Theory Aspects of Top Quark Mass Measurements

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Based on the review ”What is the Top Quark Mass?”
for Annual Reviews on Nuclear and Particle Science
arXiv:2004.12915

$\int dk \, \Pi$ Doktoratskolleg Particles and Interactions

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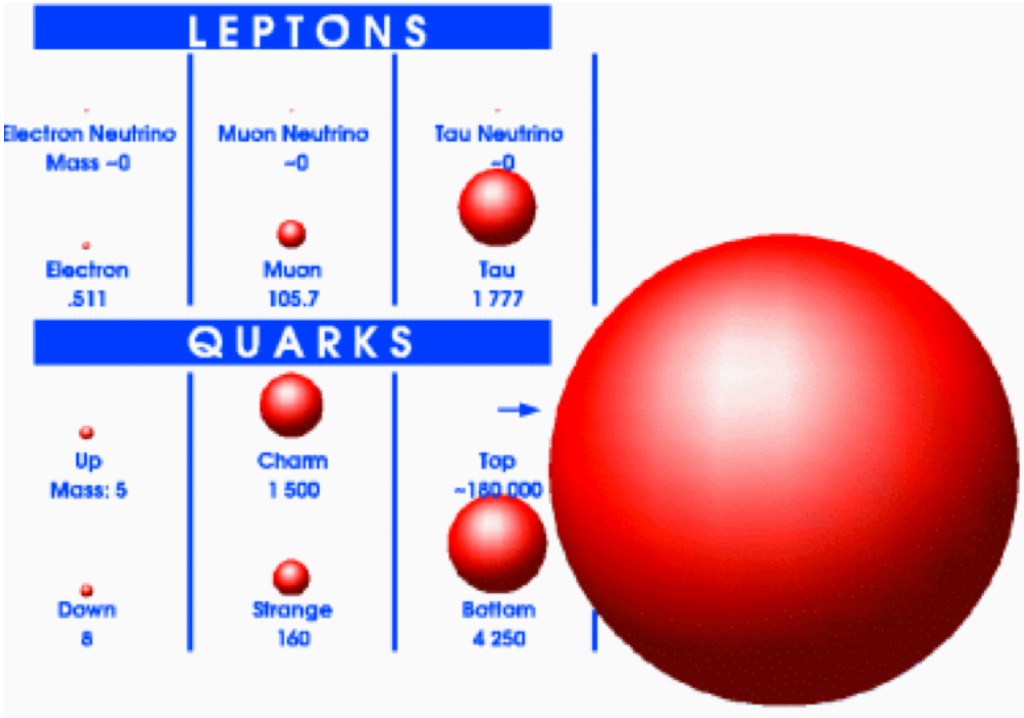
FWF Der Wissenschaftsfonds.

.. not just the heaviest SM particle



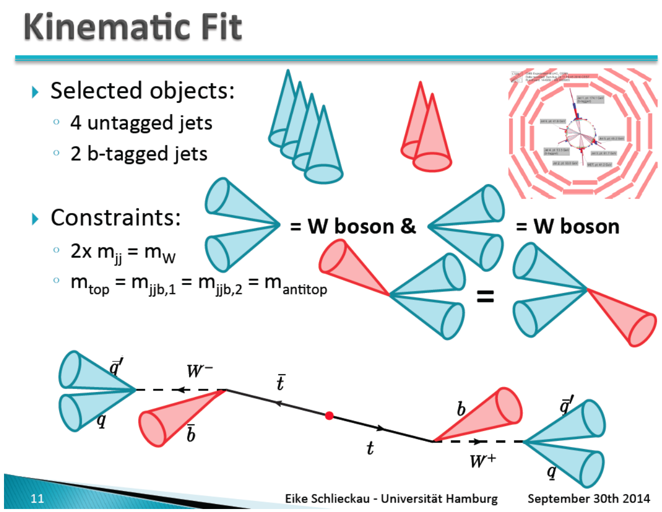
- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background it new physics searches
- One of crucial motivations for New Physics

- Very special physics laboratory: $\Gamma_t \gg \Lambda_{QCD}$
  - Top treated a particle: p_T, spin, $\boldsymbol{\sigma}_{tot}$, $\boldsymbol{\sigma}$(single top), $\boldsymbol{\sigma}$(tt+X),.. $\rightarrow q \gg \Gamma_t$
  - Quantum state sensitive low-E QCD and unstable particle effects: m_t, endpoint regions $\rightarrow q \sim \Gamma_t$
  - Multiscale problem: p_T, $m_t \gg \Gamma_t \gg \Lambda_{QCD}, \ldots$ (depends on resolution scale of observable)

Top Mass Measurements

Most precise method: Direct Reconstruction

kinematic mass determination



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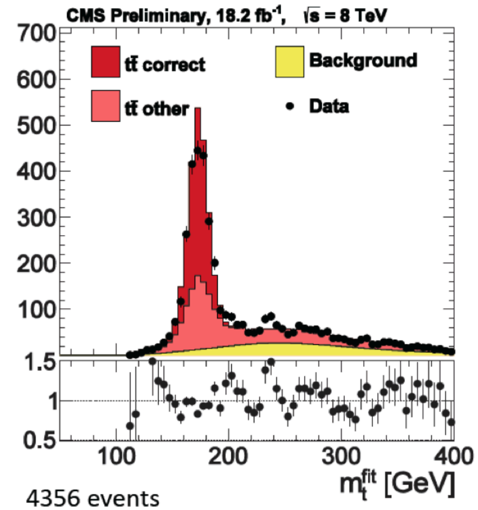


4356 events

Determination of the best-fit value of the Monte-Carlo top quark mass parameter

⊕ High top mass sensitivity

⊖ Precision of MC ?

