Energy Peak and
Its Implications on Collider Phenomenology

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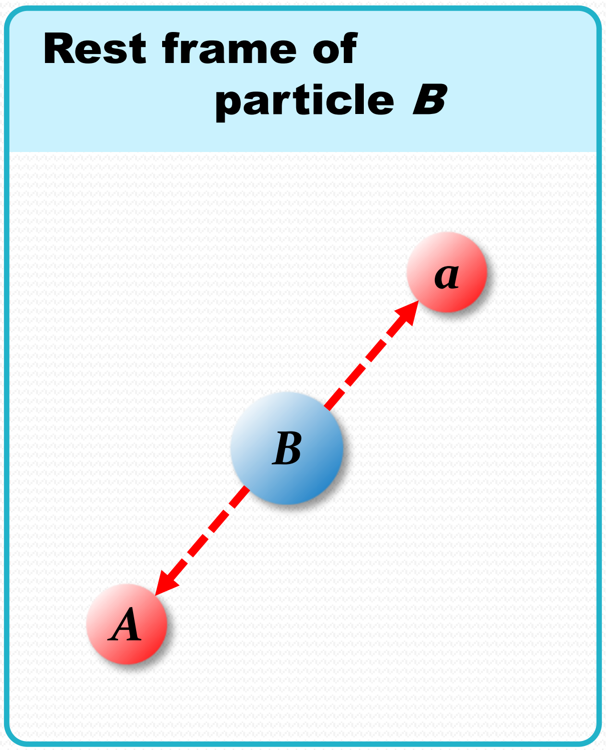
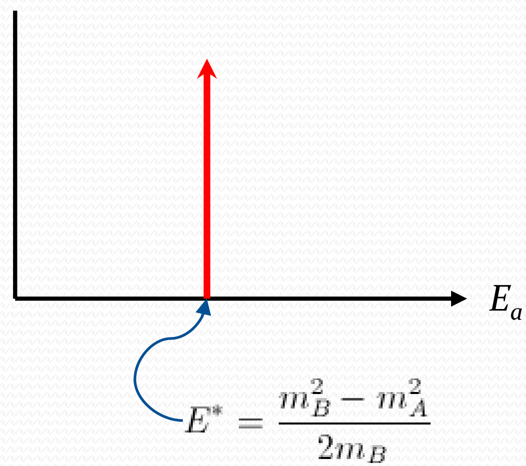
Based on 1209.0772, 1212.5230, 1309.4776, 1503.03836, and 1512.02265 in collaboration with Kaustubh Agashe, Roberto Franceschini, Sungwoo Hong, and Kyle Wardlow

Energy Peak: 2-Body Decay in the Rest Frame

A simple 2-body decay of a heavy resonance B into (massive invisible) A and massless visible a





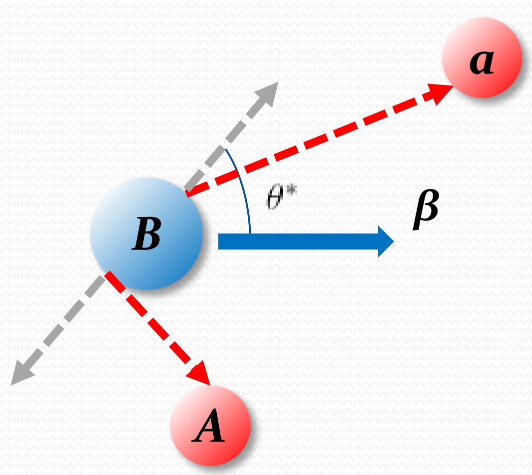
- Visible particle energy fixed, i.e., **δ-function-like** energy distribution
- E^* measured and the mass of A known, then the mass of B determined vice versa

It would be great to go onto this special frame!

Energy Peak: 2-Body Decay in the Lab Frame

A simple 2-body decay of a heavy resonance *B* into (massive invisible) *A* and massless visible *a*

Laboratory frame

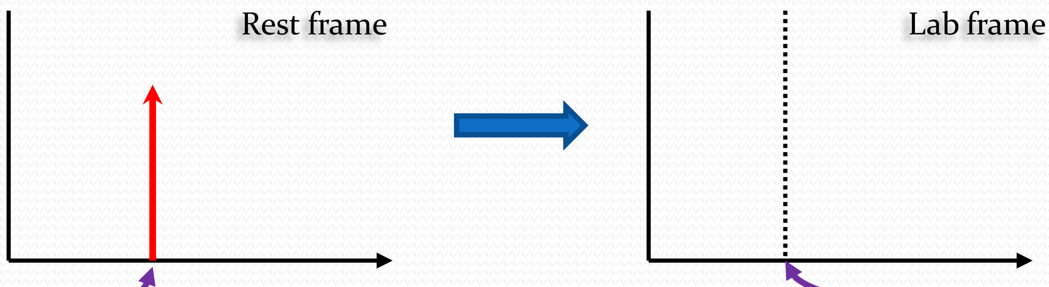


- Energy of particle *a* should be Lorentz-transformed!

$$E = E^{*}\gamma(1+\beta\cos\theta^{*}) = E^{*}(\gamma+\sqrt{\gamma^{2}-1}\cos\theta^{*})$$

- No more δ-function-like spectrum, but a function of γ, θ^{*}.

$\rightarrow$ We get a distribution.



