Theory errors and operator bases

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CPM-Snowmass Oct 6th 2020
In SMEFT framework

\[ |A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \frac{1}{\Lambda^4} \left( |A_6|^2 + 2Re(A_{SM}^* A_8) \right) + \cdots \]

interference piece, usually largest effect.
State of the art SMEFT

SMEFT Warsaw basis: \( \mathcal{O}(60) \) operators at dim-6
\( \mathcal{O}(1000) \) operators at dim-8

What’s the impact from \( 1/\Lambda^4 \) corrections?
Higher order effects so should be small... but

- they are a form of uncertainty; ‘theory error’ on extracted scale $\Lambda$

- there are instances where interference term isn’t present or is suppressed, e.g. helicity mismatch between SM and dim-6

- faster growth with energy, $E^4$ vs. $E^2$ : increasingly important when looking at high energy (e.g. tails of some kinematic distribution)

But full treatment to $1/\Lambda^4$, with all $\mathcal{O}(1100)$ operators doesn’t seem feasible
Some thoughts on how to proceed

\[ |A|^2 = |A_{SM}|^2 + \frac{2Re(A^*_SMA_6)}{\Lambda^2} + \frac{1}{\Lambda^4} \left( |A_6|^2 + 2Re(A^*_SMA_8) \right) + \cdots \]

1.) Use (dim-6)^2 piece as a proxy for higher order effects. Add it as a theory uncertainty when performing SMEFT analysis. Fully set by dim-6 operators, machinery already in place

[Shepherd et al. 1711.07484, 1812.0757, extend work in LOI with Shepherd, Lewis, Kim, Gu]

2.) Reorganize SMEFT to minimize the number of higher dimensional operators needed for as many processes as possible

[Helset, AM, Trott, 2001.01453, Hays, Helset, AM, Trott 2007.00565, part of dim 8+ LOI]
What do higher dimensional operators do?

- Change field strength normalization/inputs
- Modify existing vertices
- New multi-particle interactions

universal specific
What do higher dimensional operators do?

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- Modify existing vertices
- New multi-particle interactions

But, can reorganize so that

- Few operators
- Many operators
What do higher dimensional operators do?

Change field strength normalization/inputs

Modify existing vertices

New multi-particle interactions

With new organization: number of operators that affect 2- and 3-pt vertices is \textbf{small} and \textbf{\sim constant} with mass dimension

Makes full $1/\Lambda^4$ study possible for certain processes (can also extrapolate, generate compact all-orders results)
Fully exploiting IBP and EOM redundancies, the only SMEFT operator types that contribute to bosonic 2-pt interactions are:

\[ H^n, H^nX^2, D^2H^n \]
First hint: Misiak et al 1812.11513

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Why not e.g. \( D^4H^4 \)?

\[ (DH \sim \partial h + igA \nu + igA \bar{h}) \]

- \( (DH^\dagger)(DH)(DH^\dagger)(DH) \) – too many fields

- \( (D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H) \) – via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

... Similar arguments can be made for operators with field strengths, more derivatives
First hint: Misiak et al 1812.11513

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- \((D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H)\) ? — via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

... 

**Bosonic kinetic piece defined by two functions:**

\[ h(H)(D_\mu H^\dagger D_\mu H), g_{AB}(H) W^A_{\mu\nu} W^{B\mu\nu} \]

\[ W^A = (W^1, W^2, W^3, B) \]
**Even better:**

Number of $H^n$, $H^nX^2$, $D^2H^n$ type operators ~ doesn’t change with mass dimension

<table>
<thead>
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<tr>
<td>$g_{AB}(\phi)W^A_{\mu \nu}W^B_{\nu \mu}$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
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Consequence of group theory + Bose statistics
Verified with Hilbert series method

**contributions to $h_{IJ}$**

$$Q^{(8+2n)}_{HD} = (H^\dagger H)^{n+2} \left( D_\mu H \right)^\dagger (D_\mu H)$$

$$Q^{(8+2n)}_{H,D2} = (H^\dagger H)^{n+1} \left( H^\dagger \sigma_a H \right) \left( D_\mu H \right)^\dagger \sigma^a (D_\mu H)$$
What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

\[ D\psi, D\bar{\psi}, DX \rightarrow \text{2 fields or 1 field + 1 momentum} \]
\[ DH \rightarrow \text{1 or 2 fields or 1 field + 1 momentum} \]
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But all momentum dot products reduce to masses once we impose momentum conservation

Ex.) \[ D_\mu H(D^\mu\bar{\psi})\psi \]
\[ \sim (p_H \cdot p_{\bar{\psi}}) H\bar{\psi} \psi \]
\[ \sim \left( \frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2} \right) H\bar{\psi} \psi \]

