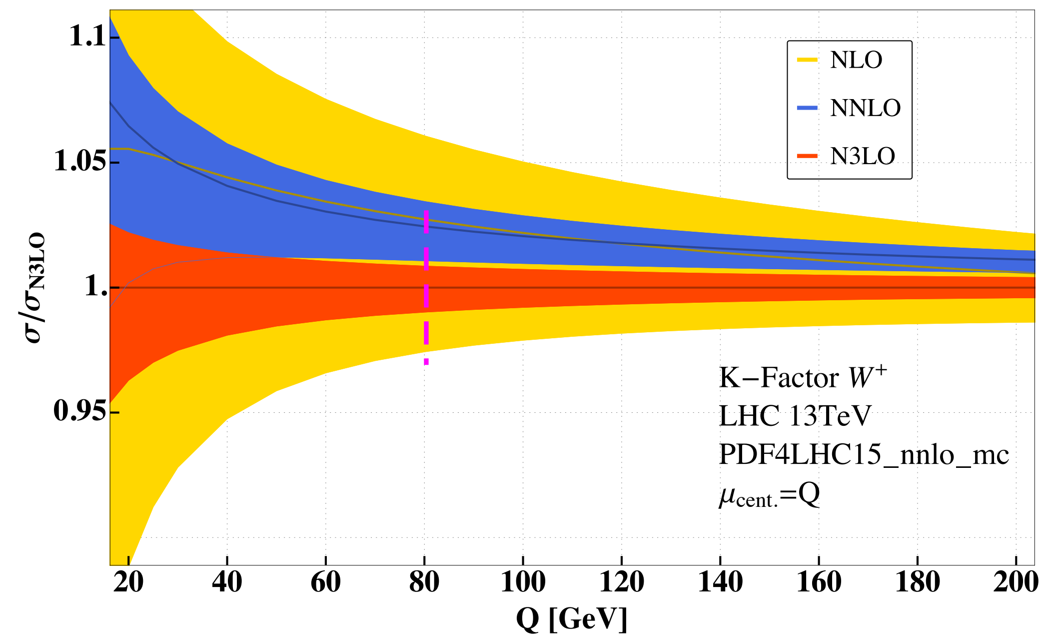
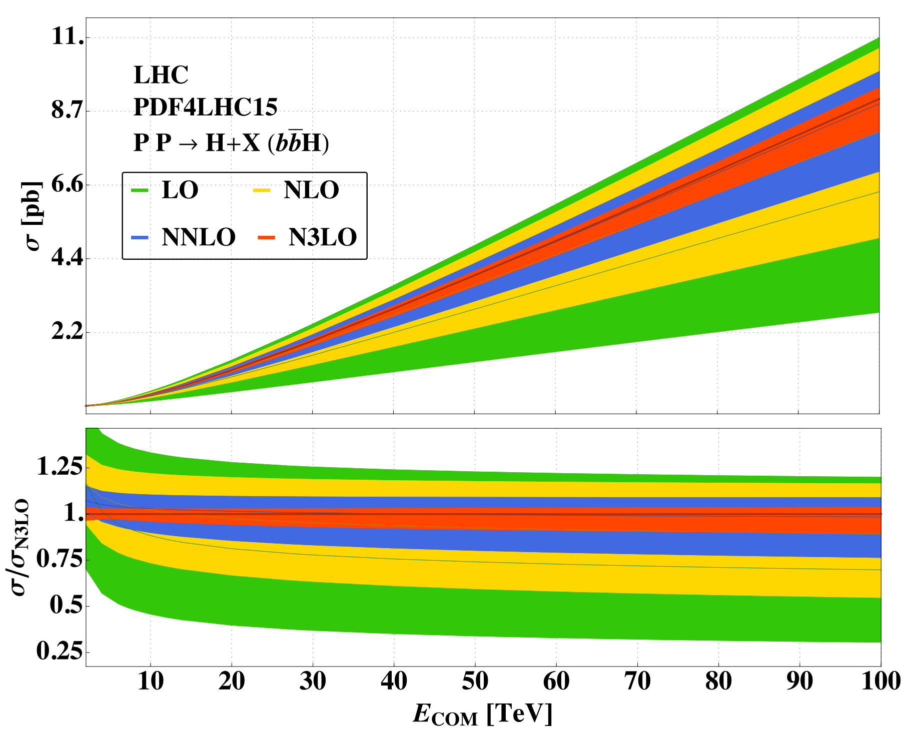