"Boost-distribution invariance" of the peak based on a simple kinematics argument

[Agashe, Franceschini, **DK**, Phys.Rev.D88 (2013) 5, 057701]

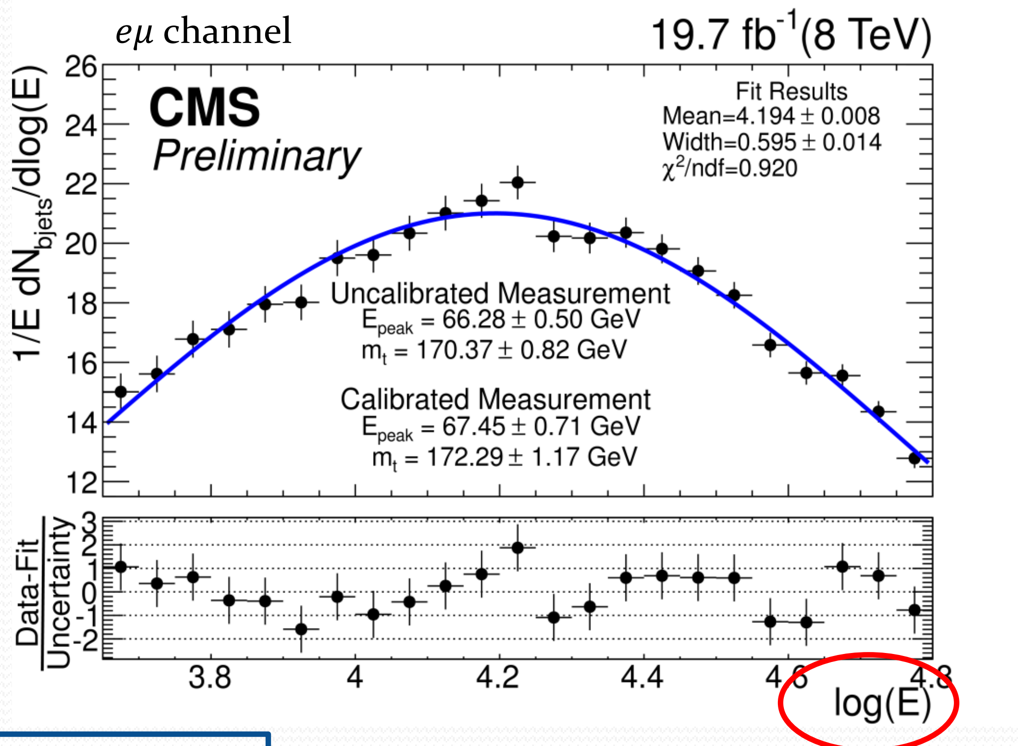
(See the back-up slides for proof.)

Energy-Peak Method for Top Quark Mass Measurement

- Based on the LO observation
- Dileptonic final state (cleanest channel)

$$t\bar{t} \rightarrow b\bar{b}e^{\pm}\mu^{\mp} + \nu_e\nu_\mu$$

- Energy spectrum should be symmetric w.r.t. E_b^* in $\log E$
  - ✓ Gaussian fit near the peak region [CMS PAS TOP-15-002]
- Best-fit top mass



$$\boxed{m_t = 172.29 \pm 1.17(\text{stat.}) \pm 2.66(\text{syst.})\ \text{GeV}}$$

The Upshot of Energy Peak

- The principle of boost-distribution-invariant "energy peak" can be understood based on a simple kinematics argument (like the usual invariant mass and the transverse mass) and has been successfully tested in a realistic application, top quark mass measurement by CMS.

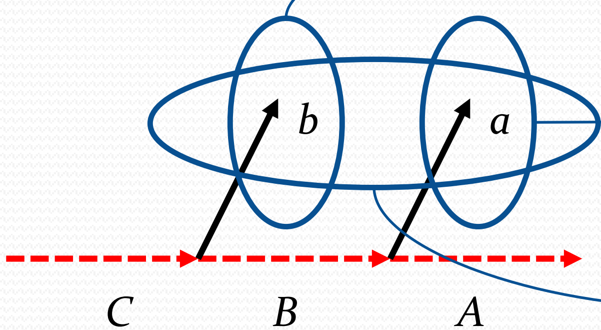
- Energy peak
  - i. relies on less model assumptions – unpolarized production of A, massless a, 2-body decay of A, non-zero value of boost distribution $g(\gamma)$ around $\gamma = 1$ – and
  - ii. does not care about the details of the other decay product B.

- For any new physics,
  - i. we don't know model details, but A are often produced via QCD, SM particles are nearly massless, the other two conditions are the case for many well-motivated models, and
  - ii. The other decay product could be invisible or a dark matter candidate!

The idea of **Energy Peak** is **well motivated** for studying **new physics** at **colliders**!

Application 1-(a): Mass Measurement of New Particles

- Two-step cascade decay: 3 unknowns (m_A, m_B, and m_C) → 3 equations needed



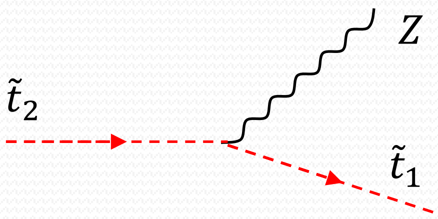
$$E_b^{\text{peak}} = \frac{m_C^2 - m_B^2}{2m_C} \qquad E_a^{\text{peak}} = \frac{m_B^2 - m_A^2}{2m_B}$$

$$m_{ab}^{\max} = \frac{\sqrt{(m_C^2 - m_B^2)(m_B^2 - m_A^2)}}{m_B}$$

- Peaks can be extracted even in the presence of background (described by a simple empirical function, see the back-up slide) → fitting full data with signal and background templates
- Example process: $\tilde{g} \to \bar{b}\tilde{b}, \tilde{b} \to b\bar{\chi}^0$ [Agashe, Franceschini, **DK**, JHEP 11 (2014) 059]. Similar processes in other SUSY scenarios, other models (e.g., UED, Little Higgs)

Application 1-(b): Mass Measurement of New Particles

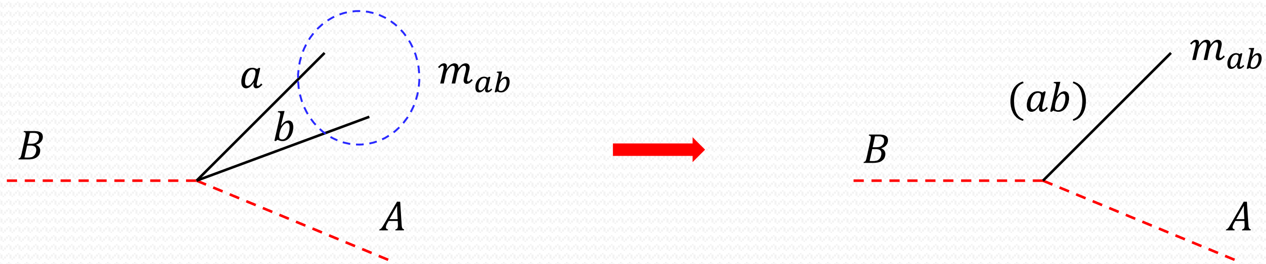
- A simple extension of earlier observations: 2-body decay into a **massive** visible particle, e.g., decay of the heavier stop to the lighter stop and a massive gauge boson Z.



