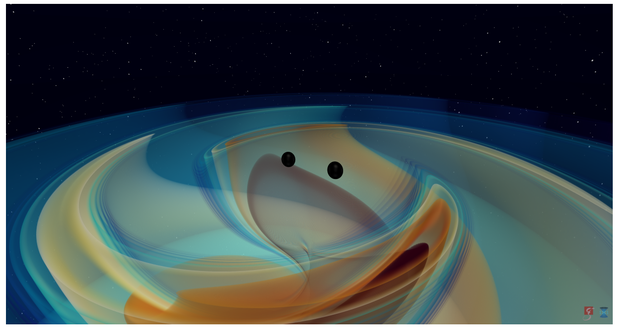
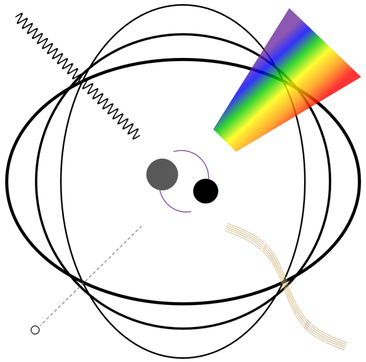
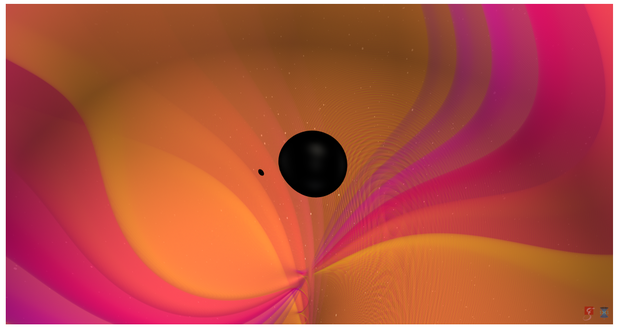
GW190521





GW190814



From Scattering Amplitudes to the Relativistic Two-Body Problem

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

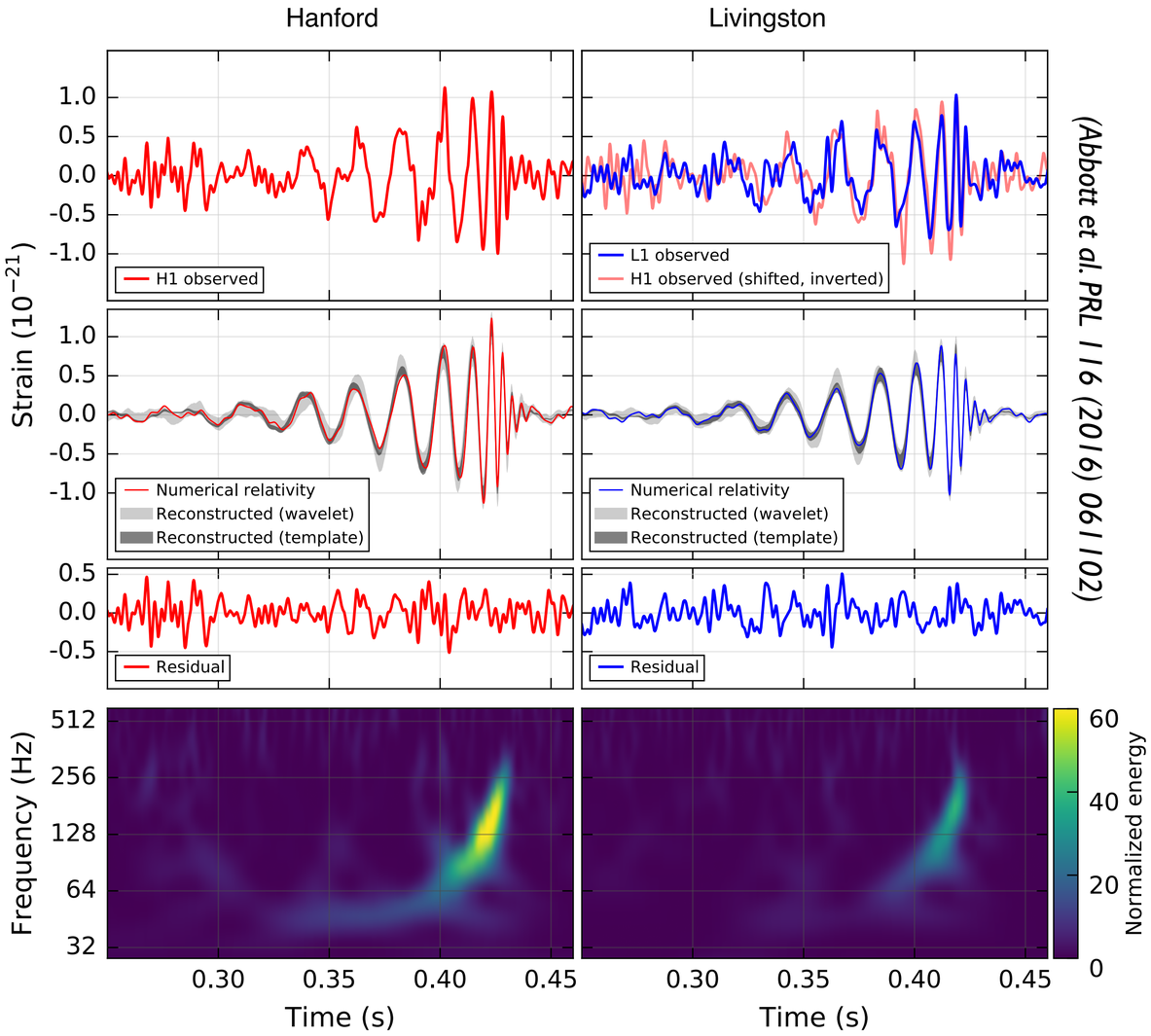
(Albert Einstein Institute)

