# Theory errors and operator bases

Adam Martin
(amarti41@nd.edu)

UNIVERSITY OF NOTRE DAME

CPM-Snowmass Oct 6th 2020

# In SMEFT framework

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}\Big(\underbrace{|A_6|^2 + 2Re(A_{SM}^* A_8)}\Big) + \cdots$$

interference piece,
usually largest effect.
State of the art
SMEFT

'Higher order'
$(1/\Lambda)$ corections

SMEFT Warsaw basis: $\mathcal{O}(60)$ operators at dim-6
$\mathcal{O}(1000)$ operators at dim-8

**What's the impact from $1/\Lambda^4$ corrections?**

# Higher order effects so should be small… but

- they are a form of uncertainty; ‘theory error’ on extracted scale $\Lambda$

- there are instances where interference term isn’t present or is suppressed, e.g. helicity mismatch between SM and dim-6

- faster growth with energy, $E^4$ vs. $E^2$ : increasingly important when looking at high energy (e.g. tails of some kinematic distribution)

But full treatment to $1/\Lambda^4$, with all $\mathcal{O}(1100)$ operators doesn’t seem feasible

# Some thoughts on how to proceed

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}\Big( |A_6|^2 + 2Re(A_{SM}^* A_8) \Big) + \cdots$$

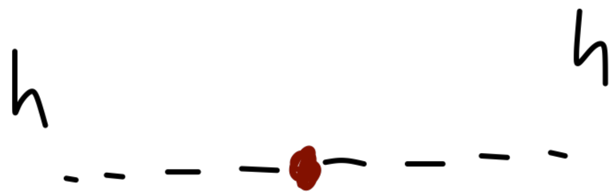
1.) Use (dim-6)$^2$ piece as a proxy for higher order effects. Add it as a theory uncertainty when performing SMEFT analysis. Fully set by dim-6 operators, machinery already in place

[Shepherd et al. 1711.07484 .1812.0757, extend work in LOI with Shepherd, Lewis, Kim, Gu]

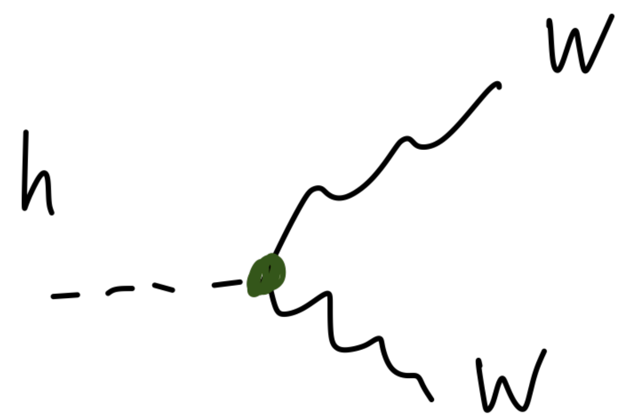
2.) Reorganize SMEFT to minimize the number of higher dimensional operators needed for as many processes as possible

[Helset, AM, Trott, 2001.01453,Hays, Helset, AM, Trott 2007.00565, part of dim 8$^+$ LOI]

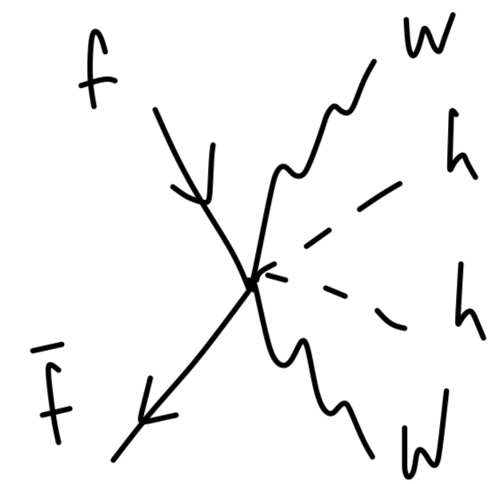
# What do higher dimensional operators do?




Change field strength normalization/inputs

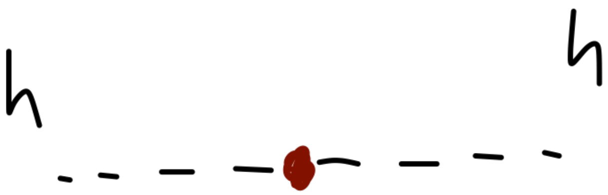



Modify existing vertices

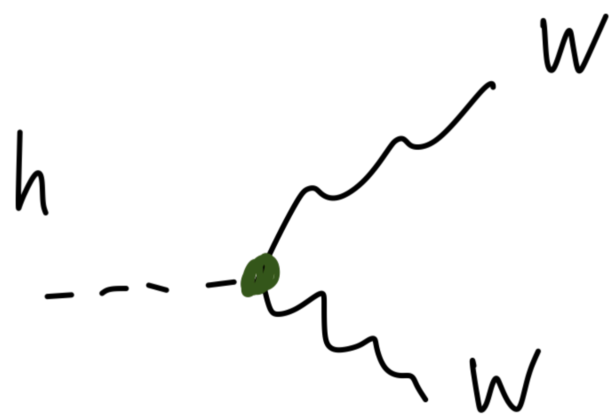



New multi-particle interactions

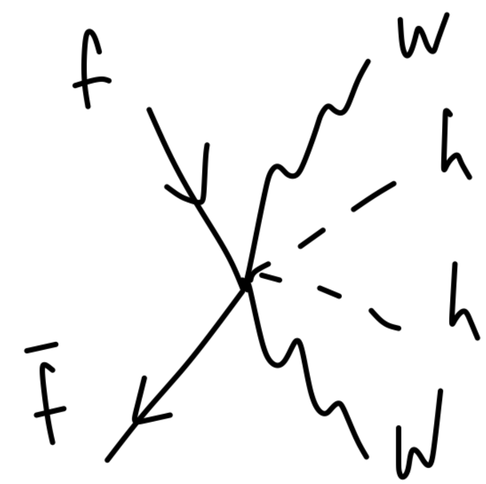
# What do higher dimensional operators do?




