Transverse instabilities in Booster at injection

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Abstract

I will analyze transverse instabilities in the frequency domain due to both broad and narrow band impedances that are in Booster at injection. I will ignore the instabilities from synchro-betatron and TMCI because they are irrelevant for Booster at injection. Ng [1, 2] has already done a lot of the necessary theoretical analysis which I will just regurgitate albeit with my modifications in this note. My conclusion is that the dampers will need a bandwidth of at least 150 MHz to damp out both the head-tail and the coupled bunch modes.

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I. INTRODUCTION

For Booster, the more important instability that I need to consider are the head-tail modes from short range wakes because of the experimental observations published in Ref. [3]. The source of these short range wakes are from the laminations of the gradient magnets. Since the wakes are short range, this implies that they have a broad band impedance. This impedance allows for the coupling between longitudinal motion to the transverse motion so that the transverse motion at the head of the bunch can be out of phase w.r.t. the tail of the bunch. In fact, there are two mechanisms that gives rise to this coupling. They are:

1. A transverse wake force $F^\perp$ that appears as a longitudinal retarding force via $\partial F^\perp/\partial z$, i.e the slope of the wake force along the bunch. See eq. 6.162 of Chao[4]. $F^\perp$ arises from the wakefield due to a transverse impedance, $Z^\perp$.

2. Chromaticity, ξ, that couples longitudinal to transverse because of the beam's non-zero momentum spread dp/p. This implies that synchrotron motion is involved for interchanging the particles between the head and tail of a bunch.

The coupling between the longitudinal and the transverse results in the appearance of the synchrotron frequency in the bunch spectrum. I will discuss head-tail modes in section II below.

Narrow band impedances can cause coupled bunch modes that include head-tail modes. Experimentally, this type of instability is also observed in Booster. The source of this instability are from the unlaminated regions in the Booster straights because from the analysis by Ng (pg. 430), laminations are not responsible for coupled bunch mode instabilities. I will examine this instability in section III.

There are two other transverse instabilities: they are transverse synchro-betatron instabilty in Booster which comes from the non-zero horizontal dispersion at the RF cavities which I will ignore. And there's also TMCI (transverse mode coupling instability which cannot be stabilized with chromaticity) that I will ignore as well.

The references I will use for the theory are Chao[4], Ng[1], and Myers[5]. And for the analysis of Booster instabilities, I will basically regurgitate a lot of Ng[1, 2] but with my modifications.

II. HEAD-TAIL MODES ONLY (BROAD BAND IMPEDANCES)

From Booster experiments done by Alexahin *et al*[3], the head-tail instability is in the horizontal plane. This is actually unexpected because naïvely, I would've expected the dominant instability to be in the vertical plane because the beam is much closer to the laminations in this plane than in the other. It turns out that the reason that the instability is in the horizontal plane is because the horzontal beta functions are, on average, $4\times$ larger [6] than the vertical beta functions in the F magnets. See Ref. [7].

I will consider the case where I have broad band impedances that only cause head-tail instabilites here. IMPORTANT: in general, a broad band impedance *cannot* drive bunched beam to instability (see pg. 379 of Ng). However, in the presence of non-zero chromaticity, as I will see below, there can be head-tail instabilities. (Ng chapter 12). Note: if the intensity is high enough, it leads to frequency shifts that are comparable to ω_s which causes azimuthal mode coupling. This can lead to mode coupling instability. See Chao section 6.5, pg 323.

The growth rate, $1/\tau_m$, of the head-tail modes[8] $m = 0, 1, 2, \ldots$, is given by (eq. 12.13 of Ng which is for *Sacherer's sinusoidal modes* approximation for the longitudinal distribution. Different approximations give different form factors. See pg. 372 etc. for Chebyschev, Legendre and Hermite modes)

$$\frac{1}{\tau_m} = -\frac{1}{m+1}\frac{q_e I_b}{4\pi\beta E_0}\int_0^\infty d\omega\, \mathrm{Re}[\beta_\perp Z_1^\perp(\omega)][h_m(\omega - \omega_\xi) - h_m(\omega + \omega_\xi)] \qquad (1)$$

where q_e is the electronic charge, $I_b = q_e N_b/T_0$ is the average current per bunch, N_b is the number of protons in a bunch, T_0 is the revolution period, β is the relativistic beta, E_0 is the total energy of the beam, $[\beta_\perp Z_1^\perp] = \sum_\ell (\beta_\perp Z_1^\perp)_\ell$, ℓ is an element in the ring and $(\beta_\perp Z_1^\perp)_\ell$ is the product of the beta function $\beta_\perp$ and the transverse impedance $Z_1^\perp$ at element ℓ (see pg. 361 of Ng) and $\omega_\xi = \xi\omega_0/\eta$ is the betatron frequency shift due to chromaticity, ξ and slip factor η. Note: I am using the US definition of the slip factor, i.e. $\eta < 0$ below transition and $\eta > 0$ above transition. Note: It looks as if the argument of h_m is independent of ω_s. It disappeared because the sum of the line spectrum was replaced by a continuous spectrum. See pg. 478 of Ng.

And h_m is proportional to the power spectrum of mode m and is given by (eq. 9.101 of

Ng)

$$h_m(\omega) = \frac{4(m+1)^2}{\pi^2} \frac{1 + (-1)^m \cos \pi y}{[y^2 - (m+1)^2]^2} \tag{2}$$

where $y = \omega \tau_L / \pi$. Note: the above comes from the approximation that the distribution on the time axis is sinusoidal like, i.e. Sacherer's sinusoidal modes (see pg. 384 of Ng). See Fig. 1.

In particular, the dipole head-tail mode is when $m = 1$. See Fig. 2.

Note: $m \in \{0, \pm 1, \pm 2, \ldots, \pm \infty\}$. See, for example, eq. 6.183 of Chao. But Eq. 1 does not make sense if $m < 0$. So I will assume that it is actually $|m|$ on the rhs for now.

As can be seen in Eq. 1, there is no instability when $\xi = 0$. However, Booster cannot operate at $\xi = 0$ because, the $m = 0$ coupled bunch mode becomes unstable in the presence of a narrow impedance like resistive wall. See next section.

Rather than running at $\xi = 0$, which I think is probably not easily maintained, the stabilizing solution is to run with ξ slightly negative. See section II B and my plots in Fig. 6. The results in that section shows that the strongest unstable modes are the rigid and dipole head-tail modes which technically can be controlled with a small negative chromaticity. However, instead of running with a small negative chromaticity at injection, we actually run with rather large negative chromaticities, about -20 units in the horizontal and -10 units in the vertical which are required to stabilize the coupled bunch mode instability. See section III.
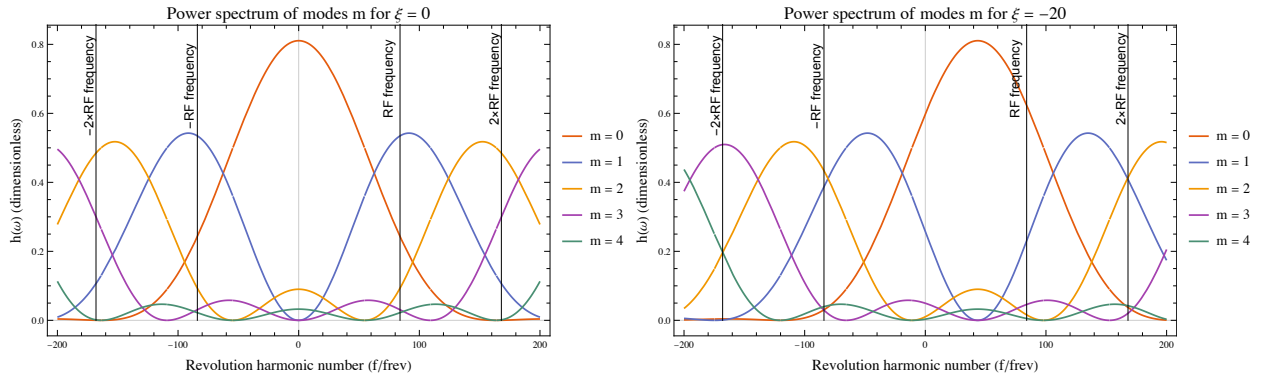

