Automated five-point QCD amplitudes at two-loops

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NLO Automation

Reasons for rapid progress mostly during '08-'11:

- Real-radiation handling available: dipole subtraction; antenna subtraction, residue subtraction

[DS: Catani, Seymore; AS: Kosower, Campbell, Cullen, Glover, Daleo, Maitre, Gehrmann; RS: Frixione, Kunszt, Signer]

- Feynman integrals available

[Collier: Denner, Dittmaier, Hofer; QCDLoop: Ellis; Looptools: vOldenborgh; AVH-OLO: vHameren]

- Advances in 1-loop reduction: OPP/Unitarity/tensor reduction based on on-shell trees/Schwinger-Dyson/ Feynman diagrams

[OPP: Ossola, Papadopoulos, Pittau; Unitarity: Bern, Dixon, Dunbar Kosower; Cachazo, Britto, Feng; Forde; Ellis, Giele, Kunszt, Menikov]

Key trigger: influx of formal ideas such as on-shell recursions, generalised unitarity, efficient trees

[OSR: Britto, Cachazo, Feng, Witten; Risager; GU: Bern, Dixon, Dunbar Kosower; Cachazo, Britto, Feng; Volovich, Roiban, Spradlin; Brandhuber, Travalini; ET: Arkani-Hamed, Boujaily, Trnka, Caron-Huot]

Automated one-loop tools

One-loop matrix elements:

Blackhat [Berger, Bern, Dixon, Febres Cordero, Forde, Kosower, Ita, Maitre]

GoSam [Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

HELAC-NLO [Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek]

MadLoop [Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau]

NJet [Badger, Biedermann, Uwer, Yundin]

OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller, Cascioli]

Rocket [Ellis, Melnikov, Zanderighi]

Recola [Actis, Denner, Hofer, Scharf, Uccirati]

many private & specialised tools

Integrals libraries:

COLLIER [Denner, Dittmaier, Hofer]

QCDLoop [Ellis]

AVH-OLO [vHameren]

Real radiation/parton shower, phase-space integration, analysis packages:

Herwig [Bellm, Bewick, Ferrario Ravasio, Gieseke, Grellscheid, Kirchgaeßer, Masouminia, Nail, Papaefstathiou, Plätzer, Rauch, Reuschle, Richardson, Seymour, Siódmok, Webster + former developers]

aMC@NLO [Frederix, Frixione, Maltoni, Stelzer, Torielli]

MCFM [Campbell, Ellis, Wiliams]

POWHEG [Alioli, Nason, Oleari, Re]

Sherpa [Gleisberg, Hoeche, Hoeth, Krauss, Schoenherr, Schumann, Siegert, Zapp]