Department of Physics, University of Maryland

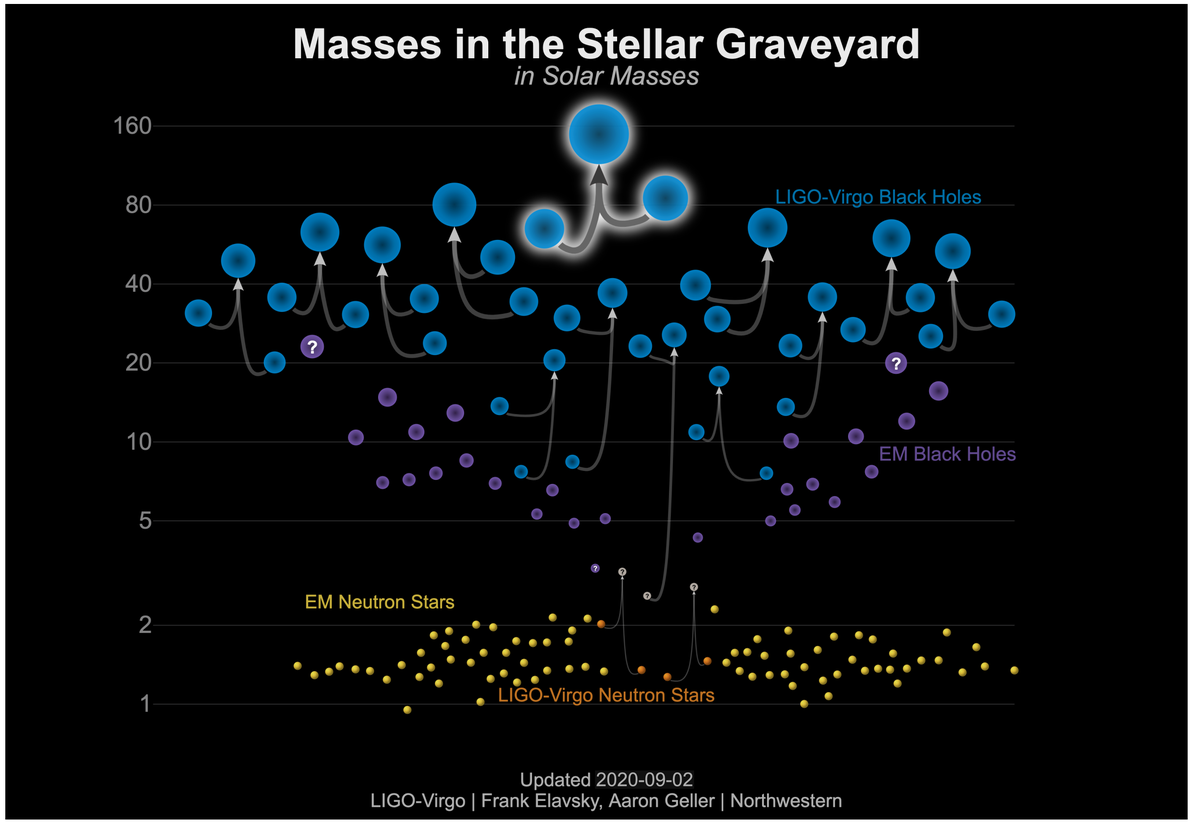
"Theory Frontier", Snowmass Community Planning (virtual) Meeting **October 6, 2020**

Gravitational Waves Ushered in New Era of Astrophysics

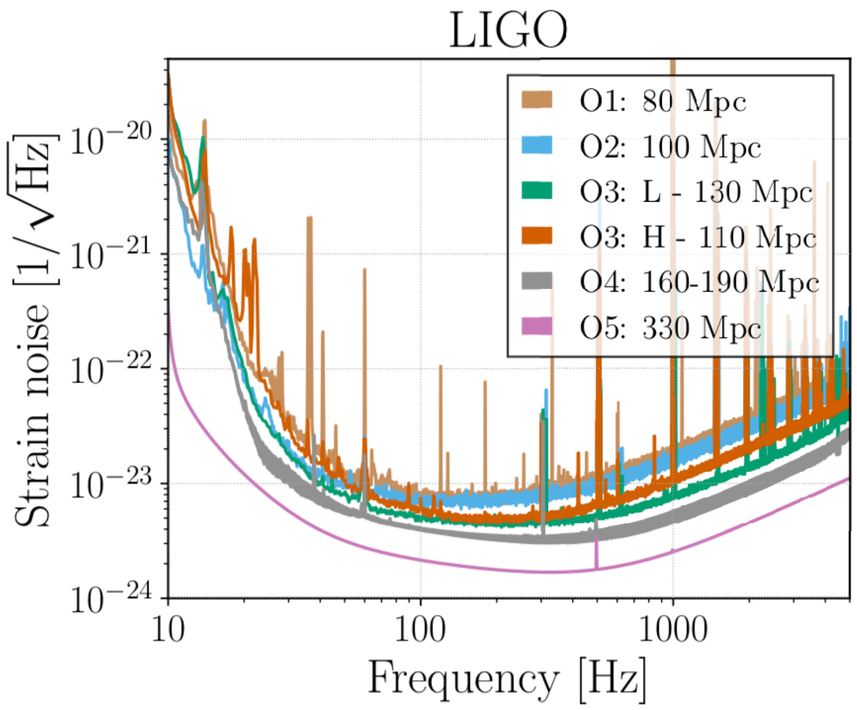
- **Discovery** of **GW** from **a binary black-hole merger** by LIGO