FIG. 1. The envelope of the power spectrum of the modes that were shown in the time domain in Fig. 2. The spectrum shifts depending on the chromaticity. These graphs are plotted with Booster injection parameters, i.e. below transition. Note: I've indicated where the RF frequencies are because the revolution harmonics are too close to be plotted individually here.
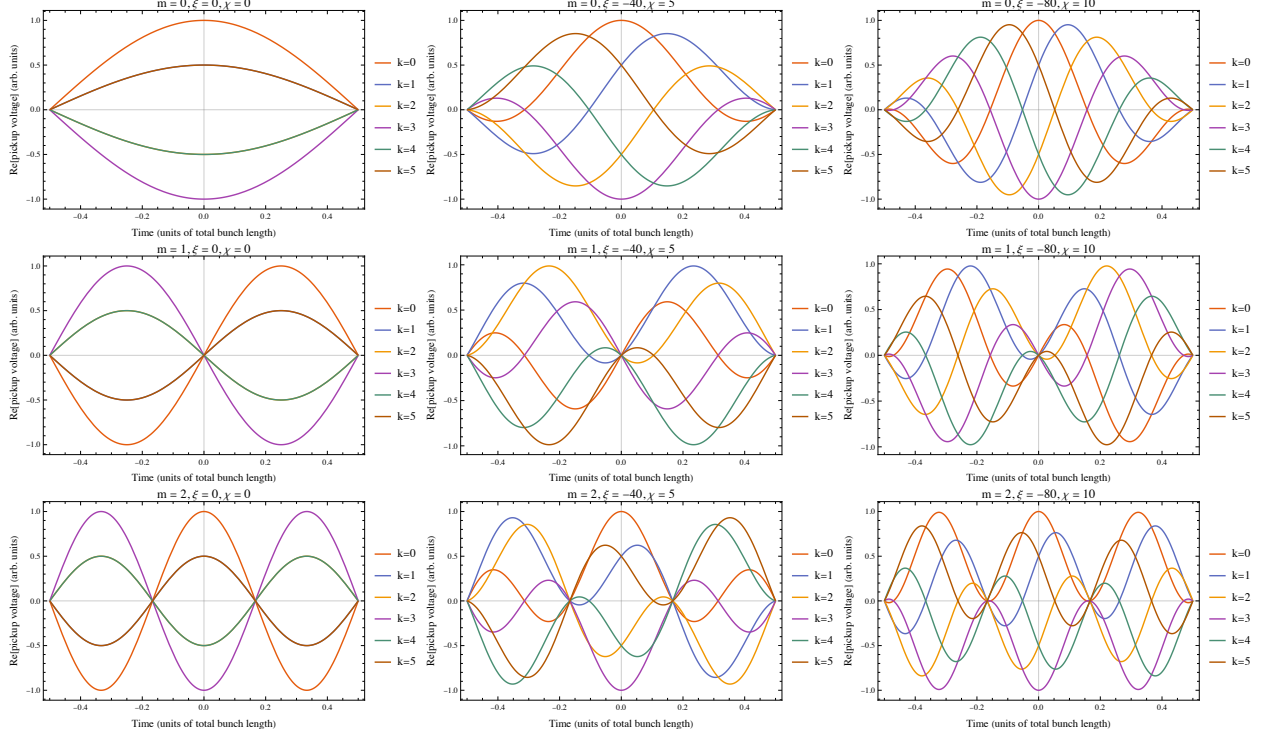
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FIG. 2. Head-tail modes using the Sacherer sinusoidal modes model with Booster parameters as seen by a transverse pickup in the time domain. The betatron tune is $Q = 6.8333$ in this calculation.

A. Laminations

A lot of the theoretical work on Booster head-tail instabilities from the laminated magnets and the lamination-free regions has already been done by Ng[2] and Ng chapter 10.3.1 of Ref. [1]. More detailed simulations with `Synergia` was done by Macridin *et al* [7, 9]. I will use Ref. [9] which is basically a summary of Ng's work because it is easier to quote the formulas using this reference.

The formulæ for calculating the longitudinal impedance per unit <u>longitudinal length</u> $Z_{||}/L$ (units: Ω/m), and horizontal impedance per unit <u>longitudinal length</u>, Z_x/L (units: Ω/m^2), and vertical transverse impedance per unit longitudinal length, Z_y/L (units: Ω/m^2) of the laminations with the dimensions shown in Fig. 3 are (eq. 6, 7 and 8 of Macridin[9]. Notice
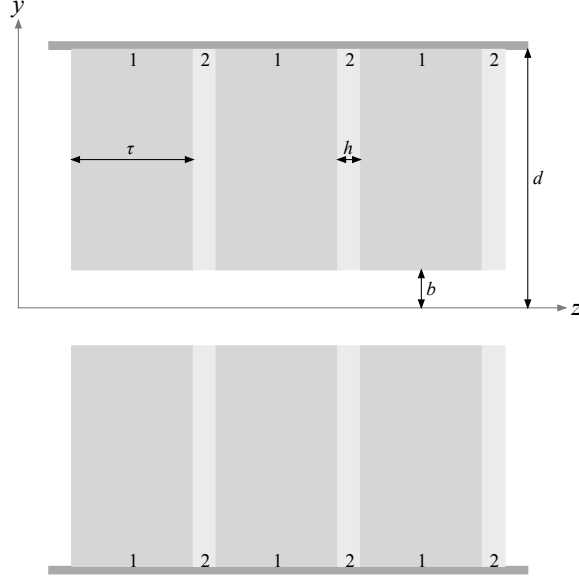
FIG. 3. This is the cross sectional view of a parallel faced beam pipe made up of laminations (labelled "1") and "cracks" (labelled "2"). The laminations are shorted by an ideal conductor.

that I've changed the symbology of the lhs of these equations [10].)