Change field strength normalization/inputs




Modify existing vertices



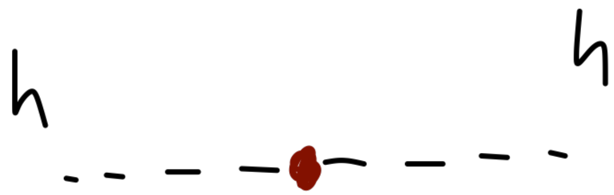

New multi-particle interactions

*universal* ← *specific*

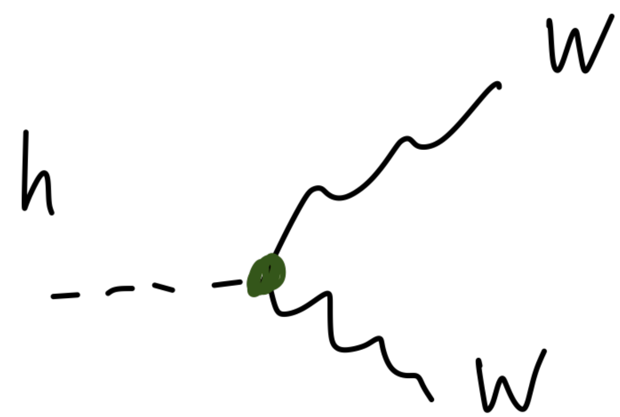
But, can reorganize so that

*Few operators* → *Many operators*

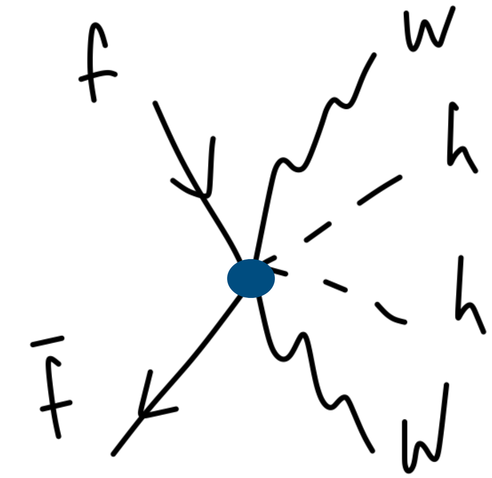
# What do higher dimensional operators do?




Change field strength
normalization/inputs




Modify existing vertices




New multi-particle
interactions

With new organization: number of operators that affect 2- and 3-pt vertices is **small** and **~constant** with mass dimension

Makes full $1/\Lambda^4$ study possible for certain processes
(can also extrapolate, generate compact all-orders results)

# First hint: Misiak et al 1812.11513

Fully exploiting IBP and EOM redundancies, the only SMEFT operator types that contribute to bosonic 2-pt interactions are:

$$H^n\,,\; H^n X^2\,,\; D^2 H^n$$

# First hint: Misiak et al 1812.11513

Fully exploiting IBP and EOM redundancies, the only SMEFT operator types that contribute to bosonic 2-pt interactions are:

$$H^n\,,\; H^nX^2\,,\; D^2H^n$$

Why not e.g. $D^4H^4$ ? $(DH \sim \partial h + igAv + igAh)$

- $(DH^\dagger)(DH)(DH^\dagger)(DH)$ ? — too many fields
- $(D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H)$ ? — via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

...

Similar arguments can be made for operators with field strengths, more derivatives

# First hint: Misiak et al 1812.11513

Fully exploiting IBP and EOM redundancies, the only SMEFT operator types that contribute to bosonic 2-pt interactions are:

$$H^n \,,\; H^n X^2 \,,\; D^2 H^n$$

Why not e.g. $D^4H^4$ ? $\quad (DH \sim \partial h + ig A v + ig Ah)$

- $(DH^\dagger)(DH)(DH^\dagger)(DH)$ ? — too many fields
- $(D_{\{\mu\nu\}}H^\dagger D_{\{\mu\nu\}}H)(H^\dagger H)$ ? — via IBP and EOM, reduces to operators with 2 derivs + operators with > 2 fields

...

Bosonic kinetic piece defined by two functions:

$$h(H)(D_\mu H^\dagger D_\mu H)\,,\; g_{AB}(H)\mathscr{W}^A_{\mu\nu}\mathscr{W}^{B\mu\nu}$$

$$\mathscr{W}^A = \left(W^1, W^2, W^3, B\right)$$

# Even better:

Number of $H^n$ , $H^n X^2$ , $D^2 H^n$ type operators ~ doesn't change with mass dimension

| | Mass Dimension | | | | |
|---|---|---|---|---|---|
| Field space connection | 6 | 8 | 10 | 12 | 14 |
| $h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$ | 2 | 2 | 2 | 2 | 2 |
| $g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$ | 3 | 4 | 4 | 4 | 4 |

Consequence of group theory + Bose statistics
Verified with Hilbert series method

**contributions to $h_{IJ}$**

$$Q_{HD}^{(8+2n)} = \left(H^\dagger H\right)^{n+2}\left(D_\mu H\right)^\dagger (D^\mu H)$$

$$Q_{H,D2}^{(8+2n)} = \left(H^\dagger H\right)^{n+1}\left(H^\dagger\sigma_a H\right)\left(D_\mu H\right)^\dagger \sigma^a (D^\mu H)$$

