Charmonia and Charmonia Hybrids

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Charmonia

Charmonia via the Potential Quark Model

standard lore (cf. talk by Qiang Zhao)

Cornell static potential +

spin-dependence from one-gluon-exchange(*) +

'scalar' confinement (**)

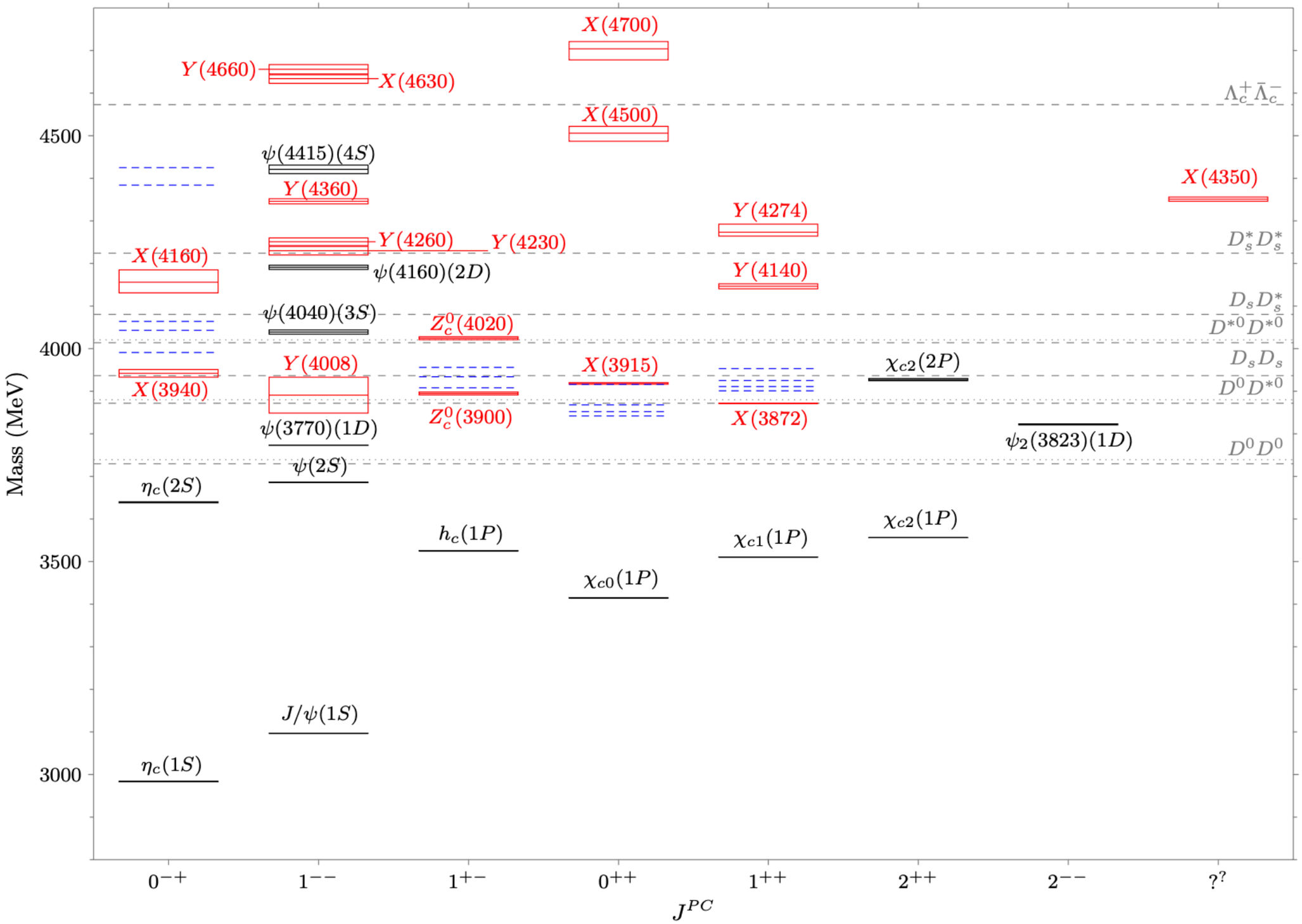
(*) need to modify the delta function in the hyperfine interaction

(**) not sensible for several reasons

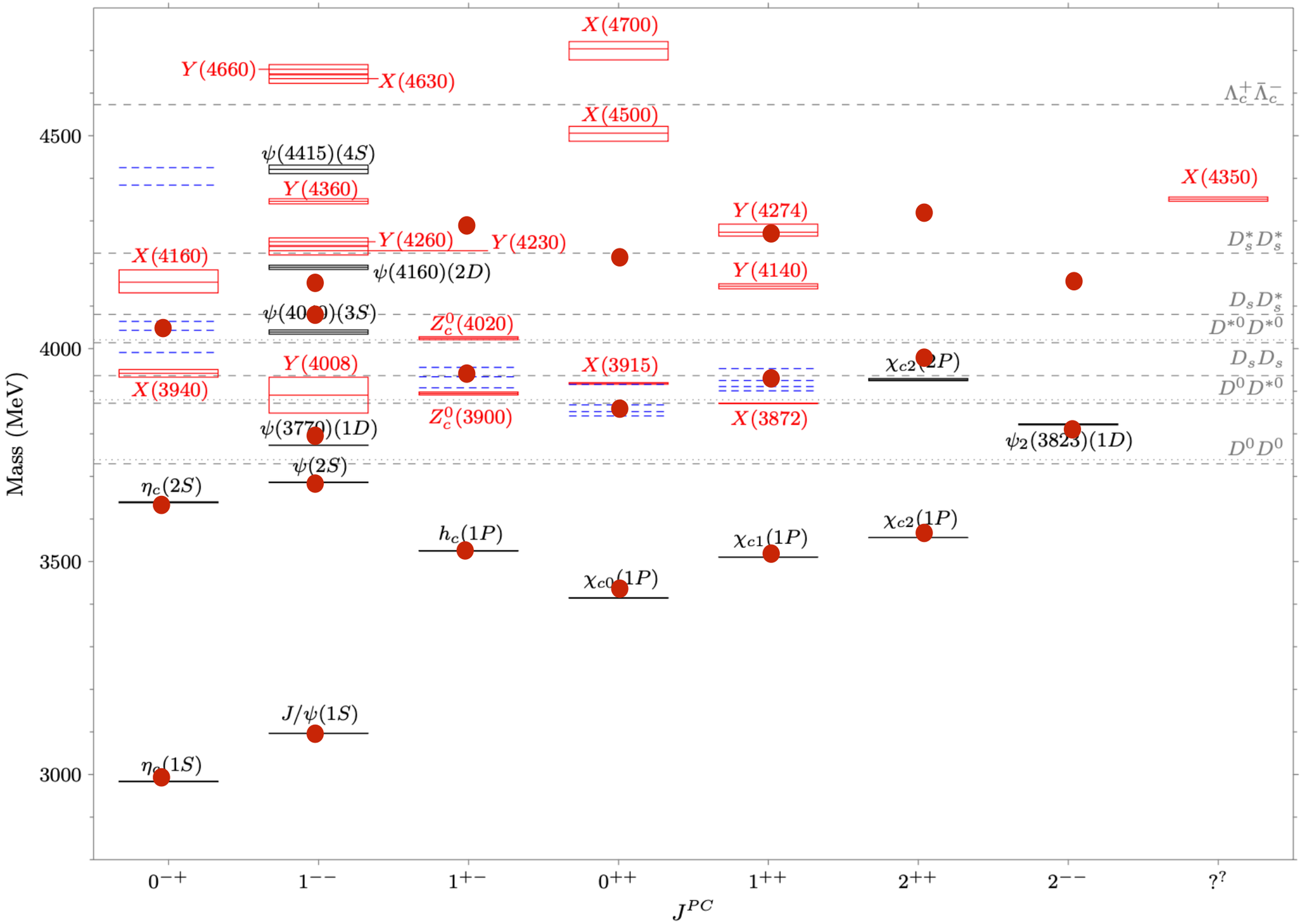
$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\,\tilde{\delta}_\sigma(r)\,\vec{\mathrm{S}}_c \cdot \vec{\mathrm{S}}_{\bar{c}}$$