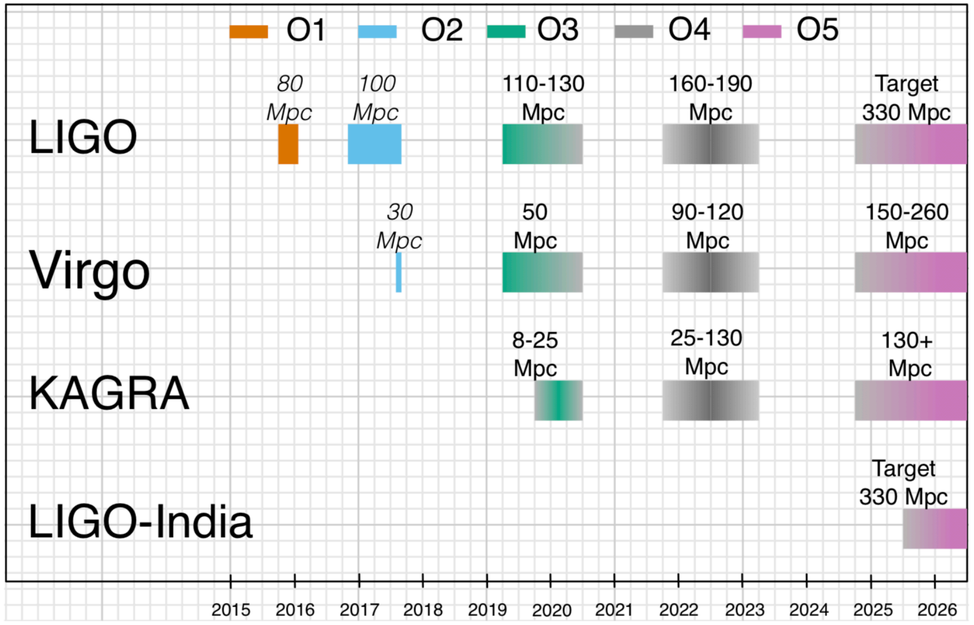
- **Since GW150914** was observed, **many more black hole binaries (BHB)** and **two binary neutron stars (BNS) discovered** by LIGO/Virgo.



Gravitational-Wave Landscape until ~2030



(Aasi et al. Living Rev. Rel. 21, 2020)



- From **several tens to hundreds** of binary detections per year.

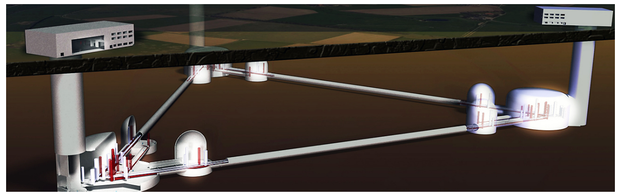
- Inference of **astrophysical properties** of BHBs, NSBHs and BNSs **in local Universe.**

(Aasi et al. Living Rev. Rel. 21, 2020)

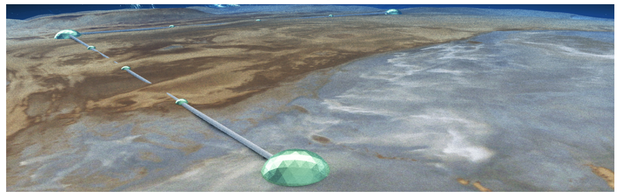
Observation run	Network	Expected BNS detections	Expected NSBH detections	Expected BBH detections
O3	HLV	1^{+12}_{-1}	0^{+19}_{-0}	17^{+22}_{-11}
O4	HLVK	10^{+52}_{-10}	1^{+91}_{-1}	79^{+89}_{-44}
		Area (deg^2) 90% c.r.	Area (deg^2) 90% c.r.	Area (deg^2) 90% c.r.
O3	HLV	270^{+34}_{-20}	330^{+24}_{-31}	280^{+30}_{-23}
O4	HLVK	33^{+5}_{-5}	50^{+8}_{-8}	41^{+7}_{-6}