- In principle, $E_{Z,\,\mathrm{peak}} \neq E_Z^*$
- We devise a suitable ansatz for the massive visible particle case to extract E_Z^*, which is inspired by the ansatz development in the massless visible particle case. [Agashe, Franceschini, Hong, **DK**, JHEP 04 (2016) 151] (See the back-up slide for the detailed expression.)

Application 1-(c): Mass Measurement of New Particles

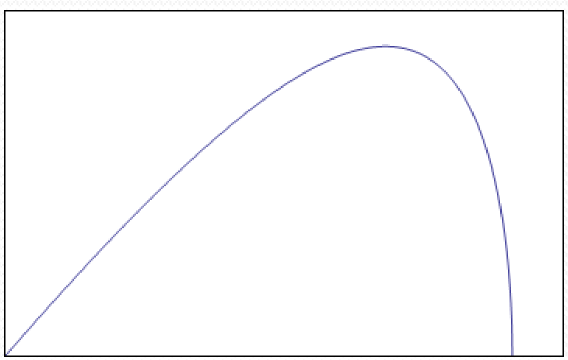
- Another simple extension of earlier observations: 3-body decay into **two** massless visible particles



- Phase-space slicing in m_{ab} space, i.e., for each of value of m_{ab} it could be understood as a two-body decay of B into a massive visible particle of m_{ab} (re-use the argument with 2-body decays).
- In principle, $(E_a + E_b)_{\text{peak}} \neq (E_a + E_b)^*$ if m_{ab} is large enough.
- We test the idea, taking an example of SUSY process, $\tilde{g} \to \bar{b}b\bar{\chi}^0$ [Agashe, Franceschini, **DK**, Wardlow, JHEP 05 (2016) 138]

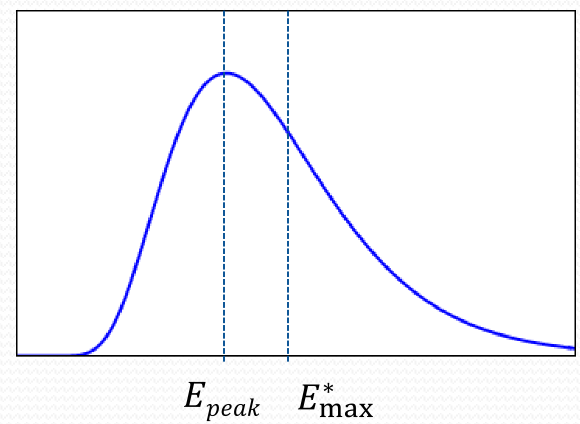
Energy Peak in 3-Body Decays

- For a 3-body decay into a **single** visible particle (i.e., $B \to AA'a$), the energy of particle a is given by a distribution even in the rest frame of particle B.
- The same argument is relevant for each value of the rest-frame energy of particle a.



Lorentz boost

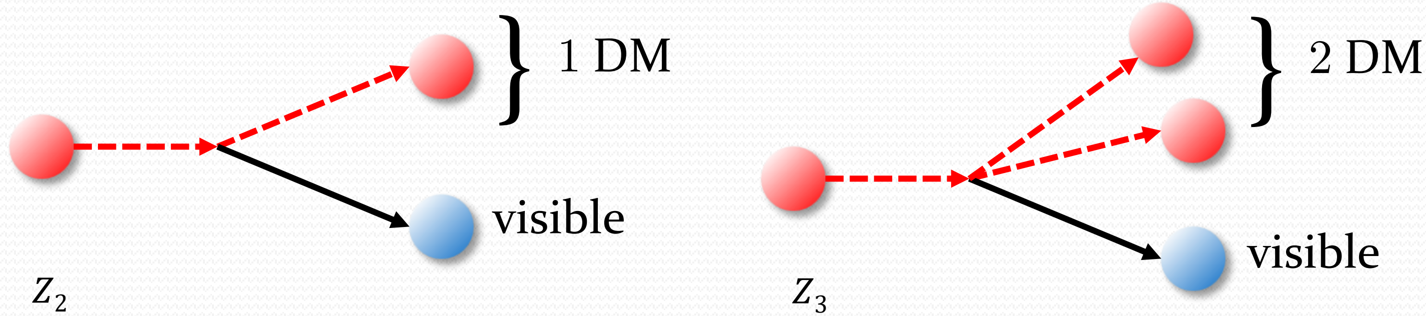
$$E^*_{\max} = \frac{m_B^2 - (m_A + m_{A'})^2}{2m_B}$$