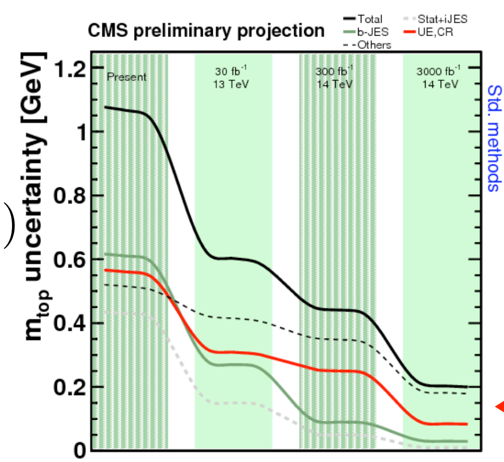
⊖ Meaning of m_t^{MC} ?

$$m_t^{\mathrm{MC}} = 172.9 \pm 0.4\,\mathrm{GeV} \quad \text{(world average)}$$

$$m_t^{\mathrm{MC}} = 172.26 \pm 0.61\,\mathrm{GeV} \quad \text{(CMS combined)}$$

$$m_t^{\mathrm{MC}} = 172.69 \pm 0.48\,\mathrm{GeV} \quad \text{(ATLAS combined)}$$

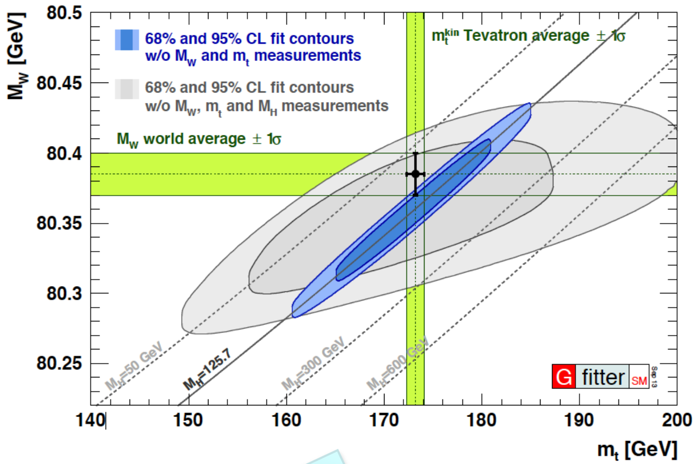
$$m_t^{\mathrm{MC}} = 174.34 \pm 0.64\,\mathrm{GeV} \quad \text{(Tevatron)}$$

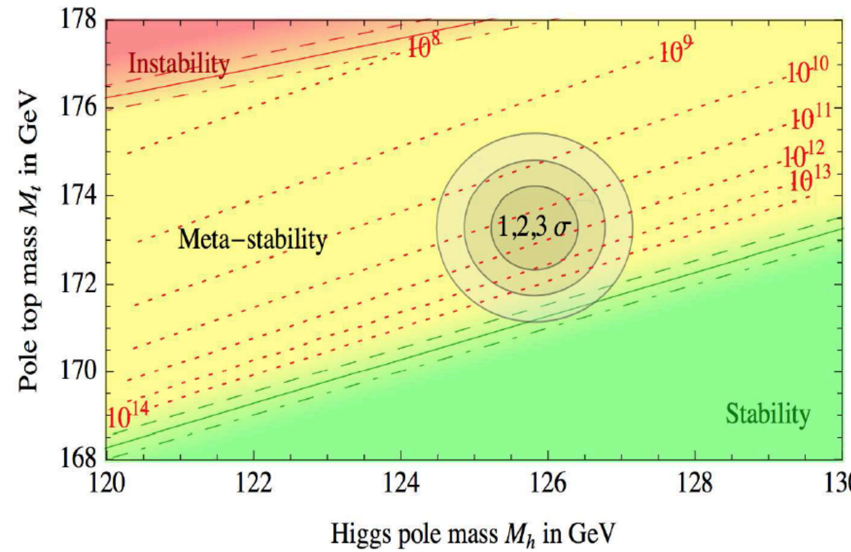


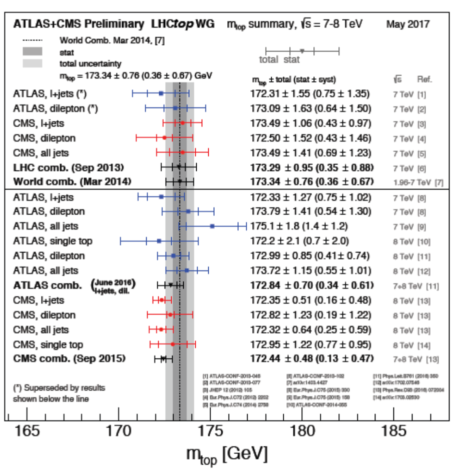
$\Delta m_t \sim 200$ MeV

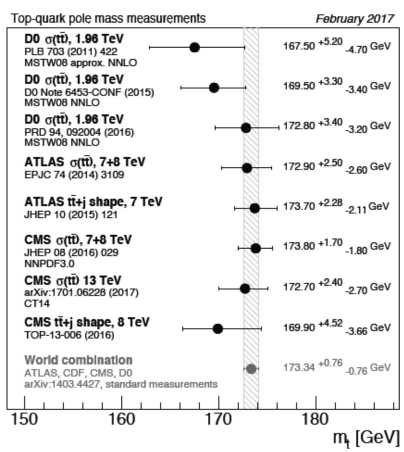
(HL-LHC projection)

Why a Precision Top Mass is Important









Aims: m_{top} wanted !