Gravitational-Wave Landscape after ~2030 on the Ground

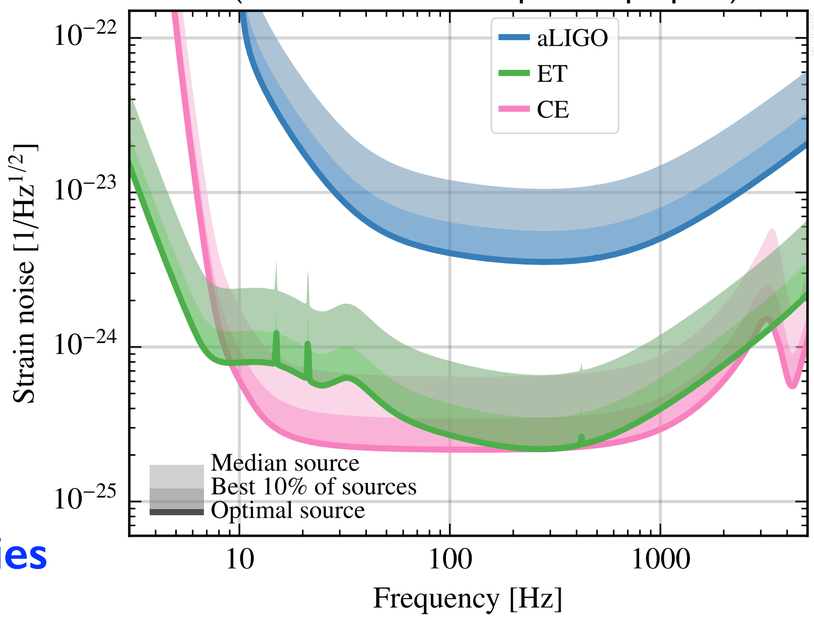
Einstein Telescope

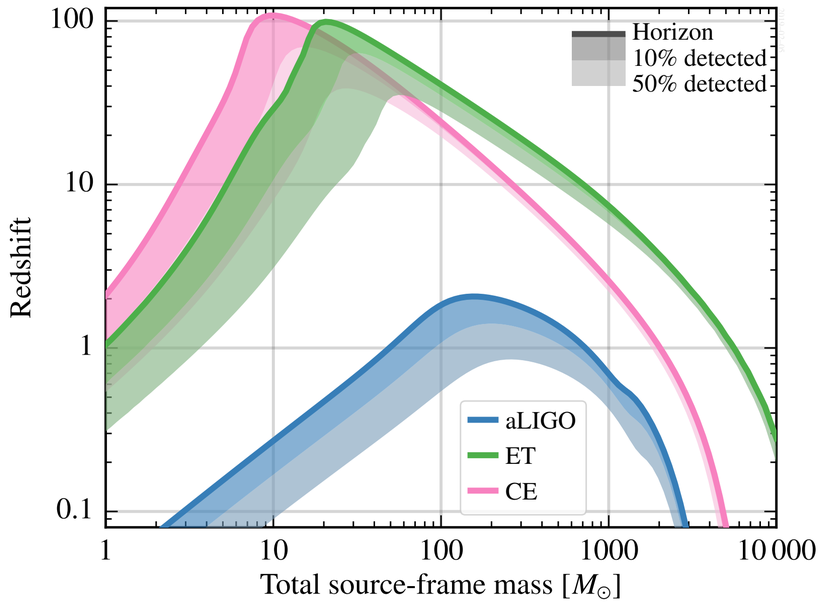


Cosmic Explorer

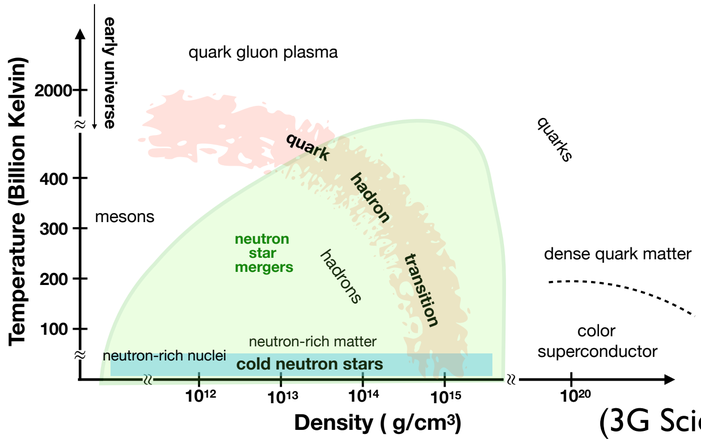


(3G Science-Case Report, in prep 20)

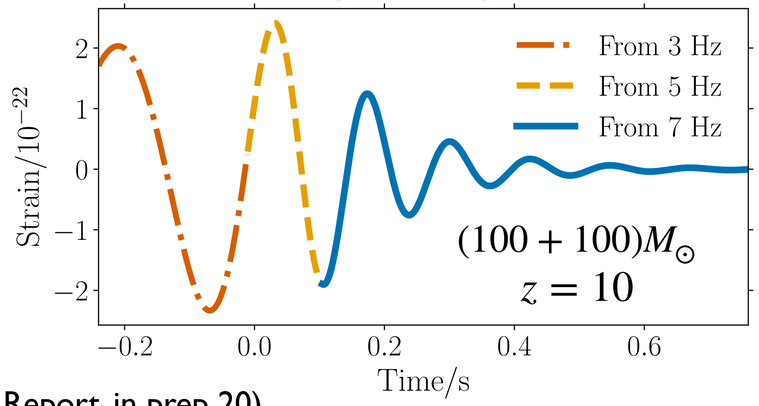




- **Understanding fundamental properties** of matter in **unexplored regimes** of **density** and **temperatures** with ET/CE and EM facilities.