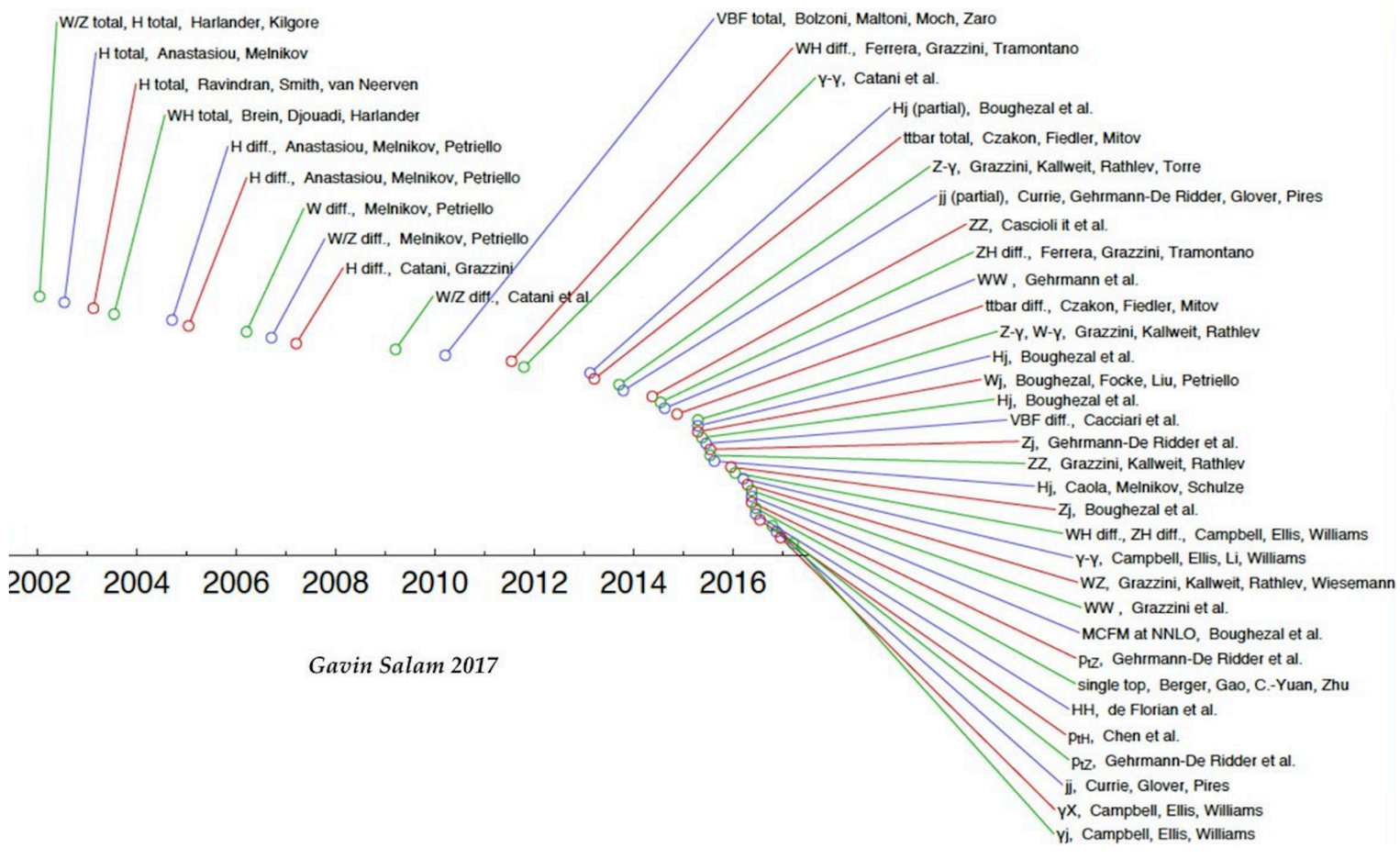
Status: NNLO QCD @ LHC

Many available 2→2 processes: jj, H+j, V+j, tt, VH, HH, VHj

First 2→3 process: $\gamma\gamma\gamma$ [Herschel, Czakon, Mitov, Poncelet '20]

Next frontier: jjj, Vjj, Hjj, γjj, ttH, ttZ etc.

A main challenge are multi-scale two-loop amplitudes.



Multi-scale two-loop amplitudes

Feynman diagrams

↓

Tensor reduction
[Tarasov 96; Anastasiou, Glover, Oleari 99]

IBPs
[Tkachov, Chetyrkin 81]

↓

Sum of master integrals

$$A = \sum_{\Gamma \in \Delta} \sum_{i \in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}.$$

↓

Differential equations
[Gehrmann, Remiddi 01]

↓

Integrated form

B. Page

Large number of Feynman integrals, complex analytic processing required.

Important ideas promise automation:

- Unitarity: trees instead of loops diagrams
- Numerical reconstruction [v Mannteuffel, Schabinger '14, Peraro '16, Abreu et al.'17]
- Integration-by-parts: FIRE, KIRA, Reduze, etc.
- Differential equations for pure Feynman integrals [Henn '13]; elliptic integrals [ongoing: Remiddi, Tancredi, Adams, Bogner, Weinzierl, Duhr, Broedel, Dulat, Penante, etc.]
- Integral function calculus, symbols [Goncharov '01; Goncharov, Spradlin, Vergu, Volovich; Brown; Del Duca, Duhr, Smirnov '10]

Two-loop planar five-parton amplitudes

Analytic form of amplitude in Euclidean phase space.