M_{top} is a renormalized QCD parameter !

- Reduce error in m_{top}^{MC}
- Improve / understand better MC
- Clarify mass scheme m_{top}^{MC} !

Content

Aims of future research on top mass measurements:

- Understand and resolve the physical and conceptual questions involved in the MC top mass interpretation problem (direct measurements).
- Better understanding of experimental and theoretical aspects of alternative top mass measurement methods.

  → based theory predictions with well-defined mass scheme

  → uncertainties comparable to direct measurements difficult for HL-LHC

Content:

- Physics of mass renormalization schemes
- Status of top mass measurements
- The controversy
- Recent insights in m_t^{MC}

Comment: Mof the problems would be resolved, if we had an e^+e^- collider where we could do measurement of the total cross section at the top-antitop threshold. But this talk will concentrate on issues related to the LHC.

e^+e^- collider: $\delta\, m_t^{\text{well-defined scheme}} \sim$ 50-100 MeV straightforward

Mass Extraction and Renormalization Schemes

The Principle of Top Mass Determinations

- Top quark is not a physical particle ("colored parton")
- Top mass defined from theoretical prescriptions (renormalization schemes)

- Different schemes are related by a perturbative series.

$$m_t^A - m_t^B = \sum_{n=1} c_n \alpha_s^n(\mu)$$

Parton level cross section formally scheme-invariant,
but can be practically scheme-dependent due to truncation

$$\hat{\sigma}(Q, m_t^A, \alpha_s(\mu), \mu; \delta m^A) = \hat{\sigma}(Q, m_t^B, \alpha_s(\mu), \mu; \delta m^B)$$

- For comparison with exp. data one has to account for non-perturbative corrections

$$\sigma^{\mathrm{exp}} = \hat{\sigma}(Q, m_t^X, \alpha_s(\mu), \mu; \delta m^X) + \sigma^{\mathrm{NP}}(Q, \Lambda_{\mathrm{QCD}})$$

Typically at LHC: $\sigma^{\mathrm{NP}} \sim \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^n , \quad n = 1$

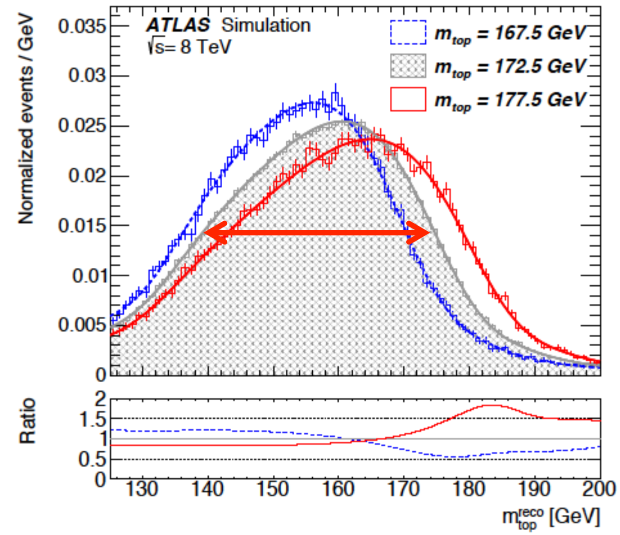
Linear effects always arise from <u>color neutralization processes</u>.
$\rightarrow$ High precision control over soft partonic and NP effects needed when mass sensitivity generated by small dynamical scales

Mass Extraction and Renormalization Schemes

- Parton level cross section and NP corrections MUST be separately consistent with QCD so that the top quark mass (as well as $\alpha_S(Q)$) can be determined reliably!
  → otherwise systematic bias: model instead of field theory parameters

- Which mass scheme is best? → Consider analogy to strong coupling α_S
  - Relevant dynamical scale Q ⇒ $\alpha_S(Q)$ frequently best choice (MSbar)
  - All quantum corrections to quark-gluon interactions from <u>scales above Q</u> are absorbed into $\alpha_S(Q)$ → IR-save definition of strong coupling
  - Multiple scale problems: factorization allows to make adequate scale choices

We seek for a scale-dependent mass scheme $m_t(Q)$ with properties similar to the strong coupling $\alpha_S(Q)$.

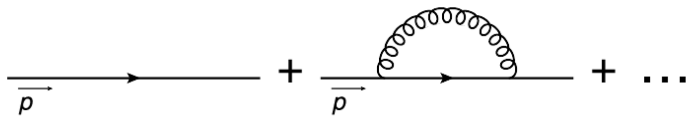


- Multi-scale issue:
  In general high mass sensitivity is associated with QCD dynamics at a low scale
  → typically: scale ~ width of distribution

Mass Extraction and Renormalization Schemes

Top Mass Renormalization Schemes

- Related to different treatments of the top self energy.



$$\sim \frac{i}{\not p - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

$$\Sigma(p, m_t^0, \mu) \sim m_t^0 \left(\frac{\alpha_s(\mu)}{\pi}\right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu)\right] + \ldots$$

Large linearly IR-sensitive contributions (soft gluons in top rest frame):
O(Λ_{QCD}) renormalon behavior at higher orders

