Inference offers a metric to constrain dynamical models of neutrino flavor transformation

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Outline

- **The problem**
- The model
- The means of attack
- The result
- The future (?)

The problem

- The astrophysics of compact objects presents a vast range of environments where neutrino flavor transformation may occur and may be important for nucleosynthesis and a detected neutrino signal.
- Flavor transformation is a complicated nonlinear problem, and presents vexing difficulties for existing numerical codes [1, 2, 3]. It can be difficult and expensive to adapt these large-scale codes to a variety of specific conditions of possible interest.
- We examine inference to augment these existing tools. Inference can efficiently probe a variety of physical conditions and parameter regimes of possible interest, and thereby guide/complement these codes.

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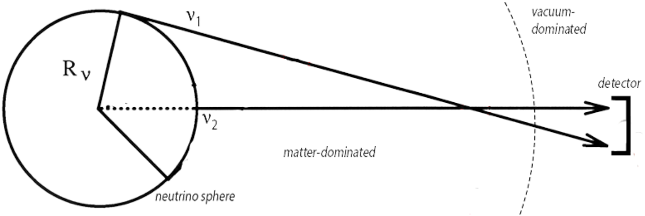
A toy steady-state, two-flavor, single-angle, forward-scattering model. Two monoenergetic neutrino beams of different energies interact with each other and with a background of weakly-charged particles. Densities dilute as some functions of position r, e.g. the distance from the neutrino sphere in a supernova. Polarization vectors[1] of each neutrino i, after decomposing the density matrices and Hamiltonians into bases of Pauli spin matrices [4, 5]:



$$\frac{\mathrm{d}\vec{P}_i}{\mathrm{d}r} = \left(\Delta_i \vec{B} + \overbrace{V(r)}^{\nu\text{-matter}} \hat{z} + \overbrace{\mu(r)}^{\nu\text{-}\nu} \sum_{j \neq i} \vec{P}_j \right) \times \vec{P}_i \tag{1}$$

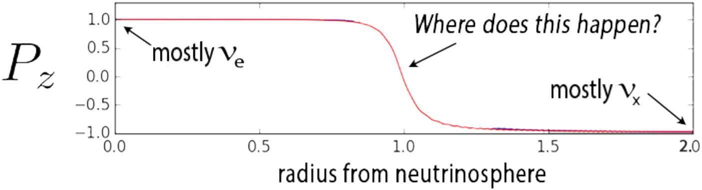
V(r) is poorly constrained by theory and could impose a signature upon a detection. Thus we chose V(r) as the focus of our parameter estimation study: $V(r) = C_m(r)/r^3$, where 1) C_m is a quadratic function with two unknown parameters (a build-up to a complicated structure representing multiple shocks).

[1] Δ_i: vacuum oscillation frequencies. $\vec{B}$: represents flavor mixing in vacuum.

Our question for an inference procedure: at what radius does the MSW resonance occur?

At $r = 0$, neutrinos are in flavor state ν_e, or: $P_z = 1$ (z-component of polarization vector $\mathbf{P}$).

In the adiabatic limit (oscillation length $<< \nabla V_m$): beginning in ν_e, efficient conversion to ν_x (MSW resonance - Mikheyev et al. 1985; Wolfenstein 1978).



OR:

What measurements must we make, and what constraints must we place on the physics within the envelope, to uniquely identify that location simultaneously with the unknown parameters governing the ν-matter potential?

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Statistical data assimilation (SDA)

- An inverse formulation [6] invented for numerical weather prediction [7, 8, 9, 10, 11, 12]: optimally combines a model with data.
- In simulations, SDA offers a means to identify the measurements and constraints required to break degeneracies and to estimate unknown model parameters to a desired precision.
- Within astrophysics, inference has been used mainly for pattern recognition [13], although its utility for model completion is gaining traction in the exoplanet community [14] and solar physics [15].

We use an optimization formulation, minimizing a cost function A_0:

$$A_0 = R_f A_{model} + R_m A_{meas}$$

- A_{model} imposes the model evolution for all D state variables;
- A_{meas} requires that any subset of the D state variables that is *measured* obey the measurements (we measure $P_{i,z}$ of each neutrino, at one location: an Earth-based detector);
- We anneal in the relative weights of R_f vs R_m, to deal with the non-convexity of the cost function surface.

In extremizing both terms simultaneously, information from A_{meas} propagates to A_{model}, to yield estimates of unknown parameters.

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Findings [16]

1. Constraining flavor only at the neutrinosphere (together with a detector measurement) permits high degeneracy of solutions. As we add successive constraints, that degeneracy dissolves.
2. Once degeneracy is eliminated, we have in the plot of action (i.e. cost) over the course of annealing a litmus test for whether a path corresponds to the true solution: *the true solution is the path of least action.*
3. Once degeneracy is eliminated, then permitting that a sufficient number of paths is searched, their action plots indicate whether the true solution exists within the parameter regime that was searched: either the path of least action is found, or it is not.

This has been a toy probe of the question: "Which physical assumptions are likely to be worth probing more extensively with sophisticated numerical codes, and which are not?"

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- **The future: Ultimately, add back-scattering.**

Thank you

Thank you! Please email ideas/comments/questions: evearmstrong.physics@gmail.com

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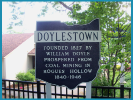

CASS

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Institute for Nuclear Theory

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