$$V_{spin-dep} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}} + \frac{4\alpha_s}{r^3}\,\mathrm{T}\right].$$

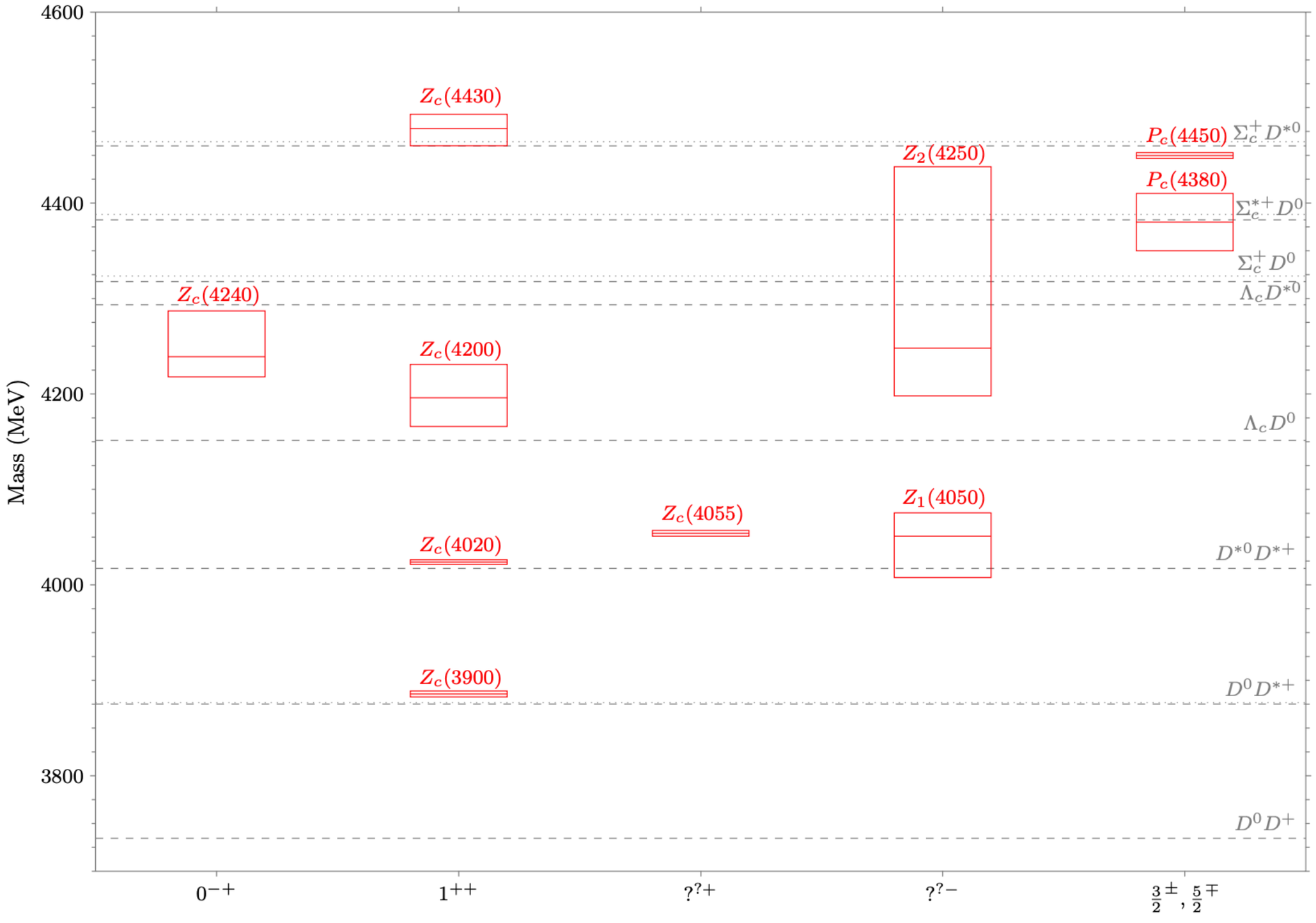
Charmonia via the Potential Quark Model



Charmonia via the Potential Quark Model



Charged Charmonia



Renovating the Potential Quark Model

the general spin-dependence is given by

$$V_{conf}(r) = -C_F \frac{\alpha(r)}{r} + br.$$

$$\begin{aligned} V_{SD}(r) \;=\; & \left(\frac{\boldsymbol{\sigma}_q}{4m_q^2} + \frac{\boldsymbol{\sigma}_{\bar{q}}}{4m_{\bar{q}}^2}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_{conf}}{dr} + \frac{2}{r}\frac{dV_1}{dr}\right) \\ & + \left(\frac{\boldsymbol{\sigma}_{\bar{q}} + \boldsymbol{\sigma}_q}{2m_q m_{\bar{q}}}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_2}{dr}\right) \\ & + \frac{1}{12m_q m_{\bar{q}}} \left(3\boldsymbol{\sigma}_q \cdot \hat{\mathbf{r}}\, \boldsymbol{\sigma}_{\bar{q}} \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}}\right) V_3 \\ & + \frac{1}{12m_q m_{\bar{q}}} \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}} V_4 \\ & + \frac{1}{2}\left[\left(\frac{\boldsymbol{\sigma}_q}{m_q^2} - \frac{\boldsymbol{\sigma}_{\bar{q}}}{m_{\bar{q}}^2}\right) \cdot \mathbf{L} + \left(\frac{\boldsymbol{\sigma}_q - \boldsymbol{\sigma}_{\bar{q}}}{m_q m_{\bar{q}}}\right) \cdot \mathbf{L}\right] V_5. \end{aligned}$$

running coupling is motivated by the persistent over-estimation of heavy meson decay constants (cf, must modify psi(r->0))

