Lattice QCD for Flavor Physics

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$|V_{ud}|$ and $|V_{us}|$

[68%CL ellipse
Without scaling $S = 2.7$

$0^+ \rightarrow 0^+$

[E. Passemar, https://indico.fnal.gov/event/44121]
$|V_{ud}|$ and $|V_{us}|$

- **LOI** “High-precision determination of $V_{us}$ and $V_{ud}$ from lattice QCD” by V. Guelpers et al. (RBC and UKQCD Collaborations)

- **LOI** “Precise Lattice QCD calculations of kaon and pion decay parameters and first-row CKM unitarity tests” by A. El-Khadra et al. (Fermilab Lattice and MILC Collaborations)

- **LOI** “Unitarity of CKM Matrix, $|V_{ud}|$, Radiative Corrections and Semi-leptonic Form Factors” by R. Gupta et al. (PNDME and NME Collaborations)
\( |V_{ud}| \) and \( |V_{us}| \)

- Lattice QCD calculations of kaon and pion decay constants and form factors have reached \( \sim 0.2\% \) precision. Now the focus is on strong isospin-breaking and QED corrections to pion and kaon leptonic and semileptonic decays.

- Lattice QCD inputs are also needed for hyperon semileptonic decays and inclusive strange hadronic \( \tau \) decays.

- To improve the \( |V_{ud}| \) determination from neutron decay, the \( \gamma W \) box diagram QED correction can be calculated on the lattice.
$|V_{ub}|$ and $|V_{cb}|$

Note: $|V_{cb}|$ is critical for the Standard-Model prediction of $\varepsilon_K$. 

Inclusive

$|V_{ub}|$: GGOU

$|V_{cb}|$: global fit in KS

Average $68\%$ C.L.

Average $\Delta \chi^2 = 1$

HFLAV

Spring 2019

$P(\chi^2) = 7.7\%$
Belle II will achieve $\sim 1\%$ for $B \rightarrow \pi \ell \nu$. 
$|V_{ub}|$ and $|V_{cb}|$

- **LOI “Precision Lattice QCD in Support of BSM Searches”** by C. DeTar et al. (Fermilab Lattice and MILC Collaborations)

- **LOI “Weak decays of b and c quarks”** by O. Witzel et al. (RBC and UKQCD Collaborations)

- **LOI “High precision determinations of $|V_{xb}|$ from a close theory-experiment collaboration”** by B. Dey et al.

- **LOI “Precision theory inputs for $|V_{cb}|$ and LFUV observables”** by A. Lytle

- **LOI “Lattice-QCD studies of inclusive B-meson decays”** by T. De Grand et al. (Fermilab Lattice and MILC Collaborations)
\[ |V_{ub}| \text{ and } |V_{cb}| \]

- Precision improvements are needed and in progress for semileptonic form factors such as \( B \to \pi \ell \nu \) and others.

- First lattice calculations of \( B \to D^* \) semileptonic form factors at nonzero recoil are expected soon.

- Close experiment-theory collaboration is needed to make the best determinations of \(|V_{ub}| \text{ and } |V_{cb}|\).
\( V_{ub} \) and \( V_{cb} \)

- Form factors for other decay channels such as \( B_c \rightarrow J/\psi \ell \nu \) and \( \Lambda_b \rightarrow \Lambda_c \ell \bar{\nu} \) can also be calculated on the lattice, and are particularly important for LFU tests.

[M. F. Sevilla, https://indico.fnal.gov/event/44442]
Methods have been developed to study inclusive decays on the lattice. The main challenge is that the Euclidean calculation occurs for spacelike kinematics.

Potential impact on $|V_{ub}|$ and $|V_{cb}|$ remains unclear. Detailed studies would be welcome.

Rare $b$ decays

Lattice inputs: form factors and decay constants. The $B \to K^*$ lattice calculation (from 2013) treated the $K^*$ as a stable particle, but there are now methods to treat the $K^*$ properly as a resonance in $B \to K\pi\ell\ell$ (these methods have already been used for $K \to \pi\pi$).

