Lattice-QCD for high-energy colliders: *progress & prospects*

Ruth Van de Water
Fermilab

Snowmass 2021 Community Planning Meeting
October 7, 2020

Motivation

- Experimental high-energy-physics community is searching for evidence of new particles and forces in numerous diverse processes spanning wide range of energies
- Revealing new physics requires reliable & precise theory
- Worldwide lattice-QCD effort targeting key hadronic parameters needed to interpret current & future experiments

quark flavor

- decay constants
- form factors
- mixing matrix elements

neutrinos

- nucleon axial-vector form factor

Higgs physics

- b, c-quark masses
- strong coupling (α_s)

muon g-2

- hadronic vacuum polarization contribution

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Aim to **reduce QCD uncertainties to at-or-below measurement errors** to maximize discovery potential of high-precision experiments!

muon g-2
- hadronic vacuum polarization contribution

Precision lattice QCD

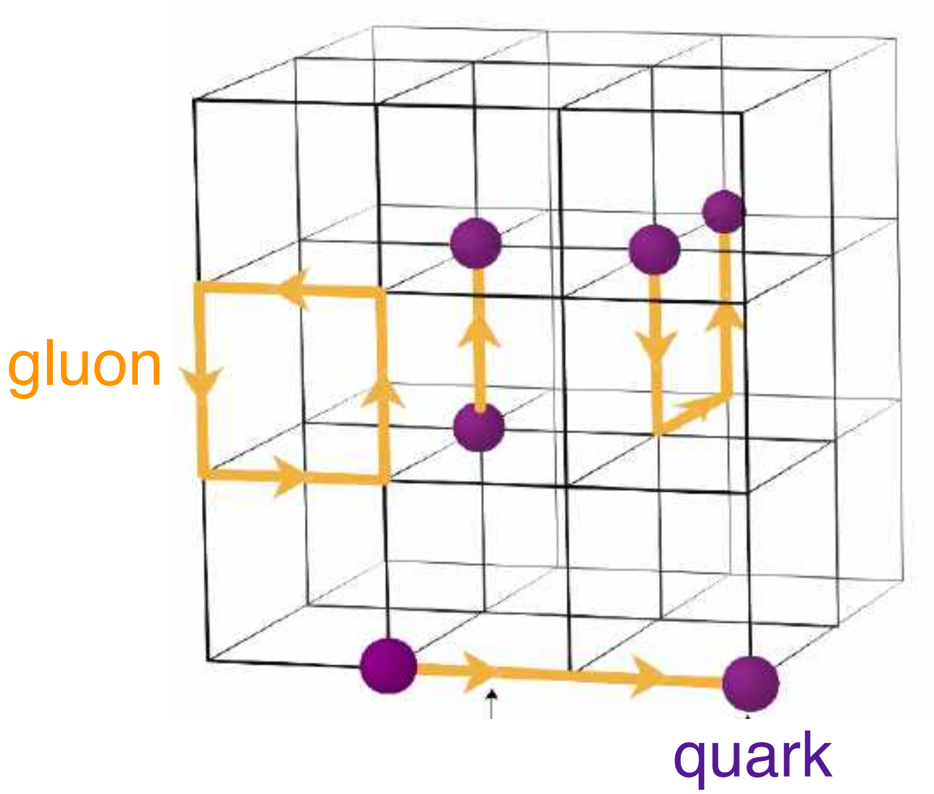
"*[An] area of striking progress has been lattice gauge theory.* ... It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. *Continuing improvements in calculational methods are anticipated in coming years.*"

– **Snowmass 2013 Executive Summary**

Lattice Quantum Chromodynamics

Systematic method for calculating hadronic parameters from **QCD first principles** with **controlled uncertainties**

- Define QCD on (Euclidean) spacetime lattice



  - Recover QCD in limits of zero lattice spacing and infinite lattice volume

- Calculate path integral numerically **Monte-Carlo methods and importance sampling**
- Run codes upon leadership-class supercomputers and dedicated clusters

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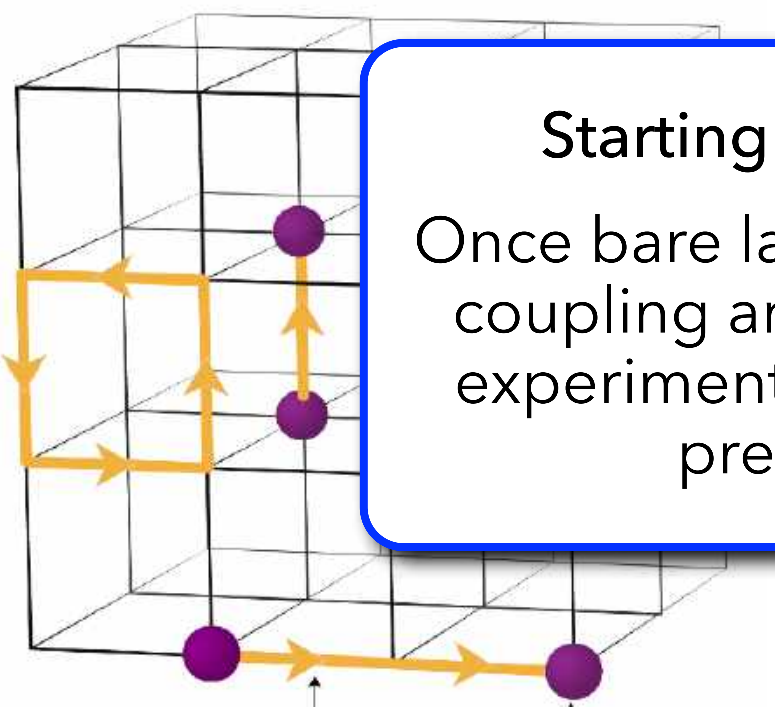

