Quark model in the singly heavy sector

- Quark model $c\bar{n}$ is still our baseline: "In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework." [Godfrey, Isgur, PR,D32,189('85)]

- The discovery of $D_{s0}^*(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is "equivalent" to the discovery of $X(3872)$ in charmonium-like system. [BABAR, PRL,90,242001('03)] [CLEO, PR,D68,032002('03)]
Quark model $c\bar{n}$ is still our baseline: "In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons—from the pion to the upsilon—can be described in a unified framework."

[Godfrey, Isgur, PR,D32,189('85)]

The discovery of $D_s^*(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is “equivalent” to the discovery of $X(3872)$ in charmonium-like system.

[BABAR, PRL,90,242001('03)]
[CLEO, PR,D68,032002('03)]
Quark model in the singly heavy sector

- Quark model $c\bar{n}$ is still our baseline: "In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework."
  [Godfrey, Isgur, PR,D32,189('85)]

- The discovery of $D_{s0}^*(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is “equivalent” to the discovery of $X(3872)$ in charmonium-like system.
  [BABAR, PRL,90,242001('03)]
  [CLEO, PR,D68,032002('03)]
### Theoretical interpretations

#### $c\bar{q}$ states

#### $c\bar{q}+$ tetraquarks or meson–meson

#### Pure tetraquarks

#### Heavy-light meson–meson molecules
Some attempts to explain $D_{s0}^*(2317)$ as a $c\bar{s}$ state

[Ortega et al., PR,D94,074037(’16) (and references therein)]

- Problem: original Quark Model prediction mass is $\sim 150$ MeV above experimental one.
- 1-loop correction to OGE potential ($\mathcal{O}(\alpha_s^2)$) reduces the mass to 2383 MeV, much closer to the experimental one.
- $^3P_0$ mechanism to couple $c\bar{s}$ states to $DK$ meson-pairs, $P_{DK} \sim 30\%$.
- Much better situation, but:
  - Still above $DK$ threshold
  - This mechanism only affects the $0^+$ sector, still problems with $1^+$
  - Coupling to $DK$ is included, but no $DK$ “dynamics”
Meanwhile, in the lattice...

- Masses larger than the physical ones if using $c\bar{s}$ interpolators only.
  

- Masses consistent with $D_0^*(2400)$ and $D_{s0}^*(2317)$ obtained when “meson-meson” interpolators are employed.
  

- Close to the physical point:
  

- More complete studies from the HadSpec collaboration:
  
  - $D_\pi$, $D\eta$ and $D_s\bar{K}$ coupled-channel scattering. A bound state with large coupling to $D\pi$ is identified with $D_0^*(2400)$.
    
    HadSpec Collab., JHEP 1610, 011 (2016)
  
  - $D_{s0}^*(2317)$: A bound state is found in the $DK$ channel, with:
    
    - $\Delta E = 25(3)$ MeV ($m_\pi = 391$ MeV)
    - $\Delta E = 57(3)$ MeV ($m_\pi = 239$ MeV)
    - Compare with experimental, $\Delta E \simeq 45$ MeV (the dependence on $m_\pi$ does not need to be monotonic!)

  HadSpec Collab., 2008.06432
Lightest $0^+$ open-charm situation and puzzles

- $D_{s0}^*(2317)$ ($S, I = (1, 0)$ $M_{D_{s0}^*(2317)} = 2317.8 \pm 0.5$ MeV (PDG)
- $D_0^*(2400)$ ($S, I = (0, 1/2)$ Not so well established:

<table>
<thead>
<tr>
<th>Collab.</th>
<th>$M$ (MeV)</th>
<th>$\Gamma/2$ (MeV)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>($B^0 \to \bar{D}^0 K^+\pi^-$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>($B^0 \to \bar{D}^0 \pi^+\pi^-$)</td>
</tr>
</tbody>
</table>

- PDG averages:
  - $D_0^{*0}(2400)$: $M = 2349 \pm 7$ MeV
  - $D_0^{*+}(2400)$: $M = 2300 \pm 19$ MeV