bb→H



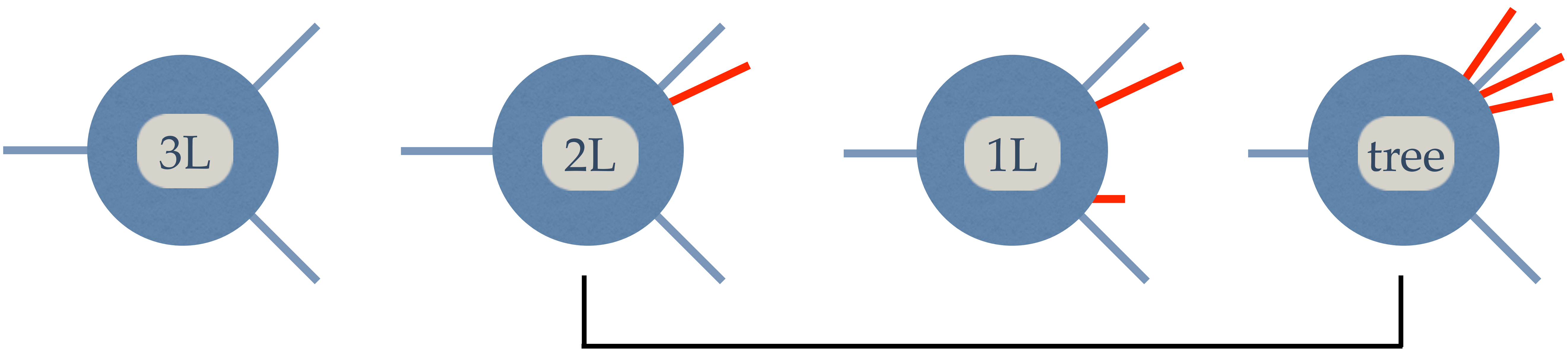
Fully inclusive N^3LO corrections for 2→1 color singlet production is possible

[Dulat, Duhr, Mistlberger + Anastasiou et al (2015-…)]

[VBF: Dreyer, Karlberg (2018)]

