From Scattering Amplitudes to the Relativistic Two-Body Problem

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Gravitational Waves Ushered in New Era of Astrophysics

- **Discovery of GW from a binary black-hole merger** by LIGO

  Hanford  
  Livingston  

  (Abbott et al. PRL 116 (2016) 061102)

- **Since GW150914** was observed, many more black hole binaries (BHB) and two binary neutron stars (BNS) discovered by LIGO/Virgo.
Gravitational-Wave Landscape until ~2030

- From **several tens to hundreds** of binary detections per year.

- Inference of **astrophysical properties** of BHBs, NSBHs and BNSs in local Universe.
Gravitational-Wave Landscape after ~2030 on the Ground

- Understanding fundamental properties of matter in unexplored regimes of density and temperatures with ET/CE and EM facilities.

- Merger and ringdown of Intermediate Black Holes (IMBHs)

- Intermediate Mass-Ratio Inspirals (IMRIs), with mass ratio $10^3$

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(3G Science-Case Report, in prep 20)

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(credit: van de Meent)

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(3G Science-Case Report, in prep 20)
Outstanding Questions in Physics and Astrophysics

- What are the properties of dynamical spacetime (gravitational waves)?

- Is General Relativity still valid in the highly dynamical, strong-field regime?

- Are Nature’s black holes the black holes predicted in the General theory of Relativity?

- How black holes and neutron stars form, which is their astrophysical environment, and how do they form binaries?

- How matter behaves under extreme density and pressure? Can dark matter make compact objects?

- What’s the origin of the most energetic phenomena in our Universe?

- Can we discover new fundamental particles (axions, ultra-light bosons)?

- Can we infer the cosmological model of our Universe through gravitational-wave observations?
Solving Two-Body Problem in General Relativity

- GR is non-linear theory.

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- Einstein’s field equations can be solved:
  - approximately, but analytically (fast way)
  - “exactly”, but numerically on supercomputers (slow way)

- Synergy between analytical and numerical relativity is crucial to provide GW detectors with templates to use for searches and inference analyses.

- Post-Newtonian (PN) (large separation, and slow motion, bound motion, i.e., early inspiral) expansion in 

\[ \frac{v^2}{c^2} \sim \frac{G M}{r c^2} \]

- Post-Minkowskian (PM) (large separation, unbound motion, i.e., scattering) expansion in \( G \)

- Small mass-ratio (gravitational self-force, GSF, i.e., early to late inspiral) expansion in \( \frac{m_2}{m_1} \)
Highly Accurate Waveform Models for GW Observations

- GR is non-linear theory.
  \[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

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- Effective-one-body (EOB) (combines results from all methods, i.e., entire coalescence)

- Key ideas of EOB theory inspired by quantum field theory.

Synergy between analytical and numerical relativity is crucial to provide GW detectors with templates to use for searches and inference analyses.
Scattering Amplitude: A New Way to Study Gravity

• Relativistic 2-body dynamics

• Classical scattering: scattering angle $\chi$

• Quantum scattering amplitude

Advantages of scattering amplitudes: on-shell, inherently gauge invariant, observables.

Advanced integration methods developed in QCD collider physics applied to classical gravity.

Generalized unitarity methods: use tree amplitudes to build higher-order (loop) amplitudes.

(Britto et al. 04, 05, Bern et al. 1994, 1995, Neil & Rothstein 13)

Double copy and color-kinematic duality: gravitons are like two gluons.

(Bern et al. 10, Monteiro et al. 15, Bjerrum-Bohr et al. 15, Luna et al. 16, 17, Goldberger & Ridgway 17)

Bound-orbit observables from unbound-orbit observables through analytical continuation.

(Kälin et al. 20)
Some Results from Interplay with Scattering Amplitude Methods & EFT

<table>
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<tr>
<th>Order</th>
<th>0PN</th>
<th>1PN</th>
<th>2PN</th>
<th>3PN</th>
<th>4PN</th>
<th>5PN</th>
<th>6PN</th>
<th>7PN</th>
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<td>1PM</td>
<td>1 + (\frac{v^2}{c^2})</td>
<td>(v^2)</td>
<td>(v^4)</td>
<td>(v^6)</td>
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<td>(v^2)</td>
<td>(v^4)</td>
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<td>(v^{10})</td>
<td>(v^{12}) + …</td>
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<td>(v^2)</td>
<td>(v^4)</td>
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<td>(v^8)</td>
<td>(v^{10}) + …</td>
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<tr>
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<td>1 + (\frac{v^2}{c^2})</td>
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<td>(v^6)</td>
<td>(v^8) + …</td>
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<tr>
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<td>1 + (\frac{v^2}{c^2})</td>
<td>(v^2)</td>
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<tr>
<td>6PM</td>
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<td>(v^2) + (v^4) + …</td>
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2-body Hamiltonian at 3PM (2 loops) for nonspinning BHs.  
(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20)

Small parameter is \(\frac{GM}{rc^2} \ll 1\), \(\frac{v^2}{c^2} \sim 1\), large separation, natural for unbound motion/scattering

\[
H(p, r) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(p, r)
\]

\[
V(p, r) = \sum_{i=1}^{\infty} c_i (p^2) \left( \frac{G}{|r|} \right)^i
\]

\[
E = E_1 + E_2 \quad \gamma = \frac{E}{m} \quad \xi = \frac{E_1 E_2}{E^2} \quad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}
\]

\[
V^{(1)}(p, q) = \int \frac{d^3r}{(2\pi)^3} M^\text{tree}(p, q) e^{-ir \cdot q}
\]

\[
c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} (1 - 2\sigma^2)
\]
Some Results from Interplay with Scattering Amplitude Methods & EFT (contd.)

• 2-body **spin-orbit (SO) Hamiltonian at 4.5PN** computed using EFT or interplay between bound and unbound orbits, and gravitational self-force results.
  (Levi et al. 20, Antonelli et al. 20)

• 2-body non-spinning **Hamiltonian at 5PN & 6PN partially** computed using EFT or interplay between bound and unbound orbits, and gravitational self-force results.
  (Foffa et al. 19, Blümlein et al. 20, Damour 20, Bini, Damour & Geralico 20)

• 2-body **Hamiltonian at 2PM (1 loop)** for spinning, precessing BHs.
  (Bini et al. 17, 18, Vines 18, Bern et al. 20)

• **Results** can be easily included into EOB formalism.
  (Damour 19, Antonelli et al. 19)
Toward High-Precision Gravitational-Wave Astrophysics

- Observing gravitational waves and inferring astrophysical/physical information hinges on our ability to make highly precise predictions of two-body dynamics and gravitational radiation.

- Crucial to improve waveform models for BBHs and binaries with matter for LIGO and Virgo upcoming runs and for future detectors (Cosmic Explorer, Einstein Telescope & LISA). Waveform accuracy would need to be improved by one or two orders of magnitude depending on the parameter space.

- Unique opportunity for theoretical particle physicists to contribute.

- Conservative dynamics

<table>
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<th>PN order</th>
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<th>3.5</th>
<th>4.5</th>
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<td>2PN</td>
<td>3PN</td>
<td>4PN</td>
<td>5PN</td>
</tr>
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<td>NLO SO</td>
<td>N2LO SO</td>
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</tbody>
</table>

N.B. Resummation methods can accelerate accuracy.

- Plus radiation!

(credit: Vines)