Can the long-distance contributions to the decays be calculated on the lattice?
Rare $b$ decays and $B_s$-$\bar{B}_s$ mixing

$L. Di Luzio, M. Kirk, A. Lenz, arXiv:1811.12884$
Neutral-meson mixing

- $B_{(s)}$-$\bar{B}_{(s)}$ mixing: short-distance dominated. Need to improve precision of dimension-6-operator matrix elements and calculate dimension-7-operator matrix elements.

- $D^0$-$\bar{D}^0$ mixing: short-distance contributions have been calculated on the lattice, but the mixing is long-distance dominated. Can the long-distance contributions be calculated on the lattice? Use similar inverse-problem method as for inclusive semileptonic decays?

- $K^0$-$\bar{K}^0$ mixing: both short-distance and long-distance contributions have been calculated on the lattice (less phase space for intermediate states allows explicit treatment of imaginary time and finite volume). Much room for precision improvements.
Rare kaon and hyperon decays, $K^0 - \bar{K}^0$ mixing, $K \to \pi \pi$

- **LOI** “Rare strange-to-down processes from lattice QCD” by A. Portelli et al. (RBC and UKQCD Collaborations)

- **LOI** “Discovering new physics in rare kaon decays” by N. Christ et al. (RBC and UKQCD Collaborations)

- **LOI** “Precise Lattice QCD calculations of kaon and pion decay parameters and first-row CKM unitarity tests” by A. El-Khadra et al. (Fermilab Lattice and MILC Collaborations)
Rare kaon and hyperon decays, $K^0 - \bar{K}^0$ mixing, $K \to \pi \pi$

$$K^+ \to \pi^+ \nu \bar{\nu}$$

NA62 at CERN aims at $O(10\%)$ measurement of $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$.

$$K^+ \to \pi^+ \ell^+ \ell^-$$

Well known from experiments, e.g. $\text{Br}(K^+ \to \pi^+ e^+ e^-) = 3.00(9) \times 10^{-7}$ \[^{[PDG]}\]

Long-distance dominated.

Also: $\Sigma^+ \to p^+ \mu^+ \mu^-$ at LHCb
Rare kaon and hyperon decays, $K^0 - \bar{K}^0$ mixing, $K \rightarrow \pi\pi$

- Long-distance contributions can be calculated on the lattice. Within 5-10 years, precision better than 10% expected.

- $K_L \rightarrow \mu^+\mu^-$: accurately measured and very challenging (but doable) for lattice QCD.

- Direct CP violation in $K \rightarrow \pi\pi$ decays: recently, first complete SM calculation of $\epsilon'_K$ using lattice QCD with 40% uncertainty.

- What can lattice QCD do for $D \rightarrow \pi\pi$, $D \rightarrow K\bar{K}$, where CP violation was recently discovered? Work is underway to extend finite-volume quantization conditions to $> 2$ particles.
Hadronic contributions to muon $g - 2$

Current discrepancy with SM of $\sim 3.7 \sigma$. New experiments at Fermilab and J-PARC will improve precision. Need more precise theory.

- LOI “Hadronic contributions to the anomalous magnetic moment of the muon” by C. Lehner et al. (RBC and UKQCD Collaborations)

- LOI “Using lattice QCD for the hadronic contributions to the muon $g-2$” by A. El-Khadra et al. (Fermilab Lattice and MILC Collaborations)
Hadronic contributions to muon $g - 2$

Ambitious goal for lattice QCD: 0.1% uncertainty in 5-10 years
Hadronic contributions to muon $g - 2$

Hadronic light-by-light scattering:

$q = p' - p, \nu$

First ab-initio result this year (RBC/UKQCD PRL124(2020)132002):

$$a_\mu^{HLbL} 10^{10} = 7.87(3.06)_{\text{stat}}(1.77)_{\text{sys}}$$

For this quantity it will suffice to reduce the uncertainty to $O(10\%)$ in the next 5-10 years.

[C. Lehner, https://indico.fnal.gov/event/45713]