Colour singlet: differential results

Consider the decay of a colour-singlet state V



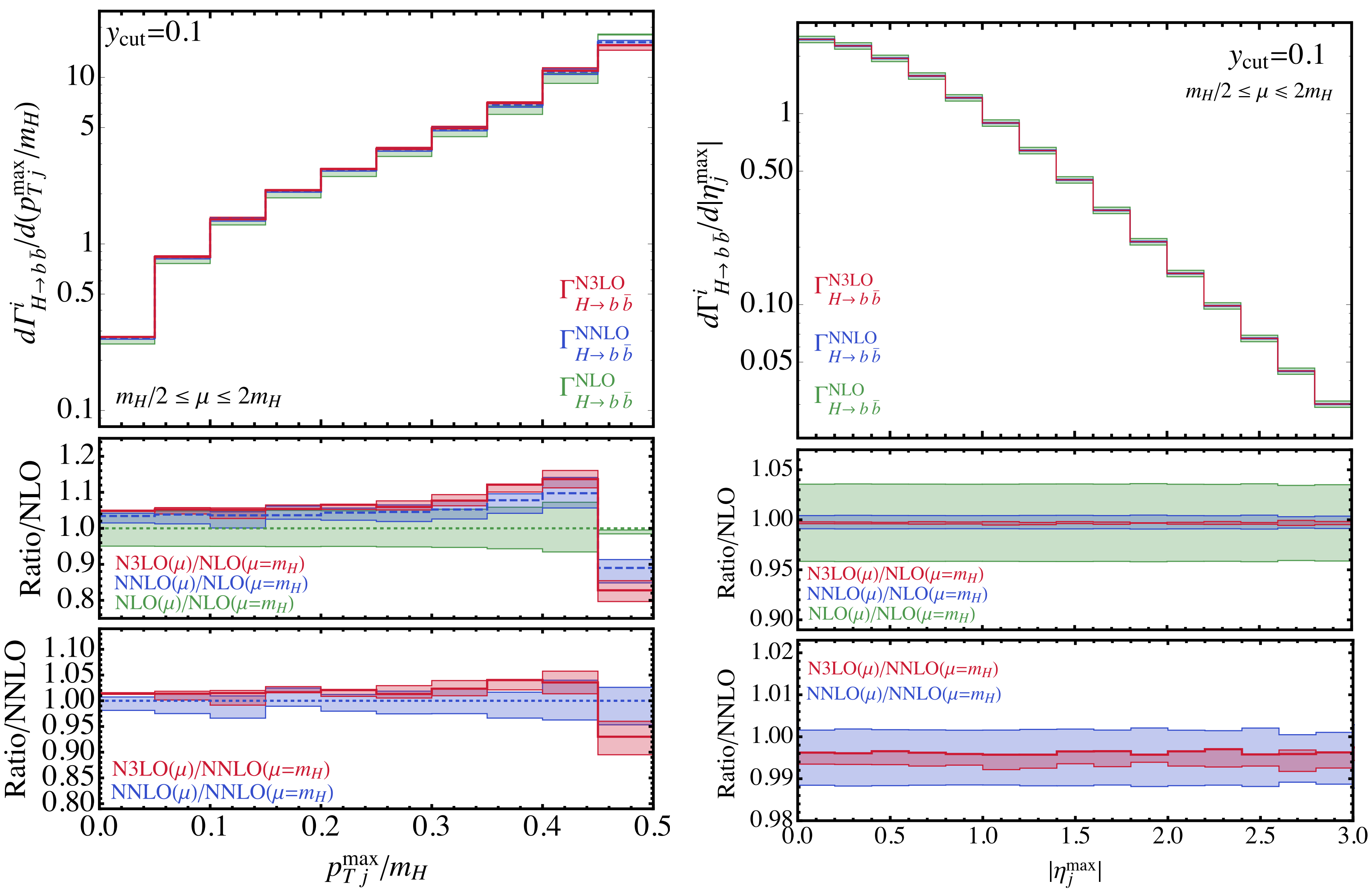
V → X+j @ NNLO

- Most of the information contained in V → X+j @ NNLO
- Born-like configuration: no kinematical dependence
- As a consequence: V → X+j @ NNLO & total rate@N^3LO enough for differential rate at N^3LO

Colour singlet: differential results

H → b b̄ @ N³LO known

[Mondini, Schiavi, Williams (2019)]



Colour singlet: differential results

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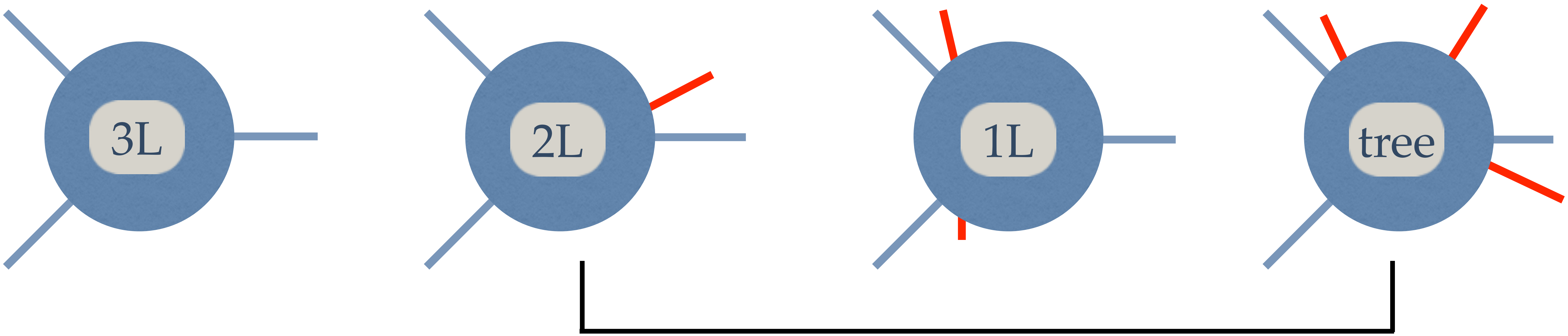