Application 2: Distinguishing Z_2 vs. Z_3 DM Models

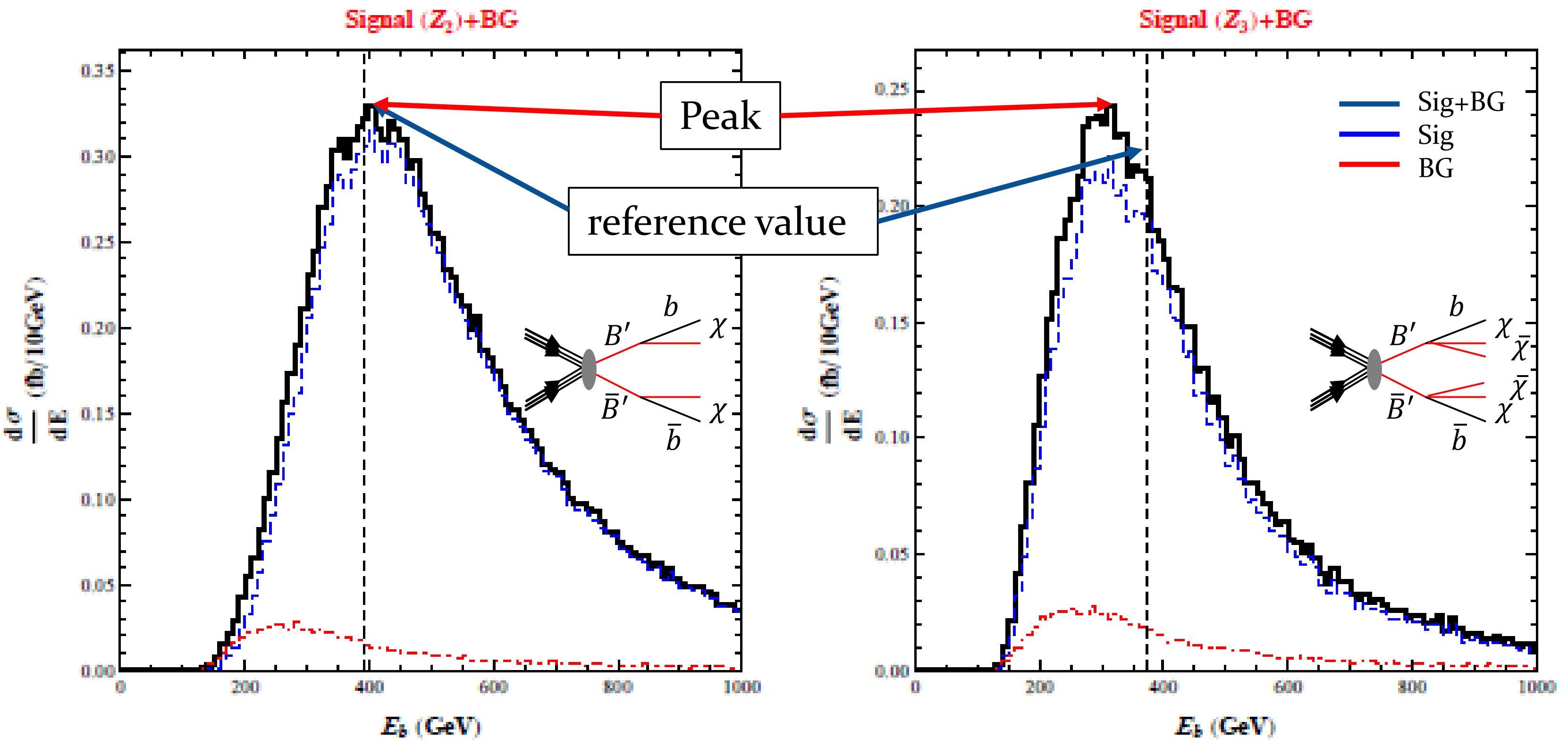
- Some (kinematic) variables (e.g., M_{T2}) can be used to extract reference values.
  - E^* for the **two-body decay**
  - $E^*_{\max}$ for the **three-body decay**

- Comparison between reference values and peak positions → distinction of DM stabilization symmetries. [Agashe, Franceschini, **DK**, Wardlow, Phys.Dark Univ. 2 (2013) 72-82]



(The same final state in the other decay side is assumed.)

Application 2: Distinguishing Z_2 vs. Z_3 DM Models



- Peak = reference value → 2 body-decay → Z_2 symmetry
- Peak < reference value → 3 body-decay → Z_3 symmetry

Future Plans & Conclusions

- The **"boost-distribution-invariant" energy peak** in the distribution of a Lorentz-variant quantity energy has been successfully tested in the top quark mass measurement.
- The Energy Peak method is **kinematics-based**, i.e., relies on **less model assumptions**, so its application to new physics property measurements is well motivated.
- Possible applications include new particle mass measurements and distinction of DM stabilization symmetries which are **major topics regarding the BSM physics program at the energy frontier**.
- Future directions
  i. Energy peak as a combinatorics-free discovery variable (i.e., kinematic cut), e.g., $\tilde{t}_1 \to b\tilde{\chi}_1^+$ and $\tilde{t}_1 \to t\tilde{\chi}_1^0$: different b-jet hardness and (possibly) polarized production of top quark [Low, Phys.Rev.D88 (2013) 9, 095018]
  ii. Phenomenology of Energy Peak at higher-energy colliders

Bonus Slides

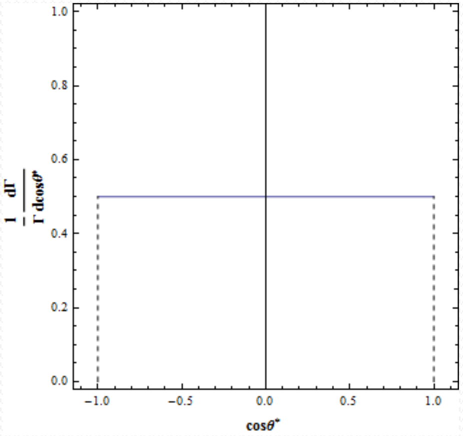
Existence of Peak: a Heuristic Proof

Step 1: flat variable

- **Unpolarized** parent particle [Assumption 1]

  → No OVERALL preference of the direction of emission

  → $\cos\theta^*$ is a flat variable .



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{1}{2}$$

- Lorentz-transformed energy of the visible particle

$$E = E^*\gamma(1+\beta\cos\theta^*) = E^*(\gamma + \sqrt{\gamma^2-1}\cos\theta^*)$$

  → E is also a **FLAT** variable.