Three puzzles

- Mass problem: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ masses much lower than the CQM expectations?
- Splittings: Why $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \approx M_{D^*} - M_D$ (within a few MeV)?
- Hierarchy: Why $M_{D_0^{*0}(2400)} > M_{D_{s0}^*(2317)}$, i.e., why $c\bar{u}$, $c\bar{d}$ heavier than $c\bar{s}$?
$D\pi, D\eta, D_s\overline{K}$ scattering amplitudes

- Coupled channel $T$-matrix: $D\pi, D\eta, D_s\overline{K}$ scattering [$J^P = 0^+, (S, I) = (0, \frac{1}{2})$].
- Unitarity: $T^{-1}(s) = V^{-1}(s) - G(s)$
- Chiral symmetry used to compute the $O(p^2)$ potential:

$$f^2 V_{ij}(s, t, u) = C_{LO}^{ij} \frac{s - u}{4} + \sum_{a=0}^{5} h_a C_a^{ij}(s, t, u)$$


- Free parameters previously fixed, not fitted (predictions!):
- Fitted to reproduce scattering lengths obtained in a LQCD simulation
Comparison with LQCD energy levels

$E_n(L)$ are provided for $D\pi$, $D\eta$, $D_s\bar{K}$ in a recent LQCD simulation. 
[G. Moir et al., JHEP 1610, 011 (2016)]

**Red Bands:** Our amplitude in a finite volume. 

Recall, no fit is performed.

$E > 2.7$ GeV is beyond the range of validity for our $T$-matrix.

Level **below threshold**, associated with a bound state.

**Second level** has large shifts w. r. t. thresholds, non-interacting energy levels:
- Strong movement of the amplitude.
- Check if there is another state (resonance).

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>Latt. Phys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>391 138</td>
</tr>
<tr>
<td>$K$</td>
<td>550 496</td>
</tr>
<tr>
<td>$\eta$</td>
<td>588 548</td>
</tr>
<tr>
<td>$D$</td>
<td>1886 1867</td>
</tr>
<tr>
<td>$D_s$</td>
<td>1952 1968</td>
</tr>
</tbody>
</table>
Spectroscopy: two-states for $D_0^*(2400)$

We also study $DK$, $D_s\eta$, $(S, I) = (1, 0)$

$D_s^0(2317)$: $M = 2315^{+18}_{-28}$ MeV.

- For lattice masses, we find a bound state (000) and a resonance (110).

- For physical masses:
  - The bound state evolves into a resonance (100) above $D\pi$ threshold.
  - The resonance varies very little, and is still a resonance (110).
  - For both states, the coupling pattern is similar.

- PDG includes only one resonance, “suspiciously” lying between both.
Comparison with experimental data: $B^- \rightarrow D^+ \pi^- \pi^-$

- $A(s, z) = A_0(s) + \sqrt{3} A_1(s) P_1(z) + \sqrt{5} A_2(s) P_2(z) + \ldots$
- $P-, D$-wave as in LHCb paper
- $S$-wave parameterization:

\[
A_0(s) = \frac{B^{-} \rightarrow D^{+} \pi^{-} \pi^{-}}{A_{0, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)} = A(s, z) = A_0(s) + \sqrt{3} A_1(s) P_1(z) + \sqrt{5} A_2(s) P_2(z) + \ldots
\]

$\langle P_0 \rangle \propto |A_0|^2 + |A_1|^2 + |A_2|^2$

$\langle P_2 \rangle \propto \frac{2}{5} |A_1|^2 + \frac{2}{7} |A_2|^2 + \frac{2}{\sqrt{5}} |A_0||A_2| \cos(\delta_0 - \delta_2)$

$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |A_0||A_1| \cos(\delta_0 - \delta_1)$

$\langle P_{\ell}\rangle(s) = \int dz |A(s, z)|^2 P_\ell(z)$
Comparison with experimental data: $B^- \rightarrow D^+\pi^-\pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

- Parameters: $B/A = -3.8 \pm 0.1$, $a = 1.2 \pm 0.1$, $\chi^2$/d.o.f. = 1.8
- This work. - - - LHCb. Bands: fit uncertainty
- Very good agreement with data & with LHCb fit
- Rapid movement in $\langle P_{13} \rangle$ [no $D_2(2460)$] between 2.4 and 2.5 GeV. Related to $D_\eta$ and $D_s\bar{K}$ openings.
- Recall: these are the amplitudes with two states in the $D^*_0(2400)$ region, and no fit of the $T$-matrix parameters is done.
**SU(3) light–flavor limit**

- **SU(3) flavor limit:** $m_i \rightarrow m = 0.49$ GeV, $M_i \rightarrow M = 1.95$ GeV.

