Theory Challenges in Heavy Flavor Decays

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CPM
Session: Theory Challenges in Precision Measurements

Snowmass 2020
(October 7, 2020, zoom)

Laundry List

- Lifetimes
- Mixing
  - Width differences
- Semileptonic decays
  - determination of $V_{\rm cb}$ and $V_{\rm ub}$
  - $R(D)$ & $R(D^*)$
  - New form factors in NP
- Non-leptonic decays
  - 2-body
    - Rates
    - Strong phases (& CPV)
  - >2-body
- Rare decays $B \to \mu\mu$
  - purely leptonic, including
  - radiative: $B \to K^*\gamma, B \to X_s\gamma$
  - 3body: $B \to K^{(*)}\ell\ell$
    - LFUV: $R(K)$ & $R(K^*)$
    - angular analysis
    - anomalies
  - of charm
- ...

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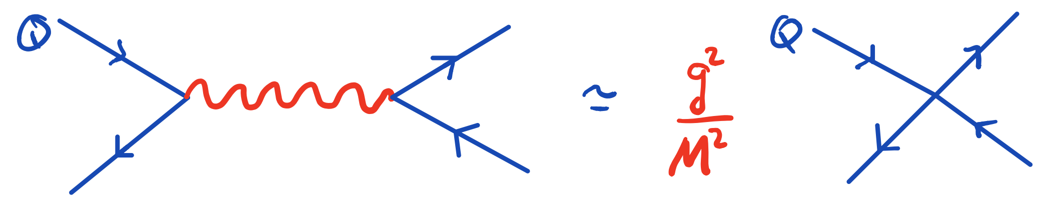
- ...

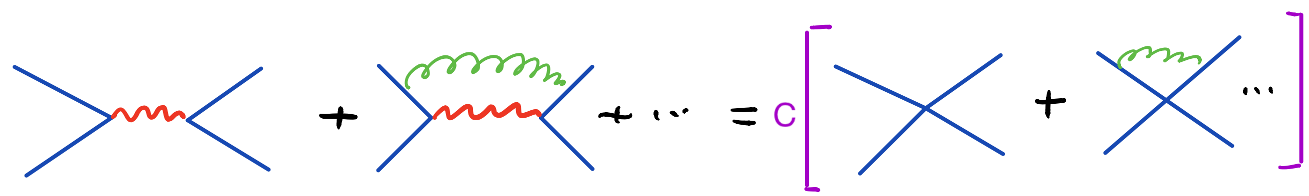
Anatomy of a calculation of decay in Heavy Flavor Physics

Multiscale problem: EFT

(i) Integrate out heavy field (t, W, Z, H) of mass $M \gg m_Q$



“Matching” beyond tree level, Wilson Coefficient $C(M)$

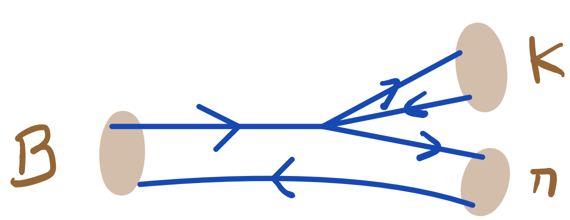


(ii) Re-sum large logs: “running”

$$C(\mu) = F(\mu/M, \alpha_s)C(M)$$

Needs anomalous dimension in (normally) one loop order higher than matching

(iii) Compute ME at $\mu \sim m_Q$



Challenges in broad terms:

- Compute matching with sufficient precision:
  - Expansion in α_s and α_{em}
  - Few percent precision means $(\alpha_s)^2$ and $(\alpha_{em})^1$
  - Matching starts at
    - Tree level: need 2-loops for precision
    - 1-loop: need 3-loops for precision !!!
  - Functional dependence on several masses (W, Z, t)

- Compute running at corresponding precision:
  - Typically one higher order than matching

$$C(\mu) = \overbrace{\Big(\underset{\substack{\uparrow \\ \text{tree level}}}{1} + \underset{\substack{\uparrow \\ \text{1-loop}}}{\alpha_s(\mu)} + \cdots\Big)}^{\text{matching}} \overbrace{\Big[1 + \underbrace{\sum_n \Big(\alpha_s \ln\frac{M}{\mu}\Big)^n}_{\text{1-loop} = \mathcal{O}(1)} + \underbrace{\alpha_s \sum_n \Big(\alpha_s \ln\frac{M}{\mu}\Big)^n}_{\text{2-loop} = \mathcal{O}(\alpha_s)} + \cdots\Big]}^{\text{running}}$$

Challenges in broad terms (continued)

- Compute matrix elements of EFT operators (and combine into decay amplitude)
  - Non-perturbative problem
  - Exclusive decay rates:
    - Lattice
      - Limited final states
        - 1 final state hadron (form factors), limited momentum range
        - 2 final state hadrons, tough
      - Limited interactions
        - Local (ie, one insertion of H_{eff})
        - EM*weak (non-local T-prod) -- tough
    - Non-systematic approaches, eg, LCSR
      - Less limited
      - Less reliable, not for precision physics
  - Inclusive decay rates:
    - HQET/OPE
      - Only semileptonic
      - Systematic (caveat: quark hadron duality at end-point)
      - Perturbative + few non-perturbative parameters
    - HQE
      - Fully inclusive: lifetimes, $\Delta\Gamma$, ...
      - Perturbative + few non-perturbative parameters
      - Relies on quark-hadron duality

Lifetimes

- Theory based on HQE, with few non-perturbative parameters:

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_q^2}\Gamma_2 + \frac{\Lambda^3}{m_q^3}\Gamma_3 + \frac{\Lambda^4}{m_q^4}\Gamma_4 + \ldots\ .$$

- Each order has perturbative expansion

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \ldots\ .$$

- Non-perturbative MEs can be independently fixed (or jointly fit) from various sources:
  - spectroscopy
  - direct calc: lattice, QCDSR
  - moments of semileptonic inclusive decay spectrum