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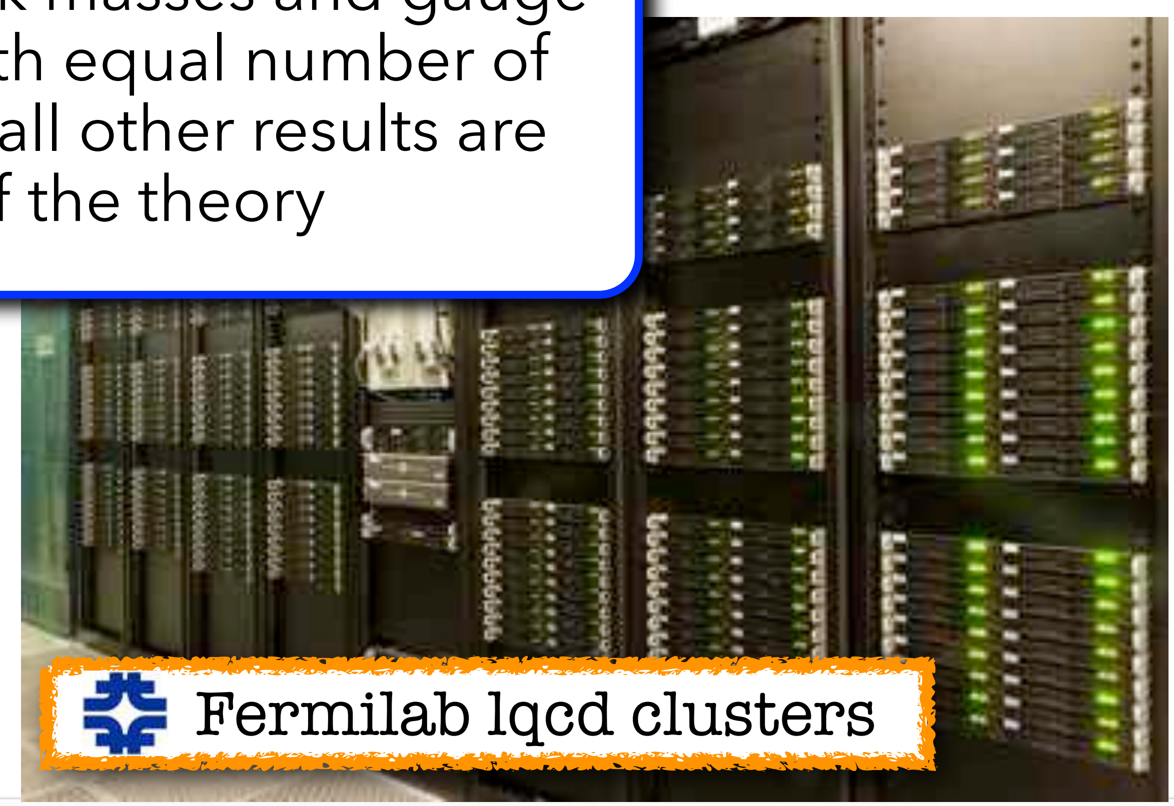
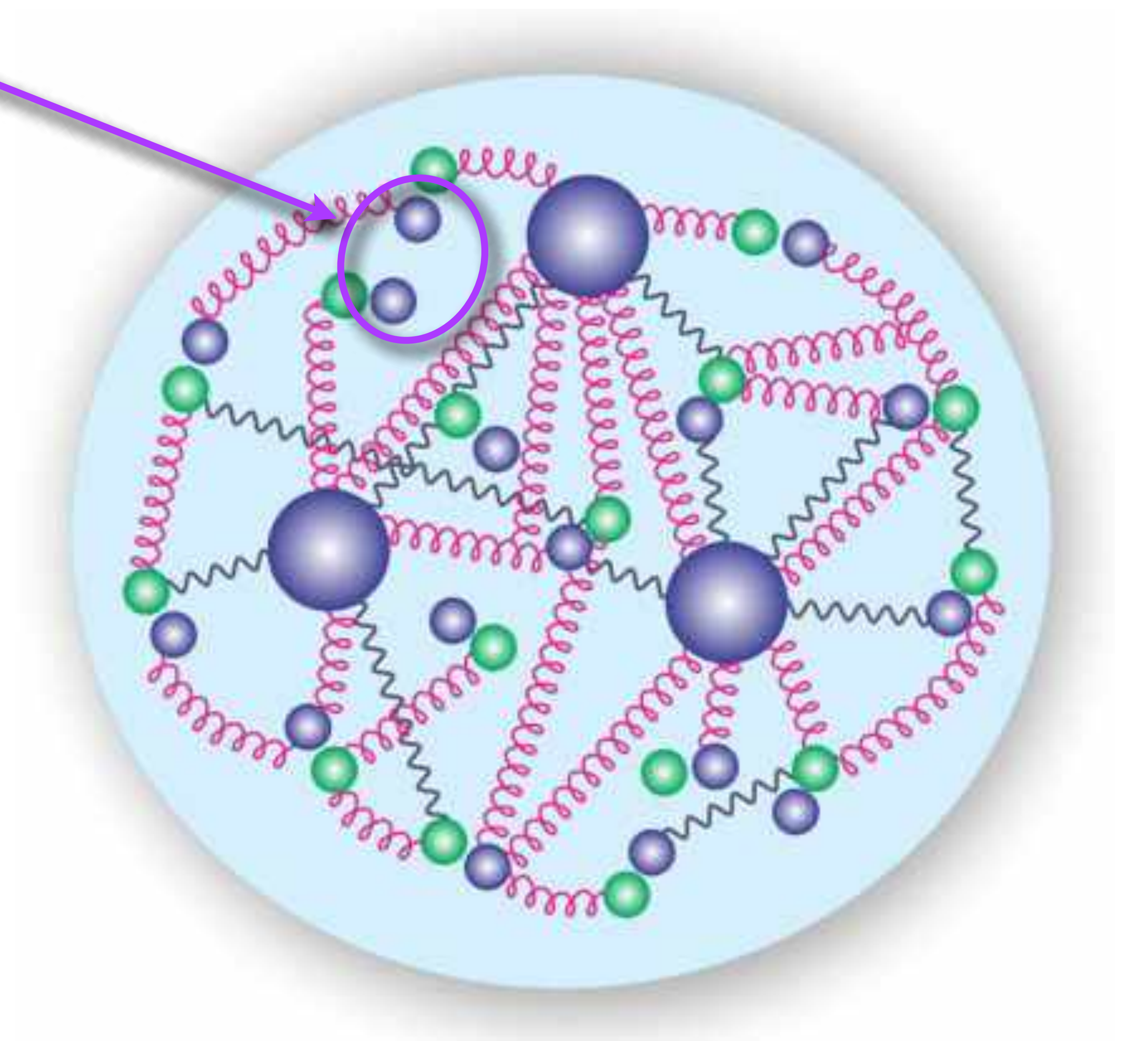


Starting point is QCD Lagrangian:

Once bare lattice quark masses and gauge coupling are fixed with equal number of experimental inputs, all other results are predictions of the theory