$$\frac{Z_{||}(\omega)}{L} = \frac{\mathcal{R}(\omega)}{2\pi b} \tag{3}$$

$$\frac{Z_x(\omega)}{L} = \frac{\mathcal{R}(\omega)}{2\pi k(\omega)} \int_0^\infty d\eta \, \frac{\eta^2 \operatorname{sech}^2 \eta b}{1 - i\frac{\mathcal{R}(\omega)}{Z_0}\frac{\eta}{k(\omega)} \tanh \eta b} \tag{4}$$

$$\frac{Z_y(\omega)}{L} = \frac{\mathcal{R}(\omega)}{2\pi k(\omega)} \int_0^\infty d\eta \, \frac{\eta^2 \operatorname{csch}^2 \eta b}{1 - i\frac{\mathcal{R}(\omega)}{Z_0}\frac{\eta}{k(\omega)} \coth \eta b} \tag{5}$$

where $Z_0 = 377 \, \Omega$ is the impedance of free space, $k(\omega) = \omega/\beta c$ with beam velocity βc and

$$\mathcal{R}(\omega) = \frac{\mathcal{R}_c(\omega) + \mathcal{R}_\ell(\omega)}{h + \tau} \approx \frac{\mathcal{R}_c(\omega)h}{h + \tau} \tag{6}$$

where $\mathcal{R}_\ell(\omega) = (1 + i)/(\delta_2(\omega)\sigma_2)$ (units: Ω) is the surface resistance of the lamination and $\mathcal{R}_c(\omega)$ is the surface resistance of the "crack" between laminations given by

$$\mathcal{R}_c(\omega) = i\frac{q(\omega)Z_0}{\omega\epsilon_1} \tan\left[q(\omega)(d - b)\right] \tag{7}$$

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with

$$q(\omega) = k_1(\omega)\sqrt{1 + \frac{\mu_2 \delta_2(\omega)}{\mu_1 h}(1 - i)\tanh\left(\frac{g_2(\omega)\tau}{2}\right)}$$

$$\delta_2(\omega) = \sqrt{\frac{2}{\omega\mu_2\sigma_2}}$$

$$g_2(\omega) \approx \frac{1+i}{\delta_2(\omega)}$$

$$k_1(\omega) = \frac{\omega\sqrt{\epsilon_{1r}\mu_{1r}}}{c}$$

(8)

1. F and D magnet impedances

The parameters that I have used for calculating the impedances of the F and D magnets are shown in Table I. The plots of the $Z_{\parallel}/L$ and $Z_{x,y}/L$ for both these laminated magnets are shown in Figures 4 and 5. The $Z_{\parallel}/L$ plot is just for my amusement because it is irrelevant for what I want to do here.

TABLE I. Parameters used for F and D magnets

Parameter	Description	Value	Units
d	dist. from outer wall to centreline	15.24×10^{-2}	m
h	size of "crack"	9.52×10^{-6}	m
τ	thickness of lamination	6.35×10^{-4}	m
b_F	aperture radius of F magnet	2.1×10^{-2}	m
b_D	aperture radius of D magnet	2.9×10^{-2}	m
ϵ_{1r}	relative permittivity of "crack" epoxy	4.75	
μ_{1r}	relative permeability of "crack" epoxy	1	
ϵ_{2r}	relative permittivity of lamination	1	
μ_{2r}	relative permeability of lamination	100	
σ_2	conductivity of lamination (assumed to be iron)	0.5×10^7	$\Omega^{-1}\text{m}^{-1}$

I'm only interested in the transverse impedances shown in Fig. 5 for transverse head-tail. And, in fact, <u>only the real part which directly affects the growth/damping of that mode.</u> The imaginary part is only relevant if I want to calculate the frequency shift of that mode. Note that the peak of the real part of the impedances are between $55 - 60$ MHz.
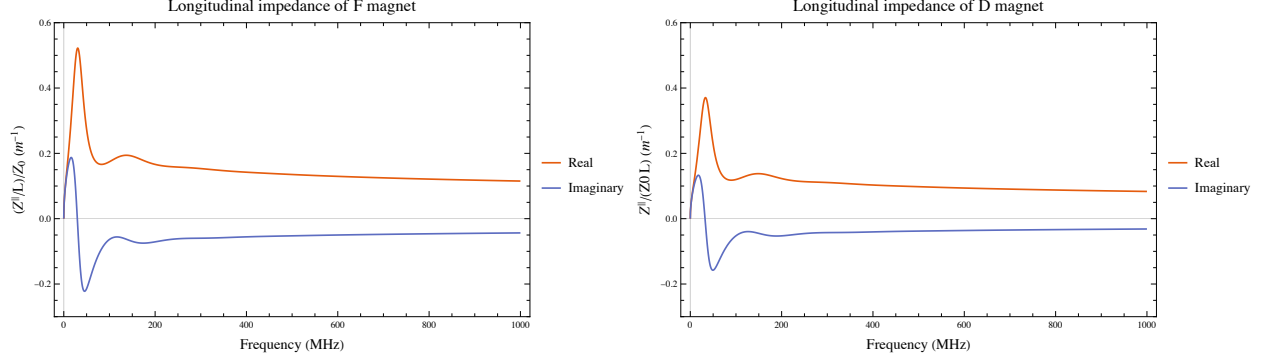
FIG. 4. The longitudinal impedance of the F and D magnets. I will not be considering these impedances because I'm working with the transverse. The F magnet plot should be compared to Macridin[9] fig. 2. I believe Macridin made a mistake in his plot because he used $k = \omega/c$ rather than $k = \omega/\beta c$.

B. Head-tail growth rates

Now, I can calculate the growth rates for different modes at 400 MeV injection using Eq. 1. Table II shows the parameters used for the following calculations and Table III shows the derived quantities required for equations 1 and 2.

But first, let me calculate $\beta_\perp Z_1^\perp$. I will work with the horizontal plane here because the result becomes the vertical by replacing x with y. So, in the horizontal plane, I have

$$\beta_\perp Z_1^\perp = \sum_\ell \left(\beta_x \frac{Z_x}{L} \right)_\ell L_g \tag{9}$$

where I have to sum over all the gradient magnets. I have to multiply by the length of the F, D magnets, L_g, because Z_x/L is the the impedance per unit length. Note: this means that $Z_1^\perp = (Z_x/L)L_g$ has units of Ω/m. Chao on pg. 69 says that $Z_1^\perp$ has dimensions of Ω/m as well. So everything is consistent.

When I expand the sum, I get

$$\beta_\perp Z_1^\perp = L_g N_g \left[\beta_{Fx} \left(\frac{Z_x}{L} \right)_F + \beta_{Dx} \left(\frac{Z_x}{L} \right)_D \right] \tag{10}$$

because there are N_g F and N_g D magnets and I have made the approximation that β_{Fx} is the same for all the F magnets and similarly for the D magnets.