1405.3601

B_1	B_2	ϵ_1	ϵ_2	
1.01 ± 0.01	0.99 ± 0.01	-0.08 ± 0.02	-0.01 ± 0.03	1997 QCD − SR
1.06 ± 0.08	1.01 ± 0.06	-0.01 ± 0.03	-0.01 ± 0.02	1998 Lattice
0.96 ± 0.04	0.95 ± 0.02	-0.14 ± 0.01	-0.08 ± 0.01	1998 QCD − SR
1.10 ± 0.20	0.79 ± 0.10	-0.02 ± 0.02	0.03 ± 0.01	2001 Lattice

- Limitation: not based on OPE in Euclidean space
  - quark-hadron duality assumed
  - OPE-like expansion (a.k.a. HQE) performed on-shell
    - no external large (euclidean) momentum
      - control?
      - organization of OPE?
  - Justification: It works! (For ratios)
  - Question: so what when it fails? What about overall normalization?

taken from 1405.3601

PDG:

	$B_d = (\bar{b}d)$	$B^+ = (\bar{b}u)$	$B_s = (\bar{b}s)$	$B_c^+ = (\bar{b}c)$
Mass (GeV)	5.27955(26)	5.27925(26)	5.3667(4)	6.2745(18)
Lifetime (ps)	1.519(5)	1.638(4)	1.512(7)	0.500(13)
$\tau(X)/\tau(B_d)$	1	1.076 ± 0.004	0.995 ± 0.006	0.329 ± 0.009

Theory:

$$\frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 1 + 0.03 \left(\frac{f_{B_d}}{190.5\,\mathrm{MeV}} \right)^2 [(1.0 \pm 0.2) B_1 + (0.1 \pm 0.1) B_2$$
$$-(17.8 \pm 0.9)\epsilon_1 + (3.9 \pm 0.2)\epsilon_2 - 0.26]$$
$$= 1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01\,.$$

$$\frac{\tau(B_s)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 1.003 + 0.001 \left(\frac{f_{B_s}}{231\,\mathrm{MeV}} \right)^2 [(0.77 \pm 0.10) B_1 - (1.0 \pm 0.13) B_2$$
$$+(36 \pm 5)\epsilon_1 - (51 \pm 7)\epsilon_2]$$
$$= 1.001 \pm 0.002\,.$$

	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Mass (GeV)	5.6194(6)	5.7918(5)	5.79772(55)	6.071(40)
Lifetime (ps)	1.451(13)	1.477(32)	1.599(46)	$1.54\left(^{+26}_{-22}\right)$
$\tau(X)/\tau(B_d)$	0.955 ± 0.009	0.972 ± 0.021	1.053 ± 0.030	$1.01\left(^{+17}_{-14}\right)$

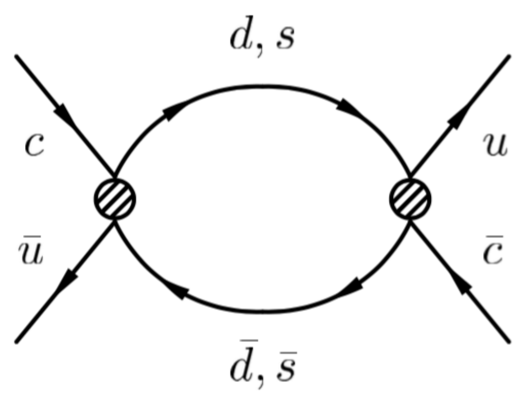
$$\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 1 - (0.8 \pm 0.5)\%_{\frac{1}{m_b^2}} - (4.2 \pm 3.3)\%^{\Lambda_b}_{\frac{1}{m_b^3}} - (0.0 \pm 0.5)\%^{B_d}_{\frac{1}{m_b^3}} - (1.6 \pm 1.2)\%_{\frac{1}{m_b^4}} = 0.935 \pm 0.054\,,$$

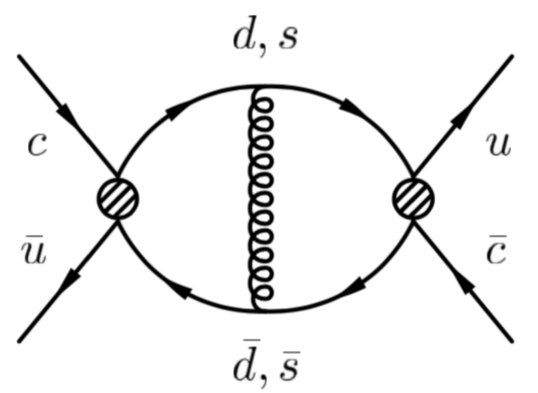
$$\frac{\tau(\Lambda_b)}{\tau(\Xi_b^+)}^{\mathrm{LHCb\,2014}} = 0.918 \pm 0.028\,, \qquad \frac{\tau(\Lambda_b)}{\bar{\tau}(\Xi_b^+)}^{\mathrm{HQE\,2014}} = 0.95 \pm 0.06\,.$$

charm

	$D^0 = (\bar{u}c)$	$D^+ = (\bar{d}c)$	$D_s^+ = (\bar{s}c)$
Mass (GeV)	1.86491(17)	1.8695(4)	1.9690(14)
Lifetime (ps)	0.4101(15)	1.040(7)	0.500(7)
$\tau(X)/\tau(D^0)$	1	2.536 ± 0.017	1.219 ± 0.017

isospin violation is large!
it's from:





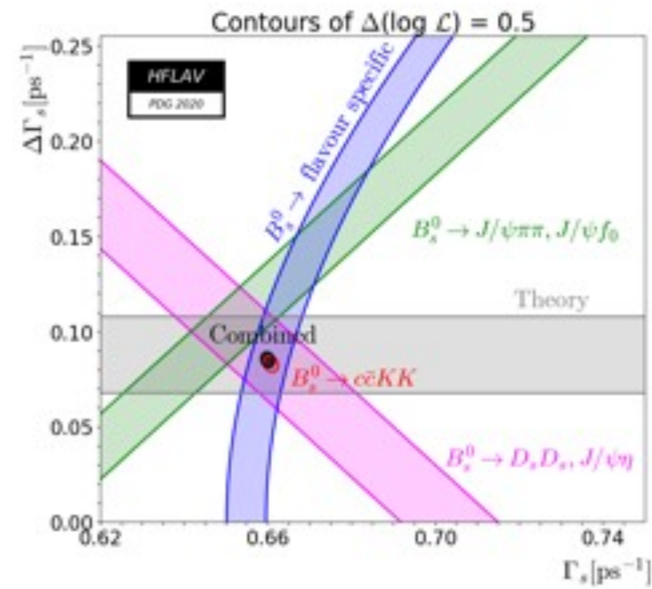
(which must stay small for beauty)