Modern lattice simulations

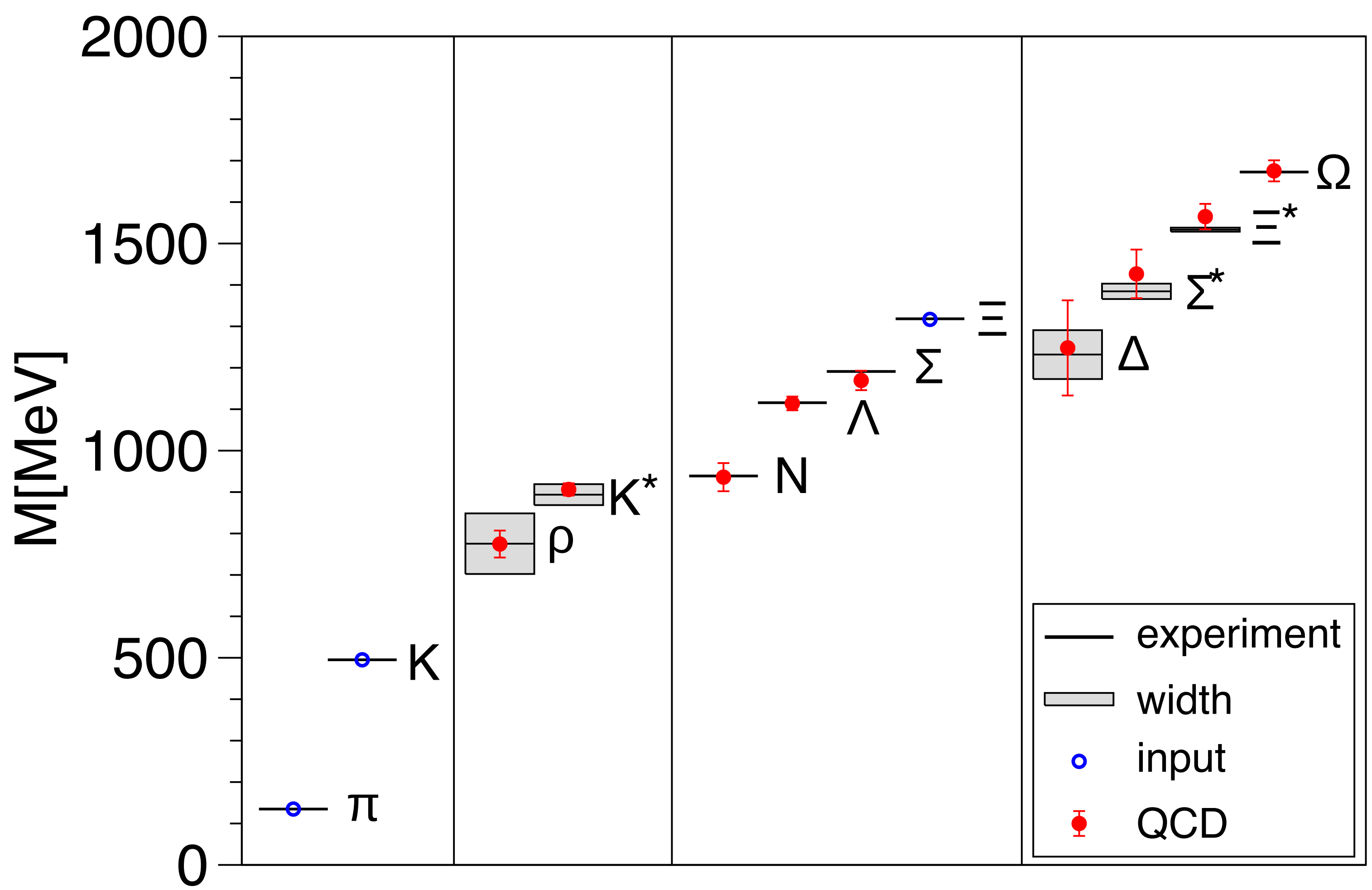
- Standard simulations include dynamical u, d, s (& c) quarks in the vacuum
    - Physical quark masses
- Control systematic errors using gauge-field ensembles with different parameters:
    - Multiple lattice spacings to extrapolate to continuum limit (a→0)
    - Multiple spatial volumes to estimate finite-size effects
- In most simulations,
    - Light-quark masses are degenerate ($m_u = m_d \equiv m_l$)
    - Quarks are electrically neutral



Validation

- Lattice QCD agrees with experiment for wide variety of hadron properties including light- and heavy-hadron spectra & proton-neutron mass difference
- Independent calculations using different methods provide corroboration for experimentally-inaccessible matrix elements and QCD parameters

Demonstrate that calculations are reliable with controlled errors!



[BMW, Science 322 (2008) 1224-1227]

Scientific impact

PHYSICAL REVIEW LETTERS **121**, 022003 (2018)

Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

T. Blum,[1] P. A. Boyle,[2] V. Gülpers,[3] T. Izubuchi,[4,5] L. Jin,[1,5] C. Jung,[4] A. Jüttner,[3] C. Lehner,[4,*] A. Portell

(RBC and UKQCD Collaborations)

PHYSICAL REVIEW D **98**, 074509 (2018)

$K^+ \to \pi^+ \nu \bar{\nu}$ decay amplitude from lattice QCD

Ziyuan Bai,[1] Norman H. Christ,[1] Xu Feng,[2,*] Andrew Lawson,[3] Antonin Portelli,[4] and Christopher

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$B^0_{(s)}$-mixing matrix elements from lattice QCD for the Standard Model and beyond

A. Bazavov, C. Bernard, C. M. Bouchard, C. C. Chang, C. DeTar, Daping Du, A. X. El-Khadra, E. D. Freeland, E. Gámiz, Steven Gottlieb, U. M. Heller, A. S. Kronfeld, J. Laiho, P. B. Mackenzie, E. T. Neil, J. Simone, R. Sugar, D. Toussaint, R. S. Van de Water, and Ran Zhou (Fermilab Lattice and MILC Collaborations)

Phys. Rev. D **93**, 113016 – Published 28 June 2016

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$\Lambda_c \to \Lambda l^+ \nu_l$ Form Factors and Decay Rates from Lattice QCD with Physical Quark Masses

Stefan Meinel

Phys. Rev. Lett. **118**, 082001 – Published 21 February 2017

Connected and Leading Disconnected Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment with a Physical Pion Mass

Thomas Blum, Norman Christ, Masashi Hayakawa, Taku Izubuchi, Luchang Jin, Chulwoo Jung, and Christoph Lehner

Phys. Rev. Lett. **118**, 022005 – Published 11 January 2017

Editors' Suggestion

Standard Model Prediction for Direct CP Violation in $K \to \pi\pi$ Decay

Z. Bai, T. Blum, P. A. Boyle, N. H. Christ, J. Frison, N. Garron, T. Izubuchi, C. Jung, C. Kelly, C. Lehner, R. D. Mawhinney, C. T. Sachrajda, A. Soni, and D. Zhang (RBC and UKQCD Collaborations)

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Phenomenology of $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ using lattice QCD calculations

