Heavy mesons

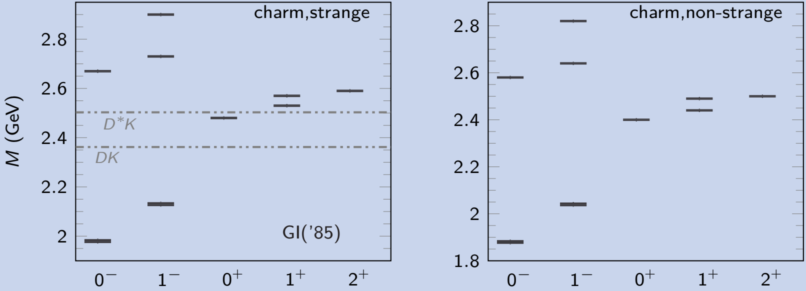
Some theoretical ideas

Miguel Albaladejo (JLab)

Snowmass meeting, September 23, 2020

Quark model in the singly heavy sector

- Quark model $c\bar{n}$ is still our baseline: *"In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework."* [Godfrey, Isgur, PR,D32,189('85)]



- The discovery of $D_{s0}^*(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is "equivalent" to the discovery of $X(3872)$ in charmonium-like system.

[BABAR, PRL,90,242001('03)]

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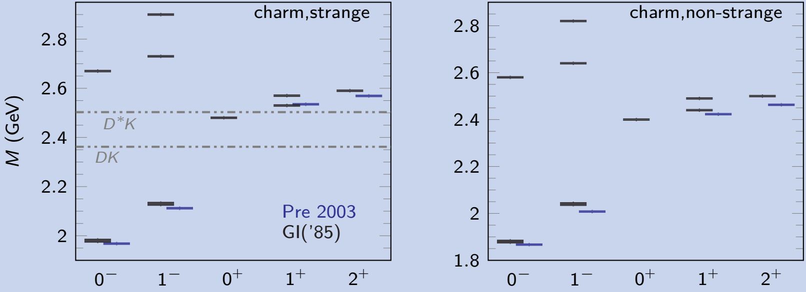

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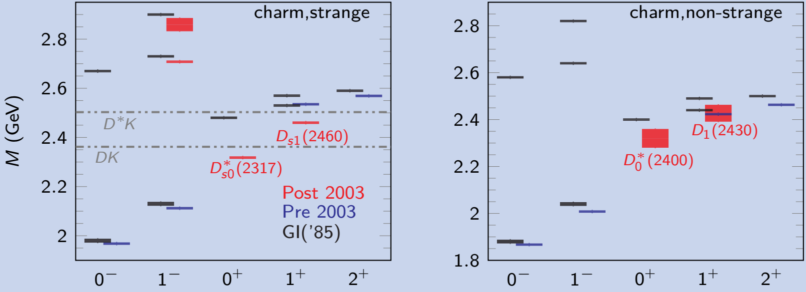

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[BABAR, PRL,90,242001('03)]

[CLEO, PR,D68,032002('03)]

Theoretical interpretations

$c\bar{q}$ states

Dai *et al.* Phys. Rev. D **68**, 114011 (2003)

Narison, Phys. Lett. B **605**, 319 (2005)

Bardeen *et al.*, Phys. Rev. D **68**, 054024 (2003)

Lee *et al.*, Eur. Phys. J. C **49**, 737 (2007)

Wang, Wan, Phys. Rev. D **73**, 094020 (2006)

Pure tetraquarks

Cheng, Hou, Phys. Lett. B **566**, 193 (2003)

Terasaki, Phys. Rev. D **68**, 011501 (2003)

Chen, Li, Phys. Rev. Lett. **93**, 232001 (2004)

Maiani *et al*,, Phys. Rev. D **71**, 014028 (2005)

Bracco *et al.*, Phys. Lett. B **624**, 217 (2005)

Wang, Wan, Nucl. Phys. A **778**, 22 (2006)

$c\bar{q}+$ tetraquarks or meson–meson

Browder *et al.*, Phys. Lett. B **578**, 365 (2004)

van Beveren, Rupp, Phys. Rev. Lett. **91**, 012003 (2003)

Heavy-light meson–meson molecules

Barnes *et al.*, Phys. Rev. D **68**, 054006 (2003)