A constraint from the Gromes relationship

$$V_{conf} = V_2 - V_1$$

Renovating the Potential Quark Model

Assuming a 'relativistic' interaction like

$$\frac{1}{2}\int d^3x d^3y\, \bar{\psi}(x)\Gamma\psi(x)\, V(x-y)\, \bar{\psi}(y)\Gamma\psi(y)$$

and taking the nonRel limit for vector-vector gives

$$V_1 = 0 \quad V_2 = V_{conf} \quad V_3 = V'_{conf}/r - V''_{conf} \quad V_4 = 2\nabla^2 V_{conf}$$

& for scalar-scalar

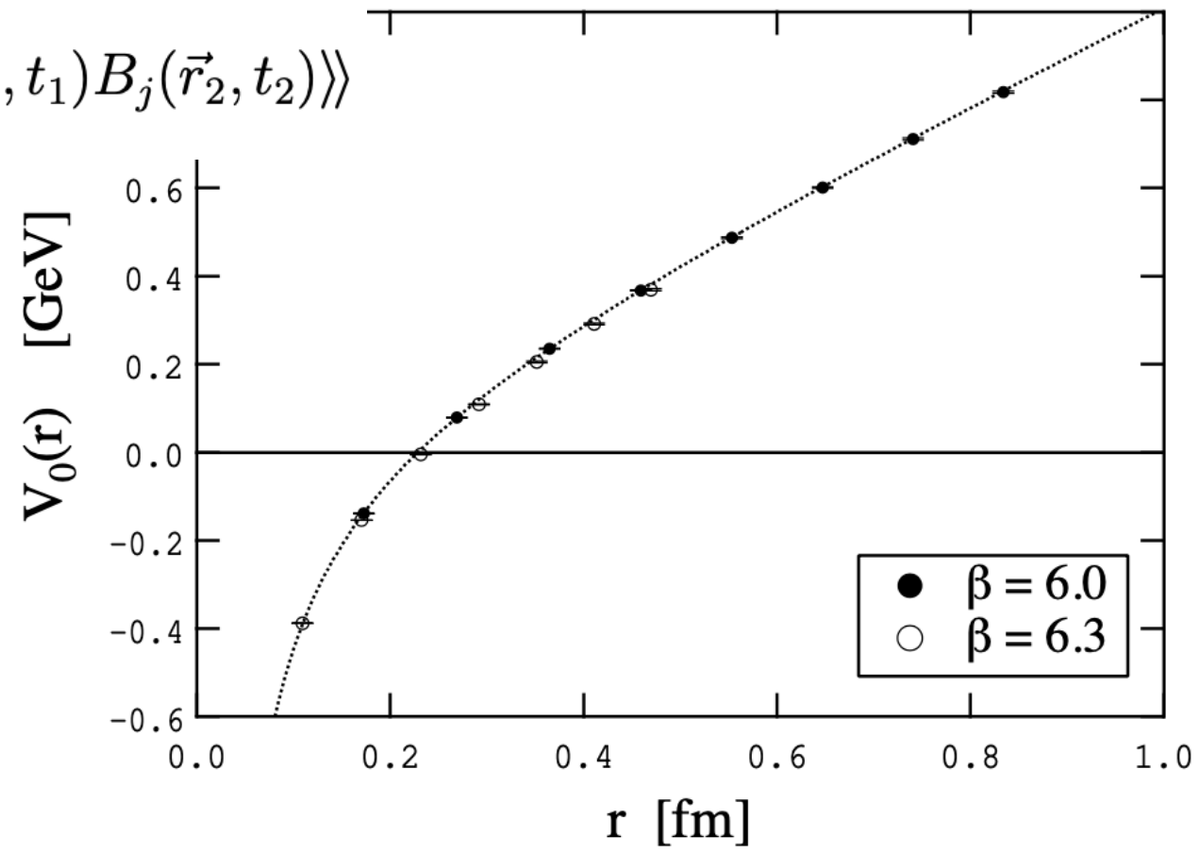
$$V_1 = -V_{conf} \quad V_2 = 0 \quad V_3 = 0 \quad V_4 = 0$$

It is the alternation in sign of the combination V1 + V2 between vector and scalar currents which, through the analysis of the heavy quarkonia spectrum, enabled Schnitzer to identify the scalar interaction as the likely structure for confinement. H. J. Schnitzer, Phys. Rev. Lett. 35, 1540 (1975).