Alakabha Datta,[a,b] Saeed Kamali,[a] Stefan Meinel[c,d] and Ahmed Rashed[a,e]

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☆ Mature program in quark-flavor physics includes fruitful collaborations with experiment ➡ now bringing same energy and expertise to bear on more complicated quantities that characterize nuclear structure

Lattice QCD for high-energy colliders

"Lattice QCD has [already] become an important tool in flavor physics. ...*The full exploitation of the experimental program requires continued support of theoretical developments.*" – **Snowmass 2013 Quark-flavor WG report**

LOIs relevant to the energy frontier

- ~20 **letters of intent** submitted to or cross-listed with energy frontier outlining **efforts and plans of several collaborations**
  - Lattice-QCD Determinations of Quark Masses and the Strong Coupling [Fermilab Lattice, MILC, & TUMQCD]
  - Parton distribution functions
  - Generalized PDFs and transverse-momentum dependent distributions
  - Hadron Spectroscopy with Lattice QCD [Bulava *et al.*]
  - Physics beyond the Standard Model, *e.g.* SUSY and new strong dynamics

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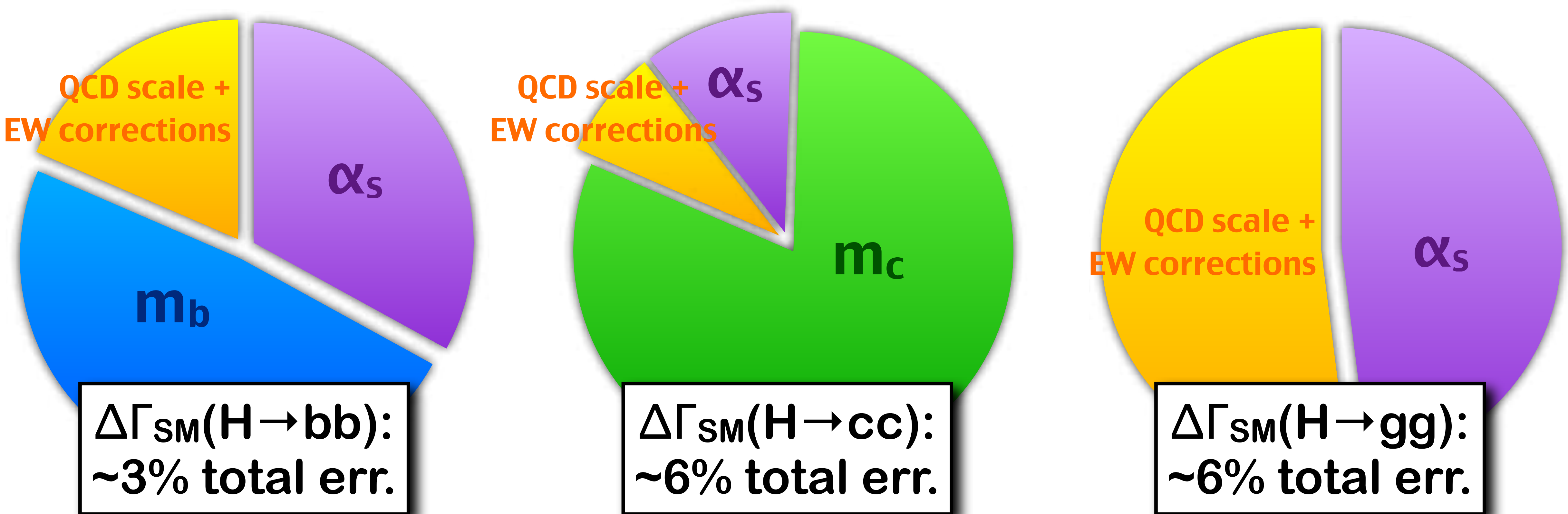
In this talk, will focus on QCD parameters and PDFs needed at the LHC and future colliders