Full reduction to approx 400 five-point integral functions h_i [Gehrmann, Henn, Lo Pesti `15], four scales $\vec{s}$

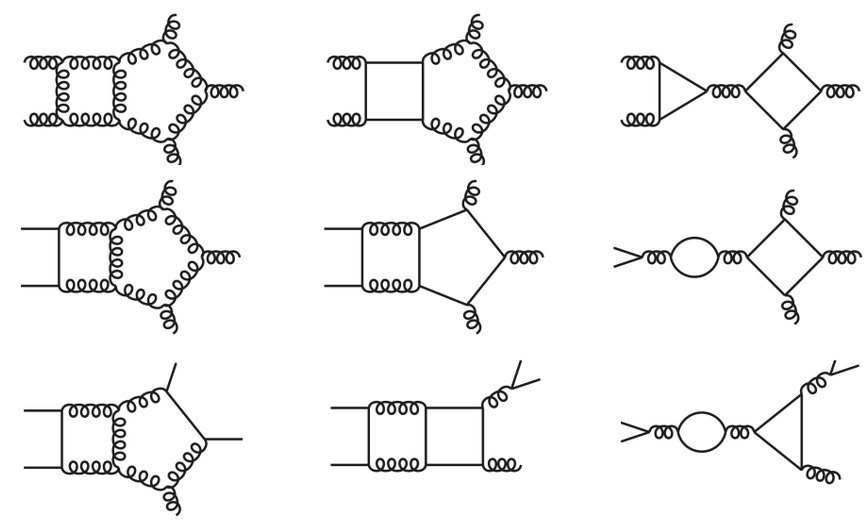
$$\mathscr{R}^{(2)}(\vec{s}) = \sum_{i \in B} r_i(\vec{s}) h_i(\vec{s})$$

Relatively compact results (compressed 1.6 Mb).

Reproduce universal IR and available literature:
[Gehrmann, Henn, Lo Presti `15, Badger, Frellesvig, Zhang `15; Dunbar, Perkins, Jehu `16; Badger, Brønnum-Hansen, Hartanto, Peraro `17; Abreu, Febres Cordero, Ita, Page, Zeng '17].

Next: integration with recent function library [Chicherin, Sotnikov '20] valid in physical phase space.

[Abreu, Febres Cordero, Dormans, Ita, Page, Sotnikov '19]



Computational paradigm

Physical properties to setup algorithm:

- Unitarity, geometry, rationality
- Structure of dimensional regularisation

Exact numerical methods:

- Use integers/rationals/finite fields instead of floating-point numbers: avoids numerical instabilities
- Suitable for implementation on computer clusters

Analytic reconstruction:

- Analytic expressions from sufficiently many exact evaluations

Tunnel complexity with (exact) numerics:

Numerical amplitude methods

- Parametrisation for amplitude integrand in terms of surface terms (= total derivatives) and master integrals.

$$\mathcal{A}^{(2)}(\ell_l) = \sum_{\Gamma\in\Delta} \sum_{i\in M_\Gamma\cup S_\Gamma} c_{\Gamma,i} \frac{m_{\Gamma,i}(\ell_l)}{\prod_{j\in P_\Gamma} \rho_j}$$

- Generalised unitarity & numerical evaluation in finite fields to determine coefficients.

$$\sum_{\text{states}} \prod_{i\in T_\Gamma} \mathcal{A}_i^{(0)}(\ell_l^\Gamma) = \sum_{\substack{\Gamma'\geq\Gamma, \\ i\in M_{\Gamma'}\cup S_{\Gamma'}}} \frac{c_{\Gamma',i}\, m_{\Gamma',i}(\ell_l^\Gamma)}{\prod_{j\in(P_{\Gamma'}\setminus P_\Gamma)} \rho_j(\ell_l^\Gamma)}$$

many recent contributions [Abreu, Febres Cordero, Dormans, Ita, Page, Sotnikov, Ruf, Klinkert, Zeng; Badger, Hartanto, Bronnum-Hansen, Peraro; Larsen, Zhang; Mastrolia, Mirabela, Ossola]

Geometric methods [Gluza, Kajda, Kosower '10; Schabinger ; Larsen, Zhang '15; Ita '15; Abreu et al '17] work well to generate Ansatz in terms of IBP relations.

Numerical amplitude methods

- Numerical formulation of dimensional regularisation (treatment of fermions)
- Reconstruction of dimensional regulator D=4-2ε
- Replace integrals by function basis (ε-expansion)
- Extract finite remainder
- Analytic reconstruction in Mandelstam variables (in finite field)
- Simplify coefficients unique partial-fraction decomposition (Leinartes-like algorithm) and reconstruct rationals from finite field