So, with Eq. 10 and the values in Tables II and III, I can create the growth rate (rate of growing by one e) graphs in Fig. 6. From these plots, I can see that modes are $m = 0, 1,$
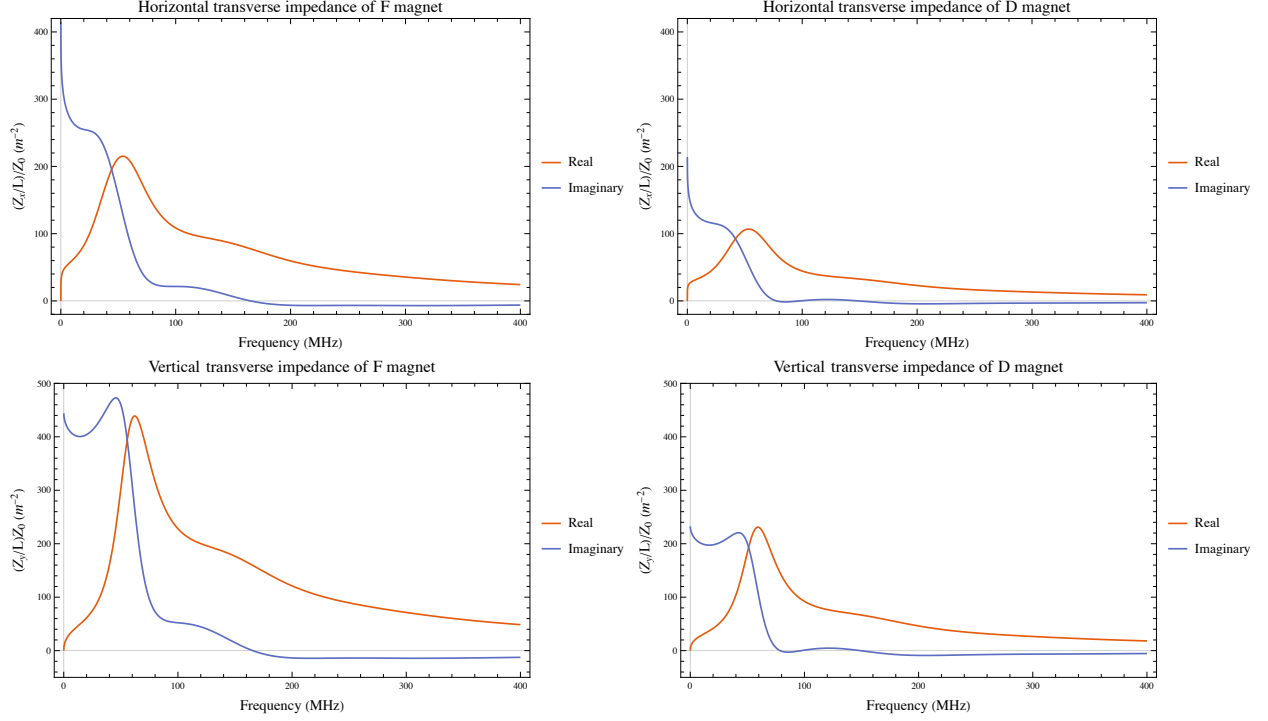
FIG. 5. The transverse impedance of the F and D magnets. Again the F magnet plot should be compared to Macridin fig. 2 which has the same mistake that I talked about in the caption of Fig. 4. Note: <u>only the real part of the transverse impedance is relevant</u> for growth or damping of the head-tail mode. The imaginary part just shifts the frequency of that mode.

i.e. the rigid and the dipole head-tail, are the most unstable if the chromaticity is positive. And the horizontal growth is stronger than the vertical if the chromaticity is positive. Also modes $m = 2, 3, 4$ are actually unstable with negative chromaticity which is interesting.

Reminder: At the start of this section, I had summarized earlier work that observed that the horizontal instability is stronger than the vertical. These simple calculations confirm both the prior observations and Macridin's simulations.

III. COUPLED BUNCH MODES (NARROW BAND IMPEDANCES)

For transverse coupled bunch modes, I must have narrow band impedances. For simplicity, I will assume that Booster is populated with $M = 84$ bunches that have the same

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TABLE II. Parameters for calculating head-tail growth rate at 400 MeV

Parameter	Description	Value	Units
N_b	number of protons in one bunch	$6 \times 10^{12}/81 = 7.4 \times 10^{10}$	
E_0	total energy at injection	$(400 + 938) = 1338$	MeV
τ_L	total bunch length	20.32[a]	ns
η	slip factor	-0.46	
r_B	radius of Booster	75.47	m
L_g	length of F or D magnet	2.9	m
β_{Fx}	horizontal beta function of F magnet	30.5	m
β_{Fy}	vertical beta function of F magnet	6.7	m
β_{Dx}	horizontal beta function of D magnet	10.5	m
β_{Dy}	vertical beta function of D magnet	18.7	m
N_g	number of F magnets (or D magnets)	48	

[a] This number came from C. Bhat's 4σ bunch length measurement FB-20170428-Ev17-19BT-3.
Technically $\tau_L = 2\sqrt{6}\sigma$ for a Gaussian bunch, but a shorter bunch length means that the instability is *worse* and so I'll leave it as is.

TABLE III. Parameters derived from Table II

Parameter	Description	Value	Units
β	relativistic beta at injection	0.7131	
ω_0	revolution frequency	2.83277×10^6	rad/s
T_0	revolution period $(2\pi/\omega_0)$	2.218×10^{-6}	s
I_b	current of one bunch $(q_e N_b/T_0)$	0.535×10^{-2}	A

intensity in each bunch. The growth rate, $\tau_{m\mu}$ is given by (eq. 10.1 of Ng)

$$\frac{1}{\tau_{m\mu}} = -\frac{q_e M I_b \omega_0}{4\pi\beta E_0} \sum_q \frac{\text{Re}\left[\beta_\perp Z_1^\perp(\omega_q)\right] h_m(\omega_q - \chi/\tau_L)}{B \sum_q h_m(\omega_q - \chi/\tau_L)} F'_m \tag{11}$$

where $\omega_q = (qM + \mu)\omega_0 + \omega_\beta + m\omega_s$, $B = M\tau_L/T_0$ is the bunching factor, τ_L is the *full* length of the bunch in temporal units, ω_0 is the revolution frequency and $\chi = \omega_\xi \tau_L = \xi\omega_0\tau_L/\eta$ is the chromaticity phase shift across the bunch; η is the slip factor; F_m is the form factor that
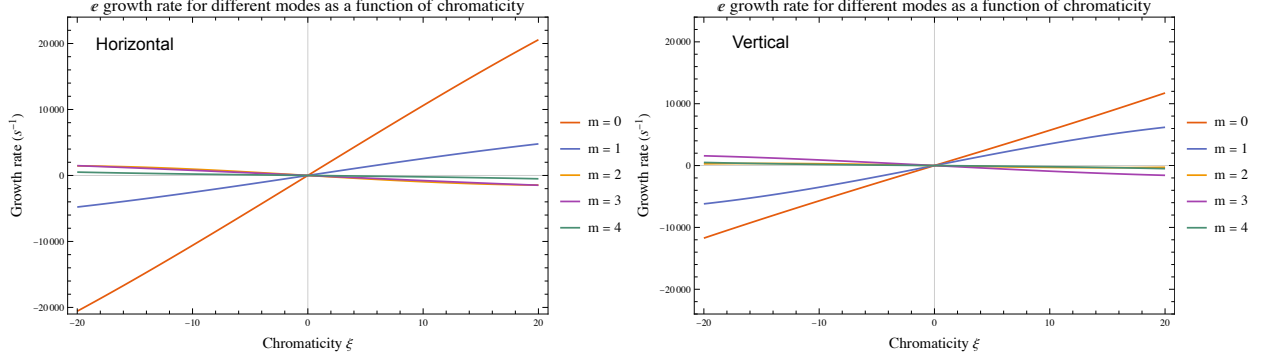
FIG. 6. The head-tail growth rate (rate of growing by one e) for head-tail modes $m = 0, \ldots, 4$ as a function of chromaticity. These results can be compared to Ng[2] fig. 17. My results are different from Ng because I have a different lamination impedance than him.

is model dependent. Again, chromaticity, ξ, is how the longitudinal dynamics is coupled to the transverse dynamics.