$\overline{\text{MS}}$ mass:

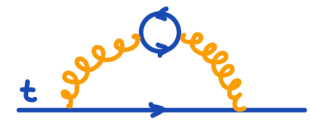
- Absorb 1/ε term into the mass ($\overline{\text{MS}}$): $$\overline{m}_t(\mu) = m_t^0 \left\{1 + \left(\frac{\alpha_s(\mu)}{\pi}\right)\left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E})\right]\right\} + \ldots$$
- All self-energy corrections from scales > μ are absorbed into $m_t(\mu)$

  → IR-save short-distance mass definition (short-distance mass)
- RG-evolution similar to α_S: $$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\overline{m}_t(\mu) = -\overline{m}_t(\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right) + \ldots$$
- Large contributions in A^{fin} can cancel with other linearly sensitive corrections in the cross section coming from soft gluons in the top rest frame
- $\overline{\text{MS}}$ only well-defined for $\mu \gtrsim m_t$ → e.g. total cross section at high energies

Mass Extraction and Renormalization Schemes

MSR mass:

- Absorb virtual top quark fluctuations into the mass as well:



Motivated by Foldy-Wouthuysen transformation:

$$m_t^{\rm MSR}(R) = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(R)}{\pi}\right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\rm fin}(m_t^0/R) \right] \right\} - R\left(\frac{\alpha_s(R)}{\pi}\right) A^{\rm fin}(1) + \ldots$$

- All self-energy corrections from scales > R are absorbed into $m_t^{\rm MSR}(R)$

  → IR-save short-distance mass definition

  → applies to $R \lesssim m_t$

Mateu, Lepenik, Preisser, AHH '17

- RG-evolution is linear in R:

$$\frac{d}{d\ln R}\, m_t^{\rm MSR}(R) = -\frac{4}{3} R \left(\frac{\alpha_s(R)}{\pi}\right) + \ldots$$

- "Non-relativistic" mass scheme numerically close to low-scale threshold masses known from top pair threshold computations at a future lepton collider:

  1S mass, PS mass, etc.

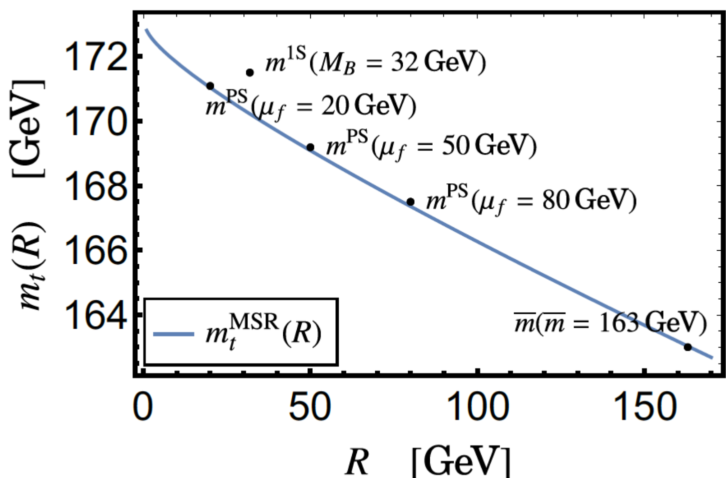
- By construction:

$$m_t^{\rm MSR}(m_t) = \overline{m}_t(m_t)[1 + \mathcal{O}(\alpha_s^2)]$$

MSR scheme is the extension of the $\overline{\rm MS}$ mass scheme for renormalization scales below m_t

→ kinematic mass scheme for

$R \sim$ (dynamic scale) $\ll m_t$



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Mass Extraction and Renormalization Schemes

Pole Mass: (canonical kinematic mass scheme)

- Absorb ALL self-energy corrections into the mass

$$m_t^{\rm pole} = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(\mu)}{\pi}\right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\rm fin}(m_t^0/\mu) \right] \right\} + \ldots$$

- RG-invariant:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\, m_t^{\rm pole} = 0$$

Realizes naive picture of a free top quark → radiation off top resolved at all scales

Mass of the LSZ state for on-shell top scattering amplitudes.

(Standard mass for most FO-NLO/NNLO calculations.)

- Large contributions in $A^{\rm fin}$ absorbed into $m_t^{\rm pole}$ as well and cannot cancel with other linearly sensitive corrections in the cross section: $O(\Lambda_{\rm QCD})$ renormalon problem !
- MSR and pole mass are numerically close for small R:

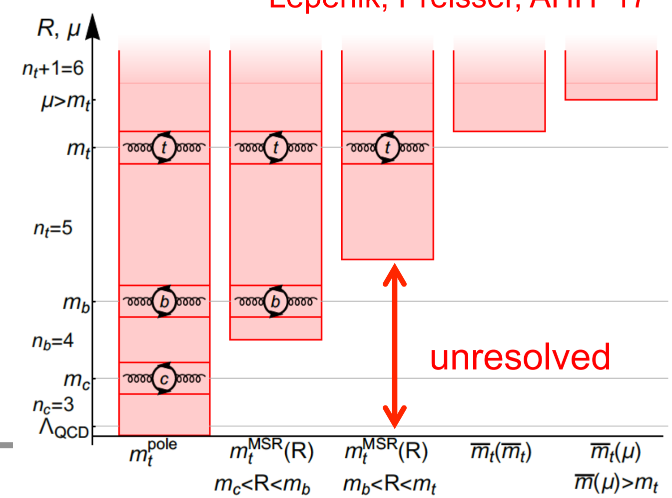
$$m_t^{\rm pole} - m_t^{\rm MSR}(R) \;=\; \frac{4}{3}\left(\frac{\alpha_s(R)}{\pi}\right) R + \ldots$$

Limit R → 0 impossible due to Landau pole.