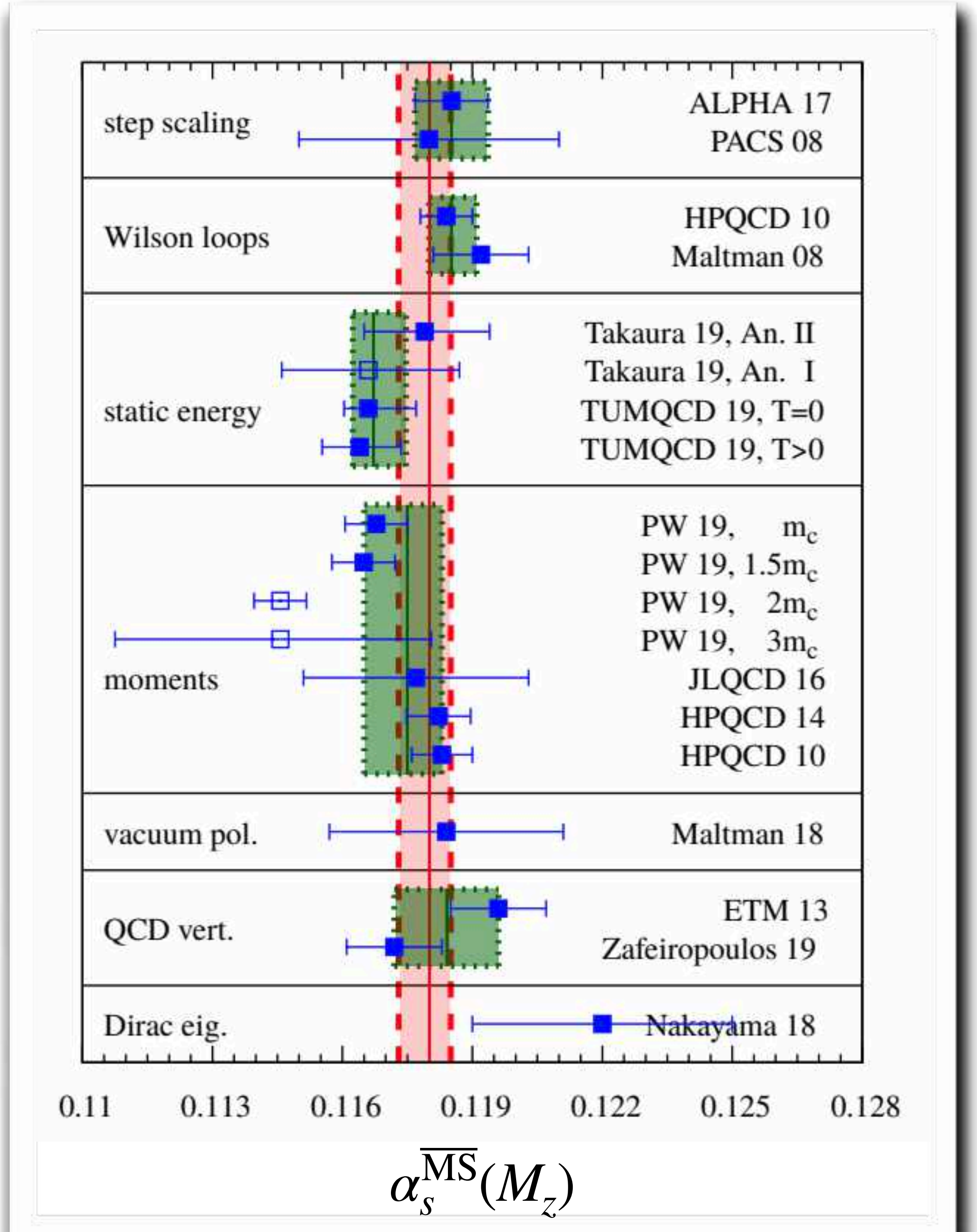
Precision Higgs physics

- **Next-generation high-luminosity colliders will measure Higgs partial widths to sub-percent precision** to look for deviations from Standard-Model expectations
  - *Full exploitation of measurements needs theory predictions with same precision*
- **Parametric errors from quark masses (m_c, m_b) & strong coupling constant (α_s) are largest sources of uncertainty in SM Higgs partial widths** for many decay modes [LHCHXSWG-DRAFT-INT-2016-008]



Quark masses and strong coupling

- **Several independent lattice-QCD methods for m_c, m_b, and α_s sensitive to different systematic uncertainties** [see [2019 FLAG Review](#) and **Komijani *et al.*** Strong coupling constant and quark masses from lattice QCD]
  - *For α_s, results are consistent, and each is more precise than determinations from fits of experimental data*
- In next five years, finer lattice spacings and improved methods will roughly halve lattice errors on m_c, m_b, and α_s, at which point the **precision will be adequate for ILC or other foreseeable future colliders**



Parton distribution functions from lattice QCD

- Parton distribution functions (PDFs) are light-cone correlation functions that cannot be directly accessed in Euclidean formulation of lattice QCD ➡ need approaches to connect matrix elements that *can* be calculated in Euclidean lattice QCD to light-cone PDFs in $\overline{\mathrm{MS}}$ scheme

- In 2013, X. Ji showed that **Euclidean-space matrix elements of fast-moving hadrons (quasi-PDFs) can be related to light-cone PDFs using Large-Momentum Effective Theory** (LaMET) [arXiv:1305.1539]

  - **quasi-PDFs $\tilde{q}$ related to PDFs via QCD factorization in the continuum:**

$$\tilde{q}_{i/H}(x, P_Z, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C^{(\tilde{q})}\left(\frac{x}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{yP_z}\right) f_{j/H}(y, \mu) + \mathcal{O}(M_h^2/P_z^2, \Lambda_{\mathrm{QCD}}^2/P_z^2)$$

where P_z is hadron momentum

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➡ Inspired flurry of new theoretical developments and lattice-QCD calculations!

Parton distribution functions from lattice QCD

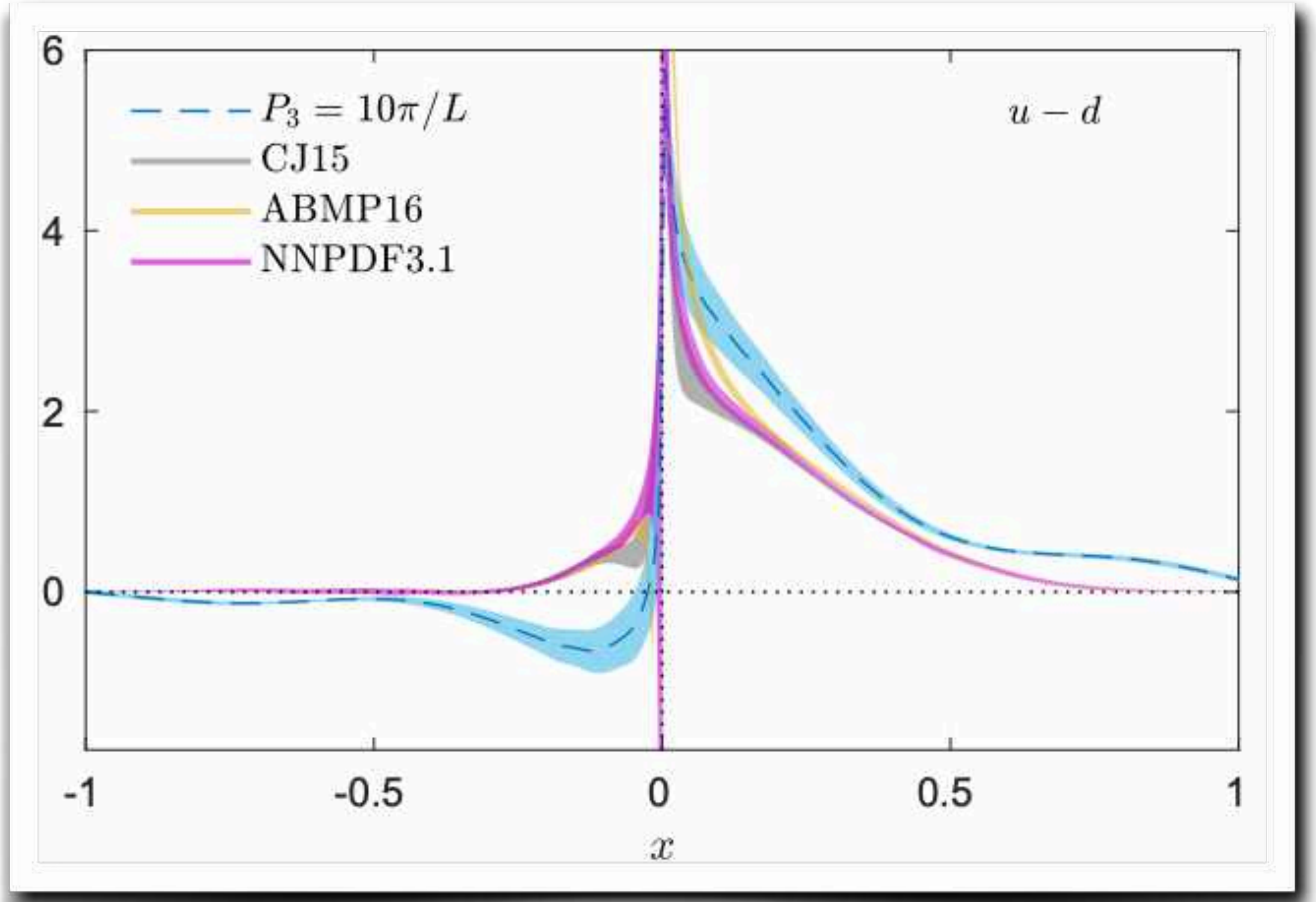
- Alternative approach to quasi-PDFs proposed by Radyushkin in arXiv:1705.01488
  - Begins with same Euclidean correlator as Ji, but instead **fixes spatial separation z between quarks and Fourier transforms over Ioffe time $\zeta = zP_z$ to obtain pseudo-Ioffe Time Distribution (pseudo-IDT)**
  - (In practice, still need several lattice momenta to provide adequate ζ range for extracting PDF)
- In fact, there exists **entire class of factorizable Euclidean-space hadronic matrix elements that can be used to extract light-cone PDFs** (called **"lattice cross sections" or LCSs** by Ma & Qiu, arXiv:1404.6860)*
  - * Similar observations made earlier in the context of pion distribution amplitudes [Braun & Muller, arXiv:0709.1348] and light-cone wave functions for exclusive B-decays [Aglietti *et al.*, hep-ph/9806277]
  - Given set of suitably chosen "lattice cross sections," can extract PDFs from global fit of lattice "data"
  - Recent lattice-QCD calculations using LCSs approach employ correlation function of two currents, in which currents are spatially separated by one or more lattice sites