# What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

$D\psi, D\bar{\psi}, DX \rightarrow$ **2 fields** or **1 field + 1 momentum**

$DH \rightarrow$ **1 or 2 fields** or **1 field + 1 momentum**

# What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

$$D\psi, D\bar{\psi}, DX \to$$ **2 fields** or **1 field + 1 momentum**

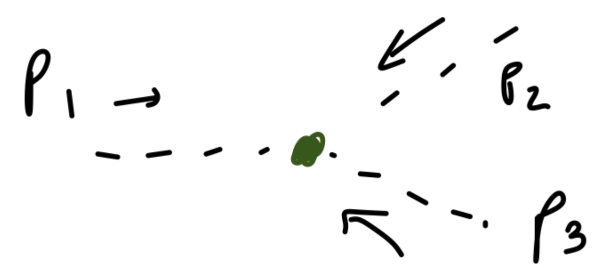
$$DH \to$$ **1 or 2 fields** or **1 field + 1 momentum**

But **all** momentum dot products reduce to masses once we impose momentum conservation

**Ex.)** $$D_\mu H \,(D^\mu \bar{\psi})\, \psi$$

$$\sim (p_H \cdot p_{\bar{\psi}})\, H\, \bar{\psi}\, \psi$$

$$\sim \left(\frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2}\right) H\, \bar{\psi}\, \psi$$




$$p_H + p_{\bar{\psi}} + p_\psi = 0$$

Just changes coefficient of $H\,\bar{\psi}\,\psi$ : not a new operator structure

# What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

$D\psi, D\bar{\psi}, DX \rightarrow$ **2 fields** or **1 field + 1 momentum**

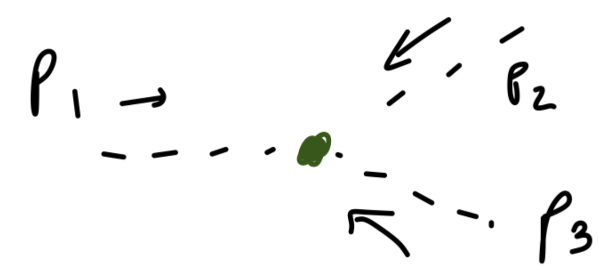
$DH \rightarrow$ **1 or 2 fields** or **1 field + 1 momentum**

But **all** momentum dot products reduce to masses once we impose momentum conservation

**Ex.)** $D_\mu H \,(D^\mu \bar{\psi})\, \psi$

$$\sim (p_H \cdot p_{\bar{\psi}})\, H\, \bar{\psi}\, \psi$$

$$\sim \left(\frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2}\right) H\, \bar{\psi}\, \psi$$




$$p_H + p_{\bar{\psi}} + p_\psi = 0$$

Just changes coefficient of $H\,\bar{\psi}\,\psi$ : not a new operator structure

True whenever $DF$ = momentum

# What about 3-pt interactions? Similar story

- 3 fields only, Lorentz invariance
- non-Higgs derivatives increase field count or introduce momentum

$D\psi, D\bar{\psi}, DX \rightarrow$ **2 fields** or ~~**1 field + 1 momentum**~~

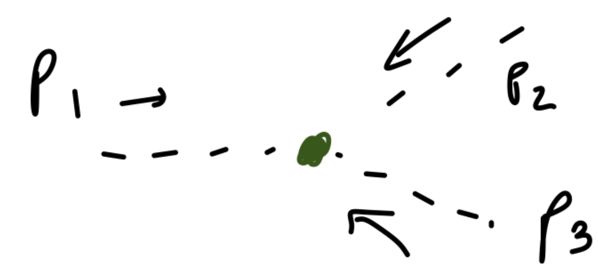
$DH \rightarrow$ **1 or 2 fields** or ~~**1 field + 1 momentum**~~

But **all** momentum dot products reduce to masses once we impose momentum conservation

**Ex.)** $D_\mu H \, (D^\mu \bar{\psi}) \, \psi$

$$\sim (p_H \cdot p_{\bar{\psi}}) \, H \, \bar{\psi} \, \psi$$

$$\sim \left( \frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2} \right) H \, \bar{\psi} \, \psi$$




$$p_H + p_{\bar{\psi}} + p_\psi = 0$$

Just changes coefficient of $H \, \bar{\psi} \, \psi$ : not a new operator structure

True whenever $DF$ = momentum

# Allowed 3-pt structures:

$$h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J,\quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\mathcal{W}^{\mu\nu}_A,\quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$ [+ versions with $G^A$]

$$Y(\phi)\bar\psi_1\psi_2,\quad L_{I,A}(\phi)\bar\psi_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I,\quad d_A(\phi)\bar\psi_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$$

Higgs-dependent 'connections'

As before, # operators small and remains ~fixed for increasing mass dimension

| | Mass Dimension | | | | |
|---|---|---|---|---|---|
| Field space connection | 6 | 8 | 10 | 12 | 14 |
| $k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$ | 0 | 3 | 4 | 4 | 4 |
| $f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$ | 1 | 2 | 2 | 2 | 2 |
| $Y^u_{pr}(\phi)\bar{Q}u+$ h.c. | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ |
| $Y^d_{pr}(\phi)\bar{Q}d+$ h.c. | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ |
| $Y^e_{pr}(\phi)\bar{L}e+$ h.c. | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ | $2\,N_f^2$ |
| $d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c. | $4\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ |
| $d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c. | $4\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ |
| $d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c. | $4\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ | $6\,N_f^2$ |
| $L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$ | $N_f^2$ | $N_f^2$ | $N_f^2$ | $N_f^2$ | $N_f^2$ |
| $L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$ | $2\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ |

# Allowed 3-pt structures:

$$h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$ [+ versions with G^A]

$$Y(\phi)\bar\psi_1\psi_2, \quad L_{I,A}(\phi)\bar\psi_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar\psi_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$$

Higgs-dependent 'connections'

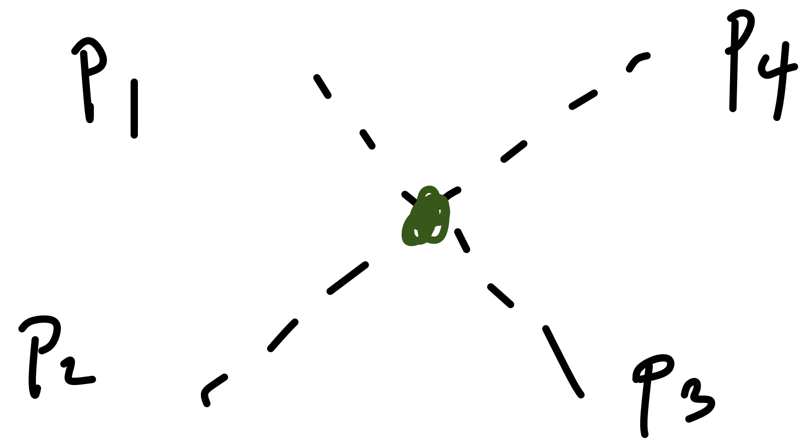
As before, # operators small and remains ~fixed for increasing mass dimension

| | Mass Dimension | | | | |
|---|---|---|---|---|---|
| Field space connection | 6 | 8 | 10 | 12 | 14 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| $L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$ | $N_f^2$ | $N_f^2$ | $N_f^2$ | $N_f^2$ | $N_f^2$ |
| $L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar\psi_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$ | $2\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ | $4\,N_f^2$ |

**Basis with minimal 2- and 3-pt operators:**

**geometric SMEFT = 'geoSMEFT'**

# 4-pt interactions: can we go 'full metric'?




Key part of 2- and 3-pt result is that special kinematics forbade

$DF \sim$ **momentum**

No longer true at $\geq$ 4-pt interactions, operators can depend on

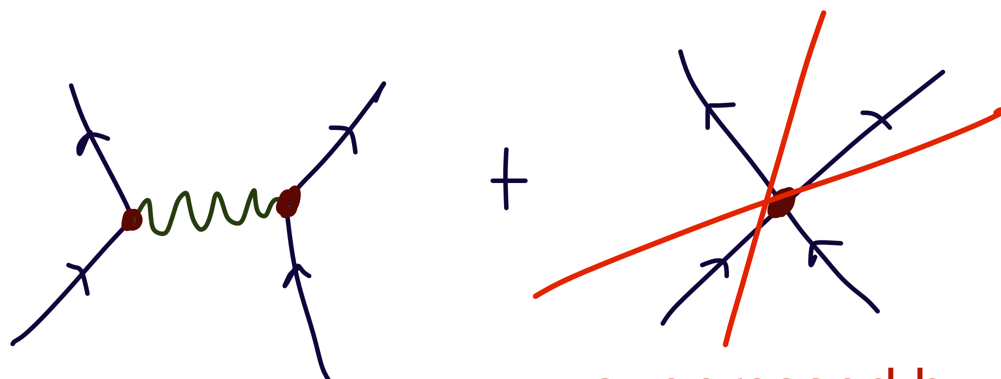
$$\mathcal{O} \sim s^n\, t^m$$

$\longrightarrow$ infinite set of higher derivative operators can contribute

## 4-pt interactions: can we go ‘full metric’?

- emphasizes the importance of on-resonance measurements for SMEFT

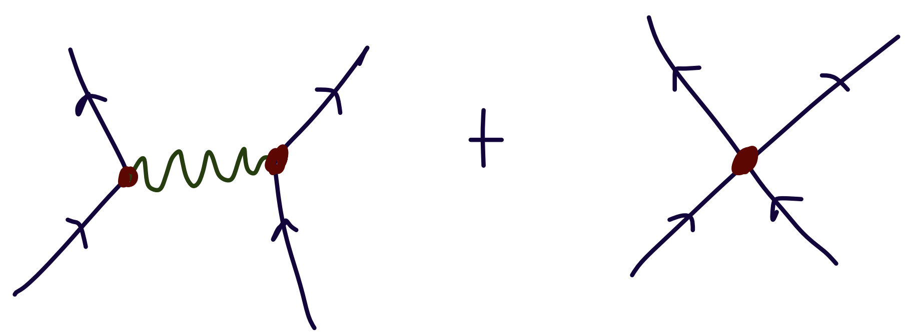
resonant:




suppressed by $\frac{\Gamma M}{v^2}$

$\mathcal{O}(10)$ operators at $1/\Lambda^4$

non-resonant:




all contribute

$\gg 10$ operators

- still may be some surprising structure for $n \geq 4$ — worth thinking about

# Can get 'all orders' expressions for $1 \to 2$ processes:

e.g) $h \to \gamma\gamma$

go to mass basis

$$\langle h A^{\mu\nu} A_{\mu\nu}\rangle \mathcal{A}^{h\gamma\gamma}_{\rm SM} \; - \langle h A^{\mu\nu} A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle \frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle \frac{\bar{e}^2}{g_2^2} + 2\langle \frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle \frac{\bar{e}^2}{g_1 g_2} + \langle \frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle \frac{\bar{e}^2}{g_1^2}\right]$$