- Irrep decomposition: $\bar{3} \otimes 8 = \overline{15} \oplus 6 \oplus 3$. $T$ and $V$ can be diagonalized:

$$V_d(s) = D^\dagger V(s)D = \text{diag} (V_{15}(s), V_6(s), V_3(s)) = A(s) \text{diag} (1, -1, -3),$$

- $\overline{15}$ is repulsive. 6 and $\bar{3}$ are attractive. “Curiously”, $\bar{3}$ admits a $c\bar{q}$ interpretation.

<table>
<thead>
<tr>
<th>State</th>
<th>Channels</th>
<th>$(S, l)$</th>
<th>$15$</th>
<th>$6$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0^*$</td>
<td>$D\pi, D\eta, D_sK$</td>
<td>$(0, \frac{1}{2})$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$D_{s0}^*(2317)$</td>
<td>$DK, D_s\eta$</td>
<td>$(1, 0)$</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
</tbody>
</table>

- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[Hadron Spectrum Collab., 2008.06432]
**SU(3) light–flavor limit**

- **SU(3) flavor limit**: $m_i \rightarrow m = 0.49$ GeV, $M_i \rightarrow M = 1.95$ GeV.

- Irrep decomposition: $\bar{3} \otimes 8 = 15 \oplus 6 \oplus \bar{3}$. $T$ and $V$ can be diagonalized:

\[
V_d(s) = D^\dagger V(s)D = \text{diag}(V_{15}(s), V_6(s), V_3(s)) = A(s) \text{diag}(1, -1, -3),
\]

- $15$ is repulsive. $6$ and $\bar{3}$ are attractive. “Curiously”, $\bar{3}$ admits a $c\bar{q}$ interpretation.

<table>
<thead>
<tr>
<th>State</th>
<th>Channels</th>
<th>$(S, I)$</th>
<th>15</th>
<th>6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0^*$</td>
<td>$D\pi, D\eta, D_sK$</td>
<td>$(0, \frac{1}{2})$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$D_{s0}^*(2317)$</td>
<td>$DK, D_s\eta$</td>
<td>$(1, 0)$</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[Hadron Spectrum Collab., 2008.06432]
The $D_0^*(2400)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds.

This two-state structure for $D_0^*(2400)$ was previously reported:


The amplitudes containing these two-poles are compatible with available LQCD simulations and experimental data.

This picture for $D_0^*(2400)$ and $D_s^*(2317)$ nicely solves simultaneously all the puzzles.
Open questions for the community

- Need of more collaboration between (and simultaneous use of!) different “subcommunities”: LQCD, molecular/tetraquarks/QM models...

- **Spectroscopy, mixing:**
  Specific example of $D_{s0}^*(2317)$, take for granted the presence of a CQM $c\bar{s}$ state.

  **Theoretical possibilities:**
  - Genuine $c\bar{s}$, (very) renormalized by $DK$ threshold. Or renormalized by $DK$ interactions themselves?
  - Or, there is a $S = 1, I = 0$ state coming from $DK$ interactions in addition to the $c\bar{s}$ state. If so, where are those two poles? Which is which?

- **Nature/size:**
  - Can we address the question of $4q, q\bar{q}$, molecule based on the size of the object?