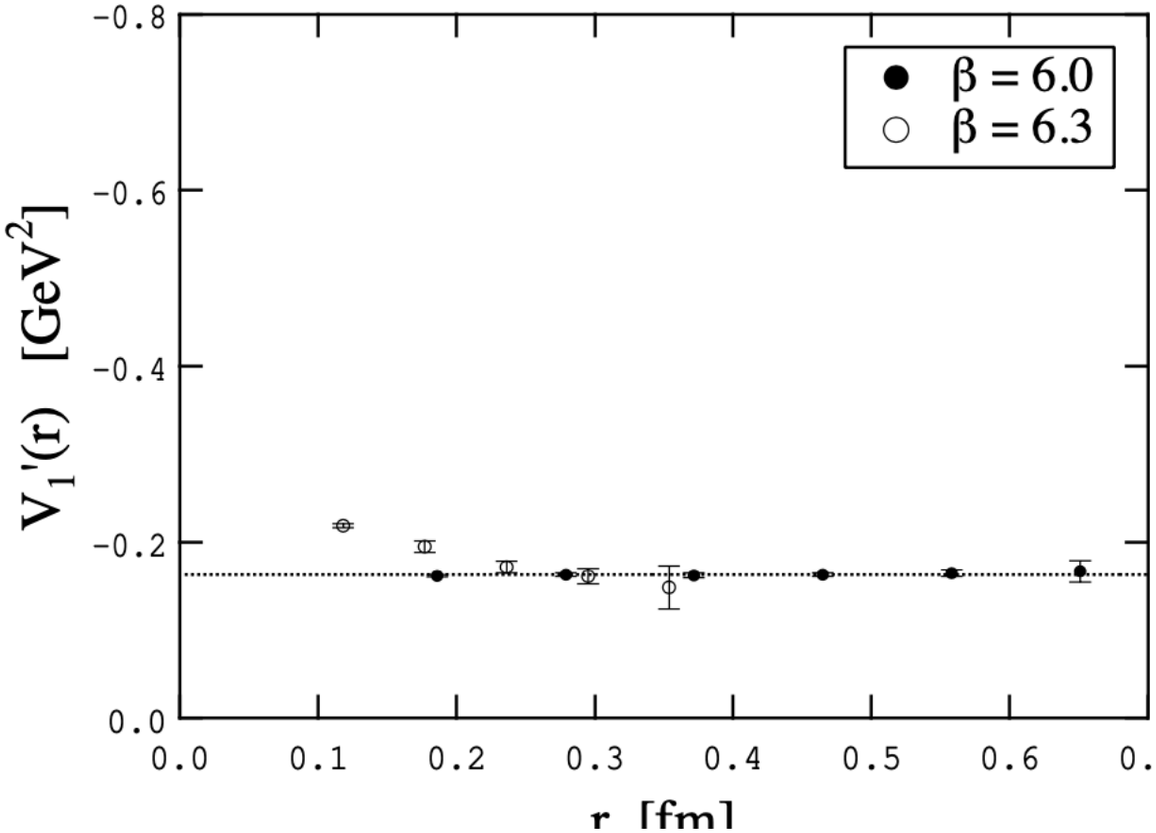
Renovating the Potential Quark Model

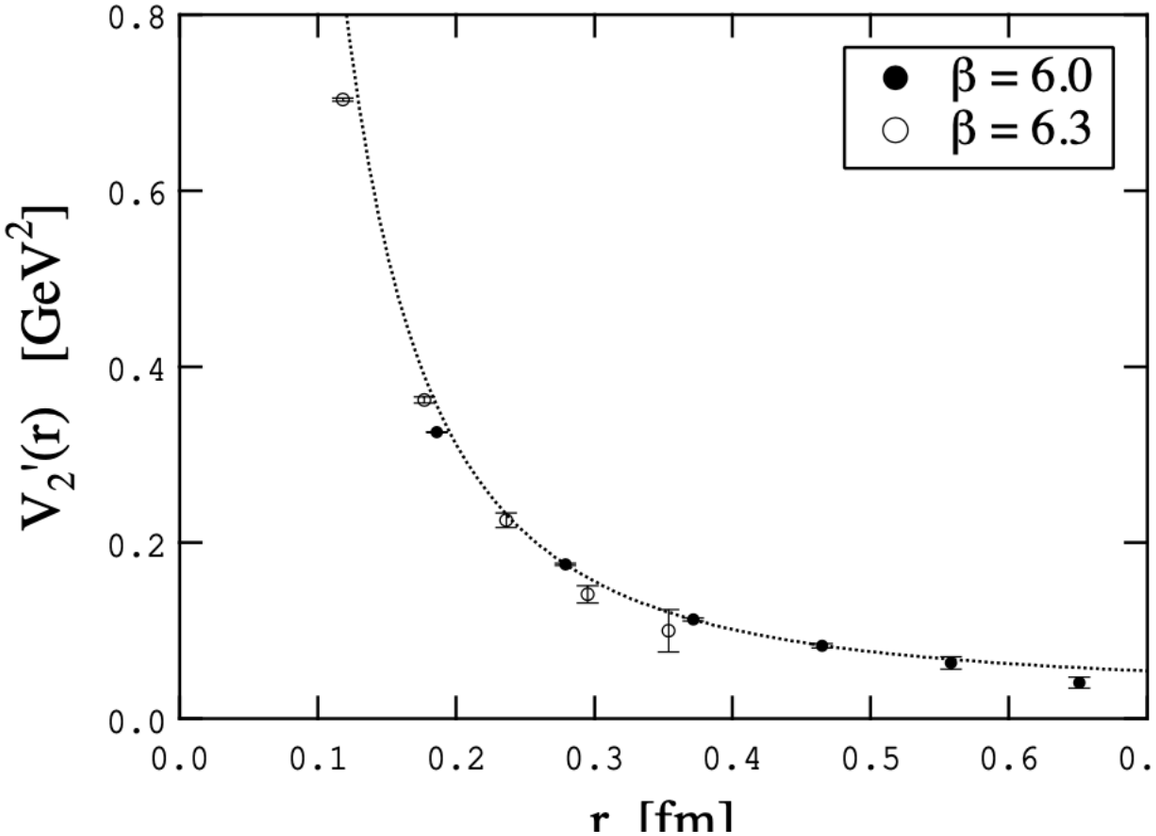
Measure the V_i on the lattice (Koma&Koma, NPB, 2007)

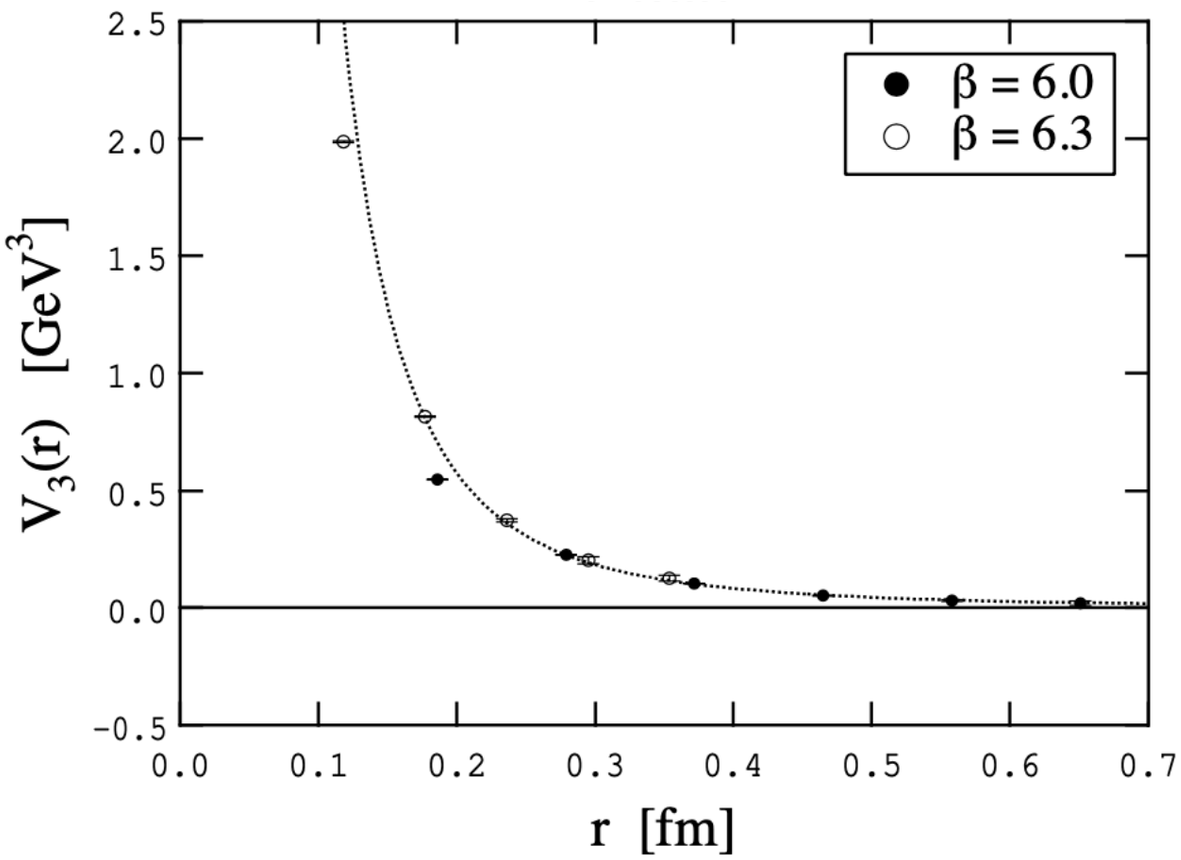
$$\frac{r_k}{r}V_1'(r) = \epsilon_{ijk}\lim_{\tau\to\infty}\int_0^\tau dt\; t\langle\langle g^2 B_i(\vec{r}_1,t_1)E_j(\vec{r}_1,t_2)\rangle\rangle \;,$$
$$\frac{r_k}{r}V_2'(r) = \epsilon_{ijk}\lim_{\tau\to\infty}\int_0^\tau dt\; t\langle\langle g^2 B_i(\vec{r}_1,t_1)E_j(\vec{r}_2,t_2)\rangle\rangle \;,$$
$$(\frac{r_i r_j}{r^2}-\frac{\delta_{ij}}{3})V_3(r)+\frac{\delta_{ij}}{3}V_4(r) = 2\lim_{\tau\to\infty}\int_0^\tau dt\;\langle\langle g^2 B_i(\vec{r}_1,t_1)B_j(\vec{r}_2,t_2)\rangle\rangle$$