H normalization

expand $g_{33}(\phi)\mathscr{W}^3_{\mu\nu}\mathscr{W}^{3\mu\nu}$ to get linear h piece

**application**: expanding, can now calculate <u>full</u> $1/\Lambda^4$ corrections and see how well (dim-6)² captures the result

defining: $$\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} = \left[\frac{g_2^2\tilde{C}^{(6)}_{HB} + g_1^2\tilde{C}^{(6)}_{HW} - g_1 g_2 \tilde{C}^{(6)}_{HWB}}{\left(g_1^2+g_2^2\right)\bar{v}_T}\right]$$

(dim-6)² estimate: $$\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \langle h|\gamma\gamma\rangle^2_{\mathscr{L}^{(6)}}$$

# Can get 'all orders' expressions for $1 \rightarrow 2$ processes:

e.g) $h \rightarrow \gamma\gamma$

Full $\mathcal{O}(1/\Lambda^4)$ result:

$$\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\left(1 + \left\langle \sqrt{h}^{44} \right\rangle_{\mathscr{L}^{(6)}}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \left(1 + 4\bar{v}_T\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\right)\left(\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}}\right)^2$$
$$+2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\left[\frac{g_2^2\tilde{C}^{(8)}_{HB} + g_1^2\left(\tilde{C}^{(8)}_{HW} - \tilde{C}^{(8)}_{HW,2}\right) - g_1 g_2 \tilde{C}^{(8)}_{HWB}}{\left(g_1^2 + g_2^2\right)\bar{v}_T}\right]$$

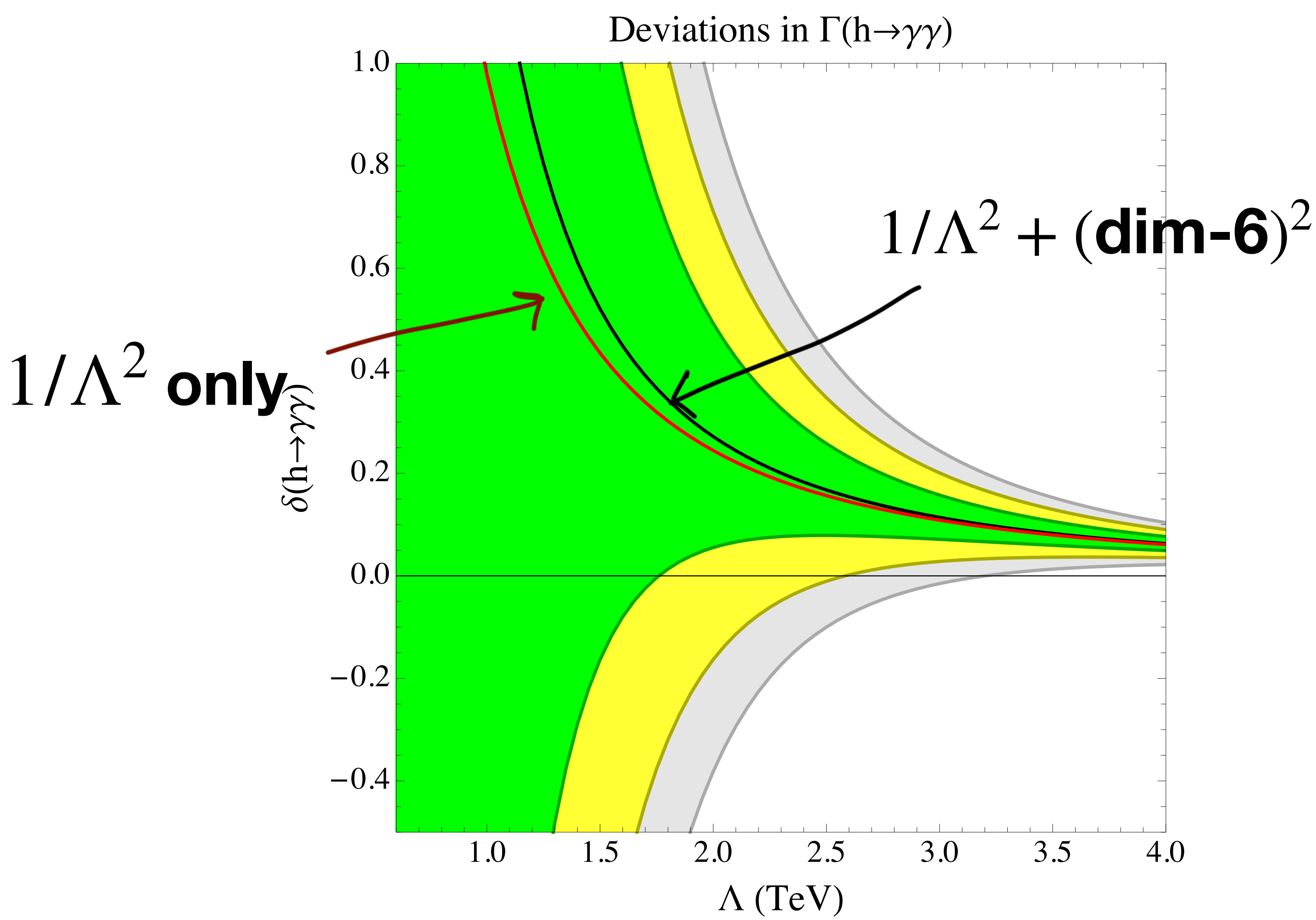
At $1/\Lambda^4$, only involves $\mathcal{O}(10)$ operators

Significant differences between full and (dim6)$^2$ result!