  → The distribution in E is a simple **rectangle/box** for any given boost factor.

$$\frac{1}{\Gamma}\frac{d\Gamma}{dE} = \frac{1}{2E^*\sqrt{\gamma^2-1}}$$

Existence of Peak: a Heuristic Proof

Step 2: *E** is always included

[Assumption 2: **massless** visible particle]

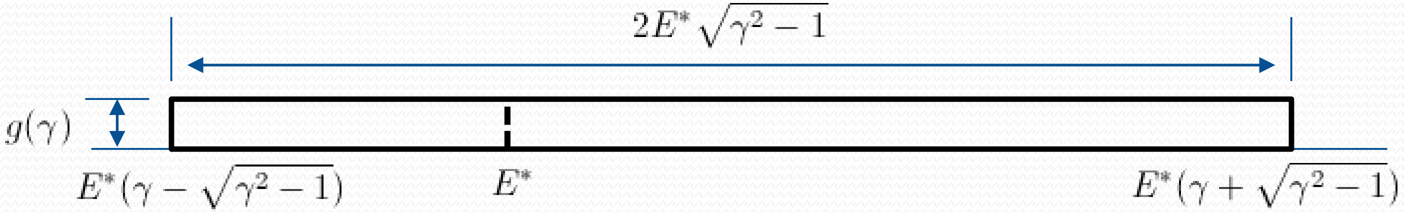
$E = E^*\gamma(1+\beta\cos\theta^*) = E^*(\gamma + \sqrt{\gamma^2-1}\cos\theta^*)$ (Cf. massive visible particle: $E = E^*\gamma + p^*\sqrt{\gamma^2-1}\cos\theta^*$)

- ❑ $\cos\theta^* \in [-1,\ 1] \ \rightarrow\ E \in [E^*(\gamma - \sqrt{\gamma^2-1}),\ E^*(\gamma + \sqrt{\gamma^2-1})]$
- ❑ Two observations

(1) Lower(upper) limit is always smaller(greater) than *E**. → Any rectangle **must contain *E*=*E****

(2) Lower limit is closer to the axis of *E*=*E** than upper limit. → **Asymmetric** w.r.t. *E*=*E**

- ❑ For any given Lorentz factor γ,

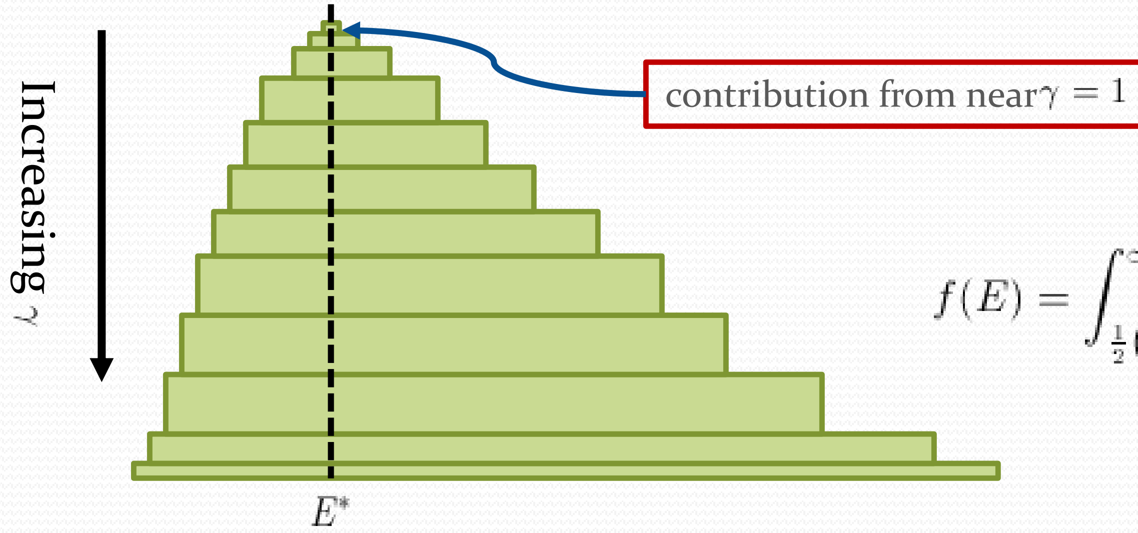


Existence of Peak: a Heuristic Proof

Step 3: "stacking up" rectangles

- Distribution in $E \rightarrow$ summing up the contributions from all relevant boost factors

  → **"Stacking up" the rectangles** weighted by boost spectrum $g(\gamma)$.
- For any generic $g(\gamma)$ with non-zero values around $\gamma = 1$ [Assumption 3], we have



$$f(E) = \int_{\frac{1}{2}\left(\frac{E}{E^*}+\frac{E^*}{E}\right)}^{\infty} d\gamma \frac{g(\gamma)}{2E^*\sqrt{\gamma^2-1}}$$

- E^* must be **the location of the peak!** [Agashe, Franceschini, **DK**, Phys.Rev.D88 (2013) 5, 057701]

Energy Peak: Formal Proof

- First derivative

$$f'(E) = \frac{\mathrm{sgn}\left(\frac{E^*}{E} - \frac{E}{E^*}\right)}{2EE^*} g\left(\frac{1}{2}\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

- Vanishing derivative gives the extrema → this is the same as solving $g = 0$.
- For simplicity, assume that $g(\gamma)$ does not vanish for any finite value of γ greater than 1.

  → This is typical for particles produced at colliders.
- Two possibilities: $g(1) = 0 \quad \text{or} \quad g(1) \neq 0$
- $g(1) = 0$: $f'(E = E^*) \propto g(1) = 0 \rightarrow f$ has a unique extremum at $E = E^*$.
- $g(1) \neq 0$: $f'(E)$ flips its sign at $E = E^*$ due to the sign function (from + to -). → the distribution has a **cusp** at $E = E^*$ which appears as a peak.