Unpolarized PDF of nucleon

Alexandrou *et al.*, arXiv:1902.00587

- Employ quasi-PDF approach
- Blue error band reflects statistical errors only
- Detailed study of systematics presented → will quantify uncertainties in future work

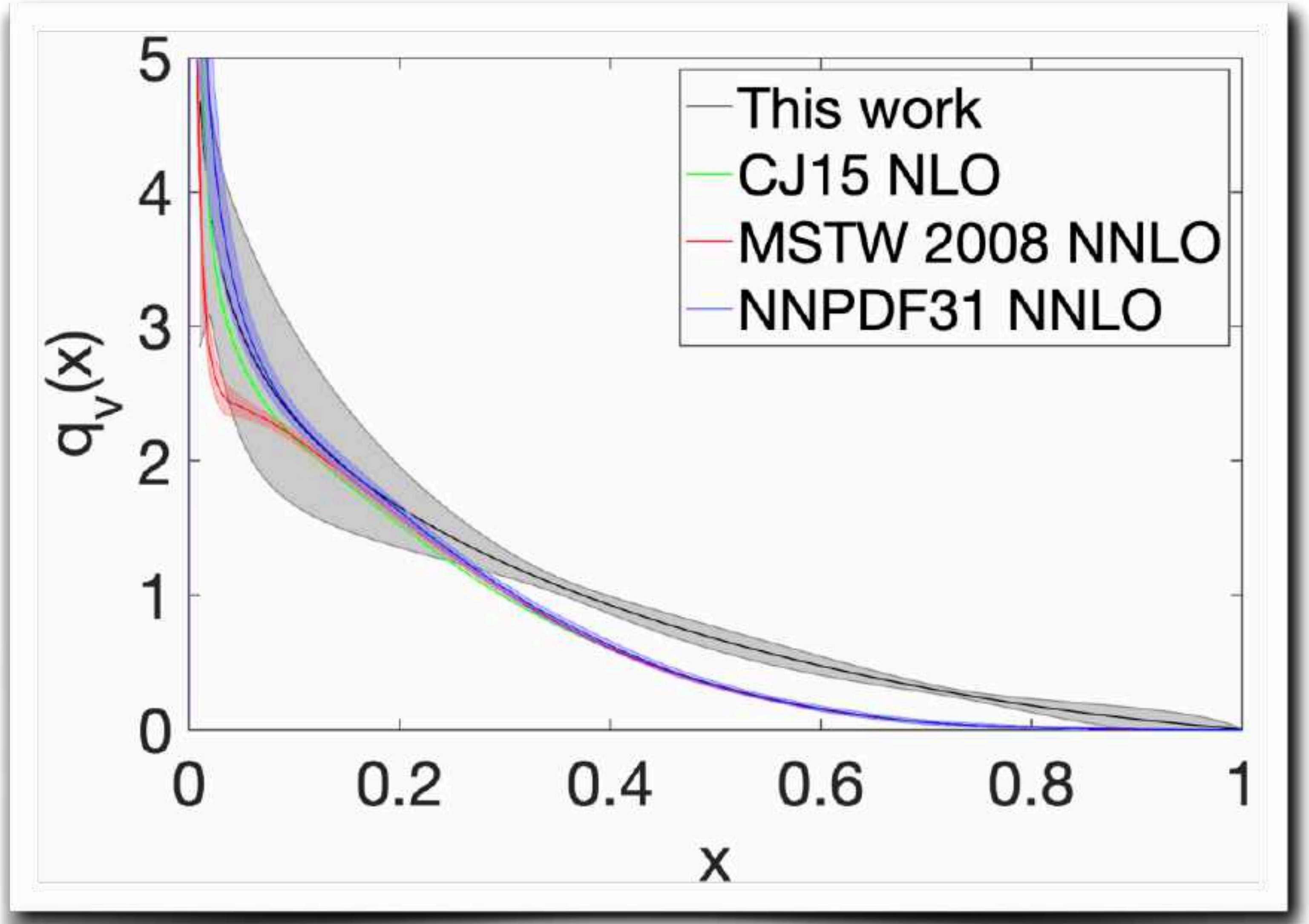
$u(x) - d(x)$



Nucleon valence distribution

Joó *et al.*, arXiv:2004.01687

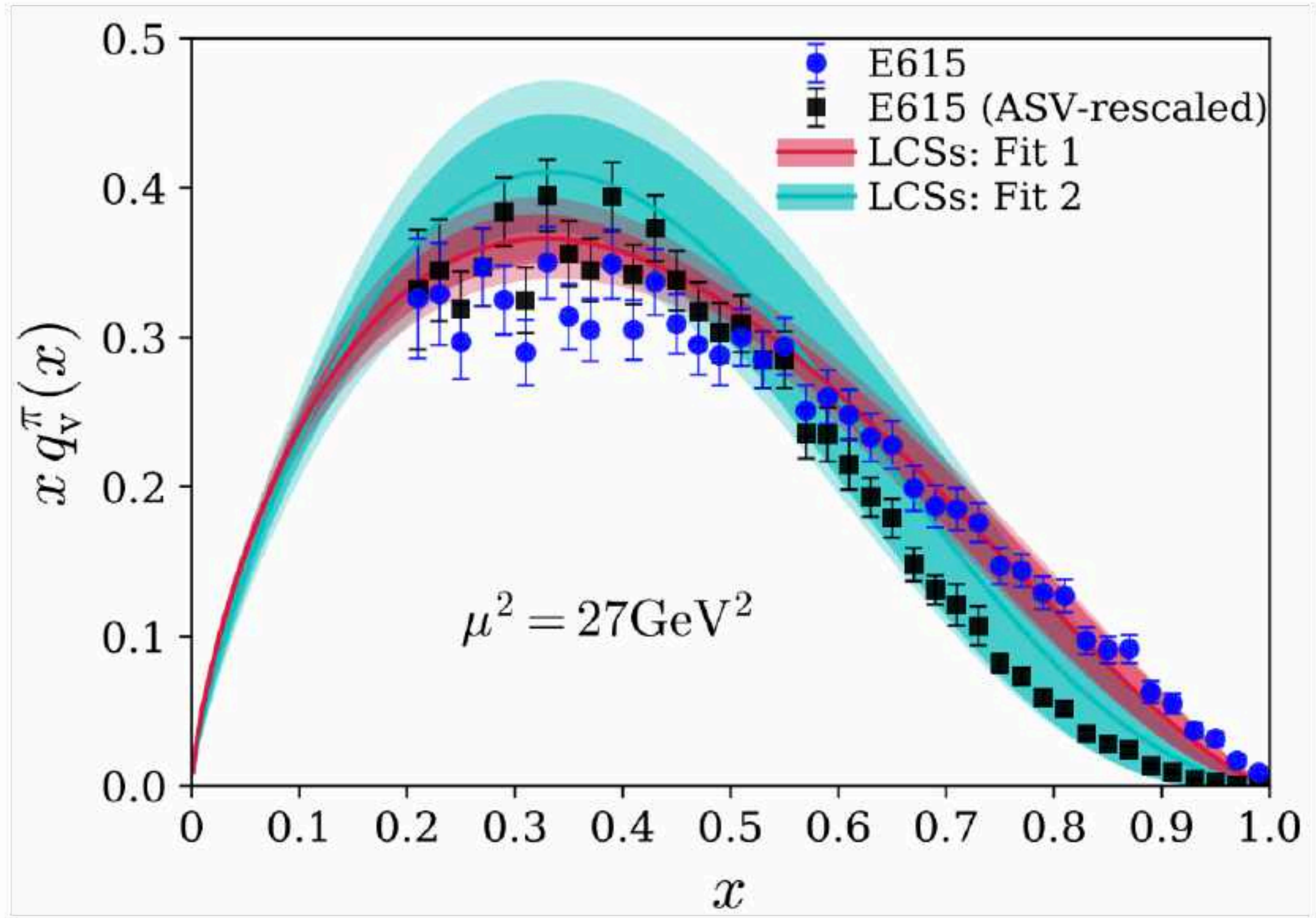
- Employ pseudo-ITD approach
- Result from single lattice spacing $a \sim 0.094$ fm