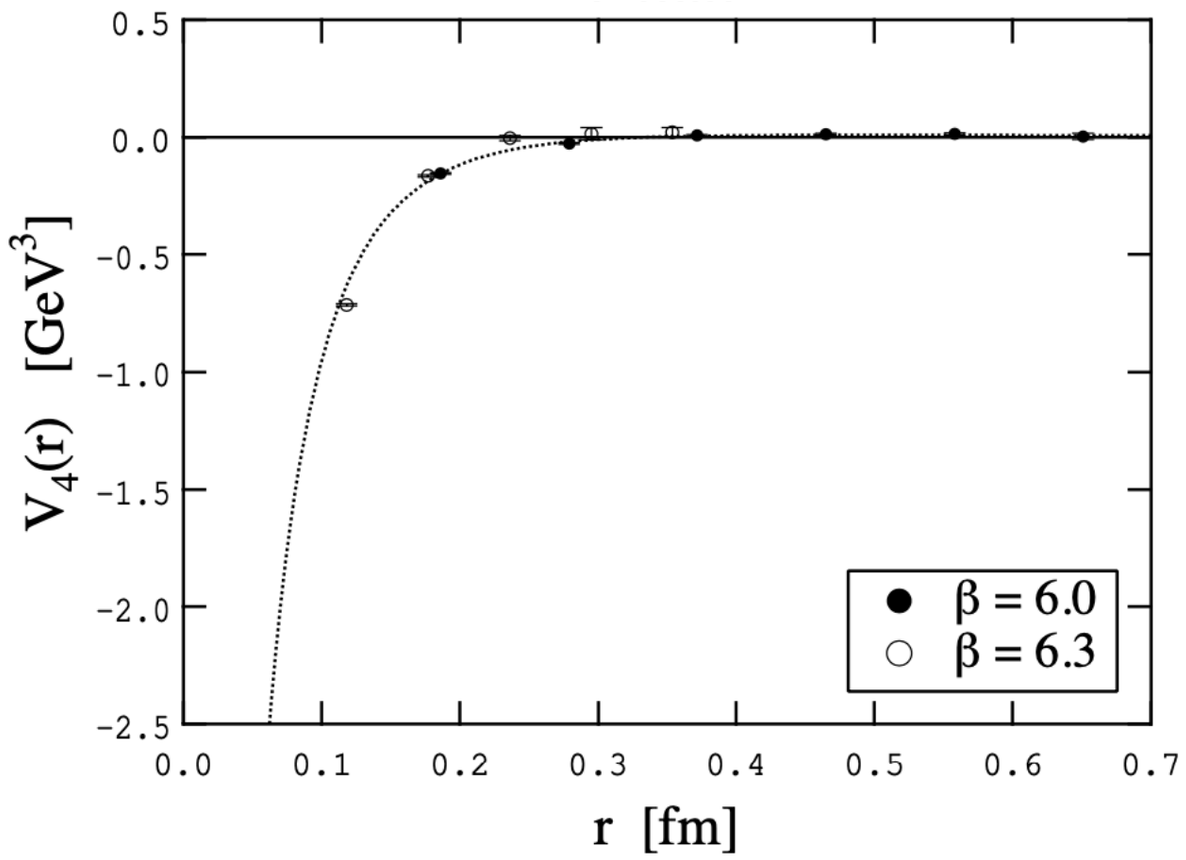


Renovating the Potential Quark Model









Renovating the Potential Quark Model

A reasonable summary

$$V_1 = -(1-\epsilon)br$$

$$V_2 = \epsilon br - C_F\alpha_S/r$$

$$V_3 = 3C_F\alpha_h/r^3$$

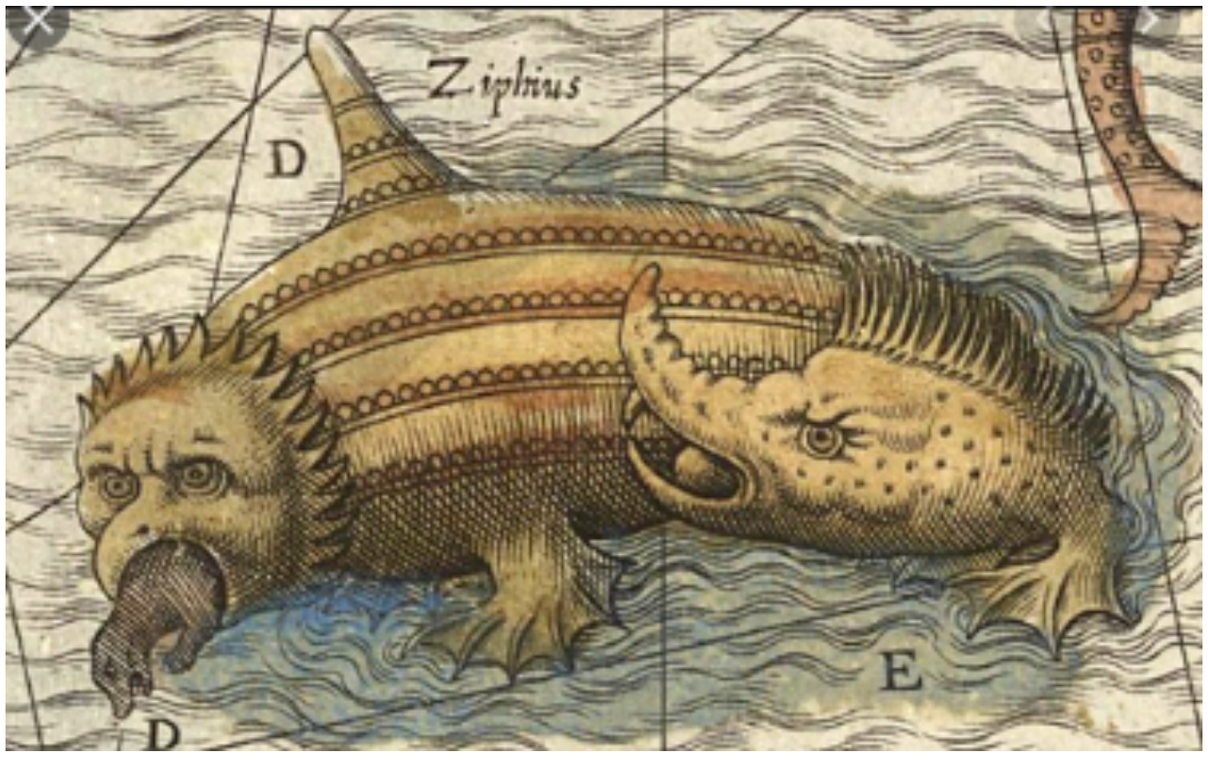