$$\frac{\tau(D^+)}{\tau(D^0)}^{\mathrm{HQE\,2013}} = 2.2 \pm 0.4^{(\mathrm{hadronic})}{}^{+0.03(\mathrm{scale})}_{-0.07} \,,$$

$$\frac{\tau(D_s^+)}{\tau(D^0)}^{\mathrm{HQE\,2013}} = 1.19 \pm 0.12^{(\mathrm{hadronic})}{}^{+0.04(\mathrm{scale})}_{-0.04}$$

which brings us to

Mixing: $\Delta\Gamma$



$$\Delta\Gamma_q = 2|\Gamma_{12}^q|\cos(\phi_q)\ , \quad \phi_q = \arg\left(-M_{12}^q/\Gamma_{12}^q\right).$$

- Again computed in HQE,

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \ldots\ .$$

from annihilation and PI diagrams (previous slide)

- B_s: Dominated by single (cc) diagram by clever use of CKM unitarity

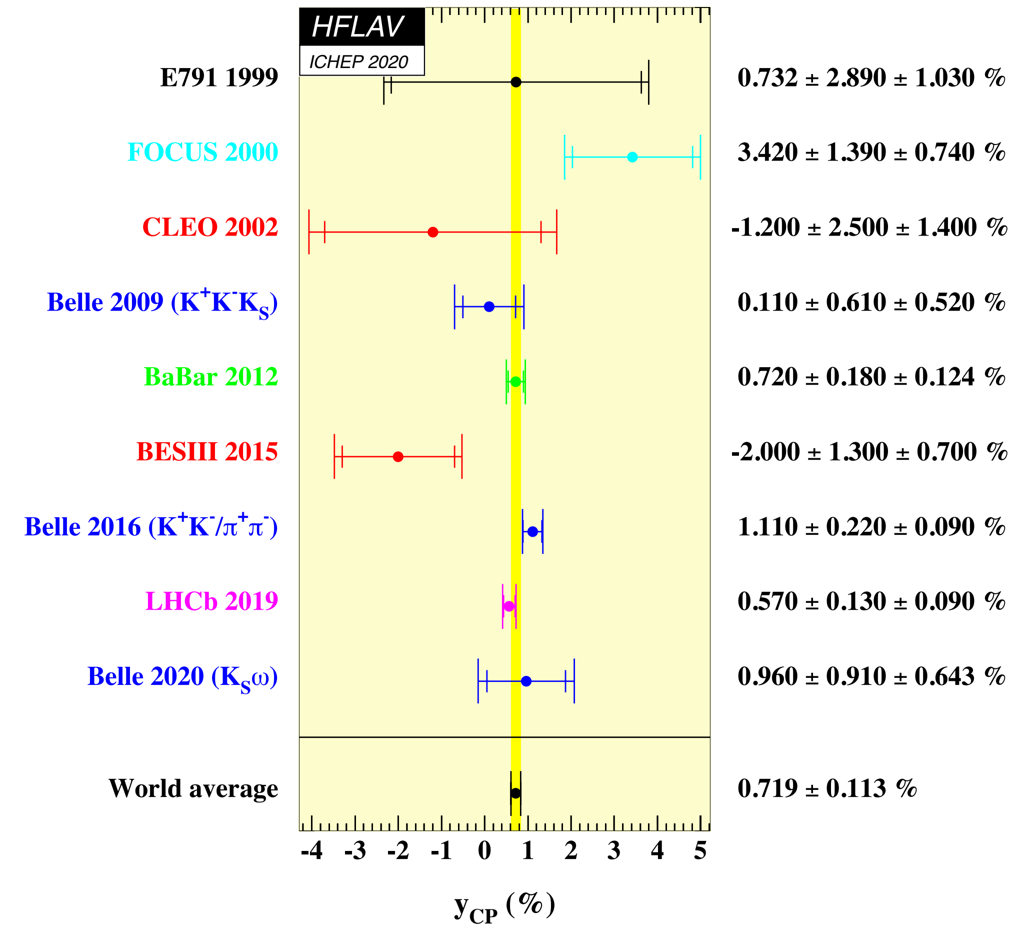
$$-\frac{\Gamma_{12}^s}{M_{12}^s} = \frac{\Gamma_{12}^{s,cc}}{\tilde{M}_{12}^s} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}}{\tilde{M}_{12}^s} + \left(\frac{\lambda_u}{\lambda_t}\right)^2\frac{\Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc} + \Gamma_{12}^{s,uu}}{\tilde{M}_{12}^s}$$

- Combining HQE with lattice MEs (1910.00970)

$$\Delta\Gamma_s = [1.86(17)B_1 + 0.42(3)B_3']f_{B_s}^2 + \Delta\Gamma_{1/m_b} = 0.092(14)\ \mathrm{ps}^{-1}\ .$$

- Caveat: duality again, but worse: overall normalization.
  - In QCD in 1+1 dim at large N_c, there is a $(1/m_Q)^1$ correction
    - The $(1/m_Q)^1$ correction is oscillatory
    - $\Gamma(m)$ smeared over m (centered at m_Q) over a region of size at least a few resonances has no $(1/m_Q)^1$ correction
  - This may happen in 3+1 at $N_c = 3$.
    - Unknown, incalculable magnitude of effect
    - Possibly light quark mass independent - common shift, to normalziation: absent in ratios

Mixing: ΔΓ



- Charm difficulties/ additional challenge

$$x = \frac{\Delta M}{\Gamma}\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma}\,.$$