Just changes coefficient of \( H\bar{\psi} \psi \): not a new operator structure
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True whenever \( DF = \text{momentum} \)
Allowed 3-pt structures:

\[
\begin{aligned}
&h_{IJ}(\phi)(D_\mu \phi)^I(D_\nu \phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^B_{\mu\nu}, \\
k_{IJ}^A(\phi)(D_\mu \phi)^I(D_\nu \phi)^J\mathcal{W}^A_{\mu\nu}, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^B_{\mu\rho}\mathcal{W}^C_{\rho}, \quad [+ \text{ versions with } G^A] \\
&Y(\phi)\bar{\psi}_1 \psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1 \gamma^\mu \tau_A \psi_2(D_\mu \phi)^I, \quad d_A(\phi)\bar{\psi}_1 \sigma^{\mu\nu} \psi_2 \mathcal{W}^A_{\mu\nu},
\end{aligned}
\]

Higgs-dependent 'connections'ᵀ

As before, # operators small and remains ≃fixed for increasing mass dimension

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Allowed 3-pt structures:

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\[ Y(\phi)\bar{\psi}_1 \psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1 \gamma^\mu \tau_A \psi_2 (D_\mu \phi)^I, \quad d_A(\phi)\bar{\psi}_1 \sigma^{\mu\nu} \psi_2 \mathcal{W}_\mu^A, \]

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Basis with minimal 2- and 3-pt operators:

geometric SMEFT = ‘geoSMEFT’
4-pt interactions: can we go ‘full metric’?

Key part of 2- and 3-pt result is that special kinematics forbade

\[ DF \sim \text{momentum} \]

No longer true at \( \geq 4 \)-pt interactions, operators can depend on

\[ \mathcal{O} \sim s^n t^m \]

\( \rightarrow \) infinite set of higher derivative operators can contribute
4-pt interactions: can we go ‘full metric’?

- emphasizes the importance of on-resonance measurements for SMEFT

\[
\theta(10) \text{ operators at } 1/\Lambda^4
\]

\[
\begin{align*}
\text{resonant:} & \quad + \\
\text{non-resonant:} & \quad + \\
\text{all contribute}
\end{align*}
\]

- still may be some surprising structure for \( n \geq 4 \) — worth thinking about
Can get ‘all orders’ expressions for $1 \to 2$ processes:

e.g. $h \to \gamma \gamma$

$$
\langle h A^{\mu \nu} A_{\mu \nu} \rangle_{SM} - \langle h A^{\mu \nu} A_{\mu \nu} \rangle \frac{\sqrt{\bar{h}}^{44}}{4} \left[ \frac{\delta g_{33}(\phi)}{\delta \phi_4} \bar{e}^2 g_2^2 + 2 \frac{\delta g_{34}(\phi)}{\delta \phi_4} \frac{\bar{e}^2}{g_1 g_2} + \frac{\delta g_{44}(\phi)}{\delta \phi_4} \frac{\bar{e}^2}{g_1^2} \right]
$$

- **$H$ normalization** expand $g_{33}(\phi)\mathbb{W}^{3\mu}_\nu \mathbb{W}^{3\mu}$ to get linear $h$ piece

**Application:** expanding, can now calculate full $1/\Lambda^4$ corrections and see how well $(\text{dim-6})^2$ captures the result

**Defining:**

$$
\langle h | \gamma \gamma \rangle_{\mathcal{L}^{(6)}} = \left[ \frac{g_2^2 \tilde{C}_{HB}^{(6)} + g_1^2 \tilde{C}_{HW}^{(6)} - g_1 g_2 \tilde{C}_{HWB}^{(6)}}{g_1^2 + g_2^2} \tilde{v}_T \right]
$$

**(dim-6)$^2$ estimate:**

$$
\left| A_{SM}^{h\gamma\gamma} \right|^2 + 2 \text{Re} \left( A_{SM}^{h\gamma\gamma} \langle h | \gamma \gamma \rangle_{\mathcal{L}^{(6)}} + \langle h | \gamma \gamma \rangle_{\mathcal{L}^{(6)}}^2 \right)
$$
Can get ‘all orders’ expressions for $1 \rightarrow 2$ processes:

e.g) $h \rightarrow \gamma\gamma$

Full $\mathcal{O}(1/\Lambda^4)$ result:

$$\left| \mathcal{A}_{SM}^{h\gamma\gamma} \right|^2 + 2 \text{Re} \left( \mathcal{A}_{SM}^{h\gamma\gamma} \right) \left( 1 + \langle \sqrt{h}^{44} \rangle \langle h \mid \gamma\gamma \rangle_{\mathcal{L}(6)} \right) + \left( 1 + 4\bar{v}_T \text{Re} \left( \mathcal{A}_{SM}^{h\gamma\gamma} \right) \langle h \mid \gamma\gamma \rangle_{\mathcal{L}(6)} \right)^2$$

$$+ 2 \text{Re} \left( \mathcal{A}_{SM}^{h\gamma\gamma} \right) \left[ \frac{g_2^2 \tilde{C}_{HB}^{(8)} + g_1^2 \left( \tilde{C}_{HW}^{(8)} - \tilde{C}_{HW,2}^{(8)} \right) - g_1g_2 \tilde{C}_{HWB}^{(8)}}{(g_1^2 + g_2^2) \bar{v}_T} \right]$$