$$V_4 = C_F\alpha_h\frac{b_h^2 e^{-b_h r}}{r}$$

$$V_5 = 0$$

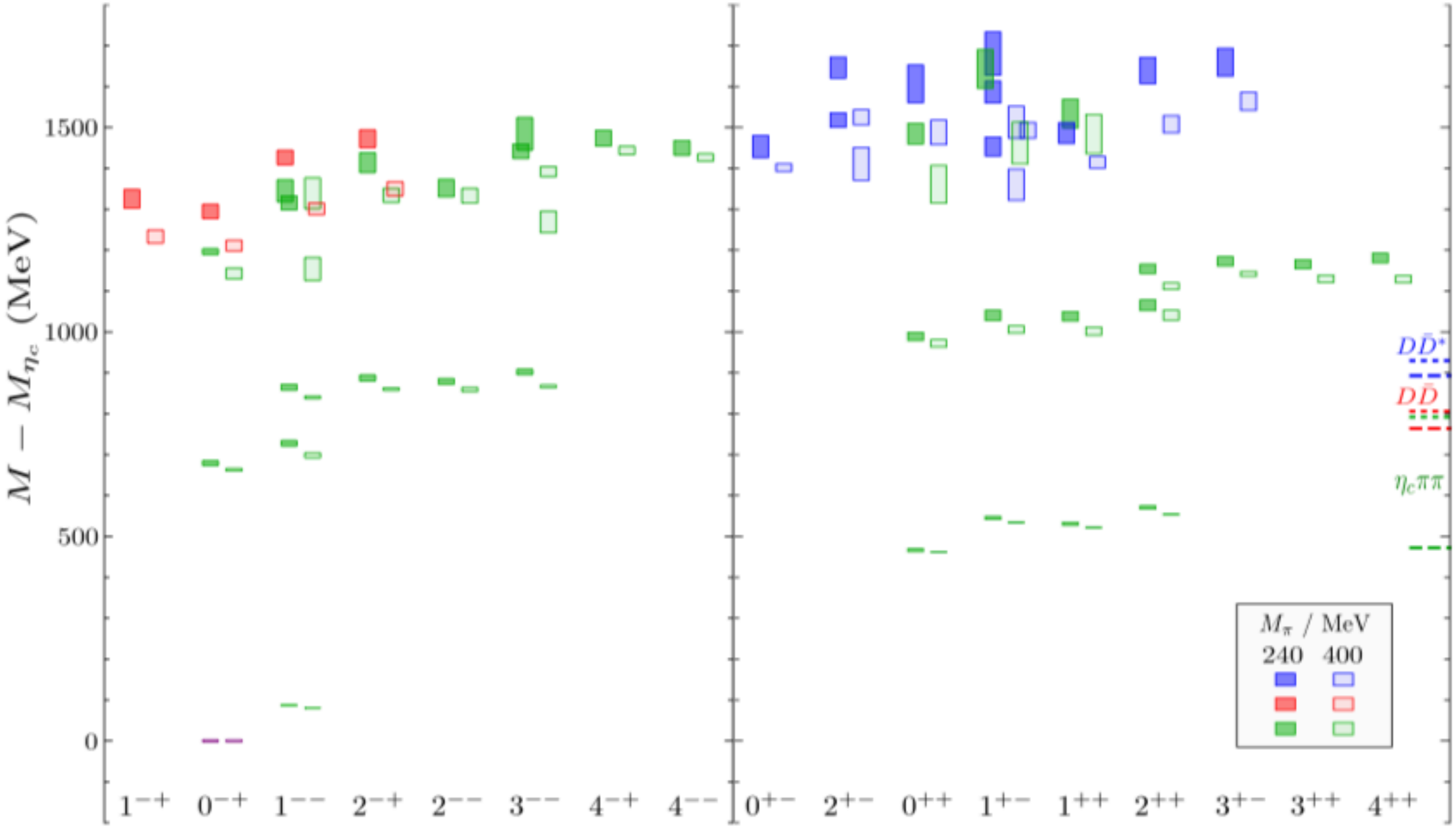
$$\epsilon \approx 1/4$$

This remains to be fully explored…

Hybrids



Lattice Gauge Theory



Cheung et al. [HadSpec], JHEP 1612, 089 (2016).