Energy Peak: Functional Properties of Generic $f(E)$

- f is a function with an argument of $\frac{1}{2}\left(\frac{E}{E^*}+\frac{E^*}{E}\right)$, i.e., even under $\frac{E}{E^*} \leftrightarrow \frac{E^*}{E}$ or $E \to \frac{E^{*2}}{E}$.

  ← clear from the expression of $f(E)$

  $$f(E) = \int_{\frac{1}{2}\left(\frac{E}{E^*}+\frac{E^*}{E}\right)}^{\infty} d\gamma \frac{g(\gamma)}{2E^*\sqrt{\gamma^2-1}}$$

- f is maximized at $E=E^*$.

  ← proven heuristically and formally

- f vanishes as E approaches 0 or ∞.

  ← the integral expression of $f(E)$ becomes trivial in those limits.

- f becomes a δ-function in some limiting case.

  ← if any of mother particles are NOT boosted, i.e., the rest frame, then f should return a δ-functionlike distribution.

Energy Peak: an Ansatz for the Energy Spectrum

$$\boxed{f(E) = \frac{1}{K_1(p)} \exp\left[-\frac{p}{2}\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right]}$$

- $K_1(p)$: modified Bessel function of the second kind of order 1
- p: fitting parameter which encodes the width of the peak
- E^* as a **fitting parameter** can be extracted by fitting!
- All four properties are satisfied. → for the last property, use the asymptotic behavior of $K_1(p)$

$$K_1(p) \overset{p\to\infty}{\to} \sim \frac{e^{-p}}{\sqrt{p}}\left(1 + \mathcal{O}\left(\frac{1}{p}\right)\right)$$

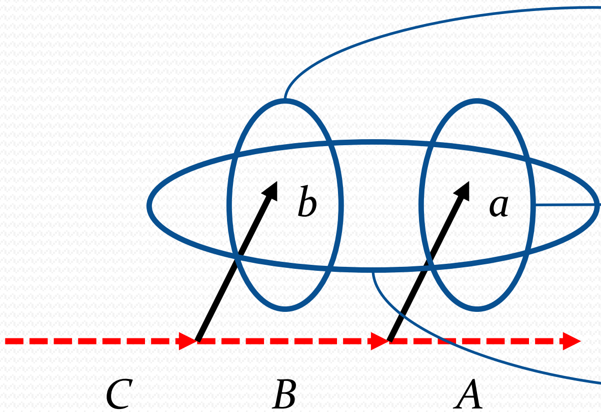
- Proposed ansatz does not develop a cusp so that it is more suitable for the case of $g(1) = 0$, e.g., pair-production of mothers (cf. the case of $g(1) \neq 0$, single production of mothers).

Top Quark Mass Measurement by CMS: Systematics

Source of uncertainty	δE_{peak} (GeV)	δm_t (GeV)
Experimental uncertainties		
Jet energy scale	0.74	1.23
b jet energy scale	0.13	0.22
Jet energy resolution	0.18	0.30
Pile-up	0.02	0.03
b-tagging efficiency	0.12	0.20
Lepton efficiency	0.02	0.03
Fit calibration	0.14	0.24
Backgrounds	0.21	0.34
Modeling of hard scattering process		
Generator modeling	0.91	1.50
Renormalization and factorization scales	0.13	0.22
ME-PS matching threshold	0.24	0.39
Top p_T reweighting	0.91	1.50
PDFs	0.13	0.22
Modeling of non-perturbative QCD		
Underlying event	0.22	0.35
Color reconnection	0.38	0.62
Total	1.62	2.66

- Statistical uncertainty of 1.17 GeV will be under control (more statistics coming up).
- Experimental uncertainty (mostly from JES) will be under control.
- Theoretical uncertainty is from modeling of hard scattering processes.
- Any chance to improve/understand the systematic uncertainty?
  - Study of **higher-order effects**

Application 1-(a): Inversion Formulas



$$E_b^{\rm peak} = \frac{m_C^2 - m_B^2}{2m_C} \qquad E_a^{\rm peak} = \frac{m_B^2 - m_A^2}{2m_B}$$

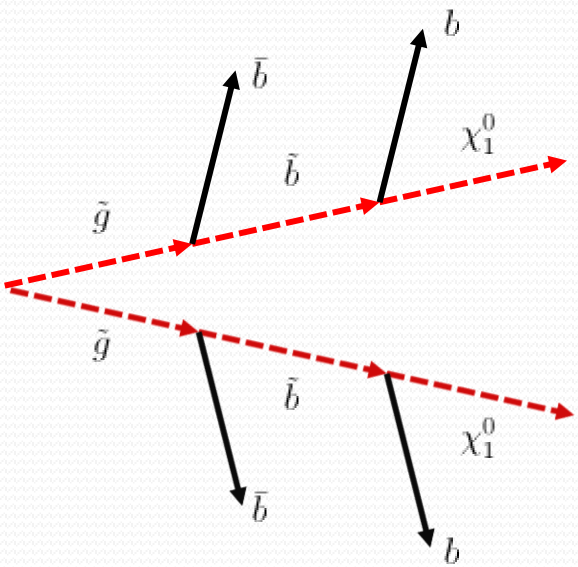
$$m_{ab}^{\rm max} = \frac{\sqrt{(m_C^2 - m_B^2)(m_B^2 - m_A^2)}}{m_B}$$