Now, let me break down the quantum numbers q, μ and m:

- $q \in \{0, \pm 1, \pm 2, \ldots, \pm \infty\}$ is the band number.

- $0 \leq \mu \leq (M - 1)$ is the coupled bunch mode number. This number tells me that the betatron phase changes by $2\pi\mu/M$ from the earlier bunch to the later bunch.

- $m \in \{0, \pm 1, \pm 2, \ldots, \pm \infty\}$ is the longitudinal azimuthal mode number (see page 301 of Ng. And in particular, $m = 1$ is the dipole head-tail mode. Note: Unlike in the description of longitudinal evolution, $m = 0$ is a valid mode for transverse evolution. See page. 302 and page 370 of Ng. Note 1: Looking through all my references, none of the authors explictly say that $m \geq 0$ on the rhs of Eq. 11. However, it doesn't make sense that $m < 0$ on the rhs so I think it is $|m|$ on the rhs of Eq. 11.

For Booster, the laminations don't contribute to coupled bunch mode instabilities. This instability comes from the unlaminated regions of the ring (see pg. 430 of Ng). If I assume that the unlaminated region behaves like a beam pipe, i.e. I have resistive wall impedance $Z_1^{\perp}\big|_{\text{RW}}$, then I have the following formula from Eq. 11 (this is eq. 10.2 of Ng)

$$\frac{1}{\tau_{m\mu}} \approx -\frac{1}{1+m} \frac{q_e M I_b \omega_0}{4\pi \beta E_0} \text{Re}[\beta_{\perp} \; Z_1^{\perp}\big|_{\text{RW}} (\omega_q)] F'_m(\omega_q, \chi; \tau_L) \tag{12}$$

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where the form factor F' is (Note: I am clarifying Ng's ambiguous notation in eq. 10.3)

$$F'_m(\omega, \xi; \tau_L) = \frac{2\pi h_m(\omega - \omega_\xi)}{\tau_L \int_{-\infty}^{\infty} h_m(\omega') \, d\omega'} \tag{13}$$

where $\omega_\xi = \xi \omega_0 / \eta$.

To continue, I will adopt Sacherer's sinusoidal modes as the longitudinal distribution so that I can calculate F'.

A.   Sacherer's sinusoidal modes

If I use Sacherer's sinusoidal modes, then I will find that

$$\int_{-\infty}^{\infty} h_m(\omega') \, d\omega' = \frac{2\pi}{\tau_L} \tag{14}$$

So Eq. 13 (again, clarifying Ng's notation of eq. 10.3 to explicitly include chromaticity) becomes

$$F'_m(\omega, \xi; \tau_L) = h_m(\omega - \omega_\xi; \tau_L) \tag{15}$$

This means that the peak of F'_m will shift to the *right* as the chromaticity becomes more negative because $\eta < 0$ at injection. See Fig. 1. Note: h_m is, of course, dependent on τ_L. I will only explicitly show it here.

B.   Resistive Wall

I am going to model the non-laminated regions of Booster as stainless steel round beam pipes. I will assume that the wall is thicker than the skin depth so that I can use eq. 1.60 of Ng

$$Z_1^{\perp}\big|_{\text{RW}}(\omega) = (1 - i\,\text{sgn}(\omega)) \frac{Lc}{\pi \omega b^3 \sigma_c \delta_{\text{skin}}} \tag{16}$$

where the skin depth is given by

$$\delta_{\text{skin}} = \sqrt{\frac{2c}{Z_0 \mu_r \sigma_c |\omega|}} \tag{17}$$

For stainless steel, the value of its permeability and conductivity are

$$\left.\begin{array}{l} \mu_r = 1 \text{ i.e. non magnetic stainless steel} \\[2mm] \sigma_c = 1.35 \times 10^6 \ \Omega^{-1}\text{m}^{-1} \end{array}\right\} \tag{18}$$

As for the length, L, I will basically just take out the 96 gradient magnets and whatever remains, I will assume that it is made of stainless steel beam pipe that has radius $b = 1.5"$. Thus,

$$L = 2\pi r_B - 96 \times L_g = 474.2 - 96 \times 2.9 = 195.8 \text{ m} \tag{19}$$

I will note that all terms in the growth rate formula, Eq. 11, are positive definite *except* for the $\text{Re}(Z_1^{\perp}|_{\text{RW}})$ term. For positive growth, I need $\text{Re}(Z_1^{\perp}|_{\text{RW}}) < 0$ because of the overall negative sign of the formula. Since $\text{Re}(Z_1^{\perp})$ scales as $1/\sqrt{\omega}$, the most unstable mode will be from the first negative ω_q mode that is closest to the singularity. When I look at plots of the form factor F', at zero chromaticity, shown in Fig. 1 , only F_0' has a value that is large around the zero frequency. Therefore, I will only worry about the $m = 0$ coupled bunch mode.

I will assume that the betatron tune is $\nu_\beta = 6.8333$. This means that the lowest negative mode is $\omega_{-1} = [(-84 + 77) + 6.8333]\omega_0 = -0.1667\omega_0$, i.e. when $q = -1$, $M = 84$, $\mu = 77$, $m = 0$. The real part of $Z_1^{\perp}|_{\text{RW}}$ is plotted in Fig. 7 and I have marked ω_{-1} in blue. The next lowest line is when $q = -2$ and ω_{-2} is at -37.9 MHz. Clearly, this line doesn't see too much of the resistive wall impedance.

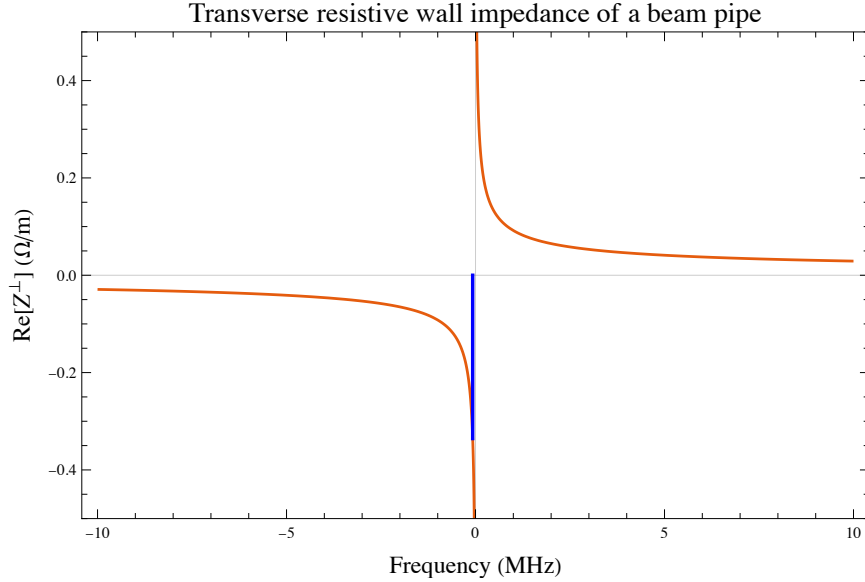Transverse resistive wall impedance of a beam pipe

FIG. 7. Here is the real part of $Z_1^{\perp}|_{\text{RW}}$. I have also plotted ω_{-1} in blue to show that it is indeed the mode that is closest to the singularity. The next mode ω_{-2} is at -37.9 MHz.