Effective Field Theory

Obtain Schrödinger-type equations for heavy quark hybrids with pNRQCD.

$$\mathcal{L} = \mathrm{tr}\left(H^{i\dagger} \left(\delta_{ij} i\partial_0 - h_{H\,ij} \right) H_j \right) \qquad (1$$

$$h_{H\,ij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r) \right) \delta_{ij} + (\delta_{ij} - \hat{r}_i \hat{r}_j) \left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r) \right]$$

$$\left[-\frac{1}{mr^2}\,\partial_r r^2 \partial_r + \frac{1}{mr^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix} \right] \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi_\Sigma^{(N)} \\ \psi_{-\Pi}^{(N)} \end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E_\Pi^{(0)} \right] \psi_{+\Pi}^{(N)} = \mathcal{E}_N\, \psi_{+\Pi}^{(N)}$$

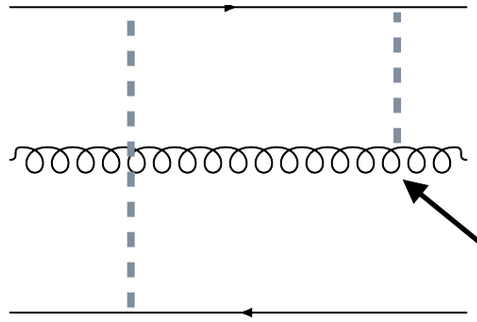
M. Berwein, N. Brambilla, J. Castella, A. Vairo, arXiv:1510.04299

R. Oncala and J. Soto, arXix: 1702.03900

N. Brambilla, W.-K. Lai, J. Segovia, J. Castella, A. Vairo, arXiv:1805.07713

A Simple Constituent Gluon Model

Construct hybrids with transverse constituent TE gluons



$$\rho_g^A = f^{ABC}\vec{A}^B(x)\cdot\vec{\Pi}^C(x)$$

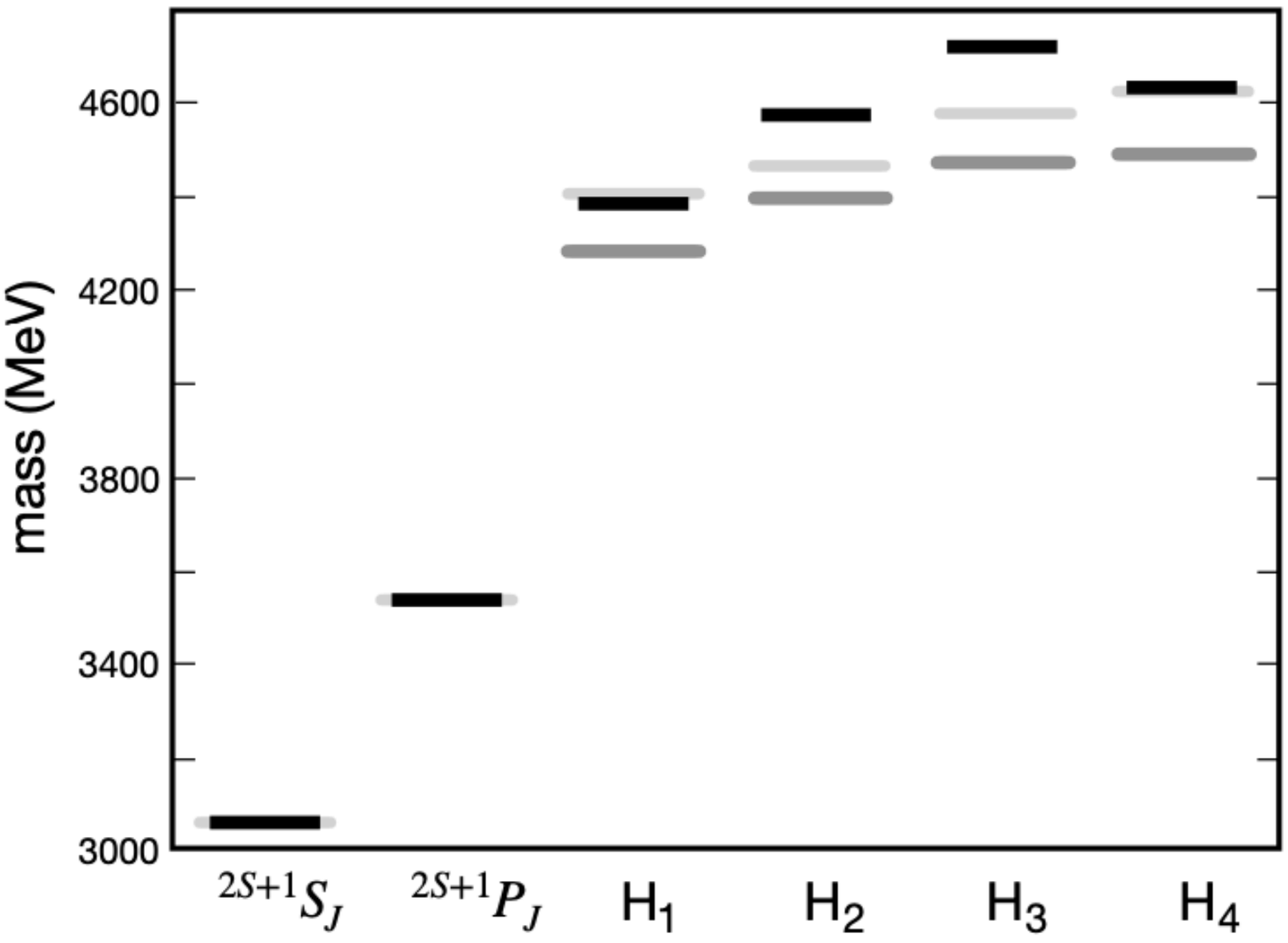
$$\rho_q^A = \psi^\dagger(x)T^A\psi(x)$$

$$V_1 = -\frac{4}{3}\left(\frac{\alpha}{R} - \frac{3}{4}bR\right) \qquad V_8 = +\frac{1}{6}\left(\frac{\alpha}{R} - \frac{3}{4}bR\right) \qquad V_{qg} = -\frac{3}{2}\left(\frac{\alpha}{R} - \frac{3}{4}bR\right)$$

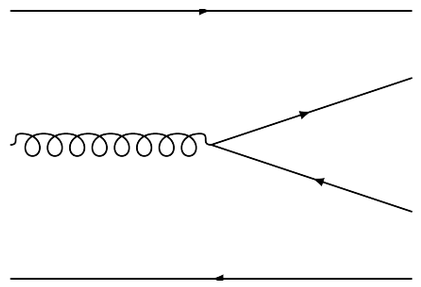
[no need for three-body if focussing on lowest multiplets]

$$\left[\;\; \mathcal{J}^{-1/2}\rho_q^A\mathcal{J}^{1/2} = \rho_q^A \qquad \mathcal{J}^{-1/2}\rho_g^C\mathcal{J}^{1/2} = \rho_{\rm eff}^C(\vec{A})\;\right]$$

A Simple Constituent Gluon Model



A Simple Hybrid Decay Model



M. Tanimoto, Phys. Lett. B 116, 198 (1982)

C. Farina et al., arXiv:2005:10850

TABLE III: Hybrid Decay Widths (MeV), computed with the $m_\pi = 236$ MeV Hybrid Spectrum. Suppressed channels are denoted as follows: $-$ = quantum number, ø= threshold, 0 = selection rule, x = negligible.