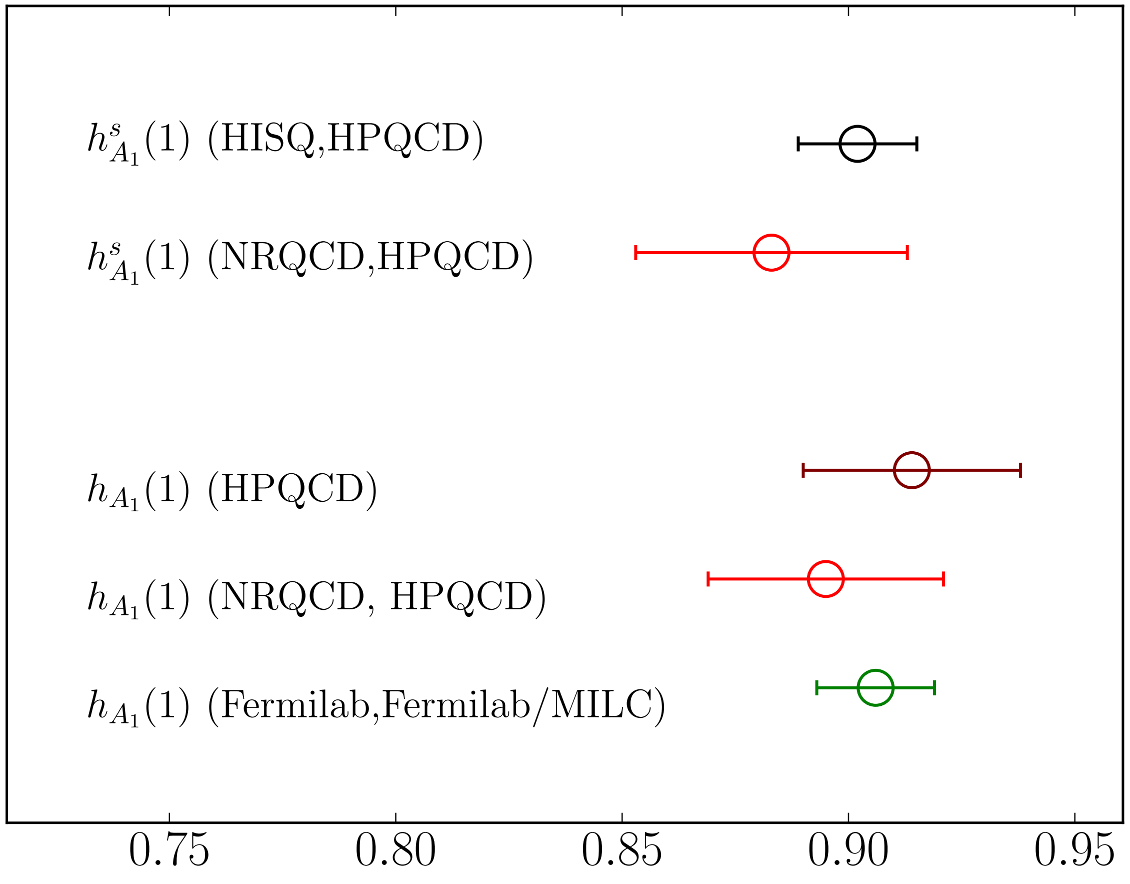
  - Leading HQE term: almost perfect GIM cancellation
  - Possibly dominated by much higher terms, eg, Γ_6
  - Alternative approaches? Sum over states using U-spin and data (Petrov 1312.5304)

Semileptonic decays

see 2006.07287 (Mainz workshop 2018)

Exclusive vs Inclusive: V_{cb}

1904.02046



Exclusive:

- Largely 2 modes: $B \to D\ell\nu$ and $B \to D^*\ell\nu$
  - but also $\Lambda_b \to \Lambda_c\ell\nu$
- Theory: form factors
  - Fixed at endpoint by HQET (+corrs); but need
    - better precision
    - form away from $q^2_{\max}$
  - Lattice: region close to $q^2_{\max}$
  - z-expansion: constrains extrapolation to small q^2
    - BGL vs CLN

A consensus of the workshop recommends that CLN no longer be used, ... (Mainz-2018)

- R(D), R(D*) derivative
  - Additional form factors

Inclusive:

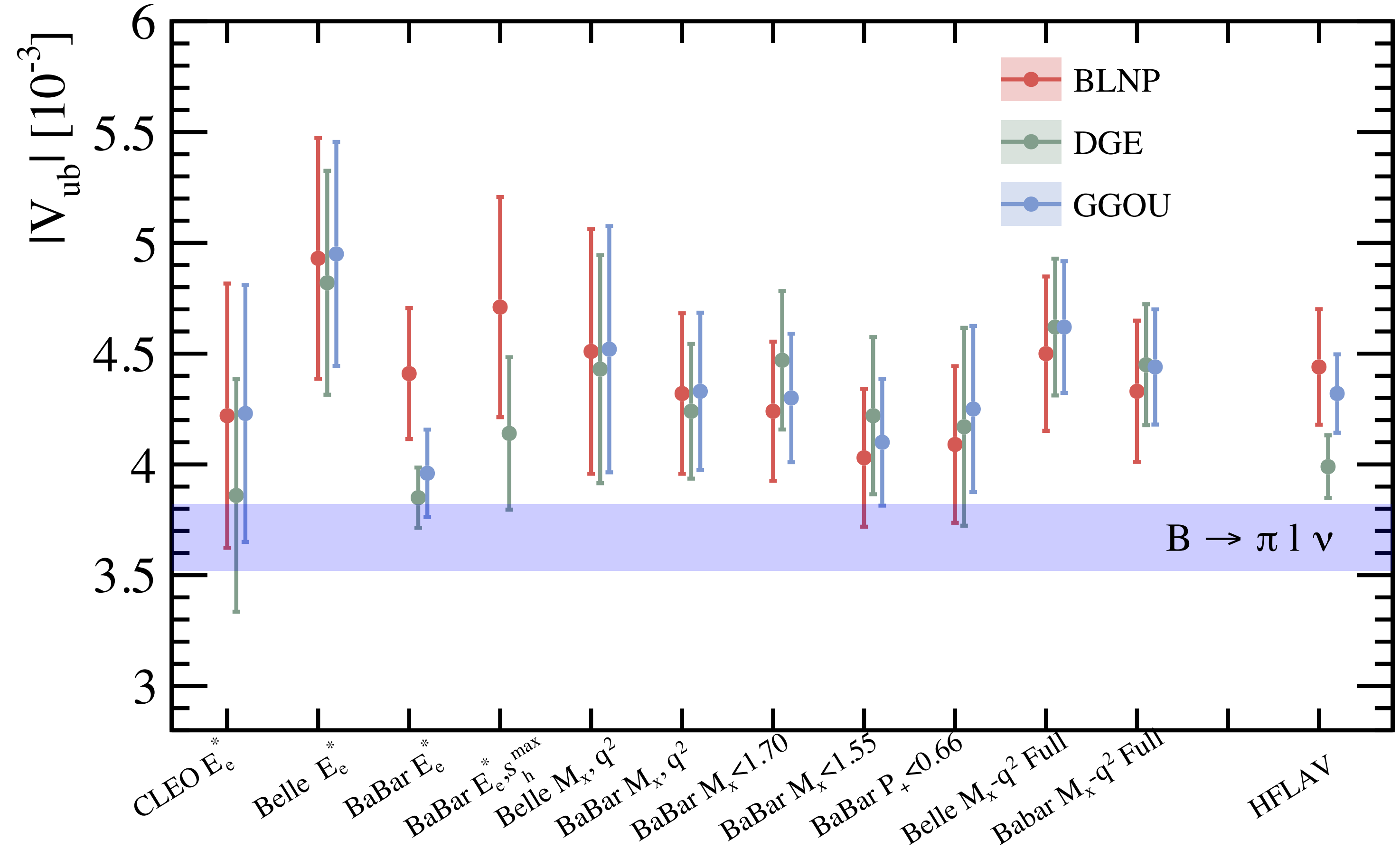
- OPE (almost) well justified
  - quark-hadron duality near endpoint
- Method of moments
- No $1/m_Q$ corrections
- Expansion to $\alpha_s \times (1/m_Q)^2, (1/m_Q)^3$