- Plan systematic study of excited-state contamination and addition of second lattice spacing to enable continuum limit



Pion distribution amplitude

Sufian *et al.*, arXiv:2001.04960

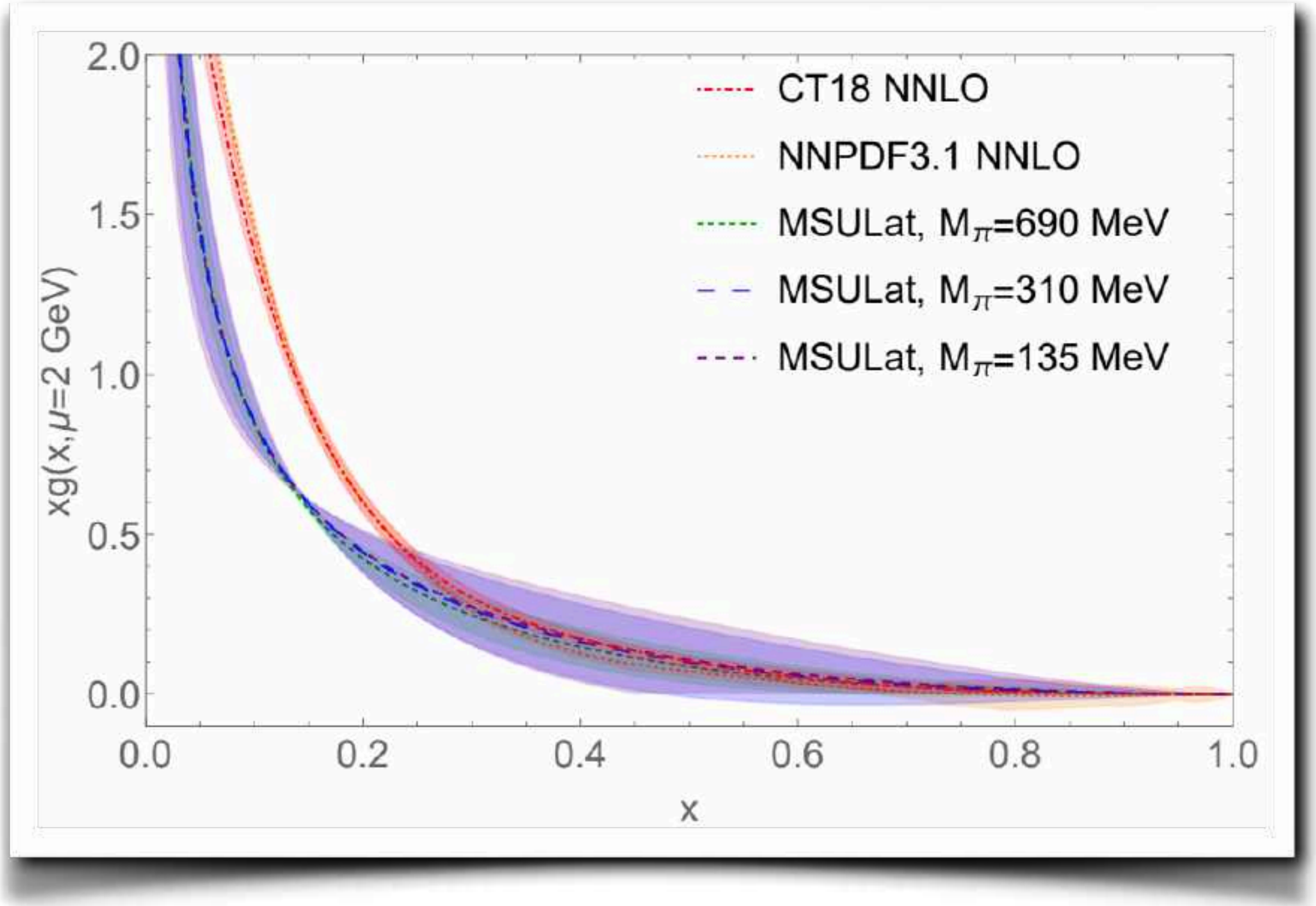
- Employ "lattice cross section" approach
- Analysis of ensemble with finer lattice spacing in future work will give access to smaller value of Ioffe time, and hence better constrain large-x region
- Finer lattice spacing will also increase data points in large Ioffe-time region due to finer momentum resolution