MSbar and pole mass differ by ~ 10 GeV:

$$m_t^{\rm pole} - \overline{m}_t(\mu) \;=\; \frac{4}{3}\left(\frac{\alpha_s(\mu)}{\pi}\right) \overline{m}_t(\mu) + \ldots$$

Lepenik, Preisser, AHH '17



Mass Extraction and Renormalization Schemes

Pole Mass Renormalon Ambiguity:

- $O(\Lambda_{QCD})$ renormalon problem related to diverging behavior of perturbative series

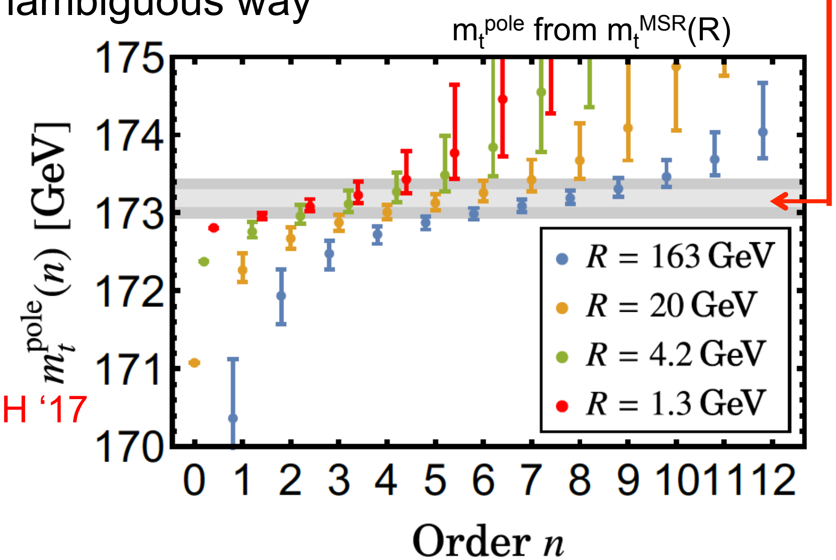
  $$\sim \alpha_s^n \, n!$$

  Strongest possible divergence behavior known.

- Pole mass scheme cannot be defined in an unambiguous way
  - Order-dependent value
  - "Asymptotic" pole mass defined at truncation order where the corrections are minimal. Ambiguous to $O(\Lambda_{QCD})$

    ambiguity: 110 MeV Beneke etal '17

    250 MeV Lepenik, Preisser, AHH '17
  - Truncation order depends the dynamical scale of the observable.



- Pert. series in QCD are always asymptotic.

  $O(\Lambda_{QCD})$ renormalons arise also from physical NP corrections

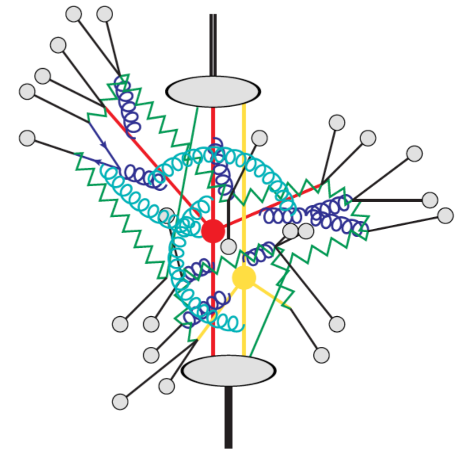
  Correct treatment of linear NP corrections essential.

  $$\sigma^{NP} \sim \left(\frac{\Lambda_{QCD}}{Q}\right)^n , \quad n = 1$$

Status of Top Mass Determinations at the LHC

Multi-Purpose Event Generators

- Backbone of all experimental analyses:
  - Pythia — Sjöstrand, etal. ‘15
  - Herwig — Bellm, etal. ‘16
  - Sherpa — Gleisberg, etal. ‘09



- Combines:
  - LO matrix elements
  - Parton shower (Markov chain):

    p_T-ordered dipole (mom. recoil local)

    coherent branching (non-global restricted)
  - Hadronization model: string, cluster

Shower cut Q_0

- Employed for theoretical predictions for all other top mass sensitive cross sections
  - Direct measurements: templates (m_t^{reco}, $M_{b\text{-jet,lepton}}$), matrix-element/idiopgram
  - b-jet and B-meson energy distribution, secondary vertices in B production
  - J/ψ method, M_{T2},

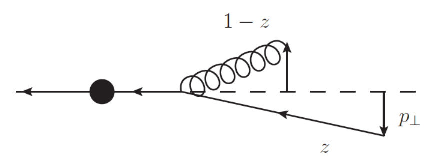
→ Measurements of MC top mass parameter m_t^{MC}

Status of Top Mass Determinations at the LHC

- Parton shower provides approximation to collinear and soft radiation
  → NLL precise for a few simple observables, but LL or less in general
  → Top quarks: theory input applies to quasi-collinear tops only!
  → No systematic treatment of finite lifetime effects
  → No systematic treatment of MPIs, color-reconnection

  Needs to be improved!