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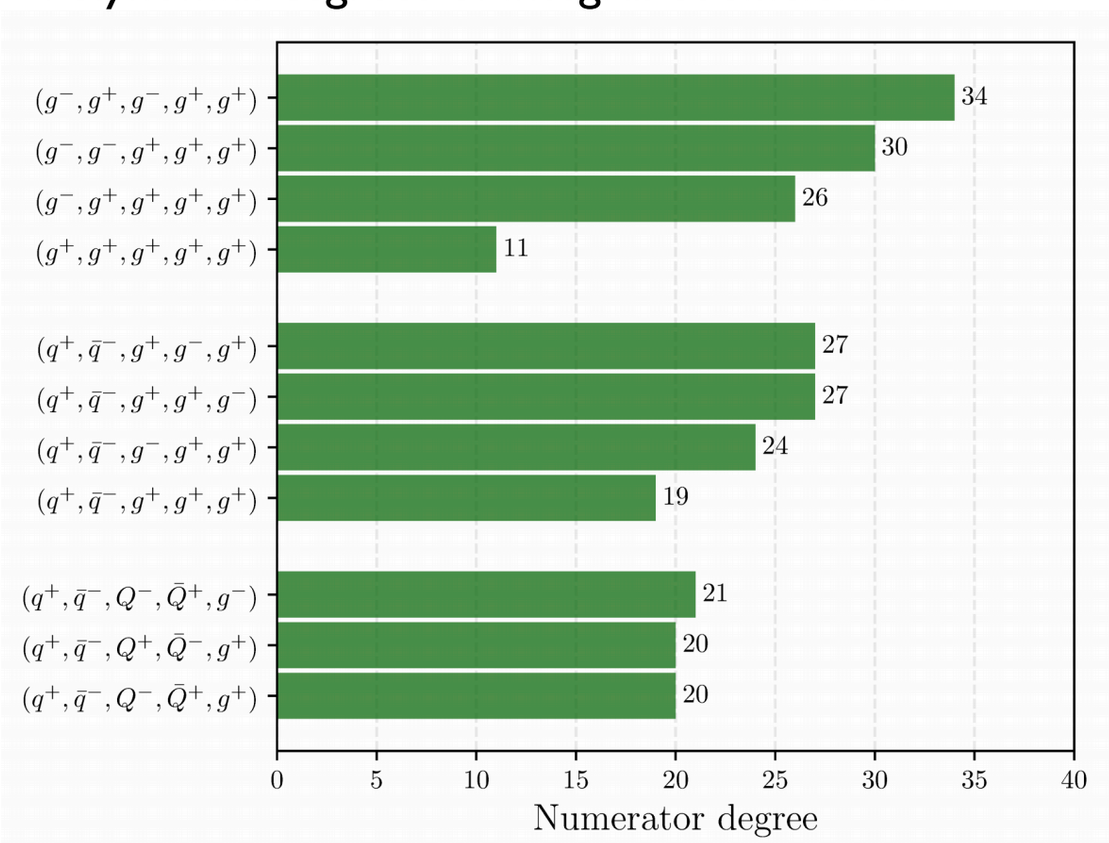
Analytic reconstruction

Polynomial degrees of integral coefficients:



Four variables, $\binom{degree}{\#\,vars}$ parameters that need to be determined in coefficients.

Approx 100k numerical evaluations to obtain full amplitude from numerics in analytic form.

Simpler than Monte-Carlo run for cross-section computation.

Improvements expected from bootstrap input [see work of De-Laurentis, Maitre '20]

Integrals

Differential equation method [Kotikov '91; Bern, Dixon, Kosower '93; Remiddi '97; Gehrmann, Remiddi '99].

$$d\vec{I}(\vec{s},\epsilon) = M(\vec{s},\epsilon)\cdot\vec{I}(\vec{s},\epsilon)$$

Challenging to obtain in analytic computation due to inversion of integration-by-parts relations.

New: numerical construction of analytic differential equation [Abreu, Page, Zeng '18]:

1) Pure basis for integrals $\vec{I}(\vec{s},\epsilon)$ [Arkani-Hamed, Bourjaily, Cachazo, Trnka '12; Henn '14]

2) Matrix $M(\vec{s},\epsilon) = \epsilon\hat{M}(\vec{s})$ factorises in regulator ϵ [Henn '12]

3) Reconstruction of analytics from $\mathcal{O}(100)$ numerical evaluations of integration-by-parts relations.