Inversion formula

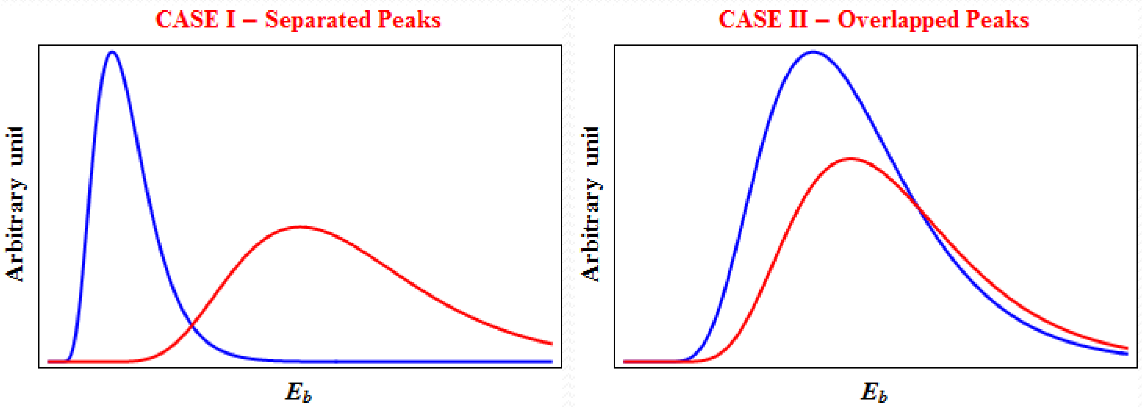
$$m_C = \frac{2\, m_{ab}^{\rm max\,4}\, E_b^{\rm peak}}{m_{ab}^{\rm max\,4} - 16\, E_b^{\rm peak\,2}\, E_a^{\rm peak\,2}} \qquad m_B = \frac{8\, m_{ab}^{\rm max\,2}\, E_b^{\rm peak\,2}\, E_a^{\rm peak}}{m_{ab}^{\rm max\,4} - 16\, E_b^{\rm peak\,2}\, E_a^{\rm peak\,2}}$$

$$m_A = \frac{4 m_{ab}^{\rm max} E_b^{\rm peak} E_a^{\rm peak}}{m_{ab}^{\rm max\,4} - 16\, E_b^{\rm peak\,2}\, E_a^{\rm peak\,2}} \sqrt{4\, m_{ab}^{\rm max\,2}\, E_b^{\rm peak\,2} + 16\, E_b^{\rm peak\,2}\, E_a^{\rm peak\,2} - m_{ab}^{\rm max\,4}}$$

Application 1-(a): Example Model – Gluino Decay



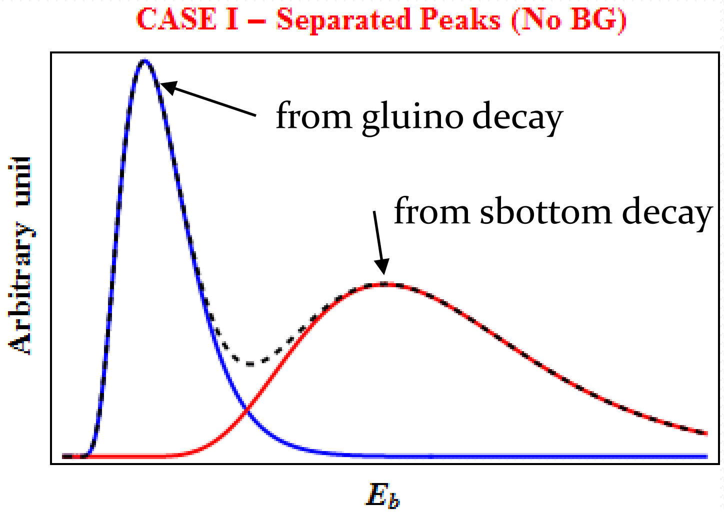
- Two visible particles (b-jets) coming from the primary (gluino) and the secondary (bottom squark) mother particles are not distinguishable!
  → *Only one combined energy distribution* is available.
  → Two cases naturally arise.



Application 1-(a): Case I – Features

- ❑ Mass hierarchy: $m_{\tilde{g}} \approx m_{\tilde{b}} \gg m_{\chi_1^0}$
- ❑ Energy of *b*-jets from the gluino decay is, on average, smaller than energy of *b*-jet from the sbottom decay.

  → The two peaks are well-separated and double-peak signature is expected.



- ❑ Typical gluinos are produced nearly at rest.
- ❑ Typical sbottoms are produced also nearly at rest.

  → Relatively narrower width of the peaks

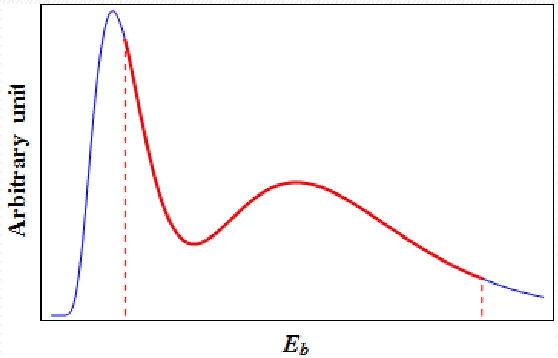
  → Less pollution from the other peak

  → helping to have a double-peak shape

Application 1-(a): Case I – Features

- Lower peak is expected to be located at the region of < 100 GeV.
- Background becomes dominant and a harder p_T cut for jets is expected.
- The lower bound of the fitting range cannot be too small.

  → Fitting still can locate the position of the lower peak by "*seeing its tail*"!



- Effective equations: Three equations are NOT sensitive to the neutralino mass.

  → Safely assume that its effect in the three equations is negligible.

  → Only two equations are needed.