Szczepaniak, Phys. Lett. B **567**, 23 (2003)

Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)

Hofmann, Lutz, Nucl. Phys. A **733**, 142 (2004)

Guo *et al.*, Phys. Lett. B **641**, 278 (2006)

Gamermann *et al.*, Phys. Rev. D **76**, 074016 (2007)

Faessler *et al.*, Phys. Rev. D **76**, 014005 (2007)

Flynn, Nieves, Phys. Rev. D **75**, 074024 (2007)

(Some) attempts to explain $D_{s0}^*(2317)$ as a $c\bar{s}$ state

[Ortega *et al.*, PR,D94,074037('16) (and references therein)]

- Problem: original Quark Model prediction mass is ~ 150 MeV above experimental one.
- 1-loop correction to OGE potential ($\mathcal{O}(\alpha_s^2)$) reduces the mass to 2383 MeV, much closer to the experimental one.
- ${}^3P_0$ mechanism to couple $c\bar{s}$ states to DK meson-pairs, $P_{DK} \sim 30\%$.
- Much better situation, but:
  - Still above DK threshold
  - This mechanism only affects the 0^+ sector, still problems with 1^+
  - Coupling to DK is included, but no DK "dynamics"

Meanwhile, in the lattice...

- Masses larger than the physical ones if using $c\bar{s}$ interpolators only.

  Bali, Phys. Rev. D **68**, 071501 (2003)

  UKQCD Collab., Phys. Lett. B **569**, 41 (2003)

- Masses consistent with $D_0^*(2400)$ and $D_{s0}^*(2317)$ obtained when "meson-meson" interpolators are employed.

  Mohler, Prelovsek, Woloshyn, Phys. Rev. D **87**, 034501 (2013)

  Mohler *et al.*, Phys. Rev. Lett. **111**, 222001 (2013)

- Close to the physical point: RQCD Collab., Phys. Rev. D **96**, 074501 (2017)

- More complete studies from the HadSpec collaboration:
  - $D\pi$, $D\eta$ and $D_s\bar{K}$ coupled-channel scattering. A bound state with large coupling to $D\pi$ is identified with $D_0^*(2400)$.

    HadSpec Collab., JHEP **1610**, 011 (2016)

  - $D_{s0}^*(2317)$: A bound state is found in the DK channel, with:
    - $\Delta E = 25(3)$ MeV ($m_\pi = 391$ MeV)
    - $\Delta E = 57(3)$ MeV ($m_\pi = 239$ MeV)
    - Compare with experimental, $\Delta E \simeq 45$ MeV (the dependence on m_π does not need to be monotonic!

    HadSpec Collab., 2008.06432

Lightest 0^+ open-charm situation and puzzles

- **D^*_{s0}(2317)** $(S, I) = (1, 0)$ $M_{D^*_{s0}(2317)} = 2317.8 \pm 0.5$ MeV (PDG)
- **D^*_0(2400)** $(S, I) = (0, 1/2)$ Not so well stablished:

	Collab.	M (MeV)	$\Gamma/2$ (MeV)	Ref.
Neu.	Belle	2308 ± 36	138 ± 33	Phys. Rev. D **69**, 112002 (2004)
Neu.	BaBar	2297 ± 22	137 ± 25	Phys. Rev. D **79**, 112004 (2009)
Neu.	FOCUS	2407 ± 41	120 ± 40	Phys. Lett. B **586**, 11 (2004)
Char.	LHCb	2360 ± 33	128 ± 29	Phys. Rev. D **92**, 032012 (2015) $(B^0 \to \bar{D}^0 K^+ \pi^-)$
Char.	LHCb	2349 ± 7	128 ± 29	Phys. Rev. D **92**, 012012 (2015) $(B^0 \to \bar{D}^0 \pi^+ \pi^-)$
Char.	FOCUS	2403 ± 38	142 ± 21	Phys. Lett. B **586**, 11 (2004)

- PDG averages:
  - $D^{*0}_0(2400)$: $M = 2349 \pm 7$ MeV
  - $D^{*+}_0(2400)$: $M = 2300 \pm 19$ MeV

Three puzzles

- Mass problem: Why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ masses much lower than the CQM expectations?
- Splittings: Why $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$ (within a few MeV)?
- Hierarchy: Why $M_{D^*_0(2400)} > M_{D^*_{s0}(2317)}$, *i.e.*, why $c\bar{u}$, $c\bar{d}$ heavier than $c\bar{s}$?