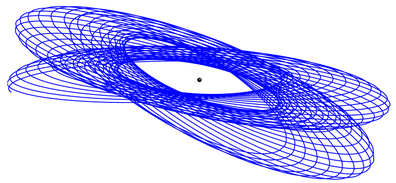


- Merger and ringdown of **Intermediate Black Holes (IMBHs)**

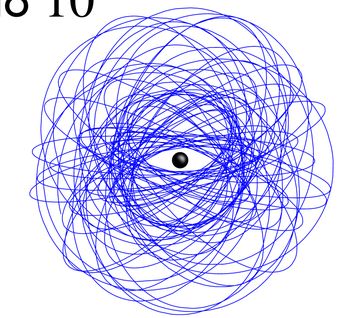


(3G Science-Case Report, in prep 20)

- **Intermediate Mass-Ratio Inspirals (IMRIs),** with mass ratio 10^3



at GW frequency ~1Hz

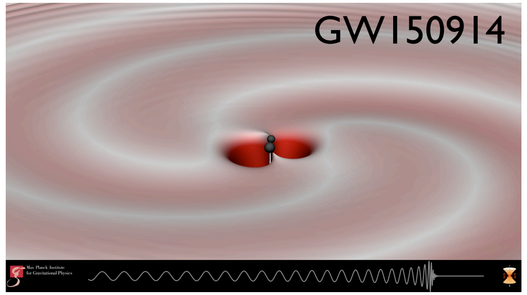


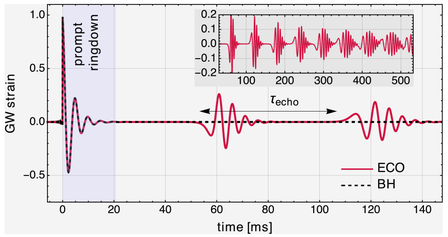
at GW frequency ~10 Hz

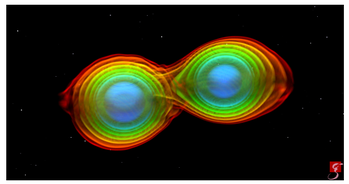
(credit: van de Meent)

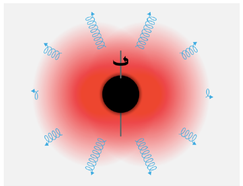
Outstanding Questions in Physics and Astrophysics

- What are the **properties** of **dynamical spacetime** (gravitational waves)?
- Is **General Relativity still valid** in the highly dynamical, strong-field regime?
- Are **Nature's black holes** the black holes predicted in the **General** theory of **Relativity**?
- How **black holes** and **neutron stars form**, which is their **astrophysical environment**, and how do they **form binaries**?
- How **matter behaves** under **extreme density and pressure**? Can **dark matter** make compact objects?
- What's the **origin** of the **most energetic phenomena** in our Universe?
- Can we **discover new fundamental particles** (axions, ultra-light bosons)?
- Can we **infer** the **cosmological model of our Universe** through gravitational-wave observations?



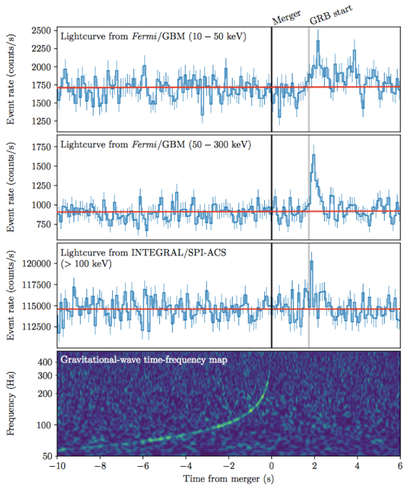






BH with boson cloud

GW170817



Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**.

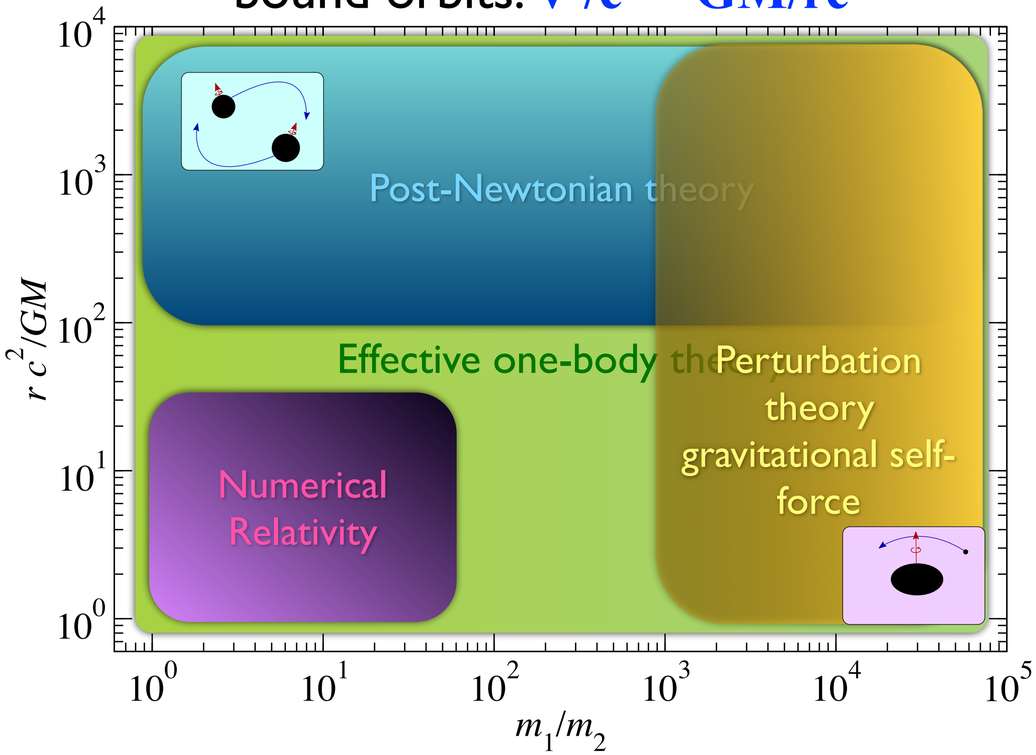
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