At $1/\Lambda^4$, only involves $\mathcal{O}(10)$ operators

Significant differences between full and (dim6)$^2$ result!
**Working to \( \frac{1}{\Lambda^4} \):**

For example, \( h \rightarrow \gamma \gamma \): Quantify effect by randomly drawing coefficients and comparing dim-6, \((\text{dim-6})^2\) and full \( \frac{1}{\Lambda^4} \) result:

- for `tree' operators: \( \mathcal{O}(1) \)
- `loop' operators: \( \mathcal{O}(0.01) \)

[Arzt'93], [Einhorn, Wudka '13], [Craig et al '20]

---

**Figure 1.** The deviations in \( h \rightarrow \gamma \gamma \) from the \( \mathcal{O}(\frac{1}{\Lambda^2}) \) (red line) and partial-square (black line) results, and the full \( \mathcal{O}(\frac{1}{\Lambda^4}) \) results (green \( \pm 1 \), yellow \( \pm 2 \), and grey \( \pm 3 \) regions).

In the left panel the coefficients determining the \( \mathcal{O}(\frac{1}{\Lambda^2}) \) and partial-square results are:
- \( C(6)_{HB} = 0.01 \)
- \( C(6)_{HW} = 0.004 \)
- \( C(6)_{HWB} = 0.007 \)
- \( C(6)_{HD} = 0.74 \)
- \( G(6)_{F} = 1.6 \)

In the right panel they are:
- \( C(6)_{HB} = 0.007 \)
- \( C(6)_{HW} = 0.007 \)
- \( C(6)_{HWB} = 0.015 \)
- \( C(6)_{HD} = 0.50 \)
- \( G(6)_{F} = 1.26 \)

---

**Figure 2.** The deviations in \( h \rightarrow Z \) from the \( \mathcal{O}(\frac{1}{\Lambda^2}) \) (red line) and partial-square (black line) results, and the full \( \mathcal{O}(\frac{1}{\Lambda^4}) \) results (green \( \pm 1 \), yellow \( \pm 2 \), and grey \( \pm 3 \) regions).

In the left panel the coefficients determining the \( \mathcal{O}(\frac{1}{\Lambda^2}) \) and partial-square results are:
- \( C(6)_{HB} = 0.01 \)
- \( C(6)_{HW} = 0.02 \)
- \( C(6)_{HWB} = 0.011 \)
- \( C(6)_{HD} = 0.53 \)
- \( G(6)_{F} = 0.13 \)

In the right panel they are:
- \( C(6)_{HB} = 0.002 \)
- \( C(6)_{HW} = 0.001 \)
- \( C(6)_{HWB} = 0.001 \)
- \( C(6)_{HD} = 0.28 \)
- \( G(6)_{F} = 1.15 \)

---

Fixing \( \frac{1}{\Lambda^2} \), \((\text{dim-6})^2\) result: contours show range of effects once full \( \frac{1}{\Lambda^4} \) effects are included.
Working to $1/\Lambda^4$:

e.g) $h \to \gamma \gamma$ : Quantify effect by randomly drawing coefficients and comparing dim-6, $(\text{dim-6})^2$ and full $1/\Lambda^4$ result:

for `tree’ operators: $\mathcal{O}(1)$ ,`

\[ \text{for \ `tree’ \ operators: } \mathcal{O}(1) \ , \ `\text{loop’ \ operators: } \mathcal{O}(0.01) \]

\[ \text{ex.} \ (H^4 H^2 \chi_{\mu\nu}^\lambda \chi_{\nu\lambda}^\mu) \]

\[ \text{ex.} \ (H^4 H) \chi_{\mu\nu}^\lambda \chi_{\nu\lambda}^\mu \]

Large effect, as only loop-level operators enter at dim-6, while tree-level operators enter at dim-8

similar story for $h \to Z\gamma$
Working to $1/\Lambda^4$:

e.g.) $Z \rightarrow \ell^+ \ell^-$

Now tree-level operators present for both dim-6 and dim-8

\[
\langle Z | \ell \ell \rangle |_{\nu^2/\Lambda^2} \sim \left( \frac{C^{(6)}}{1.0} \right) \frac{\nu^2}{\Lambda^2}
\]

\[
\langle Z | \ell \ell \rangle |_{\nu^4/\Lambda^4} \sim \left( \frac{C^{(8)}}{1.0} \right) \frac{\nu^4}{\Lambda^4}
\]

smaller impact, but still present, especially if $\Lambda$ is small
Takeaways

Study the ‘truncation error’ from higher order SMEFT effects

- Explore \((\text{dim-6})^2\) contribution as a proxy theory uncertainty

- geoSMEFT basis: basis where 2 and 3 particle vertices sensitive to a minimal \# of operators, \# ~ constant with mass dimension. Useful to inform where \((\text{dim-6})^2\) fails/succeeds at capturing \(1/\Lambda^4\) effects.