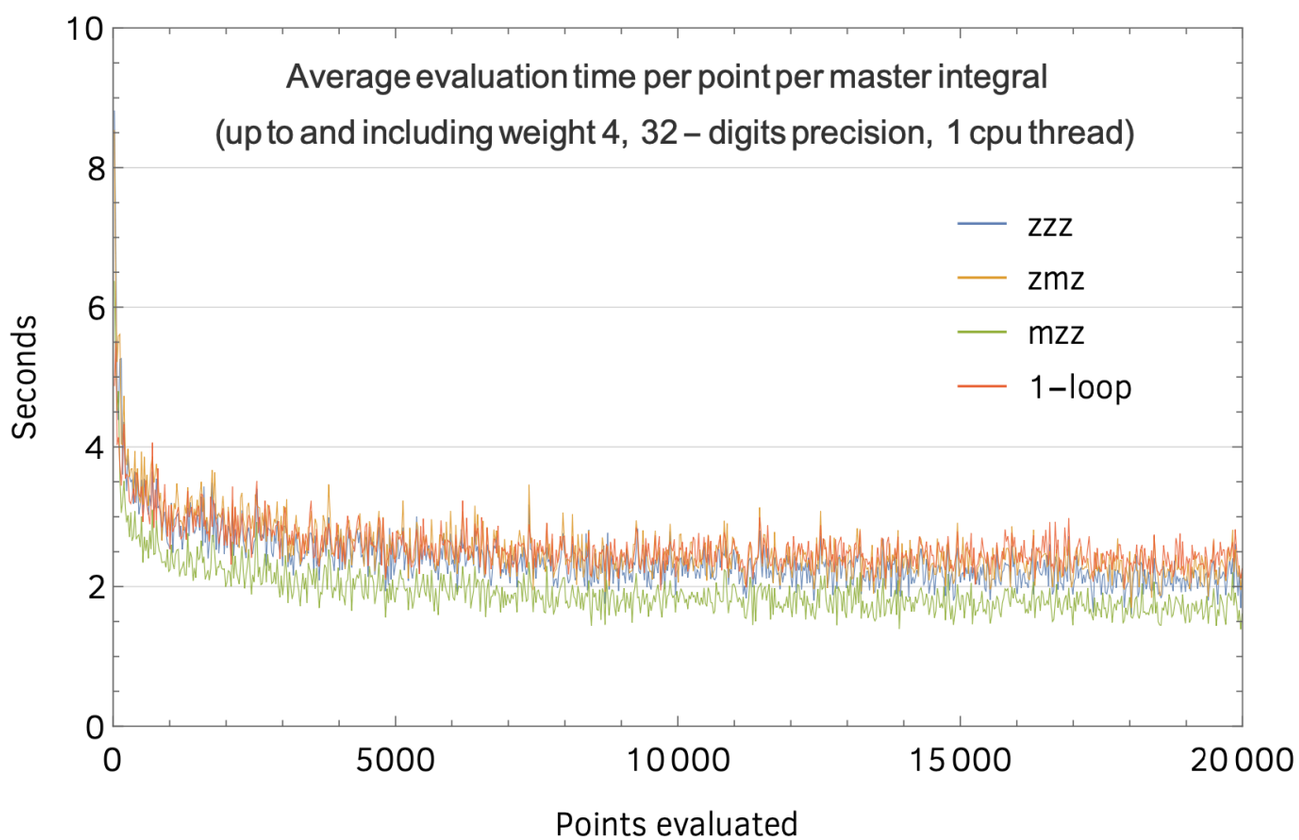
Integration

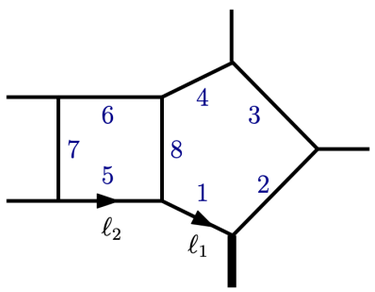
Numerical series-expansion method [Moriello '19]:

1) Solving differential equation in one variable at numerical point trivial

2) Glue local solutions to obtain global one.