  - **approximately,** but **analytically** (**fast** way)
  - **"exactly"**, but **numerically** on supercomputers (**slow** way)

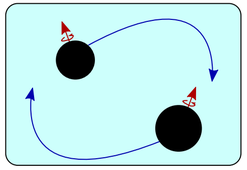
- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

bound orbits: $v^2/c^2 \sim GM/rc^2$



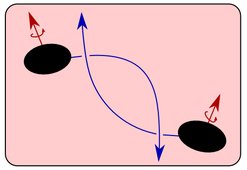
- **Post-Newtonian** (PN) (large separation, and slow motion, **bound motion**, i.e., **early inspiral**)

  expansion in $v^2/c^2 \sim GM/rc^2$



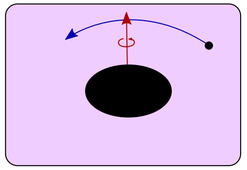
- **Post-Minkowskian** (PM) (large separation, **unbound motion**, i.e., **scattering**)

  expansion in G



- **Small mass-ratio** (gravitational self-force, GSF, i.e., **early to late inspiral**)

  expansion in m_2/m_1



Highly Accurate Waveform Models for GW Observations

- **GR** is **non-linear theory**.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

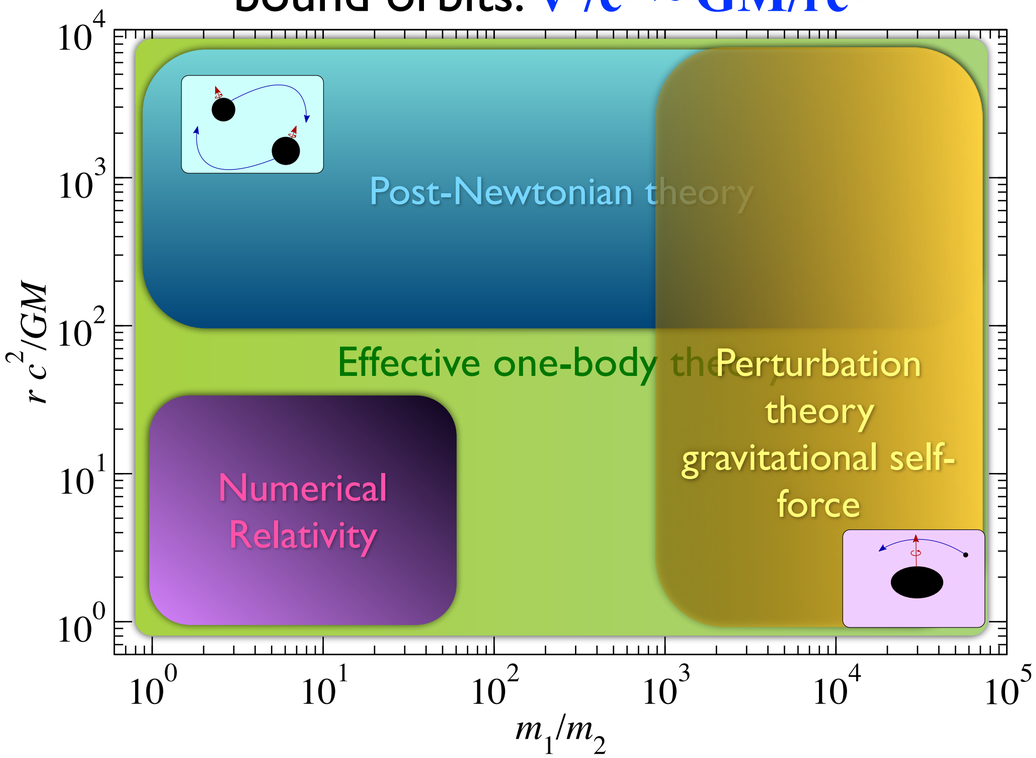
  - **approximately,** but **analytically** (**fast** way)
  - **"exactly"**, but **numerically** on supercomputers (**slow** way)

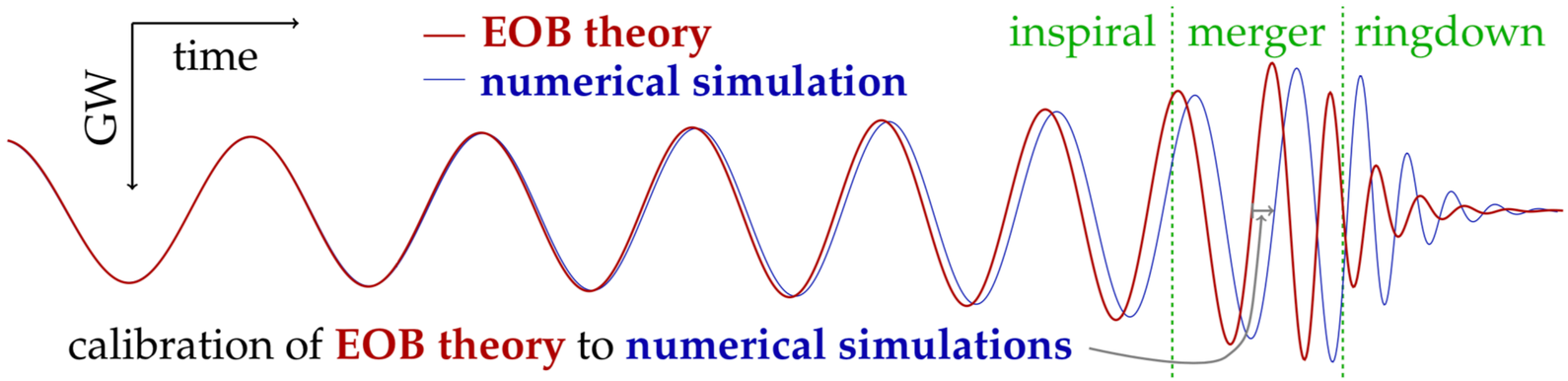
- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Effective-one-body** (EOB) (combines results from all methods, i.e., **entire coalescence**)