Lots of other possible directions:

- Top-down studies
  - \(1/\Lambda^4\) versus SM NLO

- All-orders results
  - How to pin down new coefficients, rather than treat them as nuisance parameters?

THANK YOU!
Backup
Example operator counting:

\[(H^\dagger H)^n W^2_L\] ignore Lorentz, focus on SU(2)_W reps.

\[H = (1/2) \implies H^n = (n/2) \quad W^2_L = (0 \oplus 2)\] enforced by Bose symm.

\[H^\dagger = (1/2) \implies (H^\dagger)^n = (n/2)\]

\[(H^\dagger H)^n = (0 \oplus 1 \oplus 2 \oplus \ldots n) \otimes W^2_L = (0 \oplus 2) = 2 \text{ invariants}\]

\[\text{[+1 for } B^2_L \text{ and } +1 \text{ for } W_L B_L = 4\]

To get SU(2)_W \textbf{2}, need \(\geq 4\) Higgses \(\implies\) operator dimension \(\geq 8\)
Example operator counting:

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\[
\text{[+1 for } B_L^2 \text{ and } +1 \text{ for } W_L B_L = 4]
\]

To get SU(2)_W 2, need ≥ 4 Higgses → operator dimension ≥ 8

contributions to g_{AB}:

\[
\begin{align*}
Q_{HB}^{(6+2n)} &= (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu}, \\
Q_{HW}^{(6+2n)} &= (H^\dagger H)^{n+1} W_a^{\mu\nu} W^a_{\mu\nu}, \\
Q_{HWB}^{(6+2n)} &= (H^\dagger H)^n (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu}, \\
Q_{HW,2}^{(8+2n)} &= (H^\dagger H)^n (H^\dagger \sigma^a H) (H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu},
\end{align*}
\]
What about $G_F$?

$G_F$ involves more than quadratic terms:

$$G_F^4 p t = \frac{1}{\mathcal{V}_T^2} \left( \tilde{\mathcal{C}}_{\mu cc \mu}^{(6)} + \tilde{\mathcal{C}}_{\mu \mu e}^{(6)} + \frac{\tilde{\mathcal{C}}_{4\ell, 2}^{(8+2n)}}{2^n} + \frac{\tilde{\mathcal{C}}_{4\ell, 5}^{(8+2n)}}{2^n} \right)$$

All orders result is possible even for contact terms:

$x C^{(8+2n)} (H^+ H)^{1+n} (\bar{\ell}_2^{\gamma \mu} \sigma^i \ell_2) \left( \bar{\ell}_1^{\gamma_\mu} \sigma_i \ell_1 \right) + i x C_{4\ell, 5}^{(8+2n)}\epsilon_{ijk} (H^+ H)^{n} (H^+ \sigma^i H) \left( \bar{\ell}_2^{\gamma \mu} \sigma_j \ell_2 \right) \left( \bar{\ell}_1^{\gamma_\mu} \sigma_k \ell_1 \right)$

However, since $G_F$ derived at muon mass scale ($D \sim m_\mu \ll \Lambda$) and SM term is from $L^4$, # of higher dimensional contributions is dramatically reduced.
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However, since $G_F$ derived at muon mass scale ($D \sim m_\mu \ll \Lambda$) and SM term is from $L^4$, # of higher dimensional contributions is dramatically reduced

$$
C^{(8+2n)}_{4\ell,2} (H^\dagger H)^{1+n} (\bar{\ell}_2 \gamma^\mu \sigma^i \ell_2) \left( \bar{\ell}_1 \gamma^\mu \sigma^i \ell_1 \right) \quad iC^{(8+2n)}_{4\ell,5} \epsilon_{ijk} (H^\dagger H)^n (H^\dagger \sigma^j H) \left( \bar{\ell}_2 \gamma^\mu \sigma_j \ell_2 \right) \left( \bar{\ell}_1 \gamma^\mu \sigma_k \ell_1 \right)
$$

All orders result is possible even for contact terms:

$$
\mathcal{G}^{4pt}_F = \frac{1}{V_T^2} \left( \tilde{C}^{(6)}_{\mu cc\mu} + \tilde{C}^{(6)}_{\mu \mu \mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n} \right)
$$

[Hays, Helset, Martin, Trott 2007.00565]