  → Take two well-extracted observables: higher peak and dijet edge

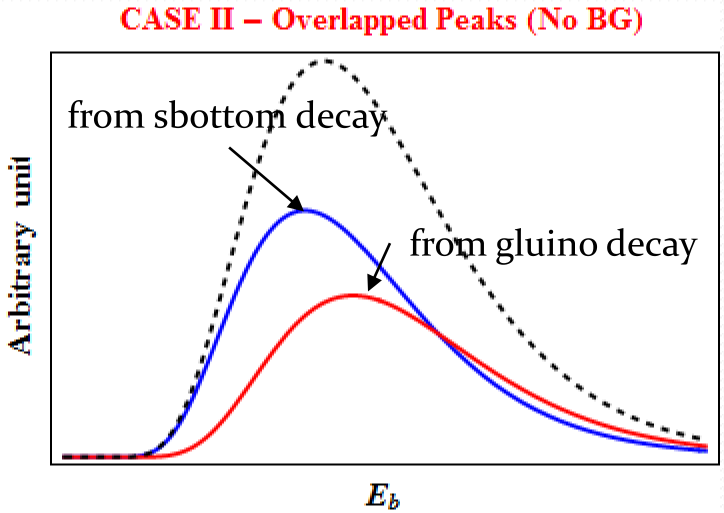
$$m_{\tilde{b}} = 2E_{b,\mathrm{hp}}$$

$$m_{\tilde{g}} = \sqrt{m_{bb}^2 + 4E_{b,\mathrm{hp}}^2}$$

Application 1-(a): Case II – Features

- Mass hierarchy: $m_{\tilde{g}} \gg m_{\tilde{b}} \gtrsim m_{\chi_1^0}$
- Energy of *b*-jets from the gluino decay is roughly equal to energy of *b*-jet from the sbottom decay.

  → The two peaks are overlapped and only a single peak is expected.



- One peak behaves like a background to the other, and vice versa.
- Fitting can extract the two peaks!
- Now the equations are sensitive to the neutralino mass as well.

  → All three mass parameters can be extracted.

Application 1-(b): Ansatz

- A possible ansatz for the massive visible particle case is given by

$$f(E) = N\left(\exp[-w \cdot \gamma_-(E)] - \exp[-w \cdot \gamma_+(E)]\right)$$

$$\gamma_+(E) \equiv \gamma^{*2}\left(\sqrt{1-\frac{1}{\gamma^{*2}}}\sqrt{\frac{E^2}{E^{*2}}-\frac{1}{\gamma^{*2}}}+\frac{E}{E^*}\right)$$

$$\gamma_-(E) \equiv \gamma^{*2}\frac{E}{E^*}\left(1-\sqrt{1-\frac{1}{\gamma^{*2}}}\sqrt{1-\frac{E^{*2}}{\gamma^{*2}E^2}}\right)$$

N: normalization parameter, w: width parameter, γ^*: parent rest-frame boost factor of the massive visible particle

- As a boundary condition, for a massless child particle (i.e., $\gamma^* \to \infty$), $\gamma_+(E)$ diverges, whereas $\gamma_-(E)$ converges to a finite value corresponding to the massless visible particle case.

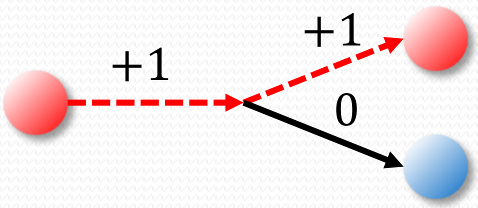
$$\lim_{\gamma^* \to \infty} \gamma_-(E) = \frac{E}{E^*} + \frac{E^*}{E} \equiv \gamma_-^{(\infty)}(E)$$

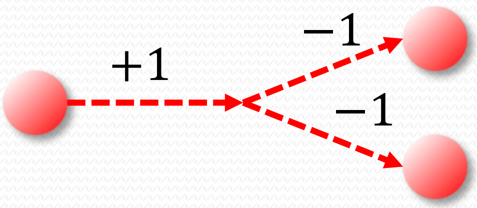
Application 2: Z_3 Primer

- Z_3 symmetry defined by the relevant transformation rule

$$\Phi \to \Phi \exp\left(\frac{2\pi i}{3} q\right) \qquad q = 0, +1, -1 (\equiv +2)$$

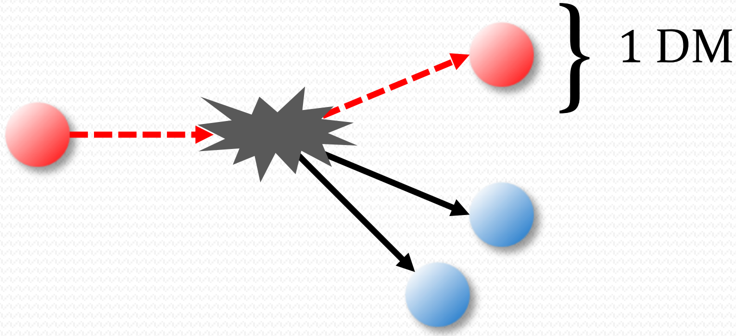
- 0: neutral → typically assigning 0 to the known particles
- $+1/-1$: non-trivially charged under Z_3
  - Z_3 : If DM assigned $+1$ → anti-DM assigned $-1(= +2)$ → DM ≠ anti-DM
  - Z_2 : If DM assigned $+1$ → anti-DM assigned $-1(= +1)$ → DM = anti-DM
- Sometimes, a new particle can decay into two Z_3-charged particles

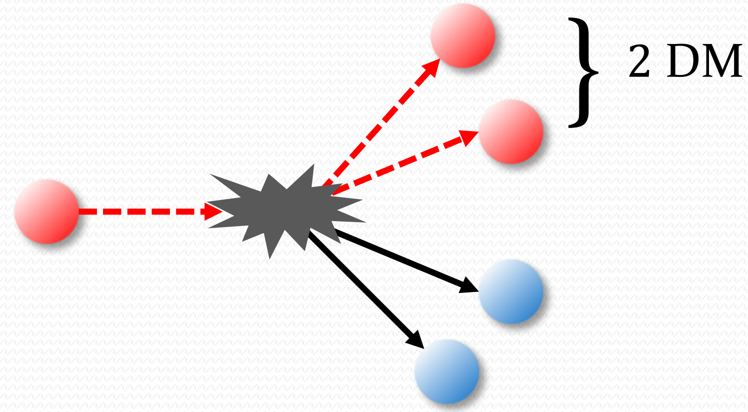




Application 2: Z_3 Primer

❑ In the final state, these lead to





❑ This simple difference makes a huge effect on the relevant variables. → Main physics for distinguishing DM stabilization symmetries coming from this simple observation!