Exclusive vs Inclusive: V_{ub}

2006.07287

Exclusive:

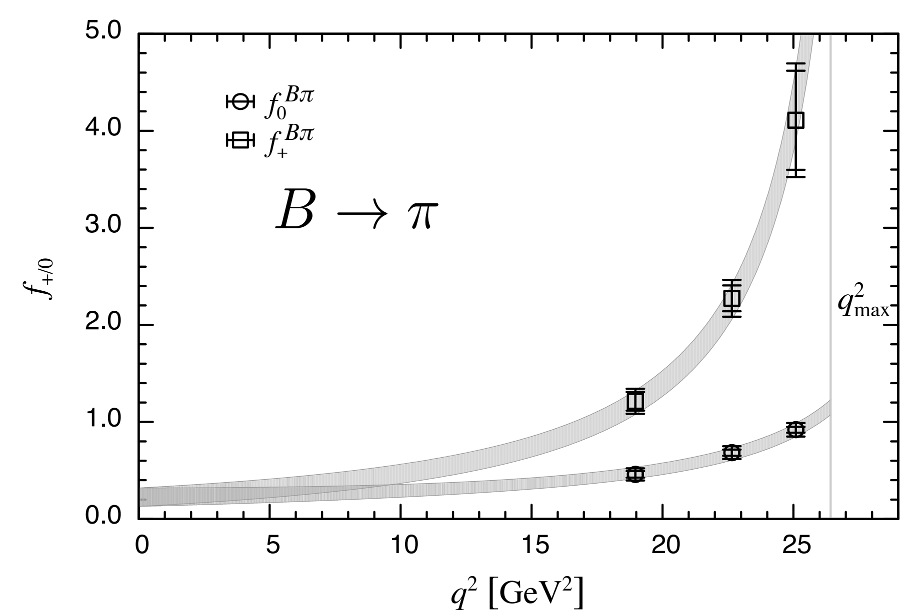
- Experimentally clean, *e.g.*, fully reconstructed
- Statistics limited
- Various processes, *ie*, $B \to \pi, B \to \rho, B_s \to K, \ldots$
- Theory: form factors
  - Lattice: near $q^2_{\max}$
  - Rate $\sim p_\pi^3 \approx (q^2_{\max} - q^2)^{3/2}$
  - Extrapolation needed (z-expansion)



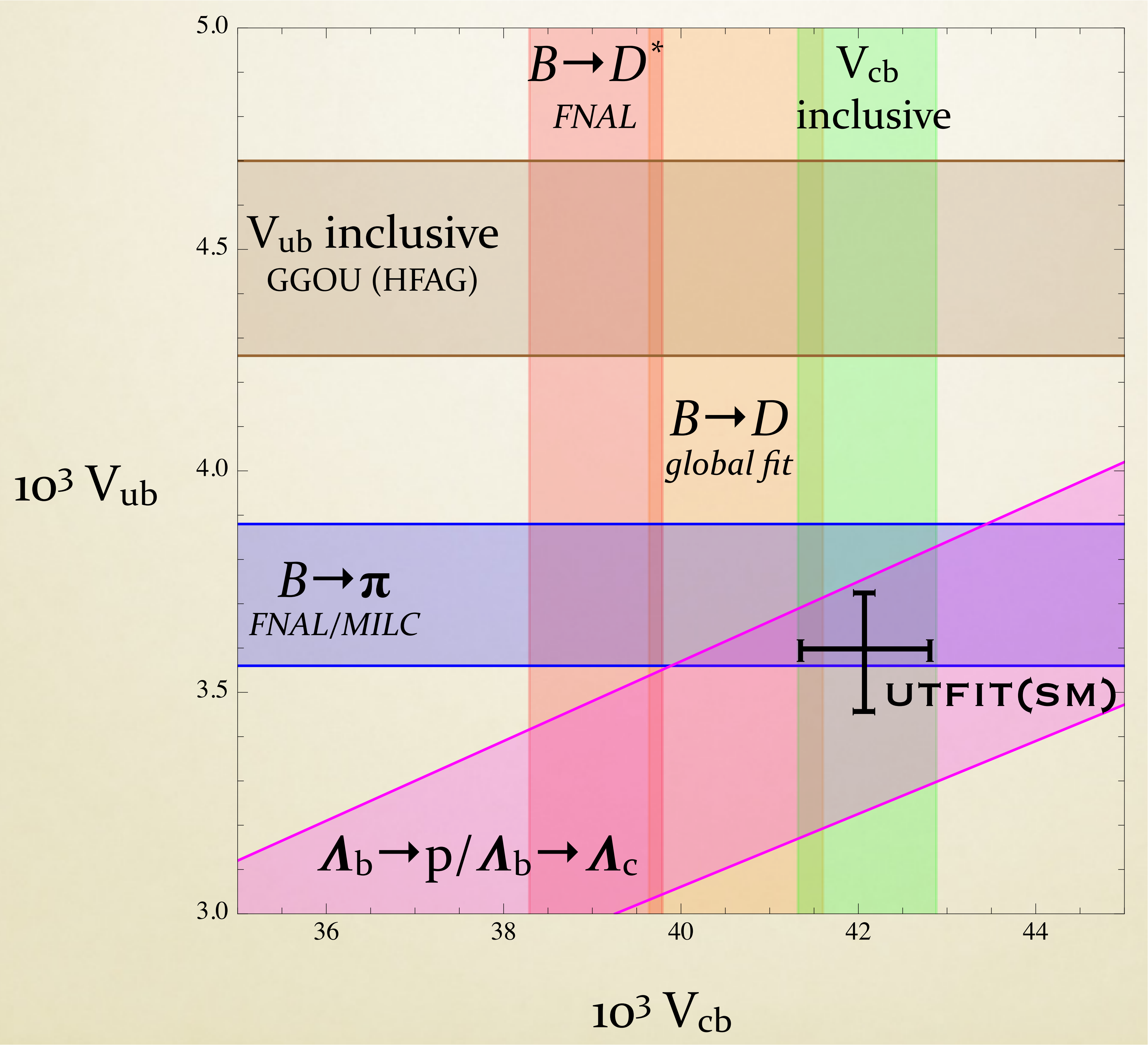
Inclusive:

- In principle, OPE as in b to c
- Large charm background
- Tight cuts needed, E_e or q^2 or M_X
- OPE breaks down ("non-local OPE")
- Some modeling required: BLNP, DGE, GGOU (ADFR, fell out of favor, not in PDG; why? Is it because it disagrees? agrees better with exclusives)
- Systematics?

1501.05373



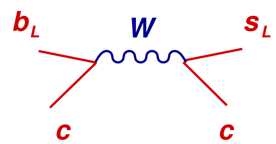
Gambino, @ Beauty 2016



Rare Decays

Effective field theory approach to $b \to s\ell\ell$ decays

- **CC** (Fermi theory):

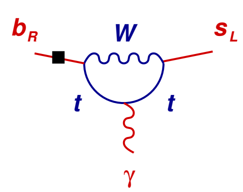


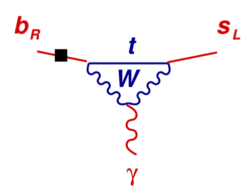
$\Rightarrow$

Wilson Coefficients convention:

$$G_F\, V_{cb}\, V_{cs}^*\, C_2\, \bar{c}_L \gamma^\mu b_L\, \bar{s}_L \gamma_\mu c_L$$

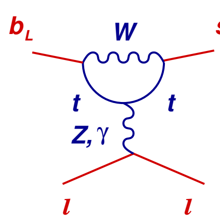
- **FCNC**:

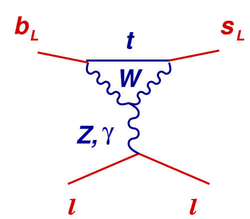


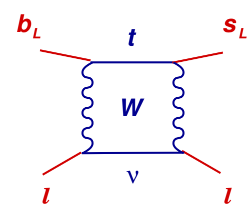


$\Rightarrow$

$$\frac{e}{4\pi^2}\, G_F\, V_{tb}\, V_{ts}^*\, m_b\, C_7\, \bar{s}_L \sigma_{\mu\nu} b_R\, F^{\mu\nu}$$







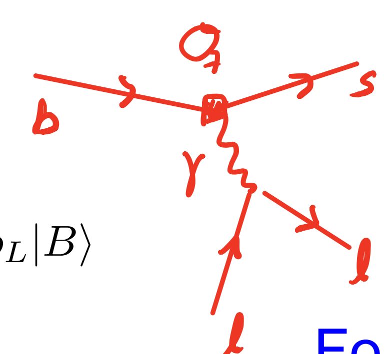
$\Rightarrow$

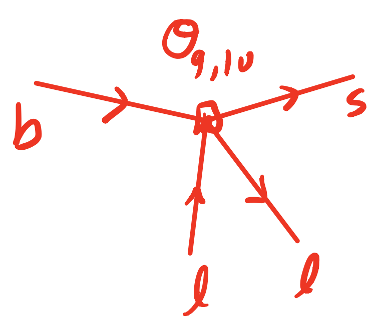
$$G_F\, V_{tb}\, V_{ts}^*\, \frac{\alpha}{4\pi}\, C_{9(10)}\, \bar{s}_L \gamma^\mu b_L\, \bar{\ell} \gamma_\mu (\gamma_5) \ell$$

  - Wilson coefficients $C_k(\mu)$ calculated in P.T. at $\mu = m_W$ and rescaled to $\mu = m_b$

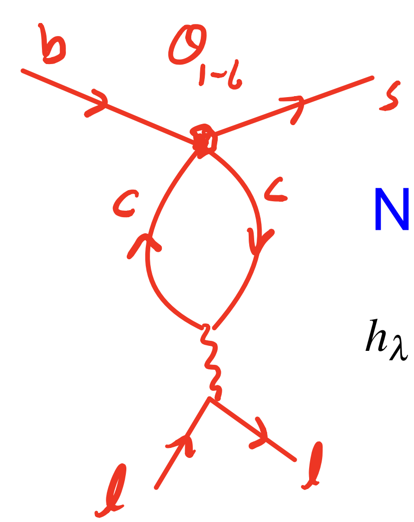
Matrix Elements (without dressing into mesons, for clarity)