3) Works for physical regions





Timing W+2-jet two-loop planar integrals [Abreu, Ita, Moriello, Page, Tschernow, Zeng '20]; recently analytics [Canko, Papadopoulos, Syrrakos '20]

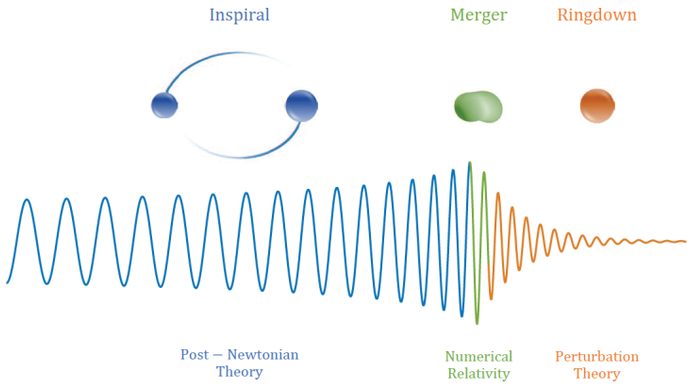
Efficient method.

Works also for elliptic integrals [Bonciani Del Duca, Frellesvig, Henn, Hidding, Maestri, Moriello, Salvatori, Smirnov '19].

Black-hole dynamics

Understand gravitational potential of two black holes for LIGO [see session 141 about gravitational waves, speakers: Goldberger, Buonanno]

Wanted: Newton potential and its corrections [problem going back to Einstein, Infeld, Hoffmann '38]



[Antelis, Moreno '16]

★ Non-linear field equations; recursive solutions in gravitational field exchanges.

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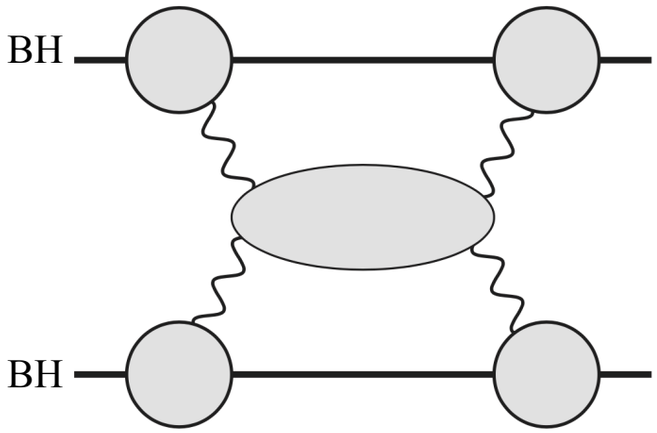
★ Classical limit ($\hbar \to 0$) of scattering amplitudes. See black-hole potential at third post Minkowskian Order [Bern, Cheung, Roiban, Shen, Solon '19]

Technology transfer: amplitude methods, minor post processing for observables: Hamiltonian, scattering angel instead of cross sections.

Great for students to get into QCD and gravity!

Graviton-wave exchange:



[Bern, Cheung, Roiban, Shen, Solon '19]

Two-loop four-graviton amplitudes

Computed two-loop four-graviton amplitudes with numerical unitarity [Abreu, Febres Cordero, Ita Jaquier, Page, **Ruf,** Sotnikov @ Caravel '20].

History: displays UV divergences of Einstein gravity [Goroff, Sagnotti '85; Van de Ven '92]

Settled dispute about applicability of amplitude computations for obtaining higher-order classical gravity predictions [Bern, Ita, Parra-Martinez, Ruf '20].

Technology transfer: same algorithms including IBP, power counting, unitarity numerical reconstruction

Integral technology & differential equations [Parra-Martinez, Ruf, Zeng '20]

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s-channel Regge limit ($s \gg -t > 0$):

$$\mathcal{R}^{(2)}_{\{-,-,+,+\}} = s^3 \left\{ 2\frac{s}{t}\pi^2 \left(\frac{i\pi}{2} - L\right)^2 - 3\pi^2 L^2 + \frac{107}{10}\pi^2 L + \frac{14191}{1350}\pi^2 - \frac{158}{45}\pi^4 - \frac{13049}{2160} + i\pi \left[-\frac{14}{3}L^3 + \frac{87}{10}L^2 - \left(8\pi^2 - \frac{17749}{450}\right) L - 20\zeta_3 + \frac{2621}{210}\pi^2 - \frac{11221}{375} \right] + \mathcal{O}(-t/s) \right\}$$

$$L = \log(-s/t)$$

Conclusion

In recent decade: automation of NLO predictions due to advances with one-loop amplitudes.

Similarly, complete numerical methodology is appearing for two-loop multi-scale amplitudes.

Examples:

1) Series expansion in differential equation approach to integrals and

2) Numerical unitarity: analytic two-loop amplitudes numerical loop computations.

Precision-physics potential for colliders and gravitational-wave studies.