Unpolarized gluon pdf

Fan, Zhang, & Lin, arXiv:2007.16113

- ✦ Employ pseudo-ITD approach
- ✦ Result from single lattice spacing $a \sim 0.12$ fm
- ✦ Future plans include analysis of ensembles with finer lattice spacings to take continuum limit
- ✦ Finer lattice spacings will also allow use of larger nucleon boost momenta (*while keeping $\mathcal{O}(P_z^2 a^2)$ discretization errors under control*) to better probe small-x behavior



Systematic uncertainties

- Lattice matrix elements suffer from statistical errors, excited-state contamination, discretization and finite-volume errors, *etc…*
- Renormalization of lattice matrix elements and calculation of continuum factorization coefficients both introduce **perturbative truncation errors** *— these will be different for each quantity and method*
- QCD factorization is only approximate ➡ uncertainty in final PDFs from omitted power corrections
  - **For quasi-PDFs**, corrections of $\mathcal{O}(M_h^2/P_z^2, \Lambda_{\mathrm{QCD}}^2/P_z^2)$ ➡ must employ large hadron momenta, for which it is challenging to control statistical noise
  - **For pseudo-ITDs**, corrections of $\mathcal{O}(M_h^2 z^2, \Lambda_{\mathrm{QCD}}^2 z^2)$ ➡ must employ small spatial separations — *and therefore fine lattice spacings*

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The proof will be in the pudding:
One approach may yield the most precise results, or methods may be competitive, but *all will provide important independent checks*

Roadmap for improved precision on PDFs

- Over the next decade, lattice theorists will improve upon exploratory calculations to yield results for PDFs with all uncertainties controlled and complete error budgets
  - For many calculations, **finer lattice spacings and lighter pion masses** will enable taking continuum and chiral limits
  - *Multiple lattice volumes* to estimate of finite-volume errors
  - Simulations with **larger boosted hadron momentum or smaller spatial separations** will reduce contamination from power-law corrections to QCD factorization
  - Dedicated studies of systematic effects such as from excited-state contamination, renormalization and matching, and the inverse Fourier transform will enable quantified uncertainties and complete error budgets

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Too soon to predict PDF errors in 5 or 10 years time, but certain that *continued dedicated effort will yield both improved precision & exciting new developments*

Parting words...

"Progress in science is based on the interplay between theory and experiment, between having an idea about nature and testing that idea in the laboratory. Neither can move forward without the other."

– **Snowmass 2013 Executive Summary**

"Theoretical particle physics is one of the last area of physics to recognize the importance of computation in forefront research and continued effort is urgently required to overcome this historical bias, and create a vibrant pool of skilled young faculty, and around them their PhD students and research groups."

– **RBC/UKQCD, SNOWMASS21-TF5_TF0-CompF2_CompF4_Boyle-030.pdf**

LOIs: *Parton Distribution Functions*

- **Parton distribution functions from lattice QCD** [Boyle *et al.*, SNOWMASS21-EF6_EF7-TF5_TF0_RBC-085]
- **Precision Moments of Strange Parton Distribution Functions from Lattice QCD** [PNDME, SNOWMASS21-EF6_EF0-TF5_TF0-CompF2_CompF0-248]
- **Charm Parton Distribution Functions from Global Analysis and Lattice QCD** [Hou *et al.*, SNOWMASS21-EF6_EF0-TF5_TF0-CompF0_CompF0-247]
- **Gluon Parton Distribution Functions from Lattice QCD** [Wang *et al.*, SNOWMASS21-EF6_EF0-TF0_TF5_Jianhui_Zhang-133]
- **Small-x parton physics on lattice** [Ji *et al.*, SNOWMASS21-EF6_EF0-TF5_TF2-165]
- See also excellent reviews by **Martha Constantinou** (The x-dependence of hadronic Parton distributions: A review on the progress of lattice QCD) and **Chris Monahan** (Recent Developments in x-dependent Structure Calculations)

LOIs: *Multidimensional hadron structure*

- **Generalized PDFs** (GPDs) and **transverse-momentum dependent distributions** (TMDs) characterize hadron's 3D spatial momentum structure ➡ **needed to understand origin of nucleon mass and spin**
- **Generalized Parton Distributions from Lattice QCD** [Alexandrou *et al.*, SNOWMASS21-EF6_EF0-TF5_TF0_Jian-Hui_Zhang-151]
- **TMD PDF in large-momentum effective theory** [Liu, Zhao, & Schäfer, SNOWMASS21-TF2_TF5-043]
- **Light-front wavefunction from lattice QCD through large-momentum effective theory** [Liu, Zhao, & Schäfer, SNOWMASS21-TF2_TF5-CompF2_CompF0-044]
- **Gluon helicity and parton orbital angular momentum contribution to the proton spin** [Hatta *et al.*, SNOWMASS21-EF6_EF0-TF0_TF5_Yong_Zhao-143]
- **Towards global fits of three-dimensional hadron structure from lattice QCD** [Monahan *et al.*, SNOWMASS21-TF5_TF0-EF6_EF0-CompF2_CompF0_Chris_Monahan-021]
- **Transverse-momentum-dependent parton distributions from lattice QCD** [Ebert *et al.*, SNOWMASS21-TF5_TF2-EF6_EF0_Yong_Zhao-063]

LOIs: *Theories beyond the Standard Model*

- **Lattice field theory for conformal systems and beyond** [Carosso *et al.*, SNOWMASS21-TF3_TF5-CompF2_CompF0_Witzel_Oliver-060]
- **Composite Higgs from strong dynamics on the lattice** [LSD Collaboration, SNOWMASS21-EF2_EF8-TF8_TF5-CompF2_CompF0_Pavlos_M._Vranas-188]
- **Composite Dark Matter from strong dynamics on the lattice** [Applequist *et al.*, SNOWMASS21-CF1_CF0-TF5_TF8-CompF2_CompF0_Pavlos_M._Vranas-166]
- **Lattice Supersymmetry: successes and opportunities** [Catterall and Giedt, SNOWMASS21-TF5_TF1-CompF2_CompF0_Catterall-088]