$\langle \bar{K}^* | \bar{s}_L \gamma^\mu b_L | B \rangle$





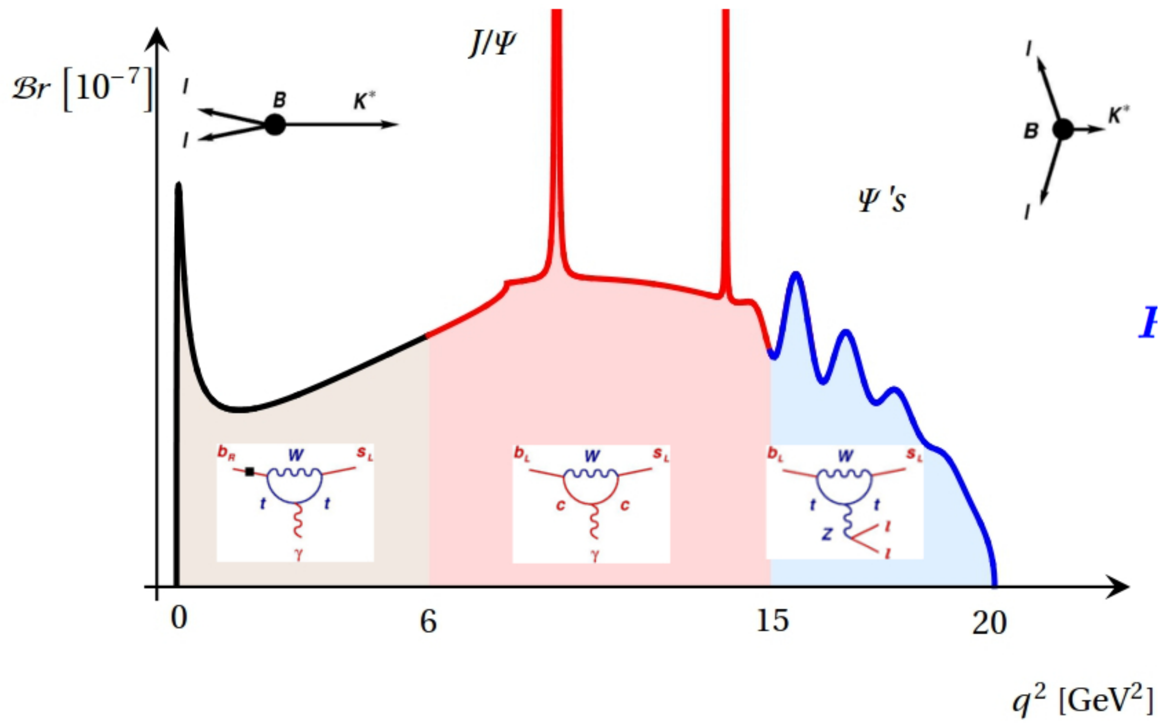
Form factors



Non-local

$$h_\lambda(q^2) = \frac{\epsilon_\mu^*(\lambda)}{m_B^2} \int d^4x e^{iqx} \langle \bar{K}^* | T\{ j^\mu_{\rm em}(x) \mathscr{H}_{\rm eff}^{\rm had}(0) \} | \bar{B} \rangle$$

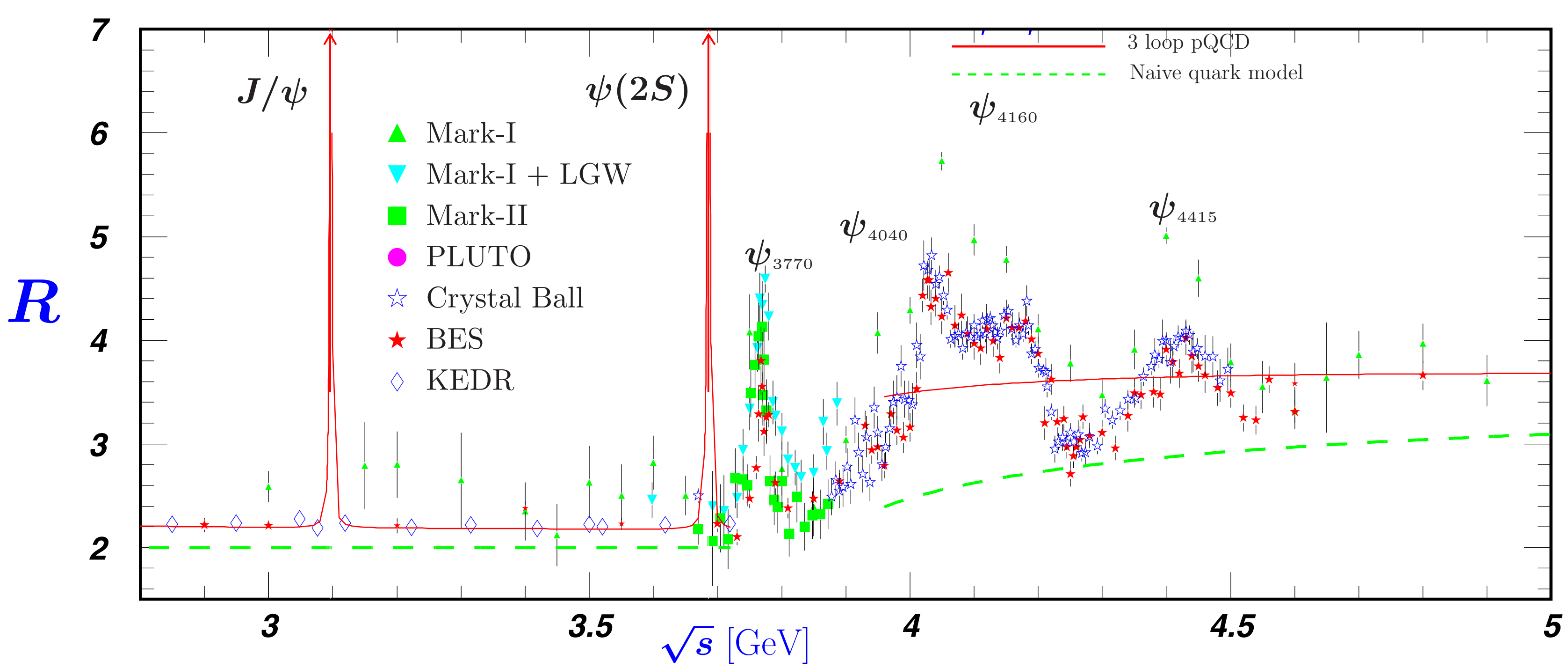
The challenge of precision: effect of non-local term