# Working to $1/\Lambda^4$ :

e.g) $h \to \gamma\gamma$ : Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)² and full $1/\Lambda^4$ result:
for `tree' operators: $\mathcal{O}(1)$ ,`loop' operators: $\mathcal{O}(0.01)$

[Arzt'93], [Einhorn, Wudka '13], [Craig et al '20]

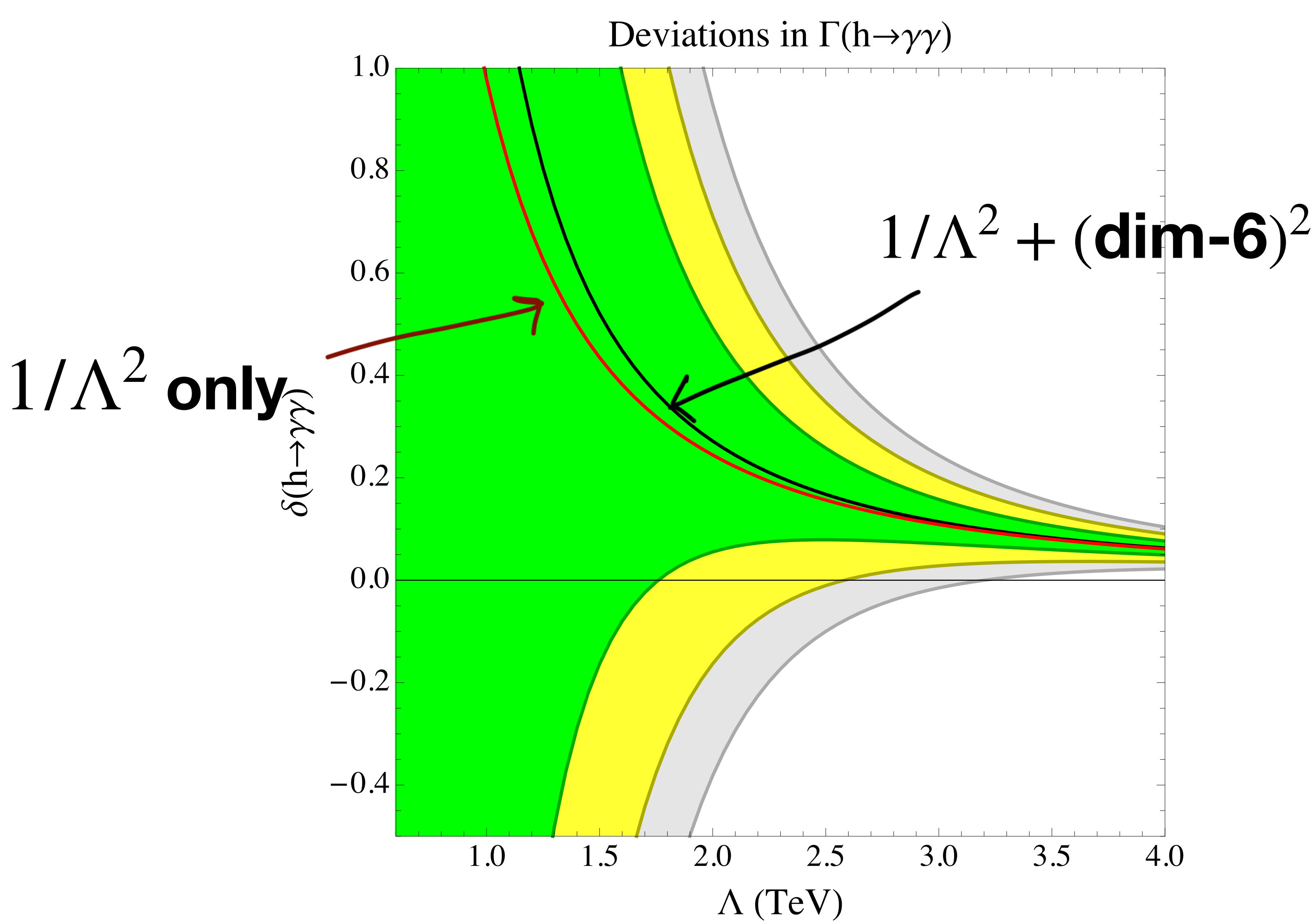



fixing $1/\Lambda^2$, (dim-6)$^2$ result: contours show range of effects once full $1/\Lambda^4$ effects are included

# Working to $1/\Lambda^4$ :

e.g) $h \to \gamma\gamma$ : Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)$^2$ and full $1/\Lambda^4$ result:
for `tree' operators: $\mathcal{O}(1)$ ,`loop' operators: $\mathcal{O}(0.01)$

ex.) $(H^\dagger H)^2 X_{\mu\nu} X^{\mu\nu}$ ex.) $(H^\dagger H) X^{\mu\nu} X_{\mu\nu}$