C. Coupled bunch growth rates

Finally, I can calculate the e growth rate $1/\tau_{0,77}$ mode when the chromaticity is zero. It is

$$\left(\frac{1}{\tau_{0,77}}\right)_x = 196 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_x = 5 \text{ ms} \tag{20}$$

$$\left(\frac{1}{\tau_{0,77}}\right)_y = 586 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_y = 1.7 \text{ ms} \tag{21}$$

where I have used the average $\beta_x = 6.8$ m and $\beta_y = 20.3$ m for the non-laminated straights. Clearly, both coupled bunch instablities are not that strong and can be stabilized with negative chromaticity. The growth rates as a function of chromaticity are shown in Fig. 8.
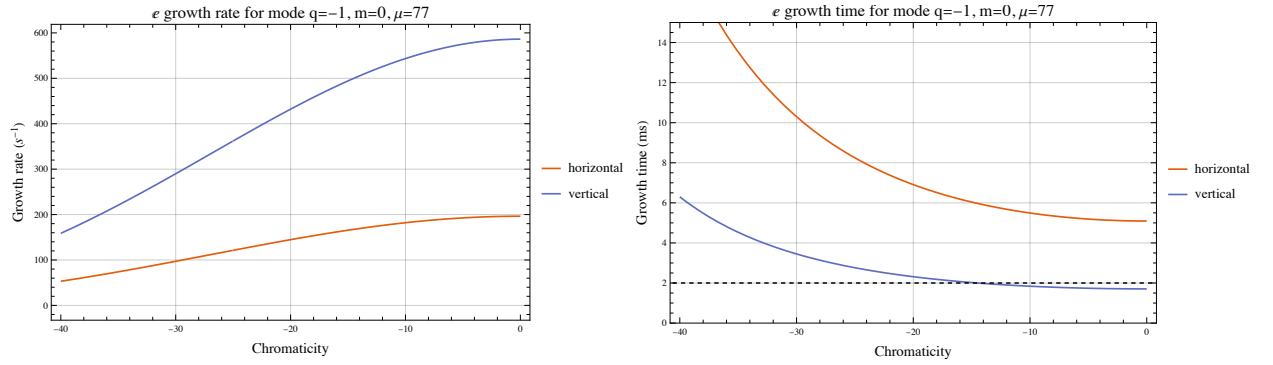

FIG. 8. The e growth rate and growth time as a function of chromaticity. From this plot, if I want a 2 ms growth time for the vertical mode, then the vertical chromaticity should be at least -15 units.

Clearly, looking at the growth rates at zero chromaticity, the growth rates are not that strong with vertical being the worse of the two. In practise, Booster does run with a large negative chromaticity of about -20 units in the horizontal and -10 in the vertical. This is opposite to what I would have expected, i.e. I would've thought that the vertical chromaticity will be larger than the vertical. I think this means that resistive wall is not the only transverse impedance that is causing the instabilities. Looking at Fig. 1, the $m = 1, 2$ head-tail modes cover the region of the RF cavity HOMs ($100 f_{\text{rev}} = 45$ MHz). My suspicion is that there are narrow band transverse impedances from the RF cavities that cause strong coupled bunch mode instabilities. Note: Lebedev[11] in his 2006 analysis thinks that it is $m = 3$ (slide 12) and that the high frequency part of the impedance between $(80 - 300)$ MHz are the major contributors to the instability (slide 23).

IV. SUMMARY

I want to run with zero chromaticity because the tune footprint is at its minimum which means that the lifetime should be improved because fewer resonance lines are crossed. However, running at zero chromaticity is not a freebie because the coupled bunch mode $m = 0$ is unstable and I suspect that the unaccounted for transverse impedances will also cause $m = 1$ and 2 coupled bunch mode instabilities, which will require transverse dampers. I can simply look at the h_m power spectrum plots in Fig. 1, and the lamination impedances, Fig. 5, the required bandwidth of the dampers should be at least 150 MHz (6 dB point of the real part of the impedance and the width of $h_0 \approx 2 \times 100 \ f_{\rm rev} = 90$ MHz).

Appendix A: PIPII parameters

The previous calculations that I did was for 400 MeV injection. Here's the analysis for 800 MeV injection. The relevant changes to the previous parameter tables for PIPII are shown in Table IV and Table V.

TABLE IV. PIPII parameters at 800 MeV

Parameter	Description	Value	Units
N_b	number of protons in one bunch	$6.7 \times 10^{12}/81 = 8.3 \times 10^{10}$	
E_0	total energy at injection	$(800 + 938) = 1738$	MeV
τ_L	total bunch length	$4 \times 3.45 = 13.8^{\rm a}$	ns
η	slip factor	-0.26	

[a] The 3.45 ns is the rms bunch length. This value came from F. Ostiguy's simulations. See email dated 21 Nov 2023.

1. Head tail growth rate

Fig. 9 are the new plots generated with the PIPII parameters shown in Tables IV and V. They can be compared to the plots in Fig. 6. In PIPII, the most offending mode still the $m = 0$ mode for positive chromaticity. It is a lot stronger than present operations because

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Parameter	Description	Value	Units
β	relativistic beta at injection	0.84	
ω_0	revolution frequency	3.34411×10^6	rad/s
T_0	revolution period $(2\pi/\omega_0)$	1.879×10^{-6}	s
I_b	current of one bunch $(q_e N_b/T_0)$	0.705×10^{-2}	A

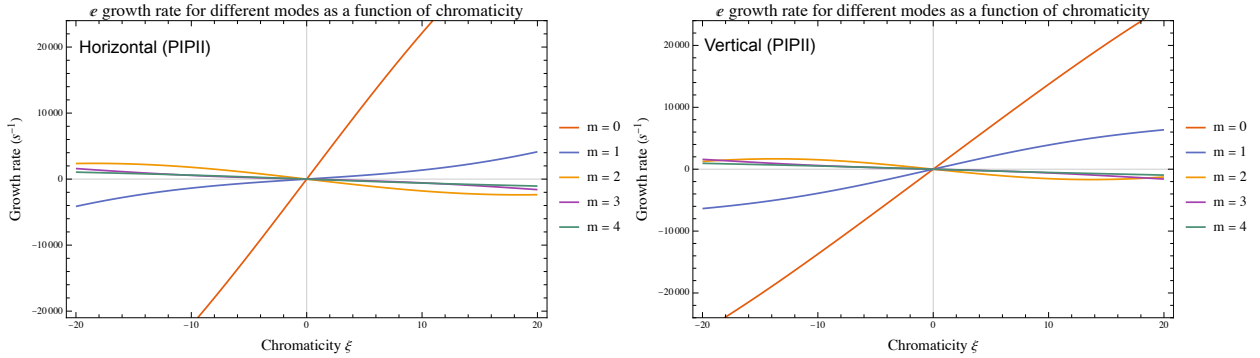of the higher intensity and shorter bunch length.

FIG. 9. The head-tail growth rate (rate of growing by one e) for head-tail modes $m = 0,\ldots,4$ as a function of chromaticity. These results can be compared to those in Fig. 6 for 400 MeV injection. Again, modes $m = 0,1$ are stable but $m > 1$ modes are unstable for negative chromaticity.