To be sure, precise form factors (FFs) needed too
In absence of non-local term, HQET +SU(3) symmetry relates FFs to semileptonic decays: double ratios good to few percent

$$\frac{\Gamma(B \to K\ell\ell)/\Gamma(B \to \pi\ell\nu)}{\Gamma(D \to K\ell\nu)/\Gamma(D \to \pi\ell\nu)}$$

Compare with R:



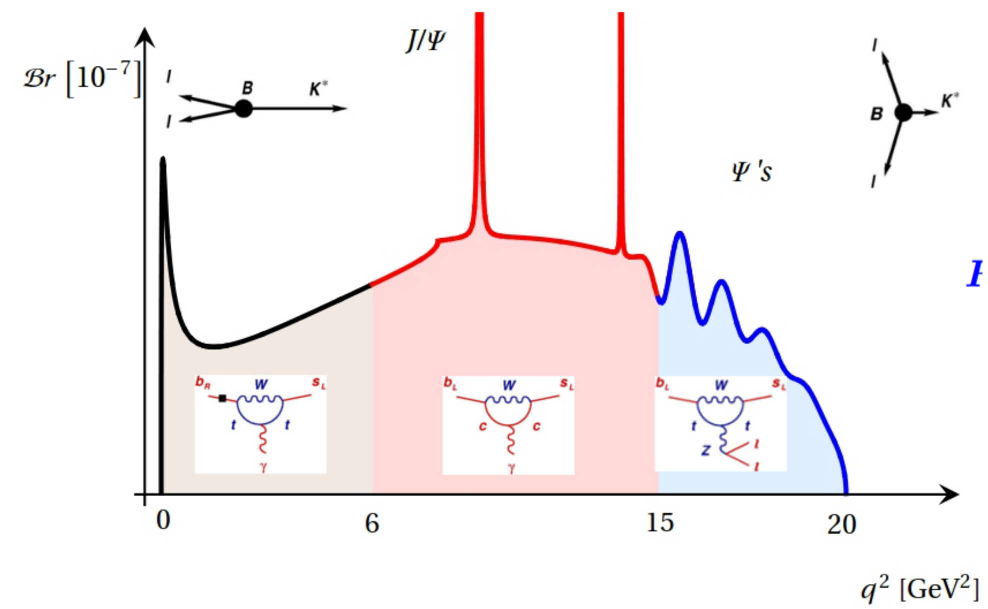
Perturbative QCD works!

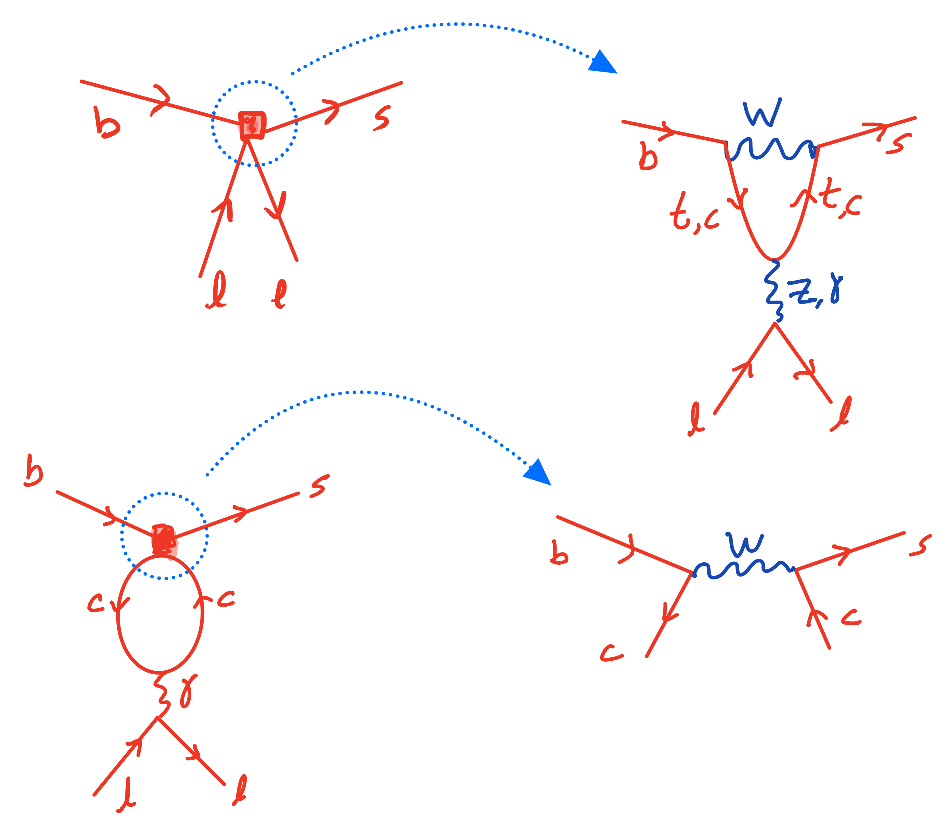
But for R we have an OPE

Not so for $O_{1\text{-}6}$ in B decay

- For small q^2 the form factor (ie local) contributions to $B \to K^*ll$ are formally more important than the non-local ones (hep-ph/0106067)

- Non-local term (aka "non-factorizable") power suppressed.
  - Dominant in resonant region
  - How small q^2 before negligible (if at all?)
  - Only estimate: LCSR. (1006.4945)
    - For $q^2 \ll 4m_c^2$
    - Dispersion relation to extend to larger q^2
    - Models resonances, no strong phases
  - Model by sum of resonances: (1709.03921)
    - Sum Breit-Wigner, data driven
    - Consider strong phases
      - Small: agree with LCSR
      - Large: quite different
  - Parametrized ignorance: (1809.03789)
    - Expand in powers of q^2
    - Fit to data
    - Order of magnitude as expected
    - "The constraining power of $B \to K^*\mu\mu$ on New Physics (NP) is lost, as some coefficients of the h_λ expansion are indistinguishable from NP contributions"





Conclusions.

Challenges: See above (tough perturbative matching/running, hard non-perturbative MEs, non-systematic approaches sneak in by force of being accustomed and lore)

Left out: the challenge of interpretation of deviations from the SM

End