pp → V+j @ NNLO

- Most of the information contained in V → X+j @ NNLO
- Born-like configuration: ~~no kinematical~~ non-trivial rapidity dependence
- As a consequence: pp→ V+j @ NNLO & (*0-p_t*) rapidity dependence @N^{3}LO enough for differential rate at N^{3}LO

Cons

3

- In principle: we can compute fully differential N^3LO predictions for colour singlet production with current technology

- In practice: need to ``loose a jet” in pp→V+j@NNLO, computationally intensive

The bad part of the story: this trick is not (immediately) generalisable to arbitrary processes

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- B lence
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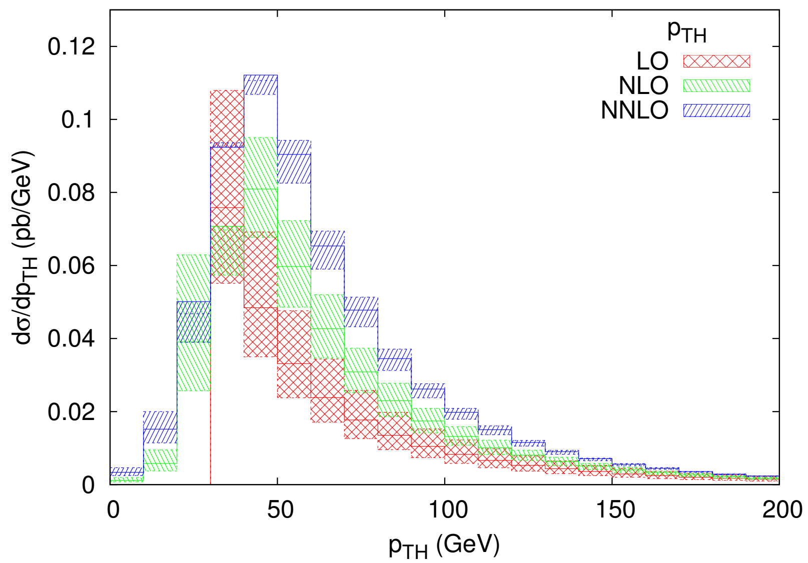
@N

Colour singlet: differential results

H+J@NNLO

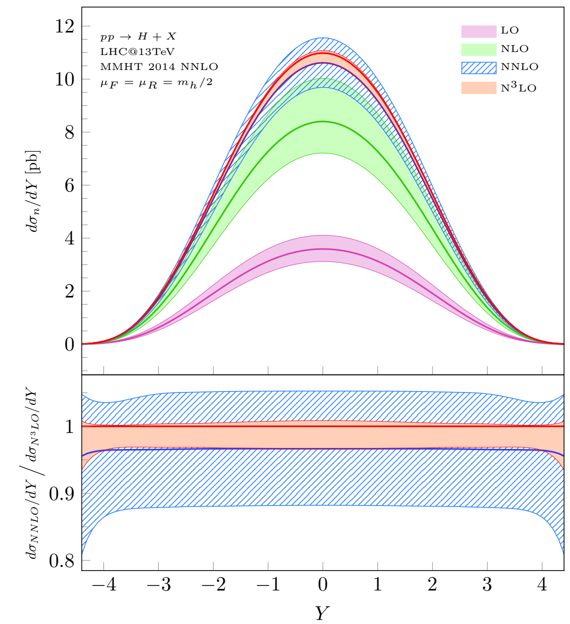
[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier (2016)]