  ![Diagram](image)

  - For $\pi\pi$ scattering, $\sigma$ meson: MA, Oller, PR,D86,034003('12)
    - $\sqrt{\langle r^2 \rangle_{\sigma}} \simeq 0.44$ fm
    - $\sqrt{\langle r^2 \rangle_{\pi}} \simeq 0.81$ fm
  - Perhaps only theoretical? Future lattice QCD calculations?
    - Briceño *et al.*, PR,D100,034511('19); PR,D100,114505('19), ...
Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$G_{ii}(s) \rightarrow G_{ii}(s) + i \frac{p_i(s)}{4\pi\sqrt{s}} \xi_i$$

$SU(3)$ limit:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),$$
$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).$$

- Physical case ($x = 0$): RS specified by $(\xi_1\xi_2\xi_3)$, $\xi_i = 0$ or 1.
- $SU(3)$ symmetric case ($x = 1$): all channels have the same threshold, so there are only two RS (000) and (111).

- To connect the lower pole with the $T_6$ virtual state,

  \[ \xi_3 = x \quad (1, 1, 0) \rightarrow (1, 1, x) \]

- To connect the lower pole with the $T_3$ bound state,

  \[ \xi_1 = 1 - x \quad (1, 0, 0) \rightarrow (1 - x, 0, 0) \]
Connecting physical \((x = 0)\) and flavor \(SU(3)\) \((x = 1)\) limits:

\[
m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),
\]

\[
M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).
\]

- The high \(D_0^*\) connects with a 6 virtual state (unph. RS, below threshold).
- The low \(D_0^*\) connects with a \(\bar{3}\) bound state (ph. RS, below threshold).
- The \(D_{s0}^*(2317)\) also connects with the \(\bar{3}\) bound state.

The low \(D_0^*\) and the \(D_{s0}^*(2317)\) are \(SU(3)\) flavor partners.

This solves the “puzzle” of \(D_{s0}^*(2317)\) being lighter than \(D_0^*(2400)\): it is not, the lower \(D_0^*\) pole \((M = 2105 \text{ MeV})\) is lighter.
Form factors in semileptonic $D \to \pi \bar{\ell} \nu_\ell$


- General definitions:
  \[
  \frac{d\Gamma(D \to \pi \bar{\ell} \nu_\ell)}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_\pi|^3 |V_{cd}|^2 |f_+(q^2)|. \quad [q^2 = 0 : f_+(0) = f_0(0)]
  \]

  \[
  \langle \pi(p')|\bar{q}\gamma^\mu Q|D(p)\rangle = f_+(q^2) \left[ \Sigma^\mu - \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu,
  \]

- "Isospin" form factors, related to $D\pi$, $D\eta$, $D_s\bar{K}$ scattering:
  \[
  \mathcal{F}^{(0,1/2)}(s) \equiv \begin{pmatrix}
  -\sqrt{3}/2 f_0^{D^0 \to \pi^-}(s) \\
  -f_0^{D^+ \to \eta}(s) \\
  -f_0^{D_s^+ \to K^0}(s)
  \end{pmatrix}, \quad \text{Im} \mathcal{F}(s) = T^*(s)\Sigma(s)\mathcal{F}(s)
  \]

- Write form factors as Omnés matrix times polynomials
  \[
  \mathcal{F}(s) = \Omega(s) \cdot \mathcal{P}(s)
  \]

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:
  \[
  \mathcal{L}_0 = f_P \left( \hat{m}\mathcal{P}_\mu^* - \partial_\mu \mathcal{P} \right) u^\dagger J^\mu,
  \]
  \[
  \mathcal{L}_0 = \beta_1 \mathcal{P} u (\partial_\mu U^\dagger) J^\mu + \beta_2 (\partial_\mu \partial_\nu \mathcal{P}) u (\partial_\nu U^\dagger) J^\mu.
  \]
- Points mostly from LQCD
- Also LCSR for $q^2 \to 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3</td>
<td>V_{ub}</td>
<td>4.51(51)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.72(19) \text{ [Excl.]}</td>
</tr>
<tr>
<td>$V_{cd}$</td>
<td>0.253(18)</td>
<td>0.220(5)</td>
</tr>
<tr>
<td>$V_{cs}$</td>
<td>0.934(35)</td>
<td>0.995(16)</td>
</tr>
</tbody>
</table>
Why is $D_0^*(2400)$ interesting?