2. Coupled bunch modes

For coupled bunch modes, the PIPII growth rates are similar to present operations. At $\xi = -1$, the $1/\tau_{0,77}$ growth rates for both planes are

$$\left(\frac{1}{\tau_{0,77}}\right)_x = 183 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_x = 5.5 \text{ ms} \tag{A1}$$

$$\left(\frac{1}{\tau_{0,77}}\right)_y = 547 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_y = 1.8 \text{ ms} \tag{A2}$$

And the growth rates as a function of chromaticity are shown in Fig. 10. For 2 ms growth rate, the chromaticity is -8 units. The reason for the smaller absolute value of chromaticty when compared to present operations is because the PIPII ω_{-1} line is at -88.7 kHz and samples a smaller $\text{Re}[Z^\perp]$ than for present operations which is at -75.1 kHz.
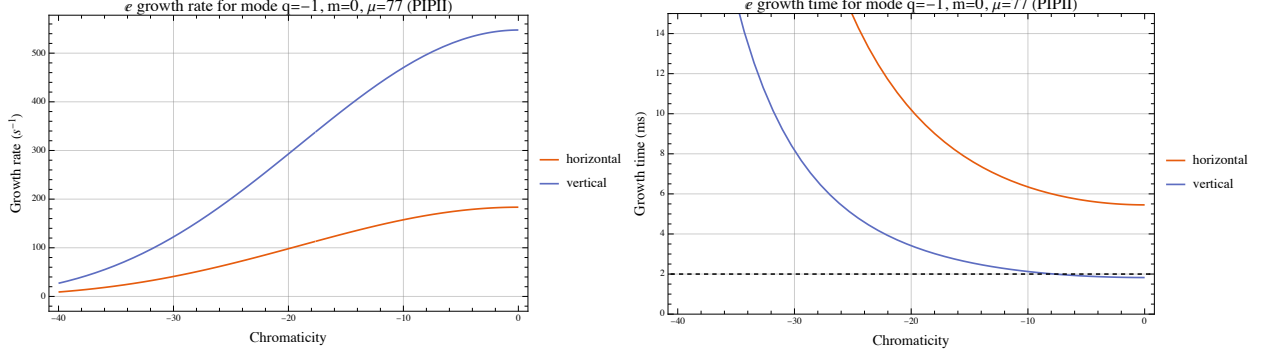
FIG. 10. The e growth rate and growth time as a function of chromaticity. From this plot, if I want a 2 ms growth time for the vertical mode, then the vertical chromaticity should between zero to -8 units.

a. *Coupled bunch growth rates when $\xi = -1$*

At $\xi = -1$, for both planes, $1/\tau_{0,77}$ growth rates for both planes are

$$\left(\frac{1}{\tau_{0,77}}\right)_x = 183 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_x = 5.5 \text{ ms} \tag{A3}$$

$$\left(\frac{1}{\tau_{0,77}}\right)_y = 546 \text{ s}^{-1} \quad \Rightarrow (\tau_{0,77})_y = 1.8 \text{ ms} \tag{A4}$$

which is basically the same result as the $\xi = 0$ case.

b. *Head-tail growth rates when $\xi = -1$*

Suppose I set the $\xi = -1$ units for both planes, then the head-tail bunch growth rate for modes $m = 0, \ldots, 4$ from Fig. 9 are shown in Table VI. Note: the $m = 1$ head-tail mode is the worst offender.

The difference power spectrum (See Eq.1)

$$\Delta\mathcal{H}_m = h_m(\omega - \omega_\xi) - h_m(\omega + \omega_\xi) \tag{A5}$$

for $\xi = -1$ and $\xi = -20$ are shown in Fig. 11. From, here I can see that $\Delta\mathcal{H}_{0,1}$ essentially disappears above 160 MHz.

TABLE VI. Head-tail growth rate when $\xi = -1$

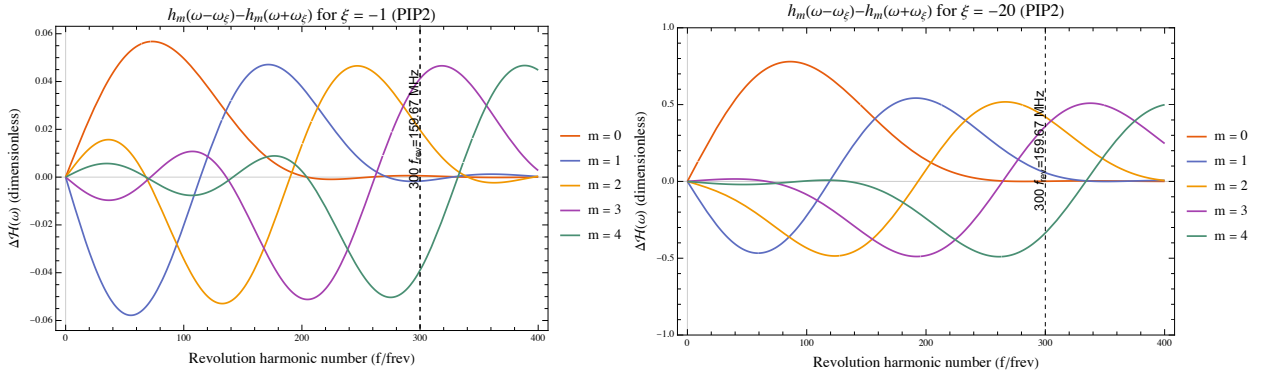| mode	H rate (s^{-1})	H time (ms)	V rate (s^{-1})	V time (ms)
0	-2305	-0.4	-1386	-0.7
1	-109	-9.1	-423	-2.4
2	203	4.9	192	5.2
3	45	22.2	48	20.9
4	64	15.5	56	17.7

FIG. 11. $\Delta\mathcal{H}_m$ for $m = 0, \ldots, 4$ are plotted here. Essentially, for $\Delta\mathcal{H}_{0,1}$, their contribution to head-tail growth in Eq. 1 essentially disappears after $300 f_{\mathrm{rev}} = 160$ MHz.

3. Experimental observation, high intensity (not PIPII parameters)

A study was done on 22 Feb 2023 to observe the spectrum of the beam. Two intensities were used for comparison: Low intensity at 1.2×10^{12} protons with $\xi = -6$ both planes which is meta-stable because it is at the threshold of instability and high intensity 4.8×10^{12} protons at $\xi = -9$ in both planes which is unstable. The spectrum of the difference signal from the damper stripline pickup between these two conditions are plotted in Fig. 12. From these observations, I can see the following:

1. The initial growth of the beam below 1 ms is from the notcher kicker.

2. As expected from theory, the horizontal instability is stronger than the vertical. For horizontal, the growth rate is about 1200 s^{-1} and for the vertical rate it is about 476 s^{-1}, i.e. about 2.5 times stronger.
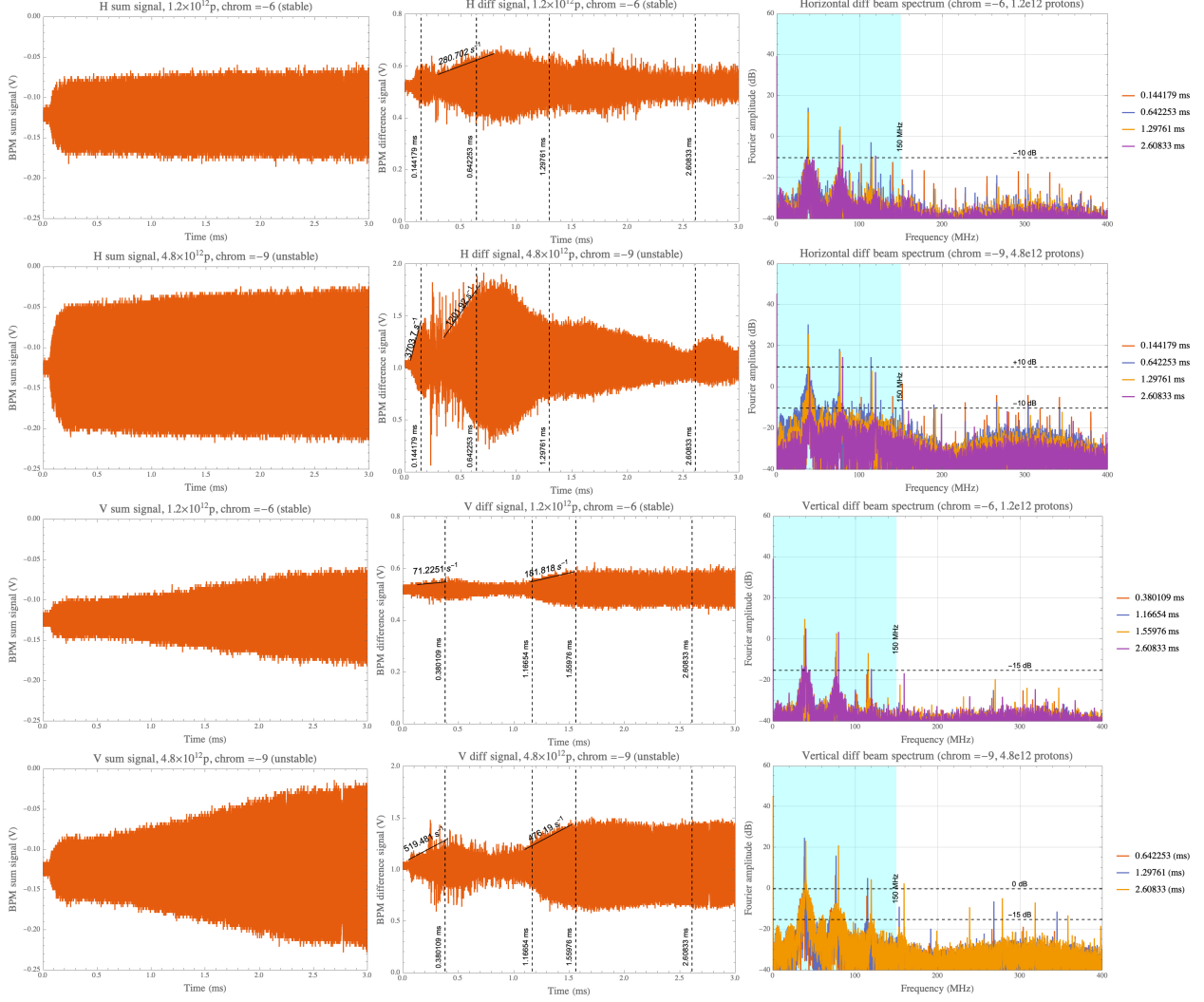
FIG. 12. This data was taken on 22 Feb 2023 with the damper stripline pickup. The two intensities were 1.2×10^{12}, $\xi = -6$ and 4.8×10^{12} protons, $\xi = -9$ both planes. The top two rows are horizontal signals and the bottom two rows are the vertical signals. The vertical dashed lines in the difference signal plots are where the Fourier transforms are taken. The spectra when the beam is stable dictates how much the damper must reduce the amplitude of the sidebands of the unstable beam. The horizontal spectrum of the unstable beam shows that the dampers have to reduce the sidebands by at least 20 dB. The cyan region in the spectra plots marks the minimum bandwidth of 150 MHz.

3. The growth time for horizontal is about 0.8 ms and for the vertical is about 2 ms. These values can be compared to the theoretical head-tail growth rates shown in Table VII.

    (a) From this table I can see that the measured growth rates are within factors of 2 of theory. Note: I have not identified the modes that are responsible for the

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growth.

4. By comparing the unstable and stable spectra, I can conclude that the dampers must
   have sufficient gain to damp the sidebands by at least 20 dB and that the bandwidth
   of the damper must be at least 150 MHz. These measurements confirm my earlier
   bandwidth conclusion in section IV.

TABLE VII. Theoretical Head-tail growth rate when $\xi = -9$ for 5×10^{12} protons with present Booster

mode	H rate (s^{-1})	H time (ms)	V rate (s^{-1})	V time (ms)
0	-7089	-0.1	-3786	-0.3
1	-1718	-0.6	-2374	-0.4
2	655	1.5	210	4.7
3	521	1.9	614	1.6
4	138	7.3	116	8.6

4. Summary

If Booster is run with $\xi = -1$ in both planes, the $m = 0$ and 1 head-tail modes are
stabilized but the other other modes are unstable. The coupled bunch mode $q = -1$,
$m = 0$, $\mu = 77$ (-75.1 kHz) is also unstable at this chromaticity. Therefore, it is necessary
to have the dampers stabilize the beam for low negative chromaticity operation, assuming
that there is insufficient Landau damping. The dampers will have to operate between DC
to at least 150 MHz to control both coupled bunch and the head-tail modes (from Fig. 11
and the > 6 dB point of the real part of the impedance in Fig. 5).

[1] K.Y. Ng. *Physics of intensity dependent beam instabilities*, chapter 9. World Scientific, 2006.

[2] K.Y. Ng. Coupling impedances of laminated magnets. Technical Report Fermilab-FN-0744,
    Fermilab, April 2004.

[3] Y. Alexahin, N. Eddy, E. Gianfelice-Wendt, V.A. Lebedev, W.L. Marsh, W. Pellico, and A.K. Triplett. Observation of instabilities of coherent transverse oscillations in the Fermilab Booster. In *IPAC2012, New Orleans, Louisiana, USA*, 2012.

[4] A.W. Chao. *Physics of collective beam instabilities in high energy accelerators*, chapter 6, pages 332–365. Wiley series in beam physics and accelerator technology, 2006.

[5] S. Myers. Instabilities and beam intensity limitations in circular accelerators. `https://cds.cern.ch/record/334182/files/sl-97-048.pdf`, 1997.

[6] Macridin says $3\times$ larger.

[7] A. Macridin, J. Amundson, P. Spentzouris, V. Lebedev, and T. Zolkin. Transverse impedance and transverse instabilities in Fermilab Booster. In *1st North American Particle Accelerator Conference*, October 2013.

[8] In this writeup, I will call the longitudinal azimuthal modes as head-tail modes.

[9] A. Macridin, P. Spentzouris, J. Amundson, L. Spentzouris, and D. McCarron. Wake functions for laminated magnets and applications for Fermilab Booster synchrotron. In *46th ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams*, 2010.

[10] Z/L is really a symbol like $B\rho$. This is a reminder that these impedances have weird units because they come from the Fourier transformation of wake fields.

[11] V. Lebedev, A. Burov, W. Pellico, and X. Yang. Coherent instabilities in Fermilab Booster. `https://beamdocs.fnal.gov/AD/DocDB/0022/002297/002/PresentationAtFNAL.pdf`, June 2006.