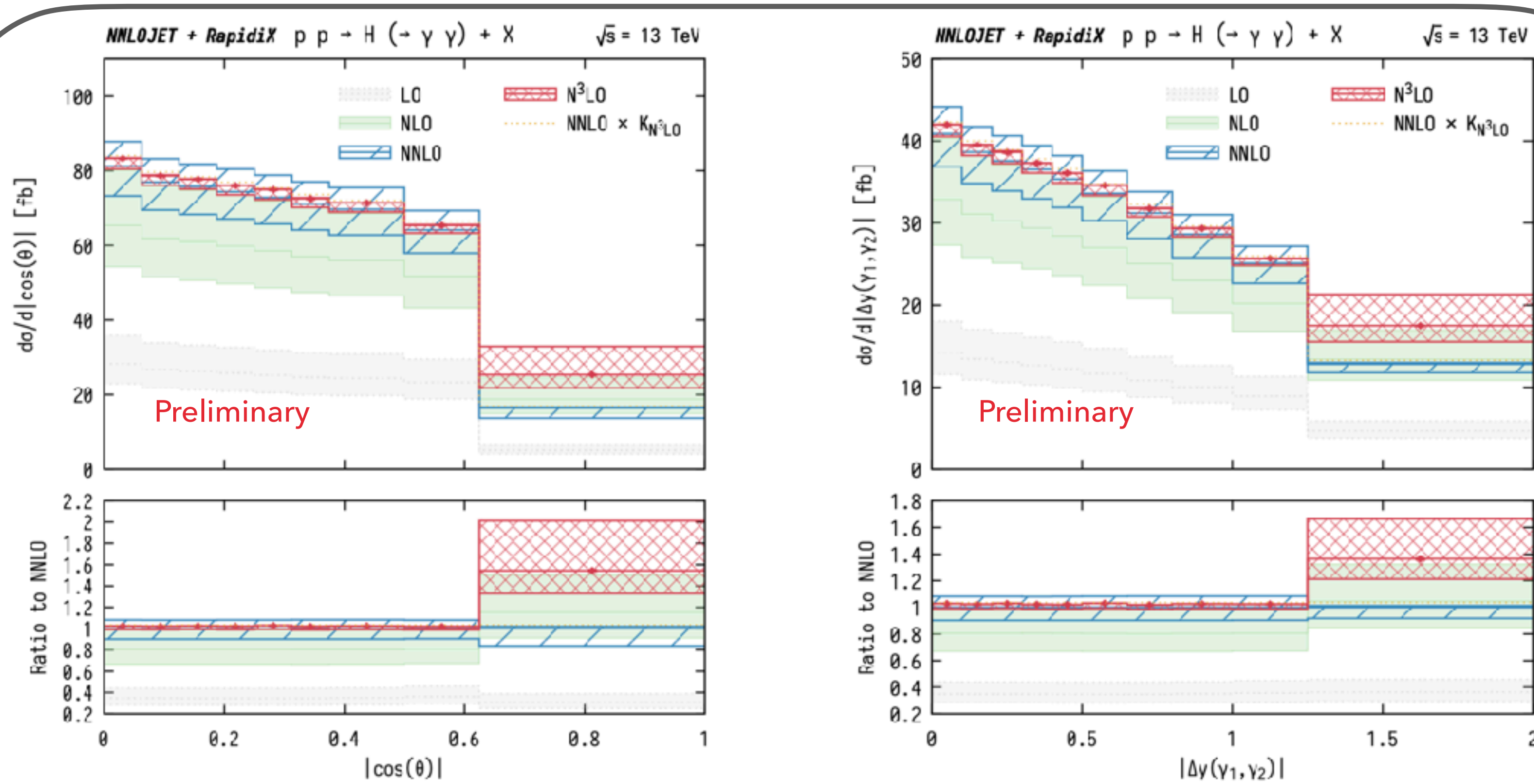


+

dσ/dy @N³LO

[Dulat, Mistlberger, Pelloni (2018)]





[Chen,Dulat,Gehrmann, Glover,Huss,**BM**,Pelloni, to appear]

[Mistlberger, talk at QCD@LHC-X]

(See also Cieri et al. (2018) for earlier work assuming trivial rapidity dependence)