- Lightest systems to test ChPT with heavy mesons, besides $D^* \to D\pi$.
- $D\pi$ interactions (where it shows up) are relevant, since $D\pi$ appears as a final state in many reactions that are being considered now (i.e., $Z_c(3900)$ and $\bar{D}^* D\pi$).
- $D_0^*(2400)$ is important in weak interactions and CKM parameters:
  

  - It determines the shape of the scalar form factor $f_0(q^2)$ in semileptonic $D \to \pi$ decays.
  - Relation to $|V_{cd}|$: $f_+(0) = f_0(0)$ and $d\Gamma \propto |V_{cd}f_+(q^2)|^2$.
  - Even more interesting: the bottom analogue $|V_{ub}|$. 
**$D\pi$, $D\eta$, $D_s\bar{K}$ energy levels in a finite volume**

- Periodic boundary conditions imposes momentum quantization
- Lüscher formalism:  
  
  \[
  \]
  \[
  \text{Nucl. Phys. B 354, 531 (1991)}
  \]

<table>
<thead>
<tr>
<th>infinite volume</th>
<th>finite volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{q} \in \mathbb{R}^3$</td>
<td>$\vec{q} = \frac{2\pi}{L} \vec{n}$, $\vec{n} \in \mathbb{Z}^3$</td>
</tr>
<tr>
<td>$\int_{\mathbb{R}^3} \frac{d^3 q}{(2\pi)^3}$</td>
<td>$\frac{1}{L^3} \sum_{\vec{n} \in \mathbb{Z}^3}$</td>
</tr>
</tbody>
</table>

- In practice, changes in the $T$-matrix: $T(s) \to \tilde{T}(s, L)$:  
  
  \[
  \tilde{G}_{ii}(s) = G_{ii}(s) + \lim_{\Lambda \to \infty} \left( \frac{1}{L^3} \sum_{\vec{n}} I_i(\vec{q}) - \int_0^\Lambda \frac{q^2 \,dq}{2\pi^2} \, I_i(\vec{q}) \right) ,
  \]
  \[
  \tilde{V}(s) = V(s) ,
  \]
  \[
  \tilde{T}^{-1}(s) = V^{-1}(s) - \tilde{G}(s, L) ,
  \]

- Free energy levels: $E_{n,\text{free}}^{(i)}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$

- Interacting energy levels $E_{n}(L)$: $\tilde{T}^{-1}(E^2(L), L) = 0$ (poles of the $\tilde{T}$-matrix)
Normalization: \(-i p_{ii}(s) T_{ii}(s) = 4\pi \sqrt{s} \left( \eta_i(s)e^{2i\delta_i(s)} - 1 \right)\).

\(G_{ii}(s) = G(s, m_i, M_i)\), regularized with a subtraction constant \(a(\mu) (\mu = 1 \text{ GeV})\).

Riemann sheets (RS) denoted as \((\xi_1 \xi_2 \xi_3)\):

\[G_{ii}(s) \rightarrow G_{ii}(s) + i \frac{p_i(s)}{4\pi \sqrt{s}} \xi_i\]
Predictions for other sectors: charm

<table>
<thead>
<tr>
<th>$(S, I)$</th>
<th>Channels</th>
<th>$M$</th>
<th>$\Gamma/2$</th>
<th>$M$</th>
<th>$\Gamma/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, \frac{1}{2})$</td>
<td>$D(\pi), D(\eta), D_S(\pi)\bar{K}$</td>
<td>✓ ✓ ✓</td>
<td>(R) $2105^{+6}_{-8}$</td>
<td>102$^{+10}_{-12}$</td>
<td>(R) $2240^{+5}_{-6}$</td>
</tr>
<tr>
<td>$(1, 0)$</td>
<td>$D(K), D_S(\eta)$</td>
<td>✓ X ✓</td>
<td>(B) $2315^{+18}_{-28}$</td>
<td>134$^{+7}_{-8}$</td>
<td>(B) $2436^{+16}_{-22}$</td>
</tr>
<tr>
<td>$(-1, 0)$</td>
<td>$D(\bar{K})$</td>
<td>X ✓ X</td>
<td>(V) $2342^{+13}_{-41}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$(1, 1)$</td>
<td>$D_S(\pi), D(K)$</td>
<td>✓ ✓ X</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

- **HQSS** relates $0^+ (D(s)P)$ and $1^+ (D(s)P)$ sectors: similar resonance pattern.