state	D^*D	D_0D	$D_{1L}D$	$D_{1H}D$	D_2D	D_0D^*	$D_{1L}D^*$	$D_{1H}D^*$	D_2D^*	$D_sD_s^*$	$D_{s0}D_s$	$D_{s0}D_s^*$	$D_{s1L}D_s$	width
1^{--} (H_1)	x	-	8	33	x	25	ø	ø	ø	x	-	ø	ø	66
0^{-+} (H_1)	x	76	ø	ø	ø	-	ø	ø	ø	34	ø	ø	ø	76
1^{-+} (H_1)	x	-	28	ø	ø	ø	ø	ø	ø	x	-	ø	ø	28
2^{-+} (H_1)	x	x	x	1	20	x	16	24	ø	x	x	x	x	60
1^{++} (H_2)	x	-	5	18	x	11	5	x	ø	x	-	4	2	45
0^{+-} (H_2)	x	x	30	5	x	17	x	ø	ø	x	x	x	1	53
1^{+-} (H_2)	x	16	4	3	5	4	x	ø	ø	x	16	x	x	48
2^{+-} (H_2)	x	16	4	3	6	7	14	5	1	x	20	4	2	82
0^{++} (H_3)	-	-	4	15	-	15	7	21	14	-	-	14	6	96
1^{+-} (H_3)	x	4	6	3	4	9	11	16	5	x	5	8	5	76
2^{++} (H_4)	x	x	0	17	x	15	23	9	14	x	x	18	0	96
1^{+-} (H_4)	x	17	7	4	x	12	2	4	3	x	31	17	8	105
2^{+-} (H_4)	x	x	23	6	2	22	10	14	9	x	x	28	29	139
3^{+-} (H_4)	x	x	0	x	9	x	x	40	13	x	x	x	0	62

A Simple Hybrid Decay Model

TABLE V: Cryptoexotic $H_1(1^{--})$ and $\psi(4^3S_1)$ Decay Modes (assuming masses of 4411 MeV).

state	DD	D^*D	D^*D^*	$D_{1L}D$	$D_{1H}D$	D_2D	D_0D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	width
$H_1(1^{--})$	0	0.078	0	8	33	0.0035	25	0	0.2	0	66
$\psi(4^3S_1)$	0.4	2.3	16	31	1	23	0	1.3	2.6	0.7	78

A Simple Hybrid Decay Model

Decays of an exotic 1^{-+} hybrid meson resonance in QCD

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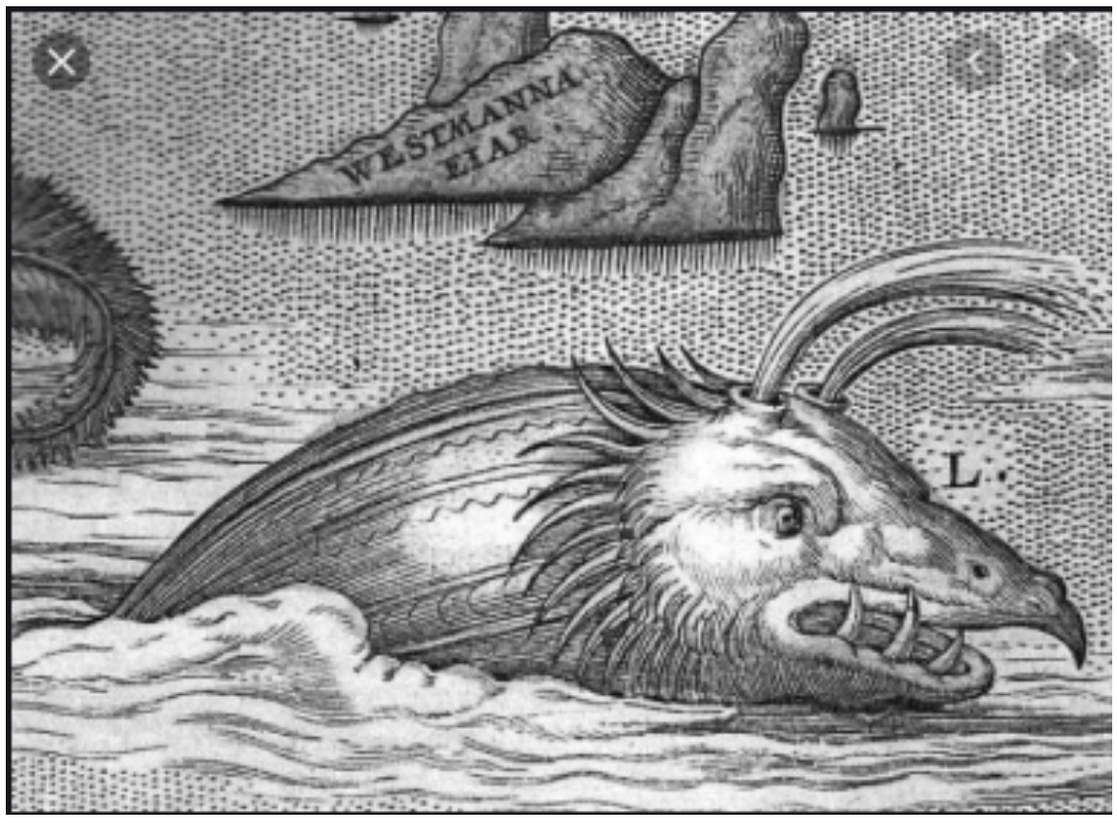
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*

(Dated: 21 September 2020)

We present the first determination of the hadronic decays of the lightest exotic $J^{PC} = 1^{-+}$ resonance in lattice QCD. Working with SU(3) flavor symmetry, where the up, down and strange quark masses approximately match the physical strange-quark mass giving $m_\pi \sim 700$ MeV, we compute finite-volume spectra on six lattice volumes which constrain a scattering system featuring eight coupled channels. Analytically continuing the scattering amplitudes into the complex energy plane, we find a pole singularity corresponding to a narrow resonance which shows relatively weak coupling to the open pseudoscalar–pseudoscalar, vector–pseudoscalar and vector–vector decay channels, but large couplings to at least one kinematically-closed axial-vector–pseudoscalar channel. Attempting a simple extrapolation of the couplings to physical light-quark mass suggests a broad π_1 resonance decaying dominantly through the $b_1\pi$ mode with much smaller decays into $f_1\pi$, $\rho\pi$, $\eta'\pi$ and $\eta\pi$. A large total width is potentially in agreement with the experimental $\pi_1(1564)$ candidate state, observed in $\eta\pi$, $\eta'\pi$, which we suggest may be heavily suppressed decay channels.

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