- **Key ideas** of EOB theory **inspired by quantum field theory.**

bound orbits: $v^2/c^2 \sim GM/rc^2$



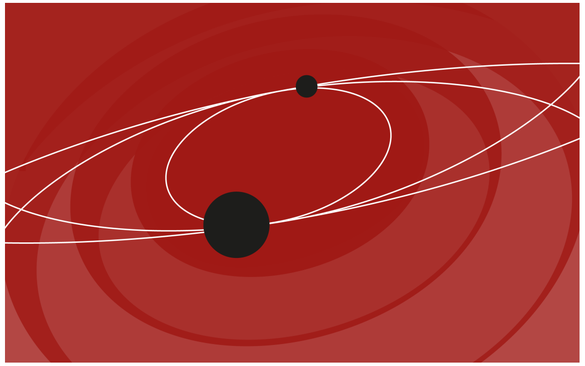


calibration of **EOB theory** to **numerical simulations**

Scattering Amplitude: A New Way to Study Gravity

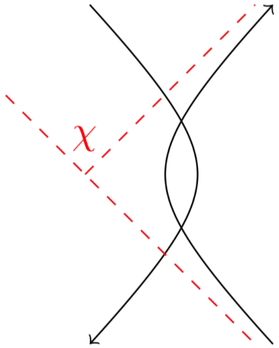
- Relativistic 2-body dynamics

(credit: Carvalho)

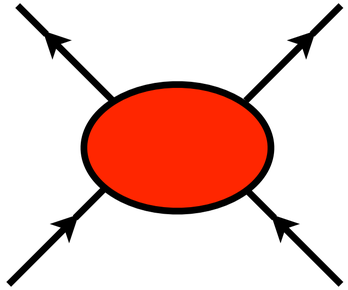


- Classical scattering: scattering angle χ

(credit: Steinhoff)



- Quantum scattering amplitude



e.g., in Born approximation: Fourier transform of potential is related to scattering amplitude

- **Advantages of scattering amplitudes:** on-shell, inherently gauge invariant, observables.
- **Advanced integration methods** developed in **QCD collider physics** applied to **classical gravity.**
- **Generalized unitarity methods**: use **tree amplitudes** to build **higher-order** (loop) **amplitudes**.

  (Britto et al. 04, 05, Bern et al. 1994, 1995, Neil & Rothstein 13)
- **Double copy** and **color-kinematic duality**: gravitons are *like* two gluons.

  (Bern et al. 10, Monteiro et al. 15, Bjerrum-Bohr et al. 15, Luna et al. 16, 17, Goldberger & Ridgway 17)
- **Bound-orbit** observables **from unbound-orbit** observables **through analytical continuation.**

  (Kälin et al. 20)

Some Results from Interplay with Scattering Amplitude Methods & EFT

(Bern et al. 19)

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}\ +\ \dots)$	G^1
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}\ +\ \dots)$	G^2
3PM			$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}\ +\ \dots)$	G^3
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8\ +\ \dots)$	G^4
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6\ +\ \dots)$	G^5
6PM						$(1$	$+\ v^2$	$+\ v^4\ +\ \dots)$	G^6
							$\vdots$		

•2-body **Hamiltonian at 3PM (2 loops)** for nonspinning BHs. *(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20)*

Small parameter is **$GM/rc^2 << 1$, $v^2/c^2 \sim 1$,** large separation, natural **for unbound motion/scattering**

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$E = E_1 + E_2 \qquad \gamma = E/m$$

$$\xi = E_1 E_2/E^2 \qquad \sigma = \frac{p_1\cdot p_2}{m_1 m_2}$$

$$V^{(1)}(\mathbf{p},\mathbf{q}) = \int \frac{\mathbf{d^3r}}{(2\pi)^3}\,\mathcal{M}^{\text{tree}}(\mathbf{p},\mathbf{q})\,\mathbf{e}^{-\mathbf{ir}\cdot\mathbf{q}}$$

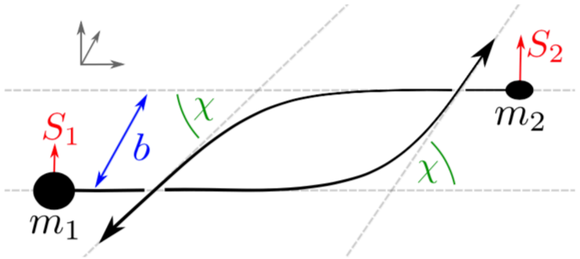
amplitude

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1 - 2\sigma^2\right)$$

Some Results from Interplay with Scattering Amplitude Methods & EFT (contd.)