Colour singlet: beyond the Higgs

- For colour-singlet production: only require rapidity-dependence at zero transverse momentum
- Factorisation → universality. ``Beam functions”
- Over the past year or so: all ingredients computed
  - p_t beam function [Luo, Zang, Zhu, Zhu (2020); Ebert, Mistlberger, Vita (2020)]
  - N-jettiness beam function*: [Behring, Melnikov, Rietkerk, Tancredi, Wever (2019); Ebert, Mistlberger, Vita (2020)]
- Provided that we have relevant loop amplitudes and enough computing power this would allow for a brute-force calculation of N^3LO QCD corrections for any colour-singlet state

**Not enough even for colour-singlet, but has the potential to be extended to arbitrary processes*

Beyond colour-singlet

- Beyond colour-singlet, new frameworks are required → better understanding of soft/collinear emission (and beyond)
- *Slicing techniques*
  - similar to what discussed so far. Introduce resolution variable that separates Born-like vs +jet
  - -: typically, this introduce some non-locality in IR cancellations → large numerical cancellation, huge computing power required
  - +: giving up locality makes everything much simpler → may be a shortcut to the result, if enough computing power
  - +: several ingredients already known / under computation *N-jettiness beam function, towards N-jettiness soft function [Baranowski (2020)]*

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  - +: several ingredients already known / under computation *N-jettiness beam function, towards N-jettiness soft function [Baranowski (2020)]*
  - The way forward (*my opinion*): sub-leading power corrections. Would allow for much better numerics / very interesting from a conceptual point of view [Moult et al. (2017-…); Boughezal, Isgro, Petriello (2018); Beneke et al (2018-…); Laenen et al. (2018-…); Neubert et al (2020-…)]

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- *Subtraction techniques*
  - allow for local cancellation of IR singularities via universal subtraction terms → interesting *per se*
  - +: theoretically appealing, much higher efficiency (slicing eventually abandoned @ NLO)
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  - +: theoretically appealing, much higher efficiency (slicing eventually abandoned @ NLO)
  - -: the price of locality: much more challenging to devise a generic framework
  - +: in the recent past, several of the universal ingredients computed [Catani, Colferai, Torrini (2019); Del Duca, Duhr, Haindl, Lazopoulos, Michel (2019-20); Bern, Dixon, Kosower (2004); Badger, Glover (2004); Duhr, Gehrmann, Jaquier (2014); Duhr, Gehrmann (2014); Li, Zhu (2013); Dixon, Hermann, Kai, Zhu (2019); Catani, de Florian, Rodrigo (2019); Badger, Buciuni, Peraro (2015); Zhu (2020)]

(Personal opinion: regulating soft/collinear emission would proceed faster than for NNLO. Big conceptual gap was between NLO and NNLO)

N^3LO: the ``real” challenges?

- Thanks to the recent progress in NNLO: good position to start thinking at N^3LO generalisations

- However, there may be surprises…

  - Top pair production: $\sigma^{N3LO} \neq$ ``R + V”.

    - Non-trivial threshold effects, starting at $O(\alpha_s^3)$ [Beneke, Ruiz-Femenia (2016), see also Melnikov, Vainshtein, Voloshin (2014)]

    - Not captured by ``standard” perturbation theory

  - Factorization breaking effects for jet production

    - Non-trivial absorbitive part of loop integrals starts playing a role at N^3LO. Could spoil universality of collinear singularity in processes with non-trivial color structure (e.g. di-jet…) [Forshaw, Siemour et al (2006-…); Catani, de Florian, Rodrigo (2011)]

    - Analysis confirmed in [Dixon, Hermann, Kai, Zhu (2019)]

This sort of issues are likely to be the real challenges for N^3LO computations

Conclusion

- N^3LO likely to be relevant for current and future precision programs at the energy frontier
- There is something we can do already now, but not immediately generalisable (either conceptually or practically)
- For ``standard" soft/collinear regularisation, the path is clear → (massively) more complicated than NNLO, but at least in principle we know where to start
- In this talk, I focused on ``standard" techniques. Recently: old ideas (LT duality…) powered by new insight and better computing → this may turn out to be a good way forward
- There may however be unexpected effects, and this may require a better understanding of the structure of gauge theories

Interesting times ahead!

Thank you very much for your attention!