- Two pole structure: higher $D_1$ pole probably affected by $\rho$ channels.

- $D\bar{K}$ $[0^+, (-1, 0)]$: this virtual state (from 6) has a large impact on the scattering length, $a_{D\bar{K}(-1,0)} \simeq 0.8$ fm. (Rest of scattering lengths are $|a| \simeq 0.1$ fm.)
### Predictions for other sectors: bottom

<table>
<thead>
<tr>
<th>$(S, I)$</th>
<th>Channels</th>
<th>$0^+$</th>
<th>$1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M$</td>
<td>$\Gamma/2$</td>
</tr>
<tr>
<td>$(0, \frac{1}{2})$</td>
<td>$B^<em>(\pi), \bar{B}^</em>(\eta), \bar{B}^*_s(K)$</td>
<td>✓ ✓ ✓</td>
<td>(R) $5537^{+9}_{-11}$</td>
</tr>
<tr>
<td>$(1, 0)$</td>
<td>$B^<em>(K), \bar{B}^</em>_s(\eta)$</td>
<td>✓ X ✓</td>
<td>(B) $5724^{+17}_{-24}$</td>
</tr>
<tr>
<td>$(-1, 0)$</td>
<td>$\bar{B}^*(K)$</td>
<td>X ✓ X</td>
<td>(V–B) thr.</td>
</tr>
<tr>
<td>$(1, 1)$</td>
<td>$\bar{B}^<em>_s(\pi), \bar{B}^</em>(K)$</td>
<td>✓ ✓ X</td>
<td>–</td>
</tr>
</tbody>
</table>

- **Heavy flavour symmetry** relates charm ($D$) and bottom ($\bar{B}$) sectors.

- $(0, \frac{1}{2})$: $B^*_0$, two-pole pattern also observed.

- $(-1, 0)$: $[B^*(K)]$: very close to threshold. Relevant prediction. Can be either bound or virtual (6) within our errors.

- $(1, 1)$: $[\bar{B}^*_s(\pi), \bar{B}^* K, 0^+]$, $X(5568)$ channel. No state is found: $15$ and $6$. If it exists, it is not dynamically generated in $B^*_s(\pi), B^* K$ interactions.


- $(1, 0)$: Our results for $B^*_s0^+$ and $B^*_s1^+$ agree with other results from LQCD:

Other famous two-poles structures rooted in chiral dynamics:

\[ \Lambda(1405) \ [\Sigma\pi, \; N\bar{K}] \quad \text{and} \quad K_1(1270) \]


Chiral dynamics:

- Incorporates the \textit{SU}(3) light-flavor structure,
- Determines the strength of the interaction,
- Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.
Conclusions of $D_0^*(2400)$ work

- We have studied $D\pi$, $D\eta$, $D_s\bar{K}$ scattering $[0^+, (S, I) = (0, \frac{1}{2})]$.

- So far only one pole reported experimentally, but we have presented a strong support for the existence of two $D_0^*(2400)$ states (different poles):
  - Successful, no-fitting comparison of our $T$-matrix with the energy levels of a recent LQCD simulation.
  - We are also able to reproduce the LHCb experimental information for $B^- \rightarrow D^+\pi^-\pi^-$, also without fitting any of the $T$-matrix parameters.
  - The lower pole ($M = 2105^{+6}_{-8}$ MeV) is lighter than $D_{s0}^*(2317)$, solving this (apparent) puzzle.
  - A $SU(3)$ study shows that $D_{s0}^*(2317)$ and the lower $D_0^*(2400)$ are flavour partners: they complete a $\bar{3}$ multiplet.
  - Predictions for other sectors (heavy vectors, bottom sector) have been also given. In particular:
    - The two-pole structure is also seen in the bottom sector.
    - A very near-threshold state (bound or virtual) is predicted for $BK$ (\bar{B}\bar{K}$).