  → Self-energy corrections are not simulated: absorbed in m_t^{MC} !
  ⇒ m_t^{MC} is close to the pole mass



- Hadronization model provides description of σ^{NP} (tuning)
  → can compensate for deficiencies of the parton shower

- <u>In such a case:</u> σ^{parton} and σ^{NP} potentially separately incompatible with QCD
  → extraction of QCD parameters (top mass, α_S) affected by systematic errors that may not at all be captured by common MC uncertainty variations

THIS IS THE CORE OF THE INTERPRETATION PROBLEM OF m_t^{MC}
OBTAINED FROM THE DIRECT MEASUREMENTS

Status of Top Mass Determinations at the LHC

Experimental Analyses

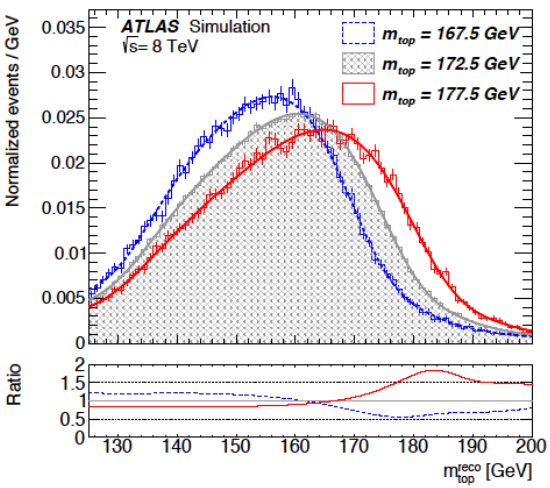
Direct Measurements:

- Template method (ATLAS), matrix element/ideogram method (CMS)

- Based on highly top mass sensitive distributions ($M_{lb\text{-}jet}$, m_t^{reco}, etc) that are dominated by parton shower and hadronization model and cannot be improved by NLO matching.

$$m_t^{\rm MC} = 172.9 \pm 0.4\,{\rm GeV} \quad ({\rm world\ average})$$
$$m_t^{\rm MC} = 172.26 \pm 0.61\,{\rm GeV} \quad ({\rm CMS\ combined})$$
$$m_t^{\rm MC} = 172.69 \pm 0.48\,{\rm GeV} \quad ({\rm ATLAS\ combined})$$
$$m_t^{\rm MC} = 174.34 \pm 0.64\,{\rm GeV} \quad ({\rm Tevatron})$$

- Mostly discussed in the context of the m_t^{MC} interpretation problem.
  Nothing to improve in the experimental analysis.
  Purely theoretical problem.



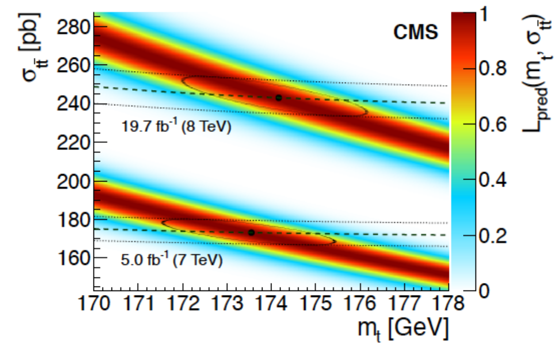
Status of Top Mass Determinations at the LHC

Pole Mass Measurements:

- Based on total and differential cross section for which the parton level calculation can be done reliably at NLO or NNLO/NNLL

- Called "pole mass measurements" only because theorists used pole mass scheme for their calculations. $\rightarrow$ misleading!

  Better: Measurements of m_t in well-defined scheme



- Total inclusive cross section:

$$m_t^{\rm pole} = 172.9^{+2.5}_{-2.6}\,\text{GeV (ATLAS, 7 and 8 TeV data)}$$

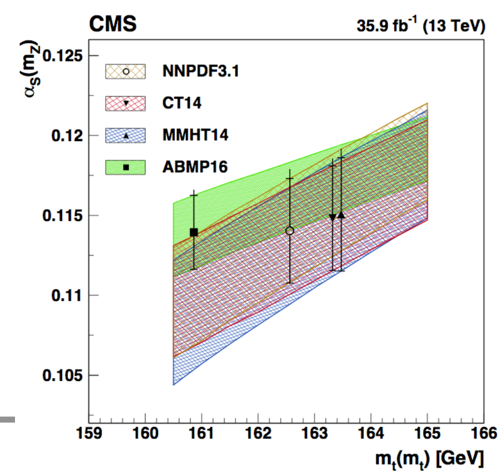
$$m_t^{\rm pole} = 173.8^{+1.7}_{-1.8}\,\text{GeV (CMS, 7 and 8 TeV data)}$$

$$m_t^{\rm pole} = 169.9^{+2.0}_{-2.2}\,\text{GeV (CMS, 13 TeV data)}$$

lower precision due to impact of norm uncertainties

(strong additional correlation to pdfs, α_S)

$\rightarrow$ reliable mass interpretation, but imprecise



CMS arXiv:1812.10505

Status of Top Mass Determinations at the LHC

- Recently also differential cross sections: M_{tt+jet}, M_{tt} + y(tt), lepton energies
  → distributions elevate top mass sensitivity due to structures

$$M_{t\bar{t}} + y(t\bar{t}) : \quad m_t^{\rm pole} = 170.5 \pm 0.8\,{\rm GeV} \quad ({\rm CMS})$$

$$M_{t\bar{t}+jet} : \quad m_t^{\rm pole} = 171.1^{+1.2}_{-1.1}\,{\rm GeV} \quad ({\rm ATLAS})$$

$$\text{leptons} : \quad m_t^{\rm pole} = 173.2 \pm 1.6\,{\rm GeV} \quad ({\rm ATLAS})$$

Important questions to address:

- Reliability of FO parton level cross sections used for m_t determination
- M_{tt}~ $2m_{top}$: ttbar in color-octet configurations, Coulomb effects

- Recent suggestion: soft-dropped boosted top quark mass masses

AHH, Mantry, Pathak, Stewart '17

- Reminder: The gold-plated platinum observable is the top threshold cross section at a future e^+e^- collider. An analogous observable does not exist for hadron colliders. [ttbar always a color singlet at a lepton collider !]