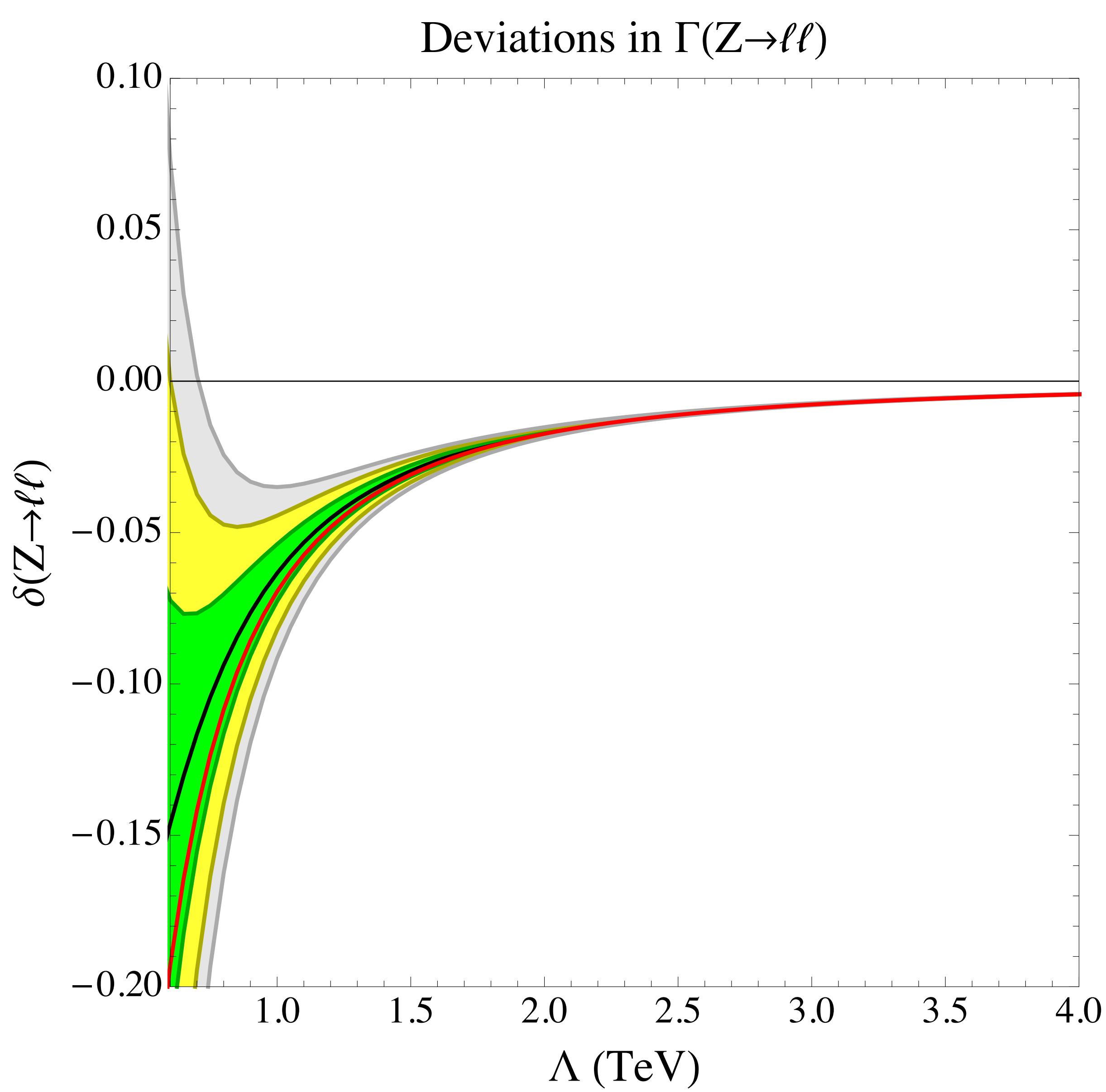



Large effect, as only loop-level operators enter at dim-6, while tree-level operators enter at dim-8

similar story for $h \to Z\gamma$

# Working to $1/\Lambda^4$ :

e.g.) $Z \to \ell^+\ell^-$




Now tree-level operators present for both dim-6 and dim-8

$$\langle Z | \ell\ell \rangle_{to\, v^2/\Lambda^2} \sim \left(\frac{C^{(6)}}{1.0}\right)\frac{v^2}{\Lambda^2}$$

$$\langle Z | \ell\ell \rangle_{to\, v^4/\Lambda^4} \sim \left(\frac{C^{(8)}}{1.0}\right)\frac{v^4}{\Lambda^4}$$

smaller impact, but still present, especially if $\Lambda$ is small

# Takeaways

Study the 'truncation error' from higher order SMEFT effects

- Explore $(\text{dim-6})^2$ contribution as a proxy theory uncertainty

- geoSMEFT basis: basis where 2 and 3 particle vertices sensitive to a minimal # of operators, # ~ constant with mass dimension. Useful to inform where $(\text{dim-6})^2$ fails/succeeds at capturing $1/\Lambda^4$ effects.

**Lots of other possible directions:**

- Top-down studies
    - $1/\Lambda^4$ versus SM NLO

- All-orders results
    - How to pin down new coefficients, rather than treat them as nuisance parameters?

**THANK YOU!**

# Backup

# Example operator counting:

$(H^\dagger H)^n\, W_L^2$ ignore Lorentz, focus on $SU(2)_W$ reps.

$H = (1/2)\ \therefore\ H^n = (n/2)$ $\quad W_L^2 = (0 \oplus 2)$ enforced by Bose symm.

$H^\dagger = (1/2)\ \therefore\ (H^\dagger)^n = (n/2)$

$$(H^\dagger H)^n = (0 \oplus 1 \oplus 2 \oplus \ldots n) \quad \otimes \quad W_L^2 = (0 \oplus 2) \quad = \text{2 invariants}$$

[+1 for $B_L^2$ and +1 for $W_L B_L$ = 4]

To get $SU(2)_W$ 2, need $\geq$ 4 Higgses $\rightarrow$ operator dimension $\geq 8$

# Example operator counting:

$(H^\dagger H)^n W_L^2$ ignore Lorentz, focus on SU(2)$_W$ reps.

$H = (1/2) \;\therefore\; H^n = (n/2)$ $\quad W_L^2 = (0 \oplus 2)$ enforced by Bose symm.