- 2-body **spin-orbit** (SO) **Hamiltonian at 4.5PN** computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

(Levi et al. 20, Antonelli et al. 20)



- 2-body non-spinning **Hamiltonian at 5PN & 6PN** *partially* computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

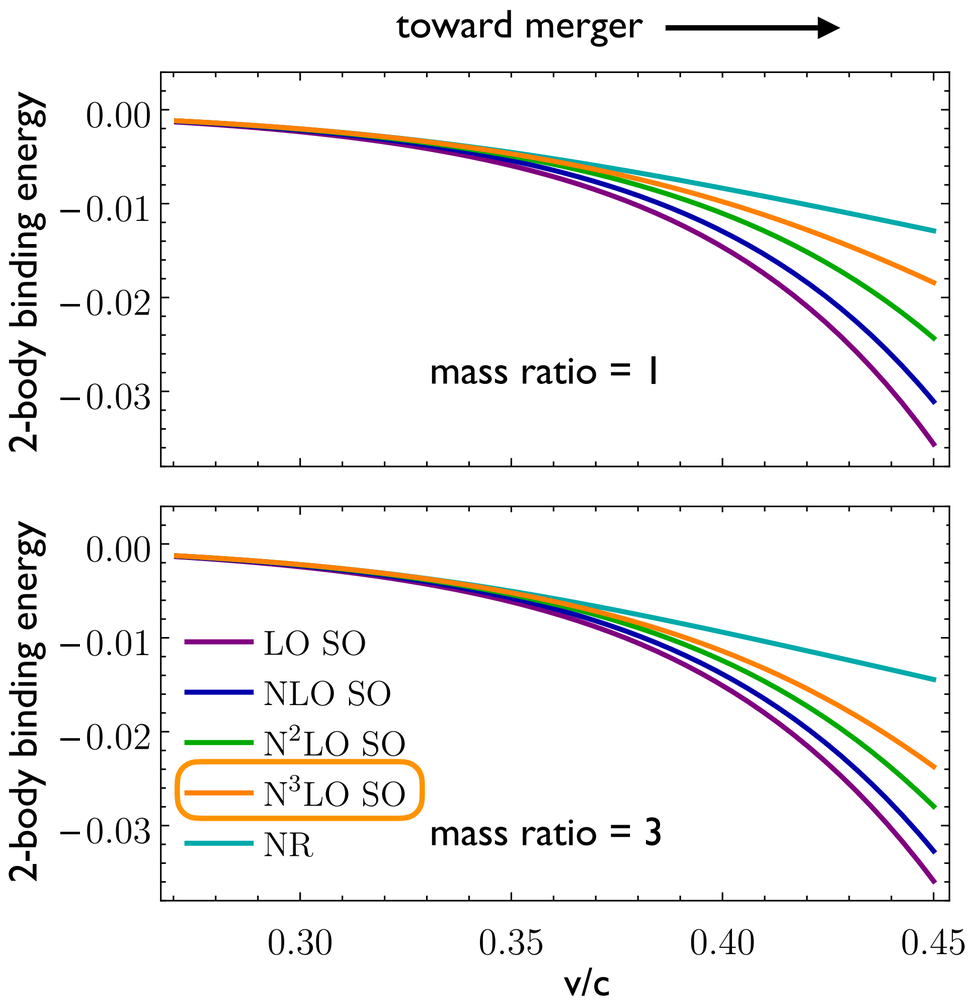
(Foffa et al. 19, Blümlein et al. 20, Damour 20, Bini, Damour & Geralico 20)

- 2-body **Hamiltonian at 2PM (1 loop)** for spinning, precessing BHs.

(Bini et al. 17, 18, Vines 18, Bern et al. 20)

- **Results** can be easily **included** into **EOB formalism**.

(Damour 19, Antonelli et al. 19)



(Antonelli et al. 20)

Toward High-Precision Gravitational-Wave Astrophysics

- **Observing** gravitational waves and **inferring astrophysical/physical information** hinges on our **ability** to make highly **precise predictions** of **two-body dynamics** and **gravitational radiation.**

- Crucial to **improve waveform models** for BBHs and binaries with matter for LIGO and Virgo **upcoming runs** and for **future detectors** (Cosmic Explorer, Einstein Telescope & LISA). **Waveform accuracy** would need to be **improved by one or two orders of magnitude** depending on the parameter space.

- Unique opportunity for **theoretical particle physicists to contribute**.

- **Conservative dynamics** →

PN order	1.5	2.5	3.5	4.5	5.5	6.5

	0	1	2	3	4	5	6
no spin	N	1PN	2PN	3PN	4PN	5PN	6PN
spin-orbit		LO SO	NLO SO	N2LO SO	N3LO SO		
spin^2			LO S2	NLO S2	N2LO S2	N3LO S2	
spin^3				LO S3	NLO S3		
spin^4					LO S4	NLO S4	
spin^5						LO S5	NLO S5
spin^6							LO S6

(credit: Vines)

need up to
1PM / tree
2PM / 1-loop
3PM / 2-loop
4PM / 3-loop
5PM / 4-loop
6PM / 5-loop
7PM / 6-loop

N.B. Resummation methods can accelerate accuracy.

- **Plus radiation!**