The Controversy

- No general consensus on
  - how to formulate the m_t^{MC} interpretation problem
  - how to quantify the associated uncertainty
  - relevance of the problem

View 1:

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + \Delta_{m_t}^{\mathrm{MC}}$$

Nason

- Δ_m^{MC} an uncertainty in addition to uncertainties quoted in direct measurements

  but much smaller than these and negligible
- Pole mass renormalon ambiguity is 110 MeV << experimental uncertainty (HL-LHC)

View 2:

$$m_t^{\mathrm{MC},Q_0} = m_t^{\mathrm{MSR}}(R_0) + \Delta_{m_t}^{\mathrm{MC}}(R_0, Q_0)$$

Hoang, Stewart

- Shower cut Q_0 of the parton showers plays an essential role because radiation with scales < Q_0 is treated unresolved

  $\rightarrow$ m_t^{MC} depends on Q_0 and the parton shower type

  $\rightarrow$ m_t^{MC} close to $m_t^{MSR}(R_0 \approx Q_0)$

  but much smaller than these and negligible
- Shower cut Q_0 acts like a IR factorization scale and should be chosen > 1 GeV

Recent Quantitative Results

- Combined analysis of direct and total cross section measurement
  $\rightarrow m_t^{pole} - m_t^{MC} < 2$ GeV

Kieseler, Lipka, Moch '16

- Calibration of m_t^{MC} using 2-jettiness for boosted tops in e^+e^- collisions

Butenschoen, Dehnadi, AHH, Preisser, Mateu, Stewart ' 16

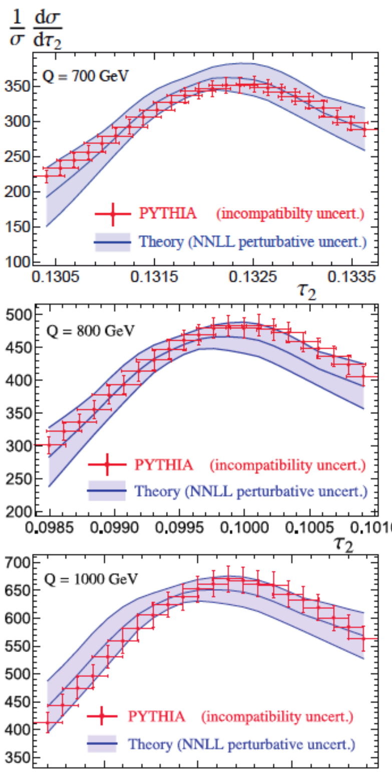
  - Numerical relation between Pythia m_t^{MC} and MSR/pole mass using 2-jettiness in e^+e^- collisions
  - Fits of NNLL+NLO+had.corr. theory predictions with Pythia 8.205 templates
  - Good agreement between Pythia and analytic calculations

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(1\,\mathrm{GeV}) + (0.18 \pm 0.23)\ \mathrm{GeV}$$

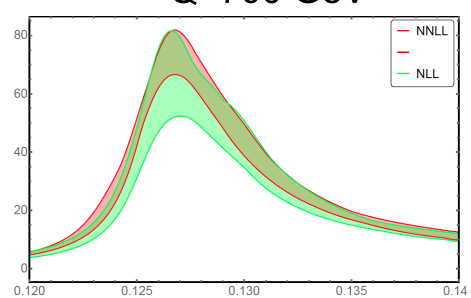
$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + (0.57 \pm 0.29)\ \mathrm{GeV}$$

  - Extended to soft-dropped groomed top jet mass distribution at LHC

Mantry, Pathak, Stewart, AHH ' 17



Q=700 GeV



Recent Quantitative Results

Why boosted top quarks as theoretically clean :

- Fat top jet mass distribution can be computed in QCD factorization (SCET) because the top decays are well separated in phase space.
- Non-perturbative corrections enter through convolutions with a soft function.
- Top decay factorizes as well and can be added through methods known from B physics

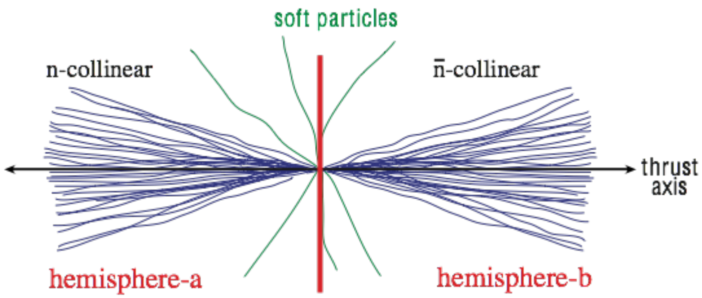
Fleming, Mantri, Stewart, AHH, 2007

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, \boxed{B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right)}\, \boxed{S_{\text{hemi}}(\ell^+, \ell^-, \mu)}$$

Ultra-collinear radiation

(soft in top rest frame)

Large-angle soft radiation



Recent Quantitative Results

- Calibration results are indirect and do not allow to scrutinize the parton shower and hadronization models individually to test whether m_t^{MC} is a perturbative quark mass definition of a model parameter.

$$m_t^{\mathrm{MC},Q_0} = m_t^{\mathrm{pole}} + \Delta^{\mathrm{pert}}(Q_0) + \Delta^{\mathrm{non-pert}}(Q_0) + \Delta^{\mathrm{MC}}$$

pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup
- (Affected by finite width effects?)