$H^\dagger = (1/2) \;\therefore\; (H^\dagger)^n = (n/2)$

$(H^\dagger H)^n = (0 \oplus 1 \oplus 2 \oplus \ldots n) \quad \otimes \quad W_L^2 = (0 \oplus 2)$ = 2 invariants

[+1 for $B_L^2$ and +1 for $W_L B_L$ = 4]

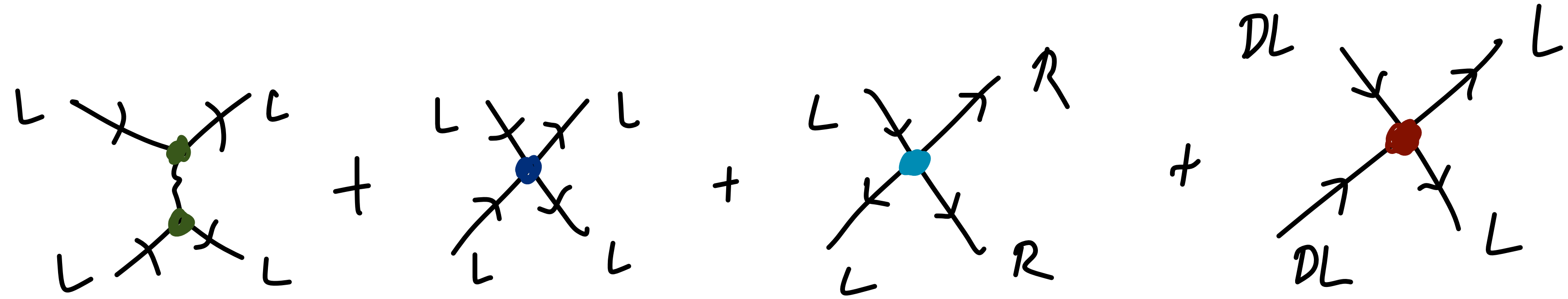
To get SU(2)$_W$ 2, need $\geq$ 4 Higgses $\rightarrow$ operator dimension $\geq 8$

**contributions to $g_{AB}$**

$$Q_{HB}^{(6+2n)} = (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW}^{(6+2n)} = (H^\dagger H)^{n+1} W_a^{\mu\nu} W_{\mu\nu}^a,$$
$$Q_{HWB}^{(6+2n)} = (H^\dagger H)^n (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW,2}^{(8+2n)} = (H^\dagger H)^n (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu},$$

# What about $G_F$?

$G_F$ involves more than quadratic terms:




However, since $G_F$ derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from $L^4$, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2}\left(H^\dagger H\right)^{1+n}\left(\bar{\ell}_2\gamma^\mu\sigma^i\ell_2\right)\left(\bar{\ell}_1\gamma_\mu\sigma_i\ell_1\right) \qquad iC^{(8+2n)}_{4\ell,5}\epsilon_{ijk}\left(H^\dagger H\right)^n\left(H^\dagger\sigma^i H\right)\left(\bar{\ell}_2\gamma^\mu\sigma_j\ell_2\right)\left(\bar{\ell}_1\gamma_\mu\sigma_k\ell_1\right)$$

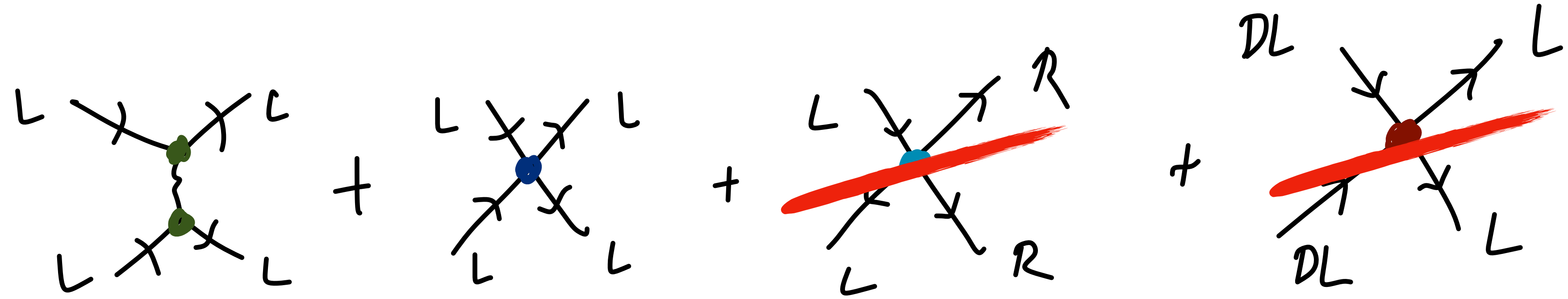
All orders result is possible even for contact terms:

$$\mathscr{G}_F^{4pt} = \frac{1}{\bar{v}_T^2}\left(\tilde{C}^{(6)}_{\mu cc\mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n}\right)$$

[Hays, Helset, Martin, Trott 2007.00565]

# What about $G_F$?

$G_F$ involves more than quadratic terms:




However, since $G_F$ derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from $L^4$, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2} \left(H^\dagger H\right)^{1+n} \left(\bar{\ell}_2 \gamma^\mu \sigma^i \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_i \ell_1\right) \qquad i C^{(8+2n)}_{4\ell,5} \epsilon_{ijk} \left(H^\dagger H\right)^n \left(H^\dagger \sigma^i H\right) \left(\bar{\ell}_2 \gamma^\mu \sigma_j \ell_2\right) \left(\bar{\ell}_1 \gamma_\mu \sigma_k \ell_1\right)$$

All orders result is possible even for contact terms:

$$\mathscr{G}_F^{4pt} = \frac{1}{\bar{v}_T^2} \left( \tilde{C}^{(6)}_{\mu c c \mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n} \right)$$

[Hays, Helset, Martin, Trott 2007.00565]