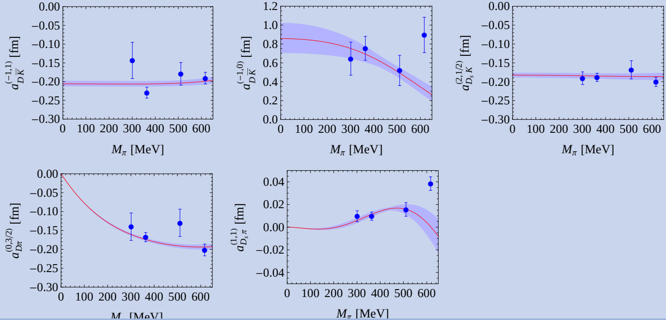
$D\pi$, $D\eta$, $D_s\overline{K}$ scattering amplitudes

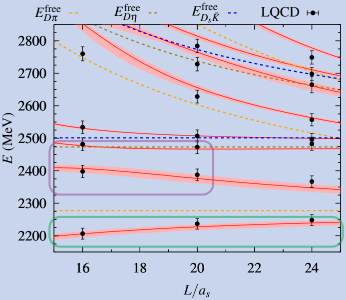
- Coupled channel T-matrix: $D\pi$, $D\eta$, $D_s\overline{K}$ scattering $[J^P = 0^+,\ (S, I) = (0, \frac{1}{2})]$.
- Unitarity: $T^{-1}(s) = V^{-1}(s) - \mathcal{G}(s)$
- Chiral symmetry used to compute the $\mathcal{O}(p^2)$ potential:

$$f^2 V_{ij}(s,t,u) = C^{ij}_{\rm LO}\frac{s-u}{4} + \sum_{a=0}^{5} h_a C^{ij}_a(s,t,u)$$

Guo *et al.*, Phys. Lett. B **666**, 251 (2008)
Liu *et al.*, Phys. Rev. D **87**, 014508 (2013)

- Free parameters previously fixed, not fitted (predictions!):
- Fitted to reproduce scattering lengths obtained in a LQCD simulation

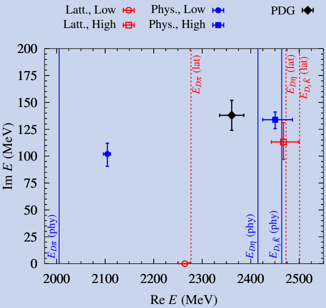




M (MeV)	Latt.	Phys.
π	391	138
K	550	496
η	588	548
D	1886	1867
D_s	1952	1968

- $\boldsymbol{E_n(L)}$ are provided for $D\pi$, $D\eta$, $D_s\bar{K}$ in a recent LQCD simulation.
  [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- **Red Bands:** Our amplitude in a finite volume.
  [M. A. *et al.*, Phys. Lett. B **767**, 465 (2017)]
- Recall, no fit is performed.
- $E > 2.7$ GeV is beyond the range of validity for our T-matrix.
- Level **below threshold**, associated with a **bound state**.
- **Second level** has large shifts w. r. t. thresholds, non-interacting energy levels:
  - Strong movement of the amplitude.
  - Check if there is another state (resonance).

Spectroscopy: two-states for $D_0^*(2400)$



Meson Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$g_{D_s\bar{K}}$
lattice	$2264^{+\ 8}_{-14}$	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
physical	2105^{+6}_{-8}	102^{+10}_{-12}	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$

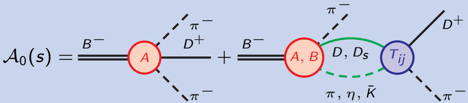
- We also study DK, $D_s\eta$, $(S,I)=(1,0)$ $D_{s0}^*(2317)$: $M = 2315^{+18}_{-28}$ MeV.

- For lattice masses, we find a bound state (000) and a resonance (110)
- For physical masses:
  - The bound state evolves into a resonance (100) above $D\pi$ threshold.
  - The resonance varies very little, and is still a resonance (110).
  - For both states, the coupling pattern is similar.
- PDG includes only one resonance, "suspiciously" lying between both.

Comparison with experimental data: $B^- \to D^+\pi^-\pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

- $\mathcal{A}(s,z) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z) + \ldots$
- P-, D-wave as in LHCb paper
- S-wave parameterization:



$$\mathcal{A}_0(s) = A\Big\{ E_\pi \Big[2 + G_1(s) \Big(\frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T^{3/2}(s) \Big) \Big] + \frac{1}{3} E_\eta G_2(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_3(s) T_{31}^{1/2}(s) \Big\} + B E_\eta G_2(s) T_{21}^{1/2}(s),$$

- Angular moments: $\langle P_\ell \rangle(s) = \int dz\, |\mathcal{A}(s,z)|^2 P_\ell(z)$

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 \;, \qquad \langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0|\,|\mathcal{A}_2|\cos(\delta_0 - \delta_2)\;,$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0|\,|\mathcal{A}_1|\cos(\delta_0 - \delta_1)\;.$$

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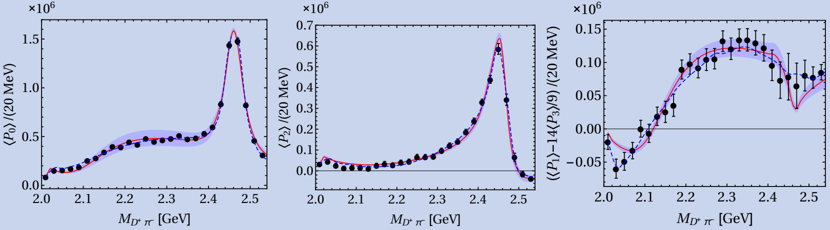


- Parameters: $B/A = -3.8 \pm 0.1$, $a = 1.2 \pm 0.1$, χ^2/d.o.f. $= 1.8$
- — This work. - - - LHCb. Bands: fit uncertainty
- Very good agreement with data & with LHCb fit
- Rapid movement in $\langle P_{13} \rangle$ [no $D_2(2460)$] between 2.4 and 2.5 GeV. Related to $D\eta$ and $D_s\bar{K}$ openings.
- Recall: these are the amplitudes with two states in the $D_0^*(2400)$ region, and no fit of the T-matrix parameters is done.

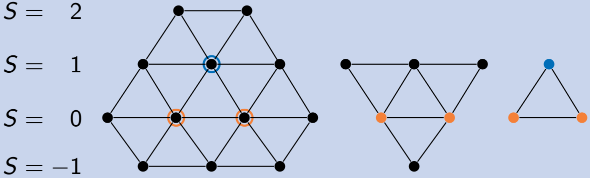
$SU(3)$ light–flavor limit

[M. A. *et al.*, Phys. Lett. B **767**, 465 (2017)]

- $SU(3)$ flavor limit: $m_i \to m = 0.49$ GeV, $M_i \to M = 1.95$ GeV.
- Irrep decomposition: $\overline{\mathbf{3}} \otimes \mathbf{8} = \boxed{\overline{\mathbf{15}} \oplus \mathbf{6} \oplus \overline{\mathbf{3}}}$. T and V can be diagonalized:

$$V_d(s) = D^\dagger V(s) D = \mathrm{diag}\left(V_{\overline{\mathbf{15}}}(s)\ , V_{\mathbf{6}}(s)\ , V_{\overline{\mathbf{3}}}(s)\right) = \boxed{A(s)\,\mathrm{diag}\,(1, -1, -3)}\ ,$$

- $\overline{\mathbf{15}}$ is repulsive. $\mathbf{6}$ and $\overline{\mathbf{3}}$ are attractive. "Curiously", $\overline{\mathbf{3}}$ admits a $c\bar{q}$ interpretation.



State	Channels	(S, I)	$\overline{\mathbf{15}}$	$\mathbf{6}$	$\overline{\mathbf{3}}$
D_0^*	$D\pi$, $D\eta$, $D_s\bar{K}$	$(0, \frac{1}{2})$	✓	✓	✓
$D_{s0}^*(2317)$	DK, $D_s\eta$	$(1, 0)$	✓	✗	✓

- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[Hadron Spectrum Collab., 2008.06432]

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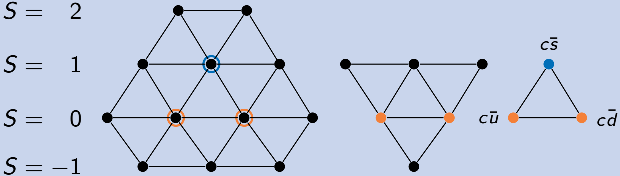

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$D_{s0}^*(2317)$	DK, $D_s\eta$	$(1, 0)$	✓	✗	✓

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[Hadron Spectrum Collab., 2008.06432]

Conclusions about $D_0^*(2400)$, $D_{s0}^*(2317)$

- The $D_0^*(2400)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds
- This two-state structure for $D_0^*(2400)$ was previously reported:

Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)

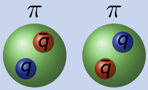
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)

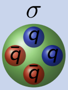
Guo *et al.*, Eur. Phys. J. A **40**, 171 (2009)

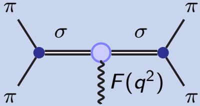
- The amplitudes containing these two-poles are compatible with available LQCD simulations and experimental data
- This picture for $D_0^*(2400)$ and $D^*(2317)$ nicely solves simultaneously all the puzzles.

Open questions for the community

- Need of more collaboration between (and simultaneous use of!) different "subcommunities": LQCD, molecular/tetraquarks/QM models...
- **Spectroscopy, mixing:**
  Specific example of $D^*_{s0}(2317)$, take for granted the presence of a CQM $c\bar{s}$ state.
  Theoretical possibilities:
  - Genuine $c\bar{s}$, (very) renormalized by DK threshold. Or renormalized by DK interactions themselves?
  - Or, there is a $S = 1$, $I = 0$ state coming from DK interactions in addition to the $c\bar{s}$ state. If so, where are those two poles? Which is which?
- **Nature/size:**
  - Can we address the question of $4q$, $q\bar{q}$, molecule based on the size of the object?







  - For $\pi\pi$ scattering, σ meson: MA, Oller, PR,D86,034003('12)
    - $\sqrt{\langle r^2 \rangle^S_\sigma} \simeq 0.44$ fm
    - $\sqrt{\langle r^2 \rangle^S_\pi} \simeq 0.81$ fm
  - Perhaps only theoretical? Future lattice QCD calculations?

    Briceño *et al.*, PR,D100,034511('19); PR,D100,114505('19), ...

Heavy mesons

Some theoretical ideas

Miguel Albaladejo (JLab)

Snowmass meeting, September 23, 2020

Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$

$SU(3)$ limit:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}})\ , \qquad (m = 0.49\ \text{GeV})\ ,$$
$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}})\ , \quad (M = 1.95\ \text{GeV})\ .$$

- Physical case ($x = 0$): RS specified by $(\xi_1\xi_2\xi_3)$, $\xi_i = 0$ or 1.
- $SU(3)$ symmetric case ($x = 1$): all channels have the same threshold, so there are only two RS (000) and (111).
- To connect the lower pole with the $T_{\mathbf{6}}$ virtual state,

$$\xi_3 = x \qquad (1,1,0) \to (1,1,x)$$

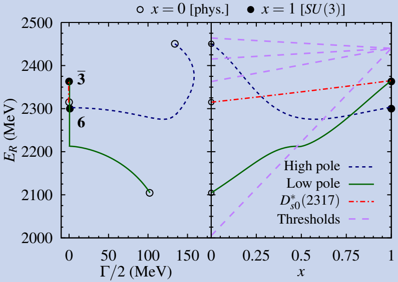
- To connect the lower pole with the $T_{\overline{\mathbf{3}}}$ bound state,

$$\xi_1 = 1 - x \qquad (1,0,0) \to (1-x,0,0)$$

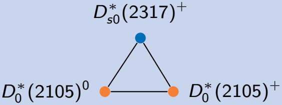
(II)

Connecting physical ($x = 0$) and flavor $SU(3)$ ($x = 1$) limits:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}) \ , \qquad (m = 0.49 \text{ GeV}) \ ,$$
$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}) \ , \qquad (M = 1.95 \text{ GeV}) \ .$$



- The high D_0^* connects with a **6** virtual state (unph. RS, below threshold).
- The low D_0^* connects with a $\overline{\mathbf{3}}$ bound state (ph. RS, below threshold).
- The $D_{s0}^*(2317)$ also connects with the $\overline{\mathbf{3}}$ bound state.



- The low D_0^* and the $D_{s0}^*(2317)$ are $SU(3)$ flavor partners.
- This solves the "puzzle" of $D_{s0}^*(2317)$ being lighter than $D_0^*(2400)$: it is not, the lower D_0^* pole ($M = 2105$ MeV) is lighter.

Form factors in semileptonic $D \to \pi\bar{\ell}\nu_\ell$

MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, D.-L. Yao, *in preparation*

- General definitions:

$$\frac{\mathrm{d}\Gamma(D\to\pi\bar{\ell}\nu_\ell)}{\mathrm{d}q^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_\pi|^3|V_{cd}|^2|f_+(q^2)| \,. \qquad [q^2=0 : f_+(0)=f_0(0)]$$

$$\langle\pi(p')|\bar{q}\gamma^\mu Q|D(p)\rangle = f_+(q^2)\left[\Sigma^\mu - \frac{m_D^2-m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_D^2-m_\pi^2}{q^2}q^\mu \ ,$$

- "Isospin" form factors, related to $D\pi$, $D\eta$, $D_s\bar{K}$ scattering:

$$\mathcal{F}^{(0,1/2)}(s) \equiv \begin{pmatrix} -\sqrt{\frac{3}{2}}f_0^{D^0\to\pi^-}(s) \\ -f_0^{D^+\to\eta}(s) \\ -f_0^{D_s^+\to K^0}(s) \end{pmatrix} \ , \qquad \mathrm{Im}\mathcal{F}(s) = T^*(s)\Sigma(s)\mathcal{F}(s)$$

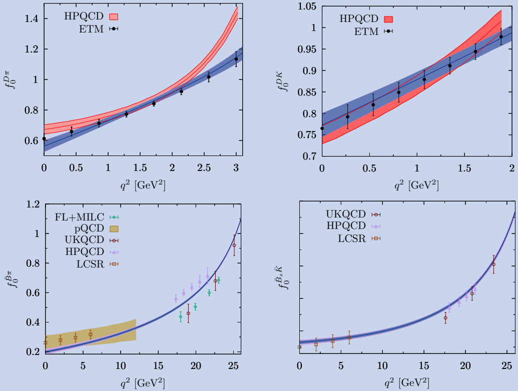
- Write form factors as Omnés matrix times polynomials

$$\mathcal{F}(s) = \Omega(s)\cdot\mathcal{P}(s)$$

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:

$$\mathcal{L}_0 = f_{\mathcal{P}}\left(\mathring{m}\mathcal{P}^*_\mu - \partial_\mu\mathcal{P}\right)u^\dagger J^\mu \ ,$$

$$\mathcal{L}_0 = \beta_1\mathcal{P}\,u\,(\partial_\mu U^\dagger)J^\mu + \beta_2(\partial_\mu\partial_\nu\mathcal{P})\,u\,(\partial^\nu U^\dagger)\,J^\mu \ .$$



	This work	Exp.
$10^3 \lvert V_{ub} \rvert$	4.51(51)	4.49(24) [Incl.] 3.72(19) [Excl.]
$\lvert V_{cd} \rvert$	0.253(18)	0.220(5)
$\lvert V_{cs} \rvert$	0.934(35)	0.995(16)

- Points mostly from LQCD
- Also LCSR for $q^2 \to 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

Why is $D_0^*(2400)$ interesting?

- Lightest systems to test ChPT with heavy mesons, besides $D^* \to D\pi$.
- $D\pi$ interactions (where it shows up) are relevant, since $D\pi$ appears as a final state in many reactions that are being considered now (*i.e.*, $Z_c(3900)$ and $\bar{D}^* D\pi$)
- $D_0^*(2400)$ is important in weak interactions and CKM parameters:

  Flynn, Nieves, Phys. Rev. D **76**, 031302 (2007)

  MA, P. Fernandez-Soler, F.K. Guo, J. Nieves, D.L. Yao, *in preparation*

  - It determines the shape of the scalar form factor $f_0(q^2)$ in semileptonic $D \to \pi$ decays.
  - Relation to $|V_{cd}|$: $f_+(0) = f_0(0)$ and $\mathrm{d}\Gamma \propto |V_{cd} f_+(q^2)|^2$.
  - Even more interesting: the bottom analogue $|V_{ub}|$.

$D\pi$, $D\eta$, $D_s\overline{K}$ energy levels in a finite volume

- Periodic boundary conditions imposes momentum quantization
- Lüscher formalism:
  Commun. Math. Phys. **105**, 153 (1986)
  Nucl. Phys. B **354**, 531 (1991)

infinite volume	finite volume
$\vec{q} \in \mathbb{R}^3$	$\vec{q} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$
$\int_{\mathbb{R}^3} \frac{\mathrm{d}^3 q}{(2\pi)^3}$	$\frac{1}{L^3} \sum_{\vec{n}\in\mathbb{Z}^3}$

- In practice, changes in the T-matrix: $T(s) \to \widetilde{T}(s, L)$:

Döring *et al.*, Eur. Phys. J. **A** 47, 139 (2011)

$$\mathcal{G}_{ii}(s) \to \widetilde{\mathcal{G}}_{ii}(s, L) = \mathcal{G}_{ii}(s) + \lim_{\Lambda\to\infty} \left(\frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}\,|<\Lambda} I_i(\vec{q}\,) - \int_0^\Lambda \frac{q^2 \mathrm{d}q}{2\pi^2} \; I_i(\vec{q}\,) \right) ,$$

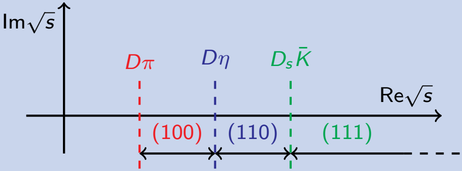
$$V(s) \to \tilde{V}(s, L) = V(s) \; ,$$

$$T^{-1}(s) \to \widetilde{T}^{-1}(s, L) = V^{-1}(s) - \widetilde{\mathcal{G}}(s, L) \; ,$$

- Free energy levels: $E^{(i)}_{n,\text{free}}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$
- Interacting energy levels $E_n(L)$: $\widetilde{T}^{-1}(E_n^2(L), L) = 0$ (poles of the $\widetilde{T}$-matrix)

- Normalization: $-ip_{ii}(s)\,T_{ii}(s) = 4\pi\sqrt{s}\left(\eta_i(s)e^{2i\delta_i(s)} - 1\right)$.
- $\mathcal{G}_{ii}(s) = G(s, m_i, M_i)$, regularized with a subtraction constant $a(\mu)$ ($\mu = 1$ GeV).
- Riemann sheets (RS) denoted as $(\xi_1\xi_2\xi_3)$:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\,\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$



Predictions for other sectors: charm

(S,I)	Channels	$\overline{\mathbf{15}}$	$\mathbf{6}$	$\overline{\mathbf{3}}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$(0,\frac{1}{2})$	$D^{(*)}\pi$, $D^{(*)}\eta$, $D_s^{(*)}\bar{K}$	✓	✓	✓	(R) 2105^{+6}_{-8} (R) 2451^{+36}_{-26}	102^{+10}_{-12} 134^{+7}_{-8}	(R) 2240^{+5}_{-6}	93^{+9}_{-9}
$(1,0)$	$D^{(*)}K$, $D_s^{(*)}\eta$	✓	✗	✓	(B) 2315^{+18}_{-28}		(B) 2436^{+16}_{-22}	
$(-1,0)$	$D^{(*)}\bar{K}$	✗	✓	✗	(V) 2342^{+13}_{-41}		–	
$(1,1)$	$D_s^{(*)}\pi$, $D^{(*)}K$	✓	✓	✗	–		–	

- HQSS relates 0^+ $(D_{(s)}P)$ and 1^+ $(D^*_{(s)}P)$ sectors: similar resonance pattern.
- Two pole structure: higher D_1 pole probably affected by ρ channels.
- $D\bar{K}$ $[0^+, (-1,0)]$: this virtual state (from **6**) has a large impact on the scattering length, $a^{D\bar{K}}_{(-1,0)} \simeq 0.8$ fm. (Rest of scattering lengths are $|a| \simeq 0.1$ fm.)

Predictions for other sectors: bottom

(S, I)	Channels	$\overline{\mathbf{15}}$	$\mathbf{6}$	$\overline{\mathbf{3}}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$(0, \frac{1}{2})$	$\bar{B}^{(*)}\pi$, $\bar{B}^{(*)}\eta$, $\bar{B}_s^{(*)}\bar{K}$	✓	✓	✓	(R) 5537^{+9}_{-11} (R) 5840^{+12}_{-13}	116^{+14}_{-15} 25^{+6}_{-5}	(R) 5581^{+9}_{-11}	115^{+13}_{-15}
$(1, 0)$	$\bar{B}^{(*)}K$, $\bar{B}_s^{(*)}\eta$	✓	✗	✓	(B) 5724^{+17}_{-24}		(B) 5768^{+17}_{-23}	
$(-1, 0)$	$\bar{B}^{(*)}\bar{K}$	✗	✓	✗	(V–B) thr.		(V–B) thr.	
$(1, 1)$	$\bar{B}_s^{(*)}\pi$, $\bar{B}^{(*)}K$	✓	✓	✗	–		–	

- Heavy flavour symmetry relates charm (D) and bottom $(\bar{B})$ sectors.
- $(0, \frac{1}{2})$: B_0^*, two-pole pattern also observed.
- $(-1, 0)$: $[B^{(*)}\bar{K}]$: very close to threshold. Relevant prediction. Can be either bound or virtual (**6**) within our errors.
- $(1, 1)$: $[\bar{B}_s\pi, \bar{B}K, 0^+]$, $X(5568)$ channel. No state is found: $\overline{\mathbf{15}}$ and **6**. If it exists, it is not dynamically generated in $B_s\pi$, $B\bar{K}$ interactions.

  M. A. *et al.*, Phys. Lett. B **757**, 515 (2016); Guo *et al.*, Commun. Theor. Phys. **65**, 593 (2016)
- $(1, 0)$: Our results for B_{s0}^* and B_{s1} agree with other results from LQCD:

  Lang *et al.*, Phys. Lett. B **750**, 17 (2015); M. A. *et al.* Eur. Phys. J. C**77**, 170 (2017)

Chiral dynamics and two-state structure(s)

- Other famous two-poles structures rooted in chiral dynamics:

$\Lambda(1405)$ $[\Sigma\pi, N\bar{K}]$

Oller, Meißner, Phys. Lett. B **500**, 263 (2001)
Jido *et al.*, Nucl. Phys. A **725**, 181 (2003)
García-Recio *et al.*, Phys. Lett. B **582**, 49 (2004)
Magas *et al.*, Phys. Rev. Lett. **95**, 052301 (2005)

$K_1(1270)$

Roca *et al.*, Phys. Rev. D **72**, 014002 (2005)
Geng *et al.*, Phys. Rev. D **75**, 014017 (2007)
García-Recio *et al.*, Phys. Rev. D **83**, 016007 (2011)

- Chiral dynamics:
  - Incorporates the $SU(3)$ light-flavor structure,
  - Determines the strength of the interaction,
  - Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.

Conclusions of $D_0^*(2400)$ work

- We have studied $D\pi$, $D\eta$, $D_s\bar{K}$ scattering $[0^+, (S,I)=(0,\frac{1}{2})]$
- So far only one pole reported experimentally, but we have presented a strong support for the existence of two $D_0^*(2400)$ states (different poles):
- Succesful, no-fitting comparison of our T-matrix with the energy levels of a recent LQCD simulation.
- We are also able to reproduce the LHCb experimental information for $B^- \to D^+\pi^-\pi^-$, also without fitting any of the T-matrix parameters.
- The lower pole ($M = 2105^{+6}_{-8}$ MeV) is lighter than $D_{s0}^*(2317)$, solving this (apparent) puzzle.
- A $SU(3)$ study shows that $D_{s0}^*(2317)$ and the lower $D_0^*(2400)$ are flavour partners: they complete a $\overline{\mathbf{3}}$ multiplet.
- Predictions for other sectors (heavy vectors, bottom sector) have been also given. In particular:
  - The two-pole structure is also seen in the bottom sector.
  - A very near-threshold state (bound or virtual) is predicted for BK ($\bar{B}\bar{K}$).