Non-perturbative contribution:

- Effects of hadronization model
- May depend on parton shower setup

Monte Carlo shift:

- Contribution arising from systematic MC uncertainties
- E.g. color reconnection, b-jet modeling, ...
- Should be covered by ‘MC uncertainty’ or better negligible

Analysed for Herwig angular-ordered parton shower

- Boosted (quasi-collinear) top quarks
- Stable top quarks
- 2-jettiness (production stage QCD dynamics only)

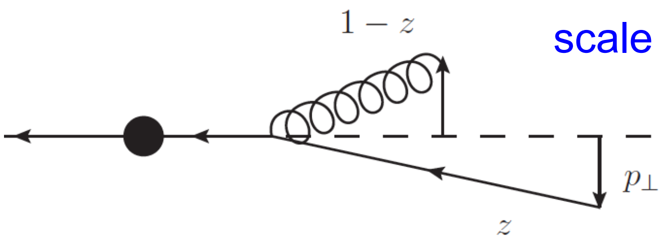
Parton showers likely to work reliably in these limits

AHH, Plätzer, Samitz 1807.06617

Recent Quantitative Results

Catani, Marchesini, Webber 1991
Gieseke, Stephens, Webber, 2003

→ Coherent branching: (basis of the Herwig parton shower)



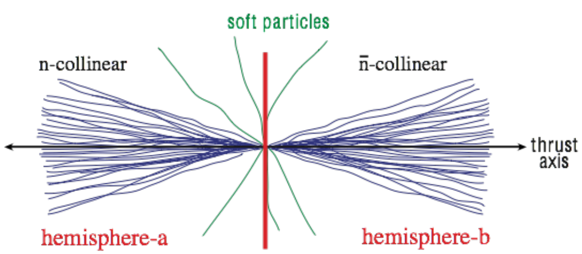
scale in α_S: $\mu^2 = p_\perp^2 + (1-z)^2 m^2$ cutoff: $p_\perp^2 > Q_0^2$

Usually not present in analytic QCD !

2-Jettiness τ_2 distribution In the peak region (for e^+e^- and boosted tops) can be analytically computed in QCD factorization (SCET) at NLL+NLO and coherent branching (CB) at NLL.

Fleming, Mantri, Stewart, AHH, 2007

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\!\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, \underbrace{B_+\!\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\!\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right)}_{\text{Ultra-collinear radiation}}\, \underbrace{S_{\text{hemi}}(\ell^+, \ell^-, \mu)}_{\text{Large-angle soft radiation}}$$



Plätzer, Samitz, AHH; arXiv:1807.06617

Recent Quantitative Results

- The following statements were strictly proven at parton level:
  - NLL precise parton shower mandatory and sufficient to control the scheme of m_t^{MC} with NLO (i.e. $O(\alpha_S)$) precision
  - Herwig 7 parton shower (coherent branching) is NLL precise for e^+e^- 2-jettiness in the resonance region (i.e. jet masses in the peak region)
  - For shower cut Q_0=0: $m_t^{MC} = m_t^{pole}$ (at NLO)
  - In realistic parton showers $Q_0 > 0$

    ⇒ the generator mass is „coherent branching mass“

    $$\boxed{m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3}\,\alpha_s(Q_0)\,Q_0 + \ldots}$$
  - $m_t^{CB}(Q_0)$ is a short-distance mass and does not have the pole mass renormalon ambiguity
  - Numerical relations:

$$m_t^{\mathrm{MSR}}(Q_0) - m_t^{\mathrm{CB}}(Q_0) = 120 \pm 70 \ \mathrm{MeV}$$

$$m_t^{\mathrm{pole}} - m_t^{\mathrm{CB}}(Q_0) = 480 \pm 260 \ \mathrm{MeV}$$

Plätzer, Samitz, AHH ‘18

Where do we stand?

- The result is a first step in a more general endeavor to understand the MC top mass m_t^{MC} and cannot yet be directly applied to the direct measurements due to the restrictions to boosted, stable tops and e^+e^- 2-jettiness.

- More work needed to generalize the analysis and to also get insights into the impact of the hadronization model $\rightarrow \Delta_m^{\text{non-pert}}$, Δ_m^{MC}

What have we learned already?

- Only for a NLL precise MC we can calculate the parton level relation of m_t^{MC} to any renormalization scheme

- Certainly m_t^{MC} is not the pole mass due to $Q_0 \neq 0$, but more closely related to the MSR mass $m_t^{MC}(R \approx Q_0)$

- The MC top mass interpretation problem is an essential issue to be worked on when considering uncertainties at the level of 0.5 GeV (i.e relevant today!)

HL-LHC top mass measurements with 200 MeV precision possible,

but much